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A BUSINESS GAME FOR TEACHING AND RESEARCH PURPOSES

PART III

DISCUSSION AND MANUAL FOR USERS

Martin Shubik

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A BUSINESS GAME FOR TEACHING AND RESEARCH PURPOSES

PART III: DISCUSSION AND MANUAL FOR USERS

by

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# A BUSINESS GAME FOR TEACHING AND RESEARCH PURPOSES\*

## PART III

### DISCUSSION AND MANUAL FOR USERS

by

Martin Shubik\*\*

#### 1. Introduction

In previous papers 1/ a general description together with a discussion of the mathematical structure of a business or oligopoly game has been given. The game was designed both for teaching and experimental purposes. This paper is designed to serve both as a discussion paper and a manual to those who wish to use the game for either purpose.

In order to facilitate the use of the program, in Section 2 a complete listing of the parameters and variables required as input is given. This is followed immediately by a sample input and in Appendix II the input subroutine is given for those who wish for details concerning formats. Utilizing the information in Section 2 as a basis for the exposition, Section 3 is devoted to a detailed discussion of the selection of parameters and initial conditions in terms of the structure of markets and the sensitivity of various aspects of the game.

Section 4 deals with problems of briefing, management of play and with the decision variables. The last section, Section 5, outlines several different teaching and experimental uses for the game.

#### 2. A Sample Game

The first subsection here lists the input symbols in the order in which they are read, describes them and gives the mathematical

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symbol used in "A Business Game for Teaching and Research Purposes, Part IIA" 2/. The second subsection provides a sample input for a two-person game.

2.1. Inputs in order of Appearance

| <u>Program Symbol</u> | <u>Mathematical Symbol</u> | <u>Description</u>  | <u>Initial Values</u>                                 |
|-----------------------|----------------------------|---|---|
| IQRT                  | -                          | Quarter number  | 1   |
| IK                    | -                          | If 0, read history from SAVTAP, otherwise from cards                        | 1   |
| IKK                   | -                          | Control for entry of new firm<br>IKK=1 for entry                            | 0   |
| ID                    | -                          | Not used  | 0   |
| GAMN(I)               | -                          | Game identification   | Game 19   |
| NPLRS                 | n                          | Number of players   | 2   |
| NPLRC                 | -                          | Highest team identification number  | 2   |
| LAG                   | -                          | Lag between final production and decision to produce                        | 0   |
| JQRT                  | -                          | Quarter number compared with IQRT   | 1   |
| GAME(I)               | -                          | Game identification compared with GAMN(I)                                   | Game 19   |
| CDVD(I)               |                            | Cumulated dividends   | 0,0   |
| CASH(J)               |                            | Initial cash  | .2+8,.2+8   |
| ICONO(J)              | i                          | Company number  | 1,2   |
| DINV(J)               | $I_1$                      | Inventory   | 200000,200000   |
| ASET(J)               | $A_1$                      | Dollar investment in plant  | 10000000,10000000                                     |
| CRD(J)                |                            | Line of Credit  | 100000000,100000000                                   |
| DBT(J)                |                            | Loans outstanding   | 0,0   |
| ADVT(M,MM)            |                            | Advertising expenditures:<br>3 periods ago<br>2 periods ago<br>1 period ago | 6000000,6000000<br>6000000,6000000<br>6000000,6000000 |
| Q(M,MM)               | $C_1$                      | Previous production rates<br>MM controls the lag.                           | 250000,250000   |
| CARY(M)               | $C_{I,i}$                  | Unit inventory carrying cost  | 10.,10.,  |

| <u>Program Symbol</u> | <u>Mathematical Symbol</u>       | <u>Description</u>  | <u>Initial Values</u>  |   |   |       |   |       |   |         |         |         |
|-----------------------|----------------------------------|---|--|---|---|-------|---|-------|---|---------|---------|---------|
| FCST(M)               | $K_i$                            | Fixed costs or overheads (administrative only)                            | 500000,500000  |   |   |       |   |       |   |         |         |         |
| R(M)                  |                                  | Ruin level  | 10000000,10000000  |   |   |       |   |       |   |         |         |         |
| BQ(M)                 |                                  | Maximum permitted production per quarter                                  | 500000,500000  |   |   |       |   |       |   |         |         |         |
| H(M,MM)               | $h_4$<br>$h_3$<br>$h_2$<br>$h_1$ | Effectiveness weights for pre-vious and current advertising (periods ago) | <table style="display: inline-table; vertical-align: middle;"> <tr> <td rowspan="4" style="font-size: 3em; vertical-align: middle;">}</td> <td>3</td> <td>- , -</td> </tr> <tr> <td>2</td> <td>- , -</td> </tr> <tr> <td>1</td> <td>.5 , .5</td> </tr> <tr> <td>current</td> <td>.5 , .5</td> </tr> </table> | } | 3 | - , - | 2 | - , - | 1 | .5 , .5 | current | .5 , .5 |
| }                     | 3                                | - , -   |  |   |   |       |   |       |   |         |         |         |
|                       | 2                                | - , -   |  |   |   |       |   |       |   |         |         |         |
|                       | 1                                | .5 , .5   |  |   |   |       |   |       |   |         |         |         |
|                       | current                          | .5 , .5   |  |   |   |       |   |       |   |         |         |         |
| ALPHA                 | $\alpha$                         | Total market size parameter   | 1.04 + 6   |   |   |       |   |       |   |         |         |         |
| BETA                  | $\beta$                          | Price sensitivity parameter   | .266666666667+4  |   |   |       |   |       |   |         |         |         |
| GAMMA                 | $\gamma$                         | Inter-firm price sensitivity parameter                                    | .3 + 1   |   |   |       |   |       |   |         |         |         |
| ETA                   | $\eta$                           | Cooperative (or institutional) advertising parameter                      | -  |   |   |       |   |       |   |         |         |         |
| THETA                 | $\theta$                         | Competitive advertising parameter   | .15 + 0  |   |   |       |   |       |   |         |         |         |
| SIG1                  |                                  | Variance of random variable in effective advertising                      | -  |   |   |       |   |       |   |         |         |         |
| SIG2                  |                                  | Variance of random variable in demand                                     | -  |   |   |       |   |       |   |         |         |         |
| AMU1                  |                                  | Adds constant amount to effective advertising                             | -  |   |   |       |   |       |   |         |         |         |
| AMU2                  |                                  | Adds constant factor to the cycle   | -  |   |   |       |   |       |   |         |         |         |
| OMEGA                 | $\omega$                         | Angular velocity of cycle   | -  |   |   |       |   |       |   |         |         |         |
| TAX                   |                                  | Tax rate  | .5 + 0   |   |   |       |   |       |   |         |         |         |
| DEP                   | $d_1$                            | Depreciation rate on fixed assets   | .25 - 1  |   |   |       |   |       |   |         |         |         |
| ROE                   | $\rho$                           | Rate of interest on dividends paid  | .1 - 1   |   |   |       |   |       |   |         |         |         |
| BU                    |                                  | Not used  | -  |   |   |       |   |       |   |         |         |         |
| ENRT                  |                                  | Level of industry return on investment required for entry                 | .1 - 1   |   |   |       |   |       |   |         |         |         |
| ANEW                  | $v$                              | Phase angle of cycle at initial point                                     | -  |   |   |       |   |       |   |         |         |         |
| AR                    | $r$                              | Rate of market growth   | .1 + 1   |   |   |       |   |       |   |         |         |         |

| <u>Program Symbol</u> | <u>Mathematical Symbol</u> | <u>Description</u>                                     | <u>Initial Values</u> |
|-----------------------|----------------------------|--|-----------------------|
| R1(K)                 | 1                          | Controls amplitude of combined trend and cycle         | 1                     |
|                       | 2                          | Not used   | -                     |
|                       | 3                          | Number of quarter                                      | -                     |
|                       | 4                          | Cycle amplitude parameter                              | -                     |
| <hr/>                 |                            |  |                       |
| CONS(M)               | 1                          | Cost per unit of increasing production                 | 20                    |
|                       | 2                          | Cost per unit of decreasing production                 | 20                    |
|                       | 3                          | Cost of a unit of added capacity                       | -                     |
|                       | 4                          | Liquidation loss                                       | -                     |
|                       | 5-12                       | discount rate<br>Not used                              |                       |
| <hr/>                 |                            |  |                       |
| NCONS(M)              |                            | Not used   | -                     |
| <hr/>                 |                            |  |                       |
| IPER                  |                            | Number of periods of profits needed for new entry      | 10                    |
| IP                    |                            | Number of periods profits have been above ENRT         | -                     |
| <hr/>                 |                            |  |                       |
| INDEX (J)             |                            |  |                       |
|                       | 1                          | If zero, QSLS not included in decision                 | 0                     |
|                       | 2                          | 1=your industry stat,<br>2=yours and adverage<br>3=all | 3                     |
|                       | 3                          | If 2, get price information                            | 2                     |
|                       | 4                          | If 2, get sales information                            | 2                     |
|                       | 5                          | If 2, get advertising information                      | 2                     |
|                       | 6                          | If 2, get production information                       | 2                     |
|                       | 7                          | If 2, get inventory information                        | 2                     |
|                       | 8                          | If 2, get profit information                           | 2                     |
|                       | 9                          | If 2, get dividend information                         | 2                     |
|                       | 10                         | If 2, get interest and cumulative dividend information | 2                     |

| <u>Program Symbol</u> | <u>Mathematical Symbol</u> | <u>Description</u>                                     | <u>Initial Values</u> |
|-----------------------|----------------------------|--|-----------------------|
| 11                    |                            | If 2, get investment information                       | 2                     |
| 12                    |                            | If 2, get lost sales                                   | 2                     |
| 13                    |                            | If 1, get individual balance sheet, if 2, get industry | 1                     |
| 14-19                 |                            | Not used   | -                     |
| 20                    |                            | If positive, more industries follow                    | 1                     |

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Current Decisions

|            |       |  |   |
|------------|-------|--|---|
| IKK        | -     | Control for entry of new firm IKK=1 for entry            |   |
| P(I)       | $p_i$ | Price  |   |
| ADVT(I,4)  | $a_i$ | Current advertising decision                             |   |
| Q(I,LUG+1) | $q_i$ | Production this period (units)                           |   |
| QSLS(I)    |       | Max units of goods offered for sale, only if INDEX (1)=0 |   |
| DVD(I)     |       | Dividends  | - |
| VEST(I)    |       | Investment   | - |

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Each ruling in the above table marks the end of a separate

READ statement.

