COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station

New Haven, Connecticut

COWLES FOUNDATION DISCUSSION PAPER NO. 128

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An Inter-City Consumption Function

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October 19, 1961

AN INTER-CITY CONSUMPTION FUNCTION

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Introduction

Friedman's theory of the consumption function [3] posits a simple proportional relationship between the "permanent components" of income and consumption. He further specifies random "transitory" components of income and consumption which are uncorrelated with the permanent components, and with each other. The statistical properties of this type of model (error in variable) have been extensively explored in the statistical literature [5,6,7]. The conclusion of basic relevance here is that ordinary least-squares regression estimates may yield seriously biased estimates of the relation between the permanent components. An alternative statement of the same result is that the parameters of the model are not identified in the absence of some extraneous information such as the relative sizes of the variances.

This basic lack of identification is largely responsible for the inconclusiveness of the several attempts to support or refute Friedman's hypothesis. It is not claimed that the analysis presented here will succeed where others have failed, but rather that this analysis is different and has advantage not shared by others. (This raises the logical question of how to combine a number of inconclusive bits of evidence.)

The "identification problem" can be partly circumvented through the introduction of an "instrumental" variable which is related to one of the components of income, say permanent income, but independent of the other, transitory income. This technique is employed in [10] where measurable, scaled variables are used as "instruments." As Madansky shows [7], the "instruments" can as well be principle of classification which do not possess natural scales of measurement. This latter approach has been used by others [1,8], but it has proven difficult to find variables, scaled or unscaled, that meet the statistical requirements of an "instrument variable."*

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* Indeed, the classification technique is obliquely suggested by Friedman where he proposes a comparison between a weighed average of "minor group" elasticities and an "overall" elasticity of consumption with respect to income.

In this paper geography is used as the instrumental variable. The primary aim is to provide some evidence on the nature of the relation between permanent income and permanent consumption. As Farrell has pointed out [2], this element of Friedman's theory is the more controversial. He argues further that it is non-essential to the remainder of the theory and, using evidence from Friend and Kravis [4], of doubtful empirical validity. The evidence provided below clearly supports this conclusion and is, I hope, more specific and pointed toward the proposition in question.

The Statistical Framework

In an abbreviated form, Friedman's theory can be stated as follows:

(1) $C_p = kY_p$; Permanent consumption is a fixed proportion of permanent income,

(2) $Y = Y_p + Y_t$; Measured income is the sum of permanent and transitory incomes,

(3) $C = C_p + C_t$; Measured consumption is the sum of permanent and transitory consumption,

(4) $\rho_{C_p C_t} = \rho_{Y_p Y_t} = \rho_{C_t Y_t} = 0$; Transitory components are uncorrelated with respective permanent components and with each other.

The objective is to test and/or estimate the relation specified in (1) given data on measured income and consumption. In the analysis below means of family consumption and income for each of 23 cities are used to estimate consumption functions. The assumptions underlying this procedure are:

a) mean permanent income varies among cities, and

b) mean transitory income does not vary among cities.

Given these assumptions:

(5) $\bar{Y}(i) = \bar{Y}_p(i) + \bar{Y}_t$,

there $\bar{Y}_t$ is constant for all cities $i = 1, 2, \ldots, 23$.

(6) $\bar{C}(i) = \bar{C}_p(i) + \bar{C}_t(i) = \lambda + k\,\bar{Y}_p(i) + \bar{C}_t(i)$.

Note that transitory consumption may vary among cities and that the last expression in (6) uses a general linear relative between C_p and Y_p to permit testing of the specification in (1). With these additional assumptions ordinary least-squares applied to the city-means provides unbiased estimates of k .

An alternative version of Friedman's theory combines the two components of income and consumption in a multiplicative fashion. Equations (2) and (3) become:

(2a) $$\log Y = \log Y_p + \log(1 + Y_t) ,$$

(3a) $$\log C = \log C_p + \log(1 + C_t) .$$

Equation (1) is unchanged and the independence specifications in (4) now apply to the logarithm of the several components. Now a least squares slope between the logarithms of measured income and consumption provide an estimate of P_y , the ratio of the variance of $\log Y_p$ to the variance of $\log Y$. If Friedman's specification of a unitary elasticity is incorrect, then the least squares slope will be the product of P_y and the income elasticity of permanent consumption. The effect of additional assumption a) and b) above is to make P_y equal to unity for inter-city variation. Consequently the least squares slope estimated from city means should provide a "pure" estimate of the income elasticity and a test of Friedman's specification.

It can be argued that this model does not adequately represent the real causes of inter-city variation of income and consumption. Some of the variation may result from "compensating differences," e.g., fuel costs and transportation expense differentials may lead to equal differentials in remuneration of individuals if there is enough geographical mobility. If so then both mean income and mean consumption will be over or understated by an equal amount. It is tacitly assumed here that income and consumption measurements are supposed to be comparable in terms of some scale of material well-being. If this "compensating difference" argument is approximately correct then it is equivalent to defining a third component of income which is consumed in its entirety. The average and marginal propensities to consume this component of income are both equal to 1. It can be shown that adding such a component to the model introduces a bias toward unity in the estimates of the linear and logarithmic slopes.

Another factor which obstructs interpretation of city means as measures of material welfare is inter-city price variation. An attempt is made to correct for this by deflating all dollar amounts by an index of relative prices. The index was formed by updating a relative price index for a "City Worker's Family Budget" which was compiled for 1945. This index was brought up to date through a set of inter-temporal price indices published for certain large cities.*

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* For more details see [9].

