INVESTMENT IN HUMANS, TECHNOLOGICAL DIFFUSION
AND THE GOLDEN RULE OF EDUCATION

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A recent paper by Richard Nelson and the present author [1]
advanced the hypothesis that education increases the pace of technological
diffusion. After a condensed restatement of the hypothesis and under-
lying theory presented in that paper, I shall develop a complete model
of education, diffusion and growth, based on a diffusion model presented
in the earlier paper, and derive from it a Golden Rule of Education.

I. The Hypothesis

A basic principle in the theory of the relation between
education and economic growth is that certain kinds of education equip a
man to perform certain jobs or functions, or enable a man to perform a
given function more effectively. Thusfar, growth theorists have applied
this principle only to completely routinized jobs, jobs which require no
adaptation to change or learning in the performance of the job. (Even a
highly routinized job may require education to master the necessary skills).
In its usual, rather general form, the theory postulates a production
function which states how maximum current output depends upon the current

services of tangible capital goods, the current number of men performing each of these jobs, possibly the current educational attainment of each of these job-holders, and possible time. Such a production function implies that the marginal productivity of education can, for given inputs, remain positive forever even if the technology is stationary.

But education is also important to the performance of functions which require adaptation to change. The function of innovating is a prime example. It is a reasonable hypothesis that the more educated are those in production management, the quicker they will be to introduce new processes and products. To put the hypothesis simply, educated people make good innovators, so that education speeds the process of technological diffusion.

There is evidence for this hypothesis in United States agriculture. Relatively well educated farmers have tended to adopt productive innovations earlier than farmers with relatively little education. Undoubtedly part of the explanation is that the greater education of the better educated farmers has increased their ability to read and understand the information on new processes and products disseminated by governments, farm journals, seed and equipment companies and so on. With this technical information in hand, the more educated farmer is better able to discriminate between promising and unpromising ideas and is hence less likely to make mistakes. The less educated farmer, for whom the information in technical literature means less, is prudent to delay the

introduction of a new technique until he has concrete, practical evidence of its profitability.

A specific model of education, diffusion and growth will now be developed.

II. The Model

I shall work with a simple model in which there is an aggregate production function. Technical progress is postulated to be Harrod neutral everywhere so that it can be described as purely labor augmenting. If we let Q denote output, K denote capital, N denote labor in production, and $\underline{t}$ denote time, our production function takes the form

(1) $$Q(t) = F[K(t), A(t) N(t)] .$$

Constant returns to scale is postulated.

It is now possible to speak meaningfully about the "level" or "index" of technology. In particular, $A(t)$ is our index of technology in practice. Undoubtedly it would be more realistic to work with a vintage model of production in which $A(t)$ is the best-practice level of technology, the average technology level embodied in the latest capital goods actually being produced. But the present non-vintage model will serve adequately to indicate the main points I wish to make.

In addition to the practiced level of technology, I introduce
the notion of the theoretical level of technology. This is defined as
the level of technology in practice that would exist if technological
diffusion were complete and instantaneous. It is a measure of the stock
of knowledge or body of production techniques that is available to
innovators. I shall suppose that the theoretical technology level advances
exogenously at a constant exponential rate λ :

$$(2) \qquad T(t) = T_o \, e^{\lambda t} \, , \qquad \lambda > 0 \, .$$

Turning to the diffusion hypothesis, I postulate that the
rate of increase of the level of technology in practice is an increasing
function of some index of per capita educational attainment, h , and of
the gap between the theoretical technology level and the technology level
in practice. Specifically,

$$(3) \qquad \overset{\circ}{A}(t) = \Phi(h) \, [T(t) - A(t)]$$

or equivalently

$$(3') \qquad \frac{\dot{A}(t)}{A(t)} = \Phi(h) \left[\frac{T(t) - A(t)}{A(t)} \right] \, , \quad \Phi(0) = 0, \ \Phi'(h) > 0 \, .$$

The quantity $(T(t) - A(t))/A(t)$ will be called the "gap".

If $\underline{h}$ is constant, some interesting results follow. One result is that, for positive $\underline{h}$, the rate of increase of the level of technology in practice, $\dot{A}(t)/A(t)$, will settle down to the value λ in the long run, independently of the level of $\underline{h}$. The reason is that if, say, $\underline{h}$ is sufficiently large that $\dot{A}(t)/A(t) > \lambda$ initially, then the gap narrows; but the narrowing of the gap reduces $\dot{A}(t)/A(t)$; the gap continues to narrow until, in the limit, $\dot{A}(t)/A(t)$ has fallen to the value λ at which point the system is in equilibrium with a constant gap.

Nevertheless $\underline{h}$ makes a difference. The asymptotic or equilibrium path of $A(t)$, say $A^*(t)$, is an increasing function of $\underline{h}$. There is an analogy here with those models of growth which make the long-run rate of output growth independent of the saving ratio, though the long-run "level" of output depends upon the magnitude of the saving ratio.