2.2.

|           |                 |           |                                   |       |    |
|-----------|-----------------|-----------|-----------------------------------|-------|----|
| IQRT      | IK              | ID        |                                   |       |    |
| 1         | 1               |           |                                   |       | 01 |
| GAMN      |                 |           |                                   |       |    |
| GAME      | 19              |           |                                   |       | 02 |
| NPLRS     | NPLRC           | LAG       | JQRT                              |       |    |
| 2         | 2               | 0         | 1                                 |       | 03 |
| GAME      |                 |           |                                   |       |    |
| GAME      | 19              |           |                                   |       | 04 |
| CDVD      |                 |           |                                   |       | 05 |
|           | CASH            |           |                                   |       |    |
|           | .2+8            | .2+8      | (Initial cash)                    |       | 06 |
| ICONO     |                 |           |                                   |       |    |
| 1         | 2               |           | (Company number)                  |       | 07 |
| DINV      |                 |           |                                   |       |    |
| 200000    |                 | 200000    | (Initial inventories)             |       | 08 |
| ASET      |                 |           |                                   |       |    |
| 10000000  |                 | 10000000  | (Initial worth of plant)          |       | 09 |
| CRD       |                 |           |                                   |       |    |
| 100000000 |                 | 100000000 | (Initial credit line)             |       | 10 |
| DBT       |                 |           |                                   |       |    |
|           |                 |           | (Initial loans outstanding)       |       | 11 |
| ADVT      |                 |           |                                   |       |    |
| 6000000   |                 | 6000000   | (Advertising 3 periods ago)       |       | 12 |
| ADVT      |                 |           |                                   |       |    |
| 6000000   |                 | 6000000   | (Advertising 2 periods ago)       |       | 13 |
| ADVT      |                 |           |                                   |       |    |
| 6000000   |                 | 6000000   | (Advertising 1 period ago)        |       | 14 |
| Q         |                 |           |                                   |       |    |
| 250000    |                 | 250000    | (Last production rate)            |       | 15 |
| C         |                 |           |                                   |       |    |
| 150       |                 | 150       | (Cost of production (variable))   |       | 16 |
| CARY      |                 |           |                                   |       |    |
| 10        |                 | 10        | (Unit inventory carrying cost)    |       | 17 |
| FCST      |                 |           |                                   |       |    |
| 500000    |                 | 500000    | (Fixed costs)                     |       | 18 |
| R         |                 |           |                                   |       |    |
| 10000000  |                 | 10000000  | (Ruin level)                      |       | 19 |
| BQ        |                 |           |                                   |       |    |
| 500000    |                 | 500000    | (Upper limit on production)       |       | 20 |
| H         |                 |           |                                   |       |    |
|           |                 |           | (Advertising weights for previous |       | 21 |
| h         |                 |           | and current effectiveness)        |       | 22 |
| H         |                 |           |                                   |       |    |
| .5        |                 | .5        |                                   |       | 23 |
| H         |                 |           |                                   |       |    |
| .5        |                 | .5        |                                   |       | 24 |
| ALPHA     | BETA            | GAMMA     | ETA                               | THETA |    |
| 1.04+6    | .266666666667+4 | .3+1      | -                                 | .15+0 | 25 |
| SIG1      | SIG2            | AMU1      | AMU2                              | OMEGA | 26 |
| TAX       | DEP             | ROE       | BU                                | ENRT  |    |
| .5+0      | .25-1           | .1-1      | -                                 | .1-1  | 27 |
| ANEW      | AR              |           |                                   |       |    |
|           | .1+1            |           |                                   |       |    |
| R1(1)     | R1(2)           | R1(3)     | R1(4)                             |       |    |
| 1.        | -               | -         | -                                 |       | 29 |



|          |          |   |         |   |                                |    |   |   |    |    |    |    |    |  |    |    |
|----------|----------|---|---------|---|--------------------------------|----|---|---|----|----|----|----|----|--|----|----|
|          | CONS(1)  |   | CONS(2) |   | (Costs of changing production) | 30 |   |   |    |    |    |    |    |  |    |    |
|          | 20       |   | 20      |   |                                |    |   |   |    |    |    |    |    |  |    |    |
|          | NCONS(1) |   |         |   | (Not used)                     | 31 |   |   |    |    |    |    |    |  |    |    |
| IPER     | IP       |   |         |   |                                |    |   |   |    |    |    |    |    |  |    |    |
| 10       | -        |   |         |   |                                | 32 |   |   |    |    |    |    |    |  |    |    |
| INDEX(1) | 2        | 3 | 4       | 5 | 6                              | 7  | 8 | 9 | 10 | 11 | 12 | 13 | 14 |  | 20 |    |
|          | 0        | 3 | 2       | 2 | 2                              | 2  | 2 | 2 | 2  | 2  | 2  | 2  | 1  |  | 1  | 33 |

The input cards are numbered above only to give an indication of the approximate size of the input deck. As the number of players and various other structural conditions are changed a few more cards of input may be needed, as is indicated by the FORMAT instructions.

The extra two lines of input given below are the decisions of the two firms for the first period of the game.

|   |      |           |            |          |        |         |
|---|------|-----------|------------|----------|--------|---------|
| J | P(I) | ADVT(I,4) | Q(I,LUG+1) | QSL(S(I) | DVD(I) | VEST(I) |
| 1 | 240  | 6000000   | 250000     |          |        |         |
| 2 | 240  | 6000000   | 250000     |          |        |         |

### 3. Parameters and Initial Conditions

In Section 2 the parameters and variables and a sample input to a game were described. In a previous paper 3/ details and some analysis of the game were given. In general, games are played in a simulated environment. This is true in particular of computerized games. The program produces a simulated environment. Depending upon the uses of the game the selection of parameters and initial conditions will be more or less guided by a desire to

reflect some aspects of "reality" in the sense of possessing some broad features of a market.

Some of the parameters control the sensitivity of the response surface, i.e., the amounts by which the payoffs to the firms change as different decision variables are changed. By judicious selection of parameters the game can be made to focus on very different problems such as inventories, learning, competition, production, advertising, and so forth. In the subsequent subsections the control of the parameters and initial conditions are discussed.

### 3.1. Sensitivity of Payoffs, Market Conditions

The demand confronting the  $i^{\text{th}}$  firm in any period  $t$  is given by:

$$(1) \quad F_i(p, a) = \frac{1}{n} \{ \alpha - \beta [p_i + \gamma(p_i - \bar{p})] \} \{ 1 + n \sqrt{\sum a_i (1 + \epsilon_i)} \} \{ \theta + (1 - \theta) \frac{na_i (1 + \epsilon_i)}{\sum a_i (1 + \epsilon_i)} \} \{ r^t (1 + \lambda \sin(\omega t + \nu) + \xi) \}$$

where the time subscript  $t$  has not been entered. This is the demand contingent upon all other firms having sufficient supplies to satisfy their demands. If they do not have adequate supplies, a rationing scheme will reassign the lost sales. The specific algorithm for the lost sales is given in a previous paper 4/. If the inventories of the  $i^{\text{th}}$  firm were at least adequate to satisfy the first round of demand, this demand may be increased. The payoff to the firm will be given by

$$(2) \quad m_i = \min[F_i, m_i] (p_i - c_i) - c_{I,i} I_i - a_i - K_i$$

where  $m_i$  is the amount available for sale. There are some corrections and adjustments to be made for dynamic features of inventory charges, advertising, etc. They are discussed elsewhere. Furthermore, in a

dynamic context it is important to note that revenues and cash flow will differ as inventories may be built up or depleted.

We commence with an examination of the five factors in equation (1). The factor  $1/n$  which appears in front of the equation of demand enables an experimenter to scale down market shares of a fixed market as numbers increase. The game, as it is currently programmed, lays emphasis upon primarily symmetric conditions, hence the  $n$  does not have a subscript and it is not possible to initially assign different shares, all other things being equal. The main implication of this is not that all firms are the same, but that they are selling goods of similar economic importance.

The factor

$$(3) \quad \alpha - \beta(p_1 + \gamma(p_1 - \bar{p}))$$

determines the influence of prices. If all firms charge the same price this becomes:

$$(3)^1 \quad \alpha - \beta p_1, \quad \text{or} \quad \alpha - \beta \bar{p}$$

a linear function as is illustrated in Figure 1. The  $\alpha$  can be described approximately as the "absolute size" of the market.

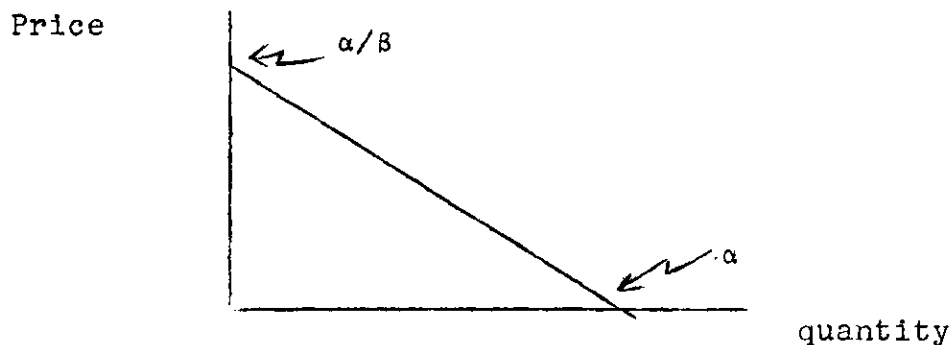


Figure 1

It is the intercept term of the linear approximation to the demand function.

The price axis intercept  $\alpha/\beta$  is the price at which demand will be zero. It may be regarded as the price above the highest reserve price for any customer. The  $\beta$  gives the overall price sensitivity of the markets as all firms vary their prices together. The concept of "industry" is not particularly well defined, however, we often refer to an industry such as the television or automobile industry where we have in mind several firms selling sufficiently similar products that the products of all are of primary strategic concern to each. Thus, we could obtain an estimate of  $\beta$  from an econometric estimate of the elasticity of demand for television sets or automobiles.

Suppose, for example, we wished to approximately simulate some of the characteristics of the automobile market we could start to select parameter values in the following manner:

Let us make the basic assumption that the actual market may be described as being in more or less a non-cooperative oligopolistic equilibrium. It has been shown elsewhere in 5/ that this will imply that the price charged by firms in the market is given by:

$$(4) \quad p = \frac{\alpha + c^1 \left(1 + \frac{n-1}{n} \gamma\right)}{2 + \left(\frac{n-1}{n}\right) \gamma}$$

We may want a three firm game, (which sets  $n=3$ ) in which, at a noncooperative equilibrium the price is \$2,000 per unit and the volume around 8,000,000. Furthermore, we may wish to have variable costs run, say approximately 40% of retail price, thus  $c^1=800$  (the  $c^1$  includes not only variable manufacturing costs, but inventory costs as well, this is shown elsewhere 6/).

The  $\gamma$  measures the interdependence between the change in the price of one firm and the demand for the goods of the others. It is directly related to the cross-elasticity between the firms. The value of  $\beta(1+\gamma(\frac{n-1}{n}))$  can be interpreted as the slope of the line  $dd'$  in Figure 2. This line is a linear approximation of the contingent demand  $d'$  faced by one firm for its product as it varies its price, given the prices of the others. It can be seen immediately that

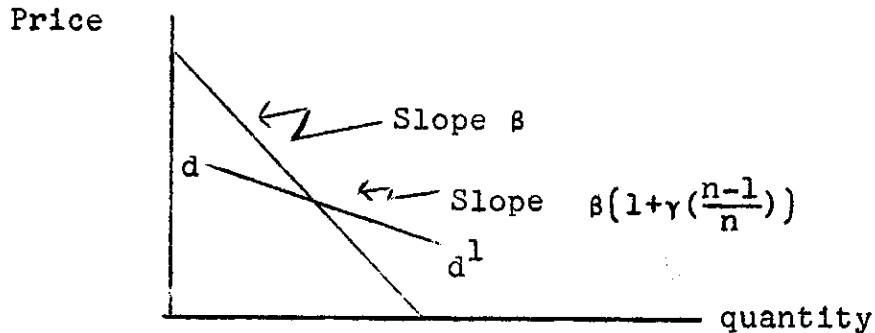


Figure 2

$\gamma=0$  implies that the firms are independent monopolists.  $\gamma=3$  gives a relatively high cross-elasticity of demand between the firms.

Returning to equation (4) we have:

$$2,000 = \frac{\frac{\alpha}{\beta} + 800(1 + \frac{2}{3} \cdot 3)}{2 + \frac{2}{3} \cdot 3} = \frac{\frac{\alpha}{\beta} + 2400}{4}$$

or  $\alpha/\beta = 5,600$

We also wish to have  $q=\alpha-\beta p$  where  $q$  will be a quarter of yearly demand, as the period in a game is a quarter.

$$2,000,000 = \alpha - \beta(2,000)$$

hence,  $\beta = 20000/36 = 555 \frac{5}{9}$   
 $\alpha = 3 \frac{1}{9} \times 10^6$ .

Briefly, we can characterize the effects of price as follows:

$\beta=0$  a completely inelastic set of independent markets.

(If one wished to study competition in an inelastic market it can be done by setting  $\beta=\epsilon$  and  $\gamma = \frac{k}{\epsilon}$  so that  $\beta\gamma=k$  where  $k$  may be some relatively large number but  $\epsilon$  is very small).

$\beta =$  intermediate size and  $\gamma=0$  gives a set of independent markets, each with a price elasticity controlled by  $\beta$ .

$\beta =$  intermediate size and (1)  $0 < \gamma \leq 3$ , (2)  $\gamma \geq 8$  give respectively moderate to mediumly heavy price cross-elasticities in case (1) and conditions for a cut-throat market for case (2).

We may next consider advertising. There is a cooperative, or institutional effect given by:

(5)  $1 + n\sqrt{\sum a_i(1+\epsilon_i)}$

This particular form swings the overall industry demand function out in a manner indicated in Figure 3. There are several reasons

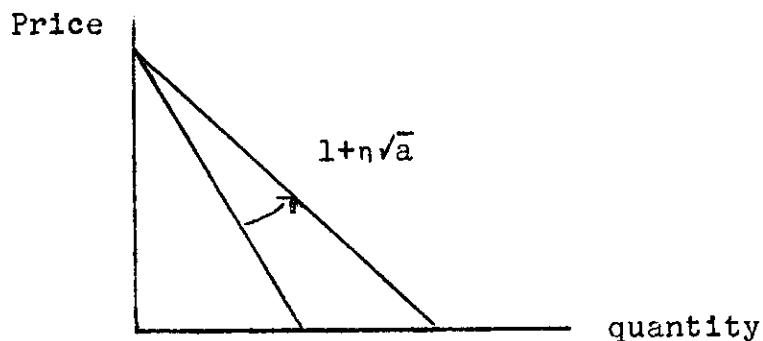


Figure 3

for the selection of this functional form for institutional advertising. It is mathematically relatively easy to handle as has been shown in a previous paper 8/. In particular, the equations

which must be solved to obtain the predictions of several theories of interest are in triangular form which enables one to solve them first for price, then for advertising budgets sequentially instead of simultaneously. From an experimental viewpoint, there is basically one clearly defined parameter to control for a cooperative advertising effect. As far as "reality" is concerned cases can be made both for and against the specific form used. A list of alternatives based upon insights, observations and measurement would provide a fruitful way to investigate the effect of advertising.

Suppose we want the market to be approximately ten percent larger at equilibrium, with advertising, than without it. We want  $(1+n\sqrt{\Sigma a_1(1+\epsilon_1)})= 1.1$ . First, we note the dynamic and stochastic aspects of advertising. The  $a_1$  in the expressions (1) and (5) are not actually the advertising decisions made currently, but are the current "effective" advertising obtained by weighting as far back as the three previous periods, thus:

$$(6) \quad a = \sum_{i=1}^n a_i = \sum_{i=1}^n \sum_{t=1}^4 h_t a_{i,t}$$

where the  $h_t$  are weightings for the relative effectiveness of advertising in different periods,  $a_{i,t}$  is the actual advertising budget of the  $i^{\text{th}}$  firm during the period  $t$ , and  $t$  ranges back over this period and the 3 preceding. Four periods were selected on the assumption that for most industries one year is a reasonable upper bound on the duration of the effect of advertising expenditure. Values which are naturally suggested for the  $h_t$  are those which produce an exponential smoothing, for

example 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ . The values given in the example in Section 2 are  $\frac{1}{2}$ ,  $\frac{1}{2}$ , 0, 0. There are two considerations to be given to selection of the weights: "realism" and the fidelity of the simulation of the phenomenon and experimental control. If all weights are used the effect of advertising is characterized by a third degree difference equation which for some experimental purposes may be more complex than is desired. In a steady state the amount advertised every period will be the same (assuming no cyclical effects), however, there will still be an important difference between static and dynamic solutions to the problem of selecting an advertising budget, that is if there is an interest rate a cost must be calculated for the lock-up of money in advertising over several periods.

In the advertising formula there are terms of the form  $(1+\epsilon_1)$  these are random variables which may be applied to make the effectiveness of advertising subject to chance. In general, unless there are well considered reasons for doing otherwise the suppressing of random variables is desirable.

As a first approximation we may want the advertising budget of a firm to be around say four percent of gross sales in which case:

$$a_1 = \frac{2,200,000}{3} \times \frac{2,000}{25} \times \frac{1}{25} \approx 60 \times 10^6$$

We wish 
$$\sqrt[n]{180 \times 10^6} = .10$$

$$\text{or } n = .10/13.4 \times 10^3 \approx 7.5 \times 10^{-5}.$$

The second advertising term stresses the competitive or product and company differentiation aspects of advertising. It is given



by:

$$(7) \quad \theta + (1-\theta) \frac{na_1(1+\epsilon_1)}{\sum a_1(1+\epsilon_1)}$$

Again we note that the  $a_1$  are the effective advertising as described before. The  $\epsilon_1$  are random variables. The  $\theta$  may be regarded as an index of consumer loyalty. It measures the percentage of customers of the firm who are not affected by the competitive aspects of advertising. The remaining percentage  $(1-\theta)$  can be moved by advertising. It is evident that a more satisfactory model of a market would have the  $\theta$  as a function of previous states of the market; here, however, as a first approximation we assume that the  $\theta$  remains fixed for the duration of the play.

The form:

$$a_1 / \frac{\sum a_1}{n} \quad \text{or} \quad \frac{na_1}{\sum a_1}$$

is a familiar one and has been used in other games 9/ and elsewhere 10/ to investigate problems in advertising. It incidentally, also reflects an advertising rule-of-thumb or piece of folk lore that market share and amount of advertising are in direct proportion to each other.

If we leave out the effect of the cyclical term we may use expression (8) to determine a value for  $\theta$  that is consistent with our selection of  $a_1=60,000,000$  and  $n=7.5 \times 10^{-5}$ . This formula was developed in a previous paper 11/.

$$(8) \quad a_1 = \frac{G^2}{4n^3} \left[ n \left( (n-1)(1-\theta) + \frac{1}{2} \right) + \sqrt{n^2 \left( (n-1)(1-\theta) + \frac{1}{2} \right)^2 + 4 \frac{n(n-1)(1-\theta)}{G}} \right]^2$$

where  $G = \beta \left( 1 + \frac{n-1}{n} \gamma \right) \left( \frac{\frac{\alpha}{\beta} - c^1}{2 + \frac{n-1}{n} \gamma} \right)^2$

Substituting in the values for the parameters we obtain an approximate value of  $\theta = .7$  (which appears to be rather high for a commodity such as an automobile, it is unlikely that "consumer loyalty" is this great).

In concluding our discussion of the parameters controlling advertising we note that:

$\theta = 0$  and  $n = 0$  gives a game that is completely cut-throat on advertising;  $\theta = 1$  and  $n = 0$  is a game in which advertising has no effect whatsoever.

The term (9) enables the experimenter to take into account a trend, cycle and random component to demand. If lagged

$$(9) \quad W = r^t (1 + \lambda \sin(\omega t + r) + \xi)$$

variables appear in advertising, production or elsewhere the various static game solutions given by Levitan and Shubik 12/ would need to be modified in a non-trivial manner, otherwise it is sufficient to multiply then by the term  $W$  as has been indicated in the previous paper.

The before tax profit of a firm in a steady state condition is given by:

$$(10) \quad \pi_1 = q_1 (p_1 - c_1) - c_{I,1} I_1 - a_1 - K_1 - d_1 A_1$$

where  $q_1$  is the quarterly amount sold

$p_1$  is price charged

$c_1$  is variable manufacturing costs

$c_{I,1}$  is inventory carrying costs per unit

$I_1$  is inventory

$a_1$  is advertising cost

$K_1$  is fixed costs (administrative)

$d_1$  is the depreciation rate

and  $A_1$  is investment in plant and other fixed assets.