These results are confirmed by the solution, in (4), of the differential equation obtained by substituting (2) into (3):

$$(4) \qquad A(t) = (A_o - \frac{\Phi}{\Phi + \lambda}) e^{-\Phi t} + \frac{\Phi}{\Phi + \lambda} T_o e^{\lambda t}$$

As (4) shows, the equilibrium path, $A^*(t)$, is

$$(5) \qquad A^*(t) = \frac{\Phi(h)}{\Phi(h) + \lambda} T_o e^{\lambda t}$$

An interesting property of this relation is that the elasticity of $A^*(t)$

with respect to $\underline{h}$ is increasing in λ :

$$(6) \qquad \frac{\partial A^*(t)}{\partial h} \; \frac{h}{A^*(t)} \; = \; \left[\; \frac{h\Phi'(h)}{\Phi(h)} \; \right]\left[\; \frac{\lambda}{\Phi(h) + \lambda} \; \right]$$

This indicates that the payoff to increased educational attainment is greater the more technologically dynamic is the economy. It suggests the possibility that society will want to invest more in education relative to tangible capital the more dynamic the technology. This is further suggested by later results.

To complete the model, I introduce the following additional relations and variables. The number of educators, E , plus the number of students, S , plus the number of production workers, N , comprise the total labor force L :

$$(7) \qquad\qquad E(t) + S(t) + N(t) = L(t)$$

The labor force and its components all grow exponentially over time at rate $\gamma > 0$. Hence

$$(8) \qquad\qquad L(t) = L_o \, e^{\gamma t}$$

$$(9) \qquad\qquad \frac{E(t)}{L(t)} = b = \text{constant} \; .$$

$$(10) \qquad\qquad \frac{S(t)}{L(t)} = s = \text{constant} \; .$$

(11)
$$\frac{N(t)}{L(t)} = n = \text{constant} .$$

In this state of balanced labor force growth, the index of per capita educational attainment, $\underline{h}$, of every post-education individual is assumed to be an increasing function of both $\underline{b}$ and $\underline{s}$:

(12)
$$h = \psi(b, s) , \qquad \psi_1 > 0, \ \psi_2 > 0$$
$$\psi(0,0) = 0$$

I interpret this model as one in which everyone attends school for some (equal) length of time; an increase in $\underline{b}$ connotes a longer period of education since it is the number of students (expressed as a proportion of the labor force) attending school at any moment of time.

Finally, I postulate golden-age growth: the ratio of capital to "augmented" or "effective" labor in production is a constant, $\underline{k}$; since N grows exponentially at rate γ and, in equilibrium, $A(t)$ grows exponentially at rate λ , as indicated by (5), we have

(13)
$$\frac{K(t)}{A^*(t)N(t)} = \frac{K(t)}{A^*(0)e^{\lambda t} N(0)e^{\gamma t}} = \frac{K(t)}{A^*(0) N(0)e^{gt}} = k , \qquad g = \lambda + \gamma .$$

The following expressions for golden-age consumption, $C(t)$, can then be obtained:

(14)
$$C(t) = F \left[K(t), A^*(t) N(t) \right] - \dot{K}(t)$$

$$= F \left[k A^*(0) N(0)e^{gt}, A^*(0) N(0)e^{gt} \right] - gK(t)$$

$$= \left[F(k, 1) - gk \right] A^*(0) N(0)e^{gt} ,$$

the last result by virtue of constant returns to scale.

IV. The Golden Rule of Education

To find the value of $N(0)$ or $\underline{n}$ which maximizes golden-age consumption it appears that we need merely maximize $A^*(0) N(0)$ with respect to $N(0)$. But we first need to determine $\underline{b}$ and $\underline{s}$ as functions of $\underline{n}$ since $\underline{h}$ and hence $A^*(0)$ is not simply a function of the sum $b + s$.

For efficiency in the education sector, $\underline{b}$ and $\underline{s}$ must be such as to maximize $\underline{h}$ for given $\underline{n}$. Hence it is necessary to maximize

(15)
$$h = \psi(b, 1 - b - n)$$

subject to a constant $\underline{n}$. Equating to zero the derivative of $\underline{h}$ with respect to $\underline{b}$ we obtain

(16)
$$\psi_1(b, 1 - b - n) - \psi_2(b, 1 - b - n) = 0$$

It will be assumed that (16) gives a unique interior maximum with

$b > 0$, $s = 1 - b - n > 0$. The second order condition, (17), is therefore
assumed to be satisfied:

(17)
$$\psi_{11} - \psi_{12} - \psi_{21} + \psi_{22} < 0$$

This implies a diminishing marginal rate of substitution between faculty
and students.