In a steady state, as it is assumed in the program that only one half of current production will be available for immediate sale, optimum inventories  $I_1 = q_1/2$ , hence we may rewrite (10) as:

$$(11) \quad \pi_1 = q_1 (p_1 - (c_1 + \frac{1}{2} c_{I,1})) - a_1 - K_1 - d_1 A_1$$

In our previous notation  $c_1^1 = c_1 + \frac{1}{2} c_{I,1}$ . A reasonable value for  $c_{I,1}$  would be 50; as  $c_1^1 = 800$  this would give  $c_1 = 750$ .

Suppose we set fixed assets  $A_1 = 8 \times 10^9$  and assume a depreciation rate of 3% per quarter, this gives a change of  $240 \times 10^6$  per quarter, where  $d_1 = .03$ . Setting administrative overheads at  $K_1 = 80 \times 10^6$  we obtain

$$\begin{aligned} \pi_1 &= \frac{2.2 \times 10^6}{3} (2,000 - 800) - 60 \times 10^6 - 240 \times 10^6 - 80 \times 10^6 \\ &= 500 \times 10^6 \end{aligned}$$

which is approximately a billion a year after taxes.

Although they do not affect the profit calculation in a steady state, two parameters must be fixed to give the values for changing the rate of production by one unit per quarter in a direction up or down.

The costs of change are not necessarily the same in each direction but may depend upon features such as union laws, unemployment codes and set-up costs.

Depending upon the selection of these two parameters and the variable costs and inventory carrying cost parameter, more or less emphasis can be laid upon production and inventory scheduling problems.

If the game is played with the possibility of adding to capacity, a parameter, CONS(3), must be assigned a value representing the cost of a unit addition to capacity. This will raise the level of BQ(M) which is the originally assigned maximum permitted production per quarter. The new capacity is available one period after it has been paid for.

There is a rate of interest which is paid on dividends and charged on bank loans. In particular, the introduction of the rate of interest enables the user to specify a goal for the firm and to set the context of the play within a general economic framework. The rate of interest may be regarded as the opportunity cost of capital elsewhere in the economy. The payment of a dividend to a stockholder releases the funds to earn at least that rate of interest.

In a dynamic context, a firm may have a possibly infinite life, it is reasonable over a finite span for it to attempt to maximize its present value which consists of the discounted income stream paid out as dividends together with its asset value. The worth of a firm can be expressed in terms of the following functional equation:

Let  $s_{i,t} \in S_{i,t}$  be the strategy of the  $i^{\text{th}}$  firm in period  $t$

$\rho$  the discount rate

$b_i$  the dividend paid in period  $t$

$V_{i,t}$  the present value of the firm

$$(12) \quad V_{i,t} = \text{Max}_{s_{i,t} \in S_{i,t}} \rho [V_{i,t+1} + b_{i,t+1}].$$

If the firm were to stay in existence indefinitely

$$V_{i,t} = \rho b_{i,t+1} + \rho^2 b_{i,t+2} + \dots + \rho^\tau V_{i,\tau}$$

where for large values of  $\tau$  the last term may be neglected and the value becomes essentially the optimal income stream.

If the firm is forced into bankruptcy or wishes to liquidate voluntarily there are two parameters which reflect financial conditions which must be taken into account.  $R(12)$  is the ruin level for liquid assets. If a firm has used its credit line up to its limit  $CRD(12)$  and has its liquid assets drop below  $R(12)$  it is declared ruined and a message is printed out to this effect. Under ruin or voluntary liquidation assets are valued at a percentage controlled by  $CONS(4)$  which reflects the liquidity problem faced when a sale of plant takes place.

Whenever an old firm is liquidated the statistics on competitive information are appropriately adjusted by the subroutine SKWEZ.

The user of the game may wish to include the possibility of having new firms enter the game during the course of play. The criterion for entry is either exogenous or computed and signalled by the subroutine ENTRY. The symbol IKK controls the entry of a new firm, if it is set a 1 the input subroutine expects to read parameters for a new firm before it reads the decisions for all the firms (see Appendix II, statement 95). The simple model for entry supplied in the subroutine is that a new firm will enter if the average profits of the industry have been above some level ENRT for a number of periods IPER.

There is a depreciation rate DEP which is applied to the fixed assets of the firm. In this game depreciation represents a usage of these assets which engenders immediate replacement expenditure.

#### 4. Decision Variables and Briefing

##### 4.1. Decision Variables

As can be seen from the program inputs and from the reproduced decision sheet, there are six possible decision variables under the control of the individual firm. They are respectively:

- price
- production level
- advertising budget
- quantity offered for sale
- dividends
- investment.

Every quarter the firms have an opportunity to select a new price for their product. As can be seen from the mathematical structure of the game, this version does not have any time effect

of price. Demand is independent of previous fluctuations. It may be desirable to impose a limit on the amount by which a price may be changed in a single period. This can be done in the briefing and focuses attention on certain problems in the theory of oligopoly concerning inventory cycles and the Edgeworth cycle 13/ in markets with price competition. In order to bring this under direct control of the program a modification would have to be made. It is noted parenthetically at this point that there are many problems connected with price which, although they are mentioned in verbal discussions, are difficult to give a satisfactory operational meaning. An example of one of these is "price" leadership.

In this simple game the firms are required to select a production rate during each quarter. Production scheduling appears in only a rudimentary form, in the sense that there are inventory carrying costs for finished goods and there are costs associated with changing the levels of production in either direction, up or down. The upper bound to permitted production is set by the parameter BQ whose size is modified by investment. The parameter LAG determines the number of periods between the decision to produce and the actual production.

The variable QSLS permits the user to run a game in which there is a direct control over inventory policy. If INDEX (1)=1 the firms are required to make a decision concerning the amount of their inventory they wish to offer to the market. If INDEX(1)=0 it is automatically assumed that everything that has been produced is available (only one half of this period's production is available for sale).

In the penumbra between economic theory and marketing the roles of different forms of advertising, distribution and promotion have scarcely been adequately differentiated. In Section 3, the various effects of the variable labelled advertising has already been discussed. The force of advertising in changing demand is channeled through three sets of parameters; ETA which controls the cooperative impact; THETA which controls the competitive impact and the array  $H(I,J)$  which specify weights indicating the effectiveness through time of an advertising commitment made at a specific period. A problem which has not yet been faced here, but which merits general consideration in gaming, concerns the labelling of decision variables. Suppose we produced a completely different scenario to a game which, like this one had been designed primarily with an economic background in mind, would there be any major change in observed behavior if the uses and verbal description given were entirely different?

The players may be required to state their policy concerning the declaration and payment of dividends during every period. As has already been noted, by introducing dividends DVD and the discount rate RHO, a tie-in of the game with general overall goal for the individual firm to maximize. This is particularly useful if the players are being paid; as a direct relation between their rewards and the sum of the value of their assets and the accumulated worth of their dividend payments can be established.



Investment enters the game in rudimentary form. If the user so desires the players may make decisions to purchase additional capacity by selecting a value for the variable VEST. The capacity will be available in the period succeeding the decision.

#### 4.2 Briefing

Unlike a computing machine, it is not possible to dump memory and start with a tabula rasa when using human beings to play in a game. Thus, there is little doubt that the effects of briefing are important and not fully understood. In the work on gaming to date, there has been considerable debate between schools favoring an "environment rich" or a strictly controlled "dust free" approach. Those interested in environment rich situations have, in general, stayed away from highly computerized games. The rules of their games are often not fully defined at the start of play; furthermore, there may be a considerable amount of verbal interchange which cannot be computerized and which may be subject to debate between the players and the referee.

The game designed and discussed here was designed in an attempt to bridge the gap between the very simple and highly controlled games favored by many experimental psychologists and the very complex and loosely structured environment rich games. By the judicious selection of a few parameters (primarily  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\theta$ ); various cost parameters, information display and initial conditions, such as starting assets, it is possible to produce many different one-person, one-variable experiments, several person, one-variable experiments and so on with increasing degrees

of complexity. The selection of parameters and the briefing can vary the stress placed upon economics, psychology or social-psychology, upon competition, learning or small group interaction. The importance of this feature of the game is that it is possible to experiment from several different viewpoints in situations which produce compatible and comparable information. By building up a set of simple different, but comparable, experiments it may be possible to gain insights into more complex situations.

The uses of the game to date have been primarily under conditions simulating many of the aspects of an oligopolistic market and the sample briefing given here is for this. A discussion of several of the detailed considerations is given immediately after the sample briefing in Exhibit I.

#### EXHIBIT I

#### INSTRUCTIONS TO PLAYERS OF BUSINESS GAME #6

You may imagine, if you wish, that the product you are producing is some major appliance, say refrigerators. There are five (5) firms in the industry, you are to make quarterly decisions about production, quantity marketed, price, advertising, and dividends.

Please make your decisions before lunch on each day. Quarterly reports will be available at the beginning of each morning.

#### GOALS FOR BUSINESS POLICY

Firms are expected to play in such a way as to get the largest possible current value of accumulated dividends and net assets. The dividends are assumed to earn compound interest for stockholders at 4% per quarter.

### DIVIDENDS

Firms are free to select any dividend policy except that dividends can only be paid to the extent that funds are available after bank loans are repaid.

### QUANTITY MARKETED

The quantity marketed by firms is bounded by starting inventory plus one half of current production.

### ADVERTISING

The effective advertising in any quarter is equal to half of the advertising in that quarter plus half of the advertising in the previous quarter.

### CYCLE

The market will be influenced by a cycle of period 8 quarters and amplitude 8%.

### RUIN

Firms must have \$10,000,000 in cash to cover operating transactions. Firms start with a \$20,000,000 line of bank credit. This will be drawn on to make up deficiencies where possible. If a firm cannot keep its cash amount above the ruin level, it will be liquidated at 60% of book value of net assets which will be credited to the dividend accounts on which interest will continue to accrue.

### END OF GAME

At the end of the game, net assets will be liquidated at book value.

### START

Initial cash is \$20 million and inventory is 200,000 units. Last production for each firm was 250,000 units; last advertising

was \$1,000,000. The cycle is on the upswing but is at the point where its effect on the market is zero.

#### PRODUCTION CAPACITY AND INVESTMENT

The initial capacity is 300,000 units per firm. Firms may invest in more plant and increase their capacities at the rate of \$80 per unit.

#### INFORMATION CONDITIONS

At the end of each quarter, you will obtain your balance sheet and detailed information on your market moves, as well as those of your competitors.