Taking the total differential of (16) we obtain

(18)
$$\frac{db}{dn} = \frac{dE}{dN} = \frac{\psi_{12} - \psi_{22}}{\psi_{11} - \psi_{12} - \psi_{21} + \psi_{22}} \quad .$$

It is reasonable to suppose that $\psi_{12} > 0$ and $\psi_{22} < 0$ so that $db/dn < 0$,
but in any case, $E(0)$ is a single-valued function of $N(0)$.

Now we can proceed to maximize $A^*(0) N(0)$ to find the consumption-
maximizing golden age path (for any given k). The problem is to maximize

(19)
$$A^*(0) N(0) = \frac{\Phi \left[\psi \left(\frac{E}{L} , 1 - \frac{E}{L} - \frac{N}{L} \right) \right] T_0 N(0)}{\Phi \left[\psi \left(\frac{E}{L} , 1 - \frac{E}{L} - \frac{N}{L} \right) \right] + \lambda}$$

with respect to $N(0)$.

The total derivative of $A^*(0) \, N(0)$ with respect to $N(0)$ is

$$(20) \qquad \frac{d(A^*N)}{dN} = \frac{dA^*}{dN} \, N + A^*$$

$$= \left(\frac{dA^*}{dh} \right) \left(\frac{dh}{dN} \right) \, N + A^*$$

$$= \left\{ T_o \left[\frac{(\Phi + \lambda)\Phi' - \Phi\Phi'}{(\Phi + \lambda)^2} \right] \right\} \left\{ \left(\frac{\psi_1}{L} - \frac{\psi_2}{L} \right) \frac{dE}{dN} - \frac{\psi_2}{L} \right\} N$$

$$+ \frac{\Phi}{\Phi + \lambda} \, T_o \; .$$

Equating this derivative to zero and noting that, for efficiency, $\psi_1 - \psi_2 = 0$ as shown in (16), we obtain the necessary condition for an interior maximum:

$$(21) \qquad - \left\{ \frac{\lambda \, \Phi'[\psi(b, \; 1 - b - n)] \; \psi_2 \, (b, \; 1 - b - n)}{\Phi[\psi(b, \; 1 - b - n)] + \lambda} \right\} n + \Phi[\psi(b, \; 1 - b - n)] = 0$$

This may be written

$$(21') \qquad n = \frac{(\Phi + \lambda) \, \Phi}{\lambda \Phi' \psi_2} \; .$$

Since $\underline{b}$ is a function of $\underline{n}$, independently of λ, (21) is of the form

(21") $\qquad - H(n, \lambda) n + J(n) = 0$

In these terms, the second-order condition that the stationary value be a maximum is

(22) $\qquad - H_n(n, \lambda) n - H(n, \lambda) + J'(n) < 0$

I omit expression of this condition in terms of the original functions. It can be stated that this condition is easily satisfied under reasonable assumptions on those functions.

I shall assume that a unique interior (local) maximum exists. (Of course, $n = 0$ and $n = 1$ could not be maxima since, for those values of n, $A^*(0) N(0) = 0)$. Hence (16), (21) and (22) characterize uniquely the consumption-maximizing golden age or Golden Rule path.

It was seen earlier that the elasticity of A^* with respect to h is increasing in λ. This suggested the hypothesis that more resources ought to be devoted to the education sector the more dynamic is the technology. Hence it is natural to ask whether the Golden Rule value of n is decreasing in λ in this model.

Differentiating (21") totally, we obtain

(23) $\qquad \dfrac{dn}{d\lambda} = \dfrac{H_\lambda(n, \lambda) n}{- H_n(n, \lambda) n - H(n, \lambda) + J'(n)}$

Since the second-order condition (22) is satisfied (by virtue of the assumption that a Golden Rule path exists), the denominator in (23) is negative. The numerator is positive since $n > 0$ and $\partial H/\partial \lambda > 0$ (as can be seen from (21)). Hence $dn/d\lambda$ is negative, as conjectured.

As for the effect of an increase of λ on Golden Rule <u>tangible</u> capital intensity, $\hat{k}$, where $f'(\hat{k}) = \lambda + \gamma$ defines $\hat{k}$, one can see that $\hat{k}$ will fall if and only if $f''(k) < 0$ (diminishing returns). The Golden Rule tangible investment-output ratio, $\hat{s} = f'(\hat{k})\ \hat{k}/f(\hat{k})$, will also fall (with $\hat{k}$) if the substitution elasticity exceeds one, and rise if the elasticity is less than one. Thus an increase of λ may or may not entail a rise of tangible investment.

IV. Concluding Remarks

A model of economic growth has been constructed which emphasizes the role of education in speeding technological diffusion and hence, in the long run, increasing the "level" of the technology in practice. As was shown, it is reasonable to expect a Golden Rule of Education to exist in this model. It was demonstrated that if a Golden Rule path exists, Golden Rule growth will require more resources in education the more technologically progressive is the economy.

REFERENCES

1. R. R. Nelson and E. S. Phelps, "Investment in Humans, Technological Diffusion and Economic Growth",