#### COSTS ARE AS FOLLOW:

1. Production Costs
  - \$170. per unit produced plus
  - \$10. per unit change to production rate per quarter
2. Inventory
  - \$2. per average unit per quarter
3. Fixed Costs
  - \$500,000 Administrative overhead
4. Advertising
  - Decision variables
5. Depreciation
  - 5% per quarter
6. Interest
  - 4% per quarter on loans at beginning of quarter
7. Taxes
  - 50% on net profit. Rebate for losses.

#### 1. Specification of Product or Market

In this game a product was suggested to the players. Although there is no clearly established evidence and no complete consensus it appears that the specification of a product provides many cues to players, especially if they have a business background and experience. It is alternatively possible to brief without specifying

a product. One way in which this may be done successfully is by giving the players several previous periods of information.

## 2. Verbal and/or Written Briefing

It has been suggested informally by Professor John Kennedy that depending upon how he verbally briefs his subjects, an experimenter can obtain almost any experimental result he wants. An oral briefing accompanied the written briefing. It was needed primarily for answering questions. The more naive the subjects or players are, the more necessary is the oral briefing.

## 3. Selection of Subjects or Players

There are many indications that the type of subjects are an important factor in understanding the play of the game. In the games run to date, which will be reported on in a subsequent publication, the subjects were administrative and scientific personnel of the IBM Research Laboratories and System Development Corporation; students at Santa Monica Junior College; and graduate students at Yale.

It appears that a pre-testing of the subjects is highly desirable. For example, prior to having a subject play in an oligopoly game he might first play in a monopoly game to determine his abilities to learn; then play in a duopoly game against a programmed strategy which reflects his own competitive behavior.

## 4. Goals of the Firm

The game has been designed so that it is possible to specify a specific economic goal for the firm as has been indicated in the briefing given above. It is also possible to define other goals such as maximize market share subject to a constraint on profits. Several alternative goals have been discussed by Shubik 14/.

## 5. Rewards to the Players

In the games played, the players have either been unpaid volunteers, subjects paid on an hourly basis, or students playing the game primarily for educational rather than experimental purposes. There is evidence that real stakes are an important determinant in the behavior of the players 15/. Local employment and gambling rules, as well as limitations of experimental funds tend to make it difficult to pay the participants in direct relation to their performance. This ties in directly with the specification of goals. One of the reasons for calculating the "beat-the-average" solution to a game is to check on the possibility that the players have converted the game into a "game of status" 16/ and are more interested in the ranking of their performance rather than their profits or the amount of money won.

## 6. Evaluation of the Firm and a Stock Market

In the evaluation of play the worth of the assets of the firm must be taken into account. A way in which this can be done both effectively and realistically, if there are many participants available is by issuing stock and having a stock market in which they can be traded 17/.

## 7. The Role of the Players

Tied in closely with the briefing on the goals of the firm and with the stakes and the existence of shares in the firm is the briefing given the players concerning their particular roles. Even if the game is played with only one player per team, a decision must be made as to whether the player is briefed that he is the owner of

his firm, the manager, or not briefed at all. Furthermore, especially if there are shares involved there may also be a board of directors to whom the management must report after a certain number of periods of play.

If the teams are composed of more than one player a decision concerning team structure must be made. Experiments have been performed where the teams have been single individuals; pairs without formal briefing on structure; teams of four with defined roles of general manager and managers of finance, marketing and production; and teams with a single decision-maker and one or two consultants 18/. It appears that this game is somewhat too simple to serve as a completely satisfactory vehicle for experimentation for study of the formal internal structure of the firm. There does not appear to be enough for an individual functional manager, such as a manager of finance, to do. Unless there are considerable time pressures members of large structured teams may tend to become bored. A game of considerably more complexity, yet based upon the game described here has been constructed by Levitan and Shubik 19/ specifically to present a sufficiently rich and detailed internal simulation of the firm that three, four or five men teams could be utilized successfully. This more complex game can be parameterized to yield the simpler game as a special case. It is not as complex as the Carnegie Tech Game 20/.

#### 8. Initial Conditions

As has been noted previously the players do not come to a game with blank and "normalized" memories. It is quite possible that human beings are designed to function optimally at a noise level

considerably above zero. Thus, beyond a certain point, attempts to create a too sterile and "dust-free" environment for the players may lessen, rather than increase the amount of control, as minor random disturbances begin to assume significance.

Given that the players will be searching for cues and clues concerning their environment, what should be supplied? One important way of providing information beyond that in the written briefing is to give the players the decisions and output for one, two or more periods of play of the game where the referee has selected these initial decisions. The number of periods provided may be made to depend upon the highest order difference relationship in the equations of the game. The initial position selected for most of the games which have been run is midway between the decisions for joint maximal behavior and non-cooperative equilibrium.

#### 9. Initial Information Conditions

The game can be played under conditions in which the players do or do not know the structure of the equations and functional relationships, and the values of parameters in the game. In the briefing exhibited above, the players are informed only of the values of some of the parameters. In Part I 21/ a different briefing is displayed.

#### 10. Information Display

The control indices INDEX(I) may be set, as has been noted in 2.1, to select alternative information displays for the players. For example, the players may obtain all balance sheets and full competitive information as is shown in Part I 22/ or they may only be given their own balance sheets and aggregated competitive



information with some aspects such as advertising suppressed.  
Exhibit II illustrates this.

EXHIBIT II

INDUSTRY AND INDIVIDUAL STATISTICS QUARTER 2

Game 2 3 Firms

Copy for Company 1

|  | Company 1   | Company 2 | Company 3      | Average  |
|--|-------------|-----------|----------------|----------|
| Price (\$)   | 270.00      |           |                | 270.00   |
| Sales (Units)                                      | 68627.      |           |                | 147876.  |
| Advertising (\$)                                   |             |           |                |          |
| Production (Units)                                 | 200000.     |           |                | 210000.  |
| Inventories (Units)                                | 381373.     |           |                | 312124.  |
| Net Profits (\$)                                   | -3335786.   |           |                | 567239.  |
| Dividends (\$)                                     | 0.          |           |                | 1666667. |
| Investment (\$)                                    | 0.          |           |                | 0.       |
| Rate of Interest                                   | 1. Per Cent |           |                |          |
| Present Value<br>of Dividends<br>Paid to Date (\$) | -0.         |           |                |          |
| Lost Sales<br>(Units)                              | 0.          |           | Industry Total | 28595.   |

| Quarter 2                           | Company 1     |
|-------------------------------------|---------------|
| Game 2 3 Firms                      |               |
| Profit and Loss Statement - (X1000) |               |
| Net Sales Billed                    | 18529.        |
| Direct Cost of Sales                | 11294.        |
| Depreciation                        | <u>250.</u>   |
| GROSS MARGIN                        | 6985.         |
| Commercial and Administrative       |               |
| Advertising                         | 10000.        |
| Inventory Charges                   | 3157.         |
| Administrative Overheads (Fixed)    | 500.          |
| Interest Expense                    | <u>-0.</u>    |
| NET PROFIT (-LOSS)                  | -6672.        |
| TAX RESERVE                         | <u>-3336.</u> |
| NET PROFIT (-LOSS) AFTER TAXES      | -3336.        |
| DIVIDENDS PAID                      | <u>0.</u>     |
| ADDITION TO NET WORTH               | -3336.        |
| Short Term Assets                   |               |
| Cash                                | 10000.        |
| Inventories (At Cost)               | <u>57206.</u> |
| TOTAL                               | 67206.        |
| Long Term Assets                    |               |
| Plant                               | <u>10000.</u> |
| TOTAL ASSETS                        | 77206.        |
| Liabilities                         |               |
| Loans Outstanding                   | <u>16042.</u> |
| Net Worth                           | 61164.        |
| Line of Credit                      | 83958.        |

11. Timing and Frequency of Play

For experimental purposes it is desirable to utilize a laboratory and to play the game at a speed of approximately twenty minutes per period for as many as fifteen periods or more. For teaching, and even for experimental purposes, where formal facilities are not available, and it is not possible to obtain players for more than a brief amount of time at any one instance, or when it is not possible to obtain sufficiently rapid computer turn-around time; the game can be played usefully at the rate of one period a day or several periods per week fitted in, for instance, with a lecture series.

There are still many open questions concerning the effects of time-compression on decision-making behavior. Games have been run under both of the conditions described above, but no convincing analysis exists at this time concerning the differences caused by the different time scales for decision-making.

12. Ruin Conditions

The game was designed so that it would be possible to study ruin and entry conditions. Furthermore, it is possible to select parameters so that most, if not all of the firms, will eventually be ruined. This can serve to focus attention on survival problems. Players must be briefed concerning their credit lines and the conditions of bankruptcy. Borrowing is automatic up to the limit of the individual's credit line. By selecting a very high credit line and a relatively potentially profitable market structure the ruin features of the game can be avoided.

13. Mechanization of Play

The running of even a moderate size experimental game involves

a considerable amount of organization and even physical labor. In this game players enter their decisions on decision sheets, these decisions must then be key punched, read into the computer which produces output, which must be separated, assembled and carried back to the players. Eventually it would be highly desirable to be able to have direct inputting of decisions by means of small input-output devices in each room of a gaming laboratory. Time and cost considerations are such that for large scale experimentation this becomes economical.

#### 14. Briefing for Teaching Purposes

Most of the comments above refer to direct experimental usage of the game. If the major use of the game is for a teaching purpose, the briefing may be tied in directly with a lecture series and with the special features such as inventory control, pricing or forecasting, which are being stressed.

#### 15. Debriefing

In experimental situations after the game is over it is often desirable to interview the players to obtain verbal information to assist in interpreting the numerical results. When using a game for teaching the post-play discussion is one of the most important parts of its use.

### 5. Teaching and Experimental Uses

#### 5.1. General Comments

The major thesis behind this work in gaming is that it is potentially a very valuable tool from both the viewpoint of teaching and experimentation (leaving aside operational gaming). Nevertheless, gaming on a large scale can be costly, hence, we are faced

with the economic problem of justifying the expenditure. Many of the aspects of costs have been discussed elsewhere 23/.

A well-designed computerized game may be looked upon as a relatively multiple purpose apparatus whose use is subject to considerable economies of scale. There are many courses and subjects in an economics department, a school of management, operations research, psychology and social-psychology, all of which with slight modifications can utilize the same game for teaching and, or research. The experience in teaching with the Carnegie Tech game has borne this out 24/. The game described here has been used in three different courses and for three different types of experimentation. Two of the courses made use of the game as an illustration of problems in oligopoly, the third used it to provide exercises in model building. The experimental uses were in multi-team competition, and in the learning behavior of individual players in a monopolistic situation and the effect of introducing debate and advisors to an individual decision-maker. In the section below several uses are sketched. The listing is by no means exhaustive nor are the uses described intensively, they are merely presented as guidelines.

## 5.2. Some Specific Uses

### 1. Inventory and Production Scheduling

By setting  $\beta=0$ ,  $\theta=1$ ,  $\eta=0$ ,  $n=1$  and adjusting the discount rate, inventory carrying costs, costs of production, costs of changing production and the trend, cycle and random factors, a pure inventory and production scheduling problem is obtained where price, advertising and oligopolistic competition have no effect. This

problem can be analytically formulated as a dynamic program. The more complex game 25/ permits problems to be formulated involving balancing of production amongst three products, raw materials purchasing and labor policy.

## 2. Econometrics

A game may be regarded as a data generator on which to test econometric techniques. The structure, behavior and parameters are known and there are none of the inaccuracy and incompleteness problems usually associated with "live" economic statistics.

## 3. Oligopoly Theory

There are many aspects of oligopoly theory for which the game can be used as a teaching aid. Specifically the concept of "solution" or behavior in an oligopolistic market can be discussed in relation to having played the game. Furthermore, solutions for several different theories of behavior are given in a previous paper 26/.

Merely in the process of constructing the game an explicit form had to be given to the advertising factors in the market, furthermore the problem of oligopolistic demand had to be considered in detail and an explicit method given for allocating oligopolistic demand. Levitan has independently investigated the mechanism of the allocation of oligopolistic demand 27/ and has devised an algorithm for the allocation of demand under assumptions which account both for different prices for the same commodity and inventory limitations and lost sales. This relates to, generalizes and extends work of Shubik on contingent demand 28/ which in turn is related to Chamberlain's treatment of oligopolistic demand 29/.

A brief list of some of the uses in relation to oligopoly theory is given below:

- (a) Competitive and cooperative behavior; "solutions"
- (b) The explicit affect of advertising
- (c) Oligopolistic demand
- (d) Oligopoly models with the independent variables as price, quantity or price and quantity
- (e) The selection of parameters to fit specific markets
- (f) The study of exit and entry

A discussion of (a), (b) and (c) has been given above. It is well known from the work of Cournot, Edgeworth and others that there are several different ways in which oligopoly models can be formulated 30/. The game can be used to stress this point made in (d),

In 3.1 an example was supplied of how to calculate values for parameters so that the game at least reflects certain salient features of specific markets. This use, listed above as (b) provides an exercise in the relating of theory, specific structure and institutional forms.

As both entry and exit are modelled in the game, explicit consideration can be given to the costs and problems of entry and bankruptcy and exit.

#### 4. Goals of the Firm

It has already been noted in the discussion on briefing that various goals for the firm can be specified. A specific use to which this game has been put has been to serve as a basis of discussion of the goals of the firms using references such as the work of Cyert and March 31/, and Shubik 32/.

## 5. Forecasting

There are two exceedingly different problems which must be faced in forecasting. They are the estimation of structural parameters and the prediction of behavior of competitors. The estimation of structure poses problems in econometrics and in operations research. The prediction of competitive behavior poses questions in marketing, oligopoly theory and psychology. It is instructive both from the viewpoint of teaching and experimentation to require the players to estimate both the parameters of the game and the moves of their competitors.

## 6. One-Person Learning

A restricted use of the game which has both the advantage and disadvantages of simplicity and control is in a monopolistic environment where there is only one firm and hence none of the difficulties associated with the prediction of competitive interaction. Among the more interesting features which may be investigated are the speeds with which players learn how to optimize (if they in fact do) when price is the independent variable, advertising is the independent variable, or when both are simultaneously of importance. Among extreme cases of interest are the situations in which price or advertising have no influence but the players are not informed of this.

There are three qualitatively highly different conditions under which monopoly games may be run. The first and most direct is where there is only one firm playing and it obtains information about its performance. The second is where there are many firms playing, but they are briefed that they are all monopolists in independent and similar markets. They obtain information about



each others performance; hence they are presented with the opportunity to learn from the experience of others. A third condition which has proved to be radically different from the other two is where the firms are in fact monopolists but receive information about each other and are not informed that they are strategically independent.

7. Model Building

An advanced use of a game of this level of complexity is in seminars on model building in the behavioral sciences, or in seminars specifically devoted to gaming and simulation. In such a use the emphasis is on having students operate the game, construct additional subroutines of their own, design experiments and write additional analysis programs.

8. Teaching of Procedures and Institutional Materials

Although games such as the Carnegie Tech game are well suited to teaching procedures and illustrating in detail many features of organizations; even a relatively simple game such as the one here is of use in teaching students an appreciation of the balance sheet and profit and loss statements and how they relate to the economic structure of the firm and the market. Even though students may not be required to do any additional modelling in their use of the game, a discussion and an examination of functional forms and parameter values when accompanied with reading on markets serves as a framework for the relating of institutional knowledge to a theoretical construct.

9. Artificial Intelligence: The Automated Player

This game is sufficiently simple that it is not

prohibitively difficult to construct an artificial player, yet it is complex enough that the task is by no means trivial. The writing of artificial player programs, the running of them against actual players and the devising of measures of performance are tasks suitable for exercises or projects for seniors or graduate students concerned with the study of behavior.

### 5.3. Some Experimental Hypotheses and Problems

#### 5.3.1. General Comments

In this subsection a few observations are made concerning the experimental use of the game. They are given merely to serve as indicators of some uses and of several problems. Several of the hypotheses noted below have been tested, most have not. There exist several problem areas where, although the importance of the problem can be recognized a satisfactory formulation of hypotheses has not yet been reached. For example, consider the affects of information display upon the players. Possibly one of the most important areas both for experimentation and development is in the display of information.

A few general open questions which need careful formulation and experimentation to answer them are:

- (1) What is the effect of organization and the size of its team upon play?
- (2) What are the various roles of learning with respect to environment and the behavior of competitors?
- (3) What differences in performance are to be expected by using administrators, businessmen, students or other specific groups as the players?

When there is more than one team in competition there is a considerable problem merely in the design of adequate measures of performance. In order to illustrate this we consider trying to devise a measure for the degree of cooperation or collusion in a market. Suppose there were two players in a cooperative game. We can portray various theoretical profit outcomes to them as is shown in Figure 4 below.

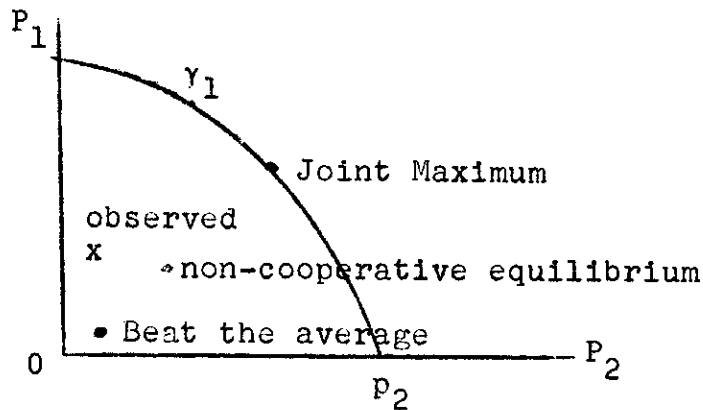


Figure 4

The axes are the profits made by the players. The curve  $p_1p_2$  is the point optimum boundary or the Pareto optimal surface 33/. The joint maximum, non-cooperative equilibrium and beat the average solutions are all indicated and lie on a 45 degree line through 0. Suppose the observed average outcome were indicated by X. What measure could we devise to illustrate the amount of cooperation in the game? The joint maximum serves as an obvious upper bound. The beat-the-average solution can be regarded as a reasonable lower bound to expect, except that under low information conditions it is possible for firms to reduce profits to below this amount by error and internal inefficiency. The non-cooperative equilibrium can be interpreted as a "middling" or inner-directed solution whereas the

others represent the poles of complete cooperation or extreme competition. How one should measure and compare "distances" from the various theoretical outcomes is an open question.

### 5.3.2 Some Specific Hypotheses

Some hypotheses considered with respect to this game have been:

- H.1. As the number of players  $n$ , is increased, ceteris paribus, the non-cooperative equilibrium point becomes the best predictor of average behavior.
- H.2. As  $n$  increases the variance in behavior decreases.
- H.3. For a fixed  $n$ , the higher the reward, the better is the non-cooperative equilibrium point as a predictor.
- H.4. For a fixed  $n$ , the more complete the information on the structural forms and parameters of the game the faster behavior converges.
- H.5. For a fixed  $n$ , games with complete information conditions converge faster than games with information only on average behavior.
- H.6. For a fixed  $n$ , the higher  $\beta$  the quicker is convergence.
- H.7. The higher inventory carrying costs, the lower are inventories.
- H.8. The higher the costs of changing production the smoother is production.
- H.9. The initial values in the preplay quarters supplied for the briefing do not affect the limiting behavior.

## Footnotes

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- 23/. Shubik, M., "Gaming, Costs and Facilities," mimeographed Yale University, April 1964.
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## Appendix I

### Variable, Parameter, and Other Symbol List

A complete listing of all symbols which appear in the computer program is given below. The meaning of the symbol is given and further identification is supplied by the notation (P) which indicates that the symbol is a parameter and will be unchanged for the run; (H) it is a state variable which may or may not need to be initialized, but will be updated by the running of the program; (D) it is a decision variable. The other symbols which do not have the further identification are controls on the program, totals of other variables and so forth.

Whenever a symbol appears with a 12 in parentheses such as DBT(12) or VLC(7,12) the 12 refers to the range of the number of firms. It is possible to run this game with as many as 12 firms. When a symbol appears with another number such as CONS(3) this refers to a specific value in the range for which CONS(I) is defined.

There are some symbols such as BU, CONS(4-12) which are not used in the current program; they have been left in so that minor changes can be made without having to recompile the whole program, as they appear in COMMON.

|              |     |   |
|--------------|-----|---|
| ADVT(12,1-3) | (H) | Advertising in three previous periods               |
| ADVT(12,4)   | (D) | Current advertising decision                        |
| ALPHA        | (P) | Total market size parameter, in units               |
| AMU1         | (P) | Adds constant amount to effective<br>advertising    |
| AMU2         | (P) | Adds constant amount to cycle and random<br>effects |

|            |  |
|------------|--|
| ANEW       | (P) Phase angle of cycle at initial point  |
| AR         | (P) Rate of market growth  |
| ACSL(12)   | Unit sales for each firm   |
| ASET(12)   | (H) Dollar investment in plant   |
| AVL        | Available Cash   |
| AVPR       | Average price  |
| BETA       | (P) Price sensitivity parameter  |
| BQ(12)     | (P) Maximum production permitted per period  |
| BU         | (P) Not used   |
| C(12)      | (P) Cost per unit of production  |
| CAPIN      | Total industry net worth   |
| CARY(12)   | (P) Inventory carrying cost per unit   |
| CASH(12)   | (H) Cash balance   |
| CDVD(1-30) | Cumulative dividends coded by team identification number (to allow firms to enter into or drop out of business and still record dividends) |
| CONS(1)    | (P) Cost per unit of increasing production   |
| CONS(2)    | (P) Cost per unit of decreasing production   |
| CONS(3)    | (P) Cost of a unit of added capacity   |
| CONS(4)    | (P) Market value of \$1 of assets in forced liquidation  |
| CONS(5-12) | (P) Not used   |
| CRD(12)    | (H) Line of credit   |
| CUMAD      | Cumulative effective advertising   |
| CUMPD      | Cumulative production  |
| DBP        | Debt payment, if negative, indicates additional borrowing  |



|            |   |
|------------|---|
| DBT(12)    | Loans outstanding   |
| DEL        | Cash outflow due to investment, debt payment,<br>and dividends        |
| DEM(12)    | Demand in units   |
| DEP        | (P) Depreciation rate   |
| DINV(12)   | (H) Inventory in units  |
| DVD(12)    | (D) Dividends   |
| ENRT       | (P) Entry level measured on industry return<br>on investment          |
| ETA        | (P) Cooperative advertising coefficient                               |
| E(12)      | Effective advertising, past advertising<br>weighted by H(I,J)         |
| FCST(12)   | (P) Fixed costs (administrative overheads)                            |
| FLPRS      | Number of players   |
| GAME(1-13) | (H) 'A' format, compared with GAMN                                    |
| GAMMA      | (P) Inter-firm price sensitivity coefficient                          |
| GAMN(13)   | (P) 'A' format, identification  |
| H(12,1-4)  | (P) Weights past advertising to get net<br>effectiveness              |
| ICONO(12)  | (H) Company number  |
| ID         | (P) Not used  |
| IK         | If 0 read history from history tape (SAVTAP),<br>otherwise from cards |
| IKK        | Control for entry of new firm, IKK=1 for entry                        |
| INDEX(1)   | If zero, QSLS not included in decision                                |

|              |  |
|--------------|--|
| INDEX(2)     | 1 = your industry stat, 2=yours and average,<br>3=all              |
| INDEX(3)     | If 2, get price information  |
| INDEX(4)     | If 2, get sales information  |
| INDEX(5)     | If 2, get advertising information                                  |
| INDEX(6)     | If 2, get production information                                   |
| INDEX(7)     | If 2, get inventory information                                    |
| INDEX(8)     | If 2, get profit information                                       |
| INDEX(9)     | If 2, get dividend information                                     |
| INDEX(10)    | If 2, get interest and cumulative dividend<br>information          |
| INDEX(11)    | If 2, get investment information                                   |
| INDEX(12)    | If 2, get lost sales   |
| INDEX(13)    | If 1, get individual balance sheet,<br>If 2, get industry          |
| INDEX(14-19) | Not used   |
| INDEX(20)    | If positive, more industries follow                                |
| IP           | (H) Number of periods profits have been<br>above ENRT              |
| IPER         | (P) Number of periods of profits needed<br>for entry               |
| IPG          | Page, for numbering output sheets                                  |
| IQRT         | (H) Quarter numbers  |
| JQRT         | (H) Compared with IQRT   |
| LAG          | (P) Periods between decision to produce and<br>finished production |

LL-LU                   Range of team numbers whose results will be  
                          printed on a page

LUG                     LAG + 1

NCONS(12)               (P) Not used

NPLRC                   (P) Highest team identification number

NPLRS                   (P) Number of players

OMEGA                   (P) Angular velocity of cycle

PROD(12)                (D) Units produced this period, = Q(12,2)

P(12)                   (D) Price

QSLS(I)                 (D) Max units of goods offered for sale, only  
                          if INDEX(1) = 0

Q(12,K)                 (H) Production in the past K periods, K less than  
                          12

Q(12,J)                 (D) Production this period (units),  
                          where J=LUG+1

R(12)                   (P) Minimum required cash balance

R1(1)                   (P) Controls amplitude of combined trend cycle  
                          and random effect

R1(2)                   (P) Not used

R1(3)                   (H) Quarter

R1(4)                   (P) Cycle amplitude parameter

ROE                     (P) Rate of interest on dividends paid and bank  
                          loans

SALOST(12)              (H) Lost sales in units

SEED                    Initial value used in calculating random number

SIG1                    (P) Amplitude of random variable in effective  
                          adv. formula

|            |   |
|------------|---|
| SIG2       | (P) Amplitude of random element in demand                 |
| S(12)      | Goods available for sale, beginning inv +<br>production/2 |
| TADVT      | Industry average advertising                              |
| TAX        | (P) Tax rate  |
| TDEMD      | (P) Total industry demand                                 |
| TDVD       | Average industry dividends                                |
| THETA      | (P) Cooperative advertising coefficient                   |
| TINV       | Average ending inventory in units                         |
| TOTLOS     | Total industry sales lost                                 |
| TPRO       | Average industry profits                                  |
| TPROD      | Average industry production                               |
| TSLS       | Average unit sales  |
| TVEST      | Average industry investment                               |
| VEST(12)   | (D) Investment  |
| VINT(12)   | Interest expense  |
| VLC(1,12)  | Sales in dollars  |
| VLC(2,12)  | Direct cost of sales                                      |
| VLC(3,12)  | Depreciation expense                                      |
| VLC(4,12)  | Gross Margin  |
| VLC(5,12)  | Inventory charges   |
| VLC(6,12)  | Net profit before taxes                                   |
| VLC(7,12)  | Income tax  |
| VLC(8,12)  | Net profit after taxes                                    |
| VLC(9,12)  | Total short term assets                                   |
| VLC(10,12) | Inventory in dollars                                      |
| VLC(11,12) | Costs due to changing production                          |
| VLC(12,12) | Total assets  |

|         |   |
|---------|---|
| VNW(12) | (H) Net worth   |
| X(12)   | (H) Addition to net worth, (temporary storage)                              |
| X(12)   | (H) Present value of dividends paid,<br>in normal order (temporary storage) |

Subroutine INPUT

This appendix contains the FORTRAN II program listing of the subroutine INPUT which contains all of the READ instructions needed to enter information for the operation of the game. (There also exists a FORTRAN IV program).

```

* LIST8
* LABEL
SUBROUTINE INPUT (NITAPE,NOTAPE)
COMMON ICONO,DINV,TINV,ASET,CASH,CRD,DBT,ADVT,TADVT,Q,C,CARY,FCST,
1R,BQ,H,P,AVPR,QLS,ACSL,TSLS, TDVD,VEST,TVEST,TPRO,TPROD,SALOST
2,TOTLOS,E,DEM,S,DIFE,CUMAD,CUIPD,CAPIN,DVD,VINT,VMW,
3ALPHA,BETA,GAMMA,ETA,THETA,OMEGA,SIG1,SIG2,AMUL,AMU2,AR,ANEN,TAX,D
4EP,ROE,BU,ENRT,
5IQRT,IK,IKK,ID,NPLRS,NPLRC,LAG,LUG,JQRT,IPER,IP,
6GAME,GAMN,CDVD,X,FORM,R1,CONS,NCONS,INDEX,VLC,GORM
DIMENSION ICONO(12),DINV(12),CASH(12),CRD(12),DBT(12),ADVT
1(12,4),Q(12,12),C(12),CARY(12),FCST(12),R(12),BQ(12),VLC(12,
212),P(12),QLS(12),ACSL(12),DVD(12),VEST(12),SALOST(12),E(12),DEM(
312),S(12),DIFE(12),ASET(12),PROD(12),VINT(12),VMW(12),H(12,4),
4GAME(13),GAMN(13),CDVD(30),X(6),FORM( 7),R1(4),CONS(12),NCONS(12),
5INDEX(20),GORM(9)
EQUIVALENCE (TOTLOS,TDEMD),(PROD,Q( 13)),(R1(2),FPLRS)
1 READ INPUT TAPE 7,47,IQRT,IK,IKK,ID,(GAMN(I),I=1,13)
47 FORMAT(4I3/13A6)
IF (IQRT)100, 54, 44
C AFTER A COMPLETE RUN OF GAMES FOR A QUARTER HAVE 2 BLANK CARDS
C WITH -1 IN COLS.2,3 OF CARD 1 ... FOR THE NEXT SET HAVE IK=0
100 REWIND NOTAPE
REWIND NITAPE
NITAPE=NOTAPE+NITAPE
NOTAPE=NITAPE-NITAPE
NITAPE=NITAPE-NITAPE
GO TO 1
44 IF (IK) 90, 91, 90
90 NTAPE = 7
GO TO 94
91 NTAPE=NITAPE
94 READ INPUT TAPE NTAPE,47, NPLRS,NPLRC,LAG,JQRT,(GAME(I),I=1,13)
IF (IQRT-JQRT) 74, 73, 74
74 WRITE OUTPUT TAPE 10,75
75 FORMAT(22HLINCORRECT STATUS TAPE.)
54 CALL RUN(NITAPE)
CALL EXIT
73 DO 76 K = 1,13
IF (GAMN(K) - GAME(K)) 74, 76, 74
76 CONTINUE
READ INPUT TAPE NTAPE,49,(CDVD(I),I=1,NPLRC)
49 FORMAT (5E15.8)
READ INPUT TAPE NTAPE,49,(CASH(J),J=1,NPLRS)
READ INPUT TAPE NTAPE,45,(ICONO(J),J=1,NPLRS)
45 FORMAT(20I3)
READ INPUT TAPE NTAPE,46,(DINV(J),J=1,NPLRS)
READ INPUT TAPE NTAPE,46,(ASET(J),J=1,NPLRS)

```

```

46  FORMAT(6F13.0)
    READ INPUT TAPE NTAPE,46,( CRD(J),J=1,NPLRS)
    READ INPUT TAPE NTAPE,46,( DBT(J),J=1,NPLRS)
    DO 92 MM = 1,3
92  READ INPUT TAPE NTAPE,46,(ADVT(M,MM),M=1,NPLRS)
    LUG = LAG + 1
    DO 93 MM = 1,LUG
93  READ INPUT TAPE NTAPE,46,(Q(M,MM),M=1,NPLRS)
97  READ INPUT TAPE NTAPE,30,( C(M),M=1,NPLRS)
30  FORMAT (6F13.2)
    READ INPUT TAPE NTAPE,30,(CARY(M),M=1,NPLRS)
    READ INPUT TAPE NTAPE,30,(FCST(M),M=1,NPLRS)
    READ INPUT TAPE NTAPE,30,( R(M),M=1,NPLRS)
    READ INPUT TAPE NTAPE,30,( BQ(M),M=1,NPLRS)
    DO 98 MM = 1,4
98  READ INPUT TAPE NTAPE,30,(H(M,MM),M=1,NPLRS)
    READ INPUT TAPE NTAPE,49,
    LALPHA,BETA,GAMMA,ETA,THETA,
    2SIG1, SIG2, AMU1, AMU2, OMEGA,
    3TAX,DEP,ROE,BU,ENRT,
    4ANEM, AR
    READ INPUT TAPE NTAPE,30,(RI(I),I=1,4)
    READ INPUT TAPE NTAPE,30,(CONS(M),M=1,12)
    READ INPUT TAPE NTAPE,48,(NCONS(M),M=1,12)
48  FORMAT (12I5)
    READ INPUT TAPE NTAPE,45, IPER, IP
    READ INPUT TAPE NTAPE,45, (INDEX(J),J=1,20)
    IF (IKK) 95, 96, 95
C   READ NEW FIRM PARAMETERS AND VARIABLES
95  NPLRC = NPLRC + 1
    NPLRS = NPLRS + 1
    READ INPUT TAPE 7,46,
    1CDVD(NPLRC),DINV(NPLRS),ASET(NPLRS),CASH(NPLRS),CRD(NPLRS),DBT(NPL
    2RS),
    3 C(NPLRS),CARY(NPLRS),FCST(NPLRS),R(NPLRS),BQ(NPLRS)
    READ INPUT TAPE 7,46, (H(NPLRS,MM),MM=1,4)
    READ INPUT TAPE 7,46, (ADVT(NPLRS,MM),MM=1,3)
77  READ INPUT TAPE 7,46, (Q(NPLRS,MM),MM=1,LUG)
78  ICONO(NPLRS) = NPLRC
96  FPLRS = NPLRS
    RI(3) = IQRT
C   READ CURRENT DECISIONS
    DO 70 I = 1,NPLRS
    READ INPUT TAPE 7,184, IKK, P(I), ADVT(I,4), Q(I,LUG+1),QSL(I),
    1DVD(I), VEST(I)
184 FORMAT (I3,6F10.0)
    IF (ICONO(I) - IKK) 71, 70, 71
71  WRITE OUTPUT TAPE 10, 72, ICONO(I)
72  FORMAT(5HOFIRM I3,18H DECISIONS MISSING)
    GO TO 54
70  CONTINUE
    RETURN
    END

```

A listing of the complete FORTRAN II or FORTRAN IV programs is available on request.

An X-Y plot program provides graphs of the performance of the industry average of the individual firms together with plots of the three theoretical solutions. Thus, each graph contains four lines. An example of the graph is given as Exhibit 1, for a 23 period experimental run. The program is designed to recompute solutions whenever a firm enters into or exits from the market.

The statistical analysis is based upon the readings  $k-1$  to  $k-r$  where  $k$  is the number of periods for which the game is played. The last play is rejected owing to terminal effects. The structure of the game<sup>1</sup> is designed to avoid terminal pathologies, nevertheless there are many reasons why the experimenter should still reject the last readings, especially if the players are aware that they are terminating the game on that play.

The statistical analysis uses the two-dimensional t-test<sup>2</sup> on each of the hypotheses that the  $r-1$  penultimate decisions of the players were drawn at random from some bivariate normal universe whose mean is the particular theoretical value derived in the analysis in Part IIA. The program, for each theory, prints out a value for which a probability can be looked up in a table of the Snedecor F-distribution. A sample output of this analysis is given as Exhibit 2.

- 
1. M. Shubik, Strategy and Market Structure (New York: John Wiley and Sons, 1959), Chs. 9 and 10.
  2. T. W. Anderson, An Introduction to Multivariate Analysis (New York: John Wiley and Sons, 1958), pp. 101-109.





EXHIBIT 2  
Output of Analytical Program

ANALYSIS OF 6 PENULTIMATE QUARTERS  
GAME 4

F RATIOS BASED ON 2 AND 22 DEGREES OF FREEDOM

|                             | PRICE  | ADVERTISING | F RATIO   |
|-----------------------------|--------|-------------|-----------|
| AVERAGE                     | 229.48 | 2033333.    |           |
| JOINT MAXIMUM               |        |             |           |
| STATIC                      | 280.50 | 3993502.    | 265.9023  |
| DYNAMIC                     | 283.83 | 3394563.    | 174.8478. |
| NON-COOPERATIVE EQUILIBRIUM |        |             |           |
| STATIC                      | 222.53 | 3673595.    | 38.0686   |
| DYNAMIC                     | 227.63 | 3317442.    | 24.6659   |
| BEAT-THE-AVERAGE            |        |             |           |
| STATIC                      | 214.80 | 3517379.    | 31.4406   |
| DYNAMIC                     | 220.13 | 3202364.    | 19.0922   |

## APPENDIX IV

### Facilities, Costs and Time Estimates

In an article referred to earlier, on the costs of gaming, a detailed discussion of the different economic criteria for costing facilities and equipment needed for a game were given. This game has been run under the two conditions of having a laboratory and immediate access to a large computer, or no formal space for the teams and a turn-about time varying between half a day to a day.

Under optimal conditions new decisions can be returned to the players in between five to ten minutes after they have been handed in. This breaks down into one or two minutes for key-punching; around one or two minutes for the actual computation and printout (this assumes that the program is already on tape) and another minute or two to separate the results and hand them back.

Eventually, with the growth of time-sharing, games of this type will be easily adopted to having decisions entered directly on an input typewriter. In which case turn-around time will be limited to decision speed. Currently approximately twenty minutes per quarter appears to be the decision cycle feasible in a laboratory.

The program has been run on several machines, including the IBM 709, 7090 and 7094. Input and output are the limiting speed factors as the actual computation is slight. On the bigger machines under a minute of running time is sufficient (even for a batch job where the inputs for several games may be run with a single loading of the game program).

For most experiments and other uses a simple input format, as is shown below, has been used. The appropriate columns are filled

## DECISION RECORD

Company \_\_\_\_\_

| Quarter | Price | Advertising<br>(000) | Production<br>(000) | Amount Offered<br>For Sale<br>(000) | Dividends<br>(000) |
|---------|-------|----------------------|---------------------|-------------------------------------|--------------------|
| 1       |       |                      |                     |                                     |                    |
| 2       |       |                      |                     |                                     |                    |
| 3       |       |                      |                     |                                     |                    |
| 4       |       |                      |                     |                                     |                    |
| 5       |       |                      |                     |                                     |                    |
| 6       |       |                      |                     |                                     |                    |
| 7       |       |                      |                     |                                     |                    |
| 8       |       |                      |                     |                                     |                    |
| 9       |       |                      |                     |                                     |                    |
| 10      |       |                      |                     |                                     |                    |
| 11      |       |                      |                     |                                     |                    |
| 12      |       |                      |                     |                                     |                    |
| 13      |       |                      |                     |                                     |                    |
| 14      |       |                      |                     |                                     |                    |
| 15      |       |                      |                     |                                     |                    |

in, collected by a laboratory assistant and handed back after key-punching. An assistant can comfortably handle as many as six teams,

if he is not also key-punching. A lab-manager, one assistant per six teams, a key-puncher and computing support are the requirements for an "on-line" laboratory played game. Scheduling will present problems if a large machine without time-sharing facilities and with an operating backlog is used.

When the game is run once or twice a day or less, the costs of operations become negligible. Machine-time of course remains the same, however time-sharing and scheduling is no longer a problem and all that is needed is a secretary and key-puncher to pick up the decisions, prepare them, and submit them.

In the current state of the development of this game, the statistical analysis is performed after all periods of the game have been run. Appendix IV indicates the present level of analysis, graphical and statistical processing.

## Appendix V

### Some Input Parameters and Initial Conditions

In Section 3 we have already sketched the method for selecting parameters to obtain a crude first order approximation to the automobile market. In Subsection 2.1. parameters were set for an unspecified market. If these parameters are utilized for the most part, with changes being made to five of them, a series of useful experimental games can be constructed. These are specified below. The

$$\text{Set } \alpha = 5.2 \times 10^5 n \text{ and } \beta = \frac{4}{3} \times 10^3 n$$

where  $n$  is the number of teams in the game. By introducing the  $n$  into the selection of the  $\alpha$   $\beta$  this keeps both the joint maximum solution and the beat-the-average solution at the same level, regardless of the numbers playing.

1. Isolated players in a pure price game (monopolists)

$$\gamma = 0, \quad \eta = 0, \quad \theta = 1$$

2. Isolated players in a pure advertising game

$$\beta = 0, \quad \gamma = 0, \quad \eta = \left(\frac{\sqrt{5}}{n}\right)(.0002), \quad \theta = 1$$

3. Isolated players with price and advertising relevant

$$\beta = \frac{4}{3} \times 10^3 n, \quad \gamma = 0, \quad \eta = \left(\frac{\sqrt{5}}{n}\right)(.0002), \quad \theta = 1$$

4. Violent competition (price only)

$$\gamma = 9, \quad \eta = 0, \quad \theta = 1$$

5. Violent competition (advertising only)

$$\gamma = 0, \quad \eta = 0, \quad \theta = .15$$

6. Violent competition in both price and advertising

$$\gamma = 9, \quad \eta = 0, \quad \theta = .15$$

For finer adjustments the size of  $\gamma$  can be changed in the range of around 5 to 12 and  $\theta$  in the range from 0 to around .3. The affects of these changes should be checked on the payoffs which may become negative and large for extreme values.

7. No price interaction, but cooperative advertising possibilities

$$\beta=0, \gamma=0, \eta=\left(\frac{\sqrt{5}}{n}\right)(.0002), \theta=1$$

8. Violent competition on price with cooperative advertising possibilities

$$\gamma=9, \eta=\left(\frac{\sqrt{5}}{n}\right)(.0002), \theta=1$$

For emphasis on production scheduling and inventory problems the parameters for costs of production, costs of changing production and inventory carrying costs should be enlarged. For emphasis on dividend policy, the discount rate should be enlarged.