Perhaps the largest deficiency of this index derives from the date of the basic inter-city study. The composition of a "City Worker's Family Budget" probably changed radically between 1945 and 1950 because of higher per-capita income, changes in age composition of families, and most important, because of post-war shortages of durable consumer goods. The only alternative, however, is to compile new indices for 1950 from basic price and quantity data; this would require a separate study in itself.

The treatment of family size as a factor influencing income and consumption remains an unsolved problem in both the theoretical and empirical literature. The average family size for each city has been included in the analysis below without any pretense of solving the problem, but with the hope that making a rough allowance for family size will improve the validity of the income coefficients. The sample is too small and the range of variation too narrow to provide any basis for choice of alternative forms of the family size variable.

To summarize, this analysis is based on the notion that truly transitory income differences among families are averaged out to zero or, at most, a constant in means over all sampled families in a given city. To the extent that this is not true, a bias toward zero is introduced into the slope estimates. This possible bias may be more or less offset by the bias toward unity which may result from "compensating difference" in incomes of different areas. Differences in price levels are taken into account by deflating dollar magnitudes by an inter-city price index. Finally an attempt is made to measure the consumption-income relation net of the influence of family size.

The Basic Data and Definition of Variables

Mean values of income, consumption and family size for 23 larger cities were used in the empirical analysis. The "sample" of cities consists of all cities included in the 1950 B.L.S. Survey of Consumer Expenditure and for which price data could be obtained. Table I lists the cities together with the sample size and 1949-50 price index for each city. The means were taken from the published tables in Vol. I of the Study of Consumer Expenditures [11].

The means of receipts and expenditure categories as reported in the statistical volumes were variously combined to yield alternative measures of income and consumption. The categories in the published table are:

Receipts	Expenditure
1) Income after tax	5) Current consumption
2) Other money receipts	6) Gifts
3) Decrease in assets	7) Increases in assets
4) Increase in liabilities	8) Decreases in liabilities
	9) Personal insurance
	10) Balancing difference (usually negative)

$$\sum_{1)}^{4)} = \text{Total Receipts} \equiv \sum_{5)}^{10)} = \text{Total Expenditures}$$

Given these categories the following variables were used

TABLE I

List of Cities, Sample Sizes and Relative Prices

City	Total families in Sample	Families with 2 or more persons	Relative Prices*
Atlanta	196	178	93.5
Baltimore	285	262	96.0
Birmingham	197	170	93.0
Boston	262	222	97.0
Chicago	391	335	98.0
Cincinnati	233	198	91.0
Cleveland	296	268	93.5
Indianapolis	217	185	91.5
Kansas City	214	182	88.5
Los Angeles	423	325	95.5
Milwaukee	209	179	97.5
Minneapolis	207	169	97.5
New Orleans	184	161	87.5
New York	451	387	96.0
Norfolk	197	175	91.5
Philadelphia	306	276	92.5
Pittsburgh	327	304	95.5
Portland Me.	128	116	92.0
Portland Ore.	236	158	93.0
St. Louis	328	287	95.5
San Francisco	289	226	96.0
Scranton	209	185	89.5
Seattle	226	172	101.0
	6011	5118	

*Washington, D.C. = 100
Average of Index for December 1949 and December 1950

Y_1 = Income after tax, = 1)

Y_2 = Y_1 - Balancing difference, = 1) - 10)

C_1 = Current consumption, = 5)

C_2 = Y_1 - Net increase in net worth = 1) + 3) + 4) - 7) - 8)

C_3 = C_1 + Gifts = 5) + 6)

= C_2 + Other Money Receipts - Personal Insurance .

Of the two income measures the first, Y_1 , is the nominal disposable income reported by the families and was obtained as the sum of incomes from several sources and for each family member. Y_2 incorporates the assumption that under-reporting of income was the main source of the discrepancy between receipts and expenditures. The differences among consumption measures are more substantial. C_1 , on reported consumption, includes all purchases of "... goods and services for family living..." whether paid for or not and including finance charges.* This measure explicitly includes purchases of

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* For description of all receipt and expenditure categories see [11] p. XXIX.

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durable goods in current consumption. C_2 embodies the assumption that all income which was not used to increase net worth was necessarily consumed. More simply, it assumed that saving is best measured by the change in net worth and that consumption should be deduced via the $Y = C + S$ "identity."

This measure also includes consumer durables in consumption since the changes in assets refer only to financial assets and real property. The third consumption measure, C_3 , was suggested by Modigliani and Ando [8]. Used together with Y_2 it implies a saving concept, Y_2-C_3 which Modigliani and Ando decide is the best measure obtainable from the categories reported in the Wharton-B.L.S. volumes.

Despite the many desirable features of the Modigliani-Ando measures, Y_1 , C_1 and C_2 are somewhat closer to the direct responses of the household and are not so much tainted by after the fact "doctoring." As it turns out, the results of primary interest are not very sensitive to the difference in definition encounted here. The inclusion of durable goods purchases in current consumption does not here pose the same problems as it does with observations on individual households. Given the averaging over at least 100 households, the mean purchases of durables is probably a fair approximation to current consumption. The main reason for systematic discrepancies would be cycle-related variation in durable purchases. If the averaging succeeds in eliminating inter-city variation of transitory income there will in any case be no complications due to correlations between transitory income and purchases of durable goods.

Table II shows means and standard deviations for the several variables described above, for family size (N), for three consumption: income ratios, and for per capita measures of income and consumption. Means are shown separately for all families and for families with 2 or more persons.

TABLE II

Means and Standard Deviations of Variables

(weighted by City sample size)

	Variable	All Families Mean	All Families Std.dev.	2 or More Person Families Mean	2 or More Person Families Std.dev.
Basic variables	Y_1	4254	442	4631	459
	Y_2	4375	450	4764	463
	N (family size)	2.93	.22	3.27	.14
	C_1	4152	387	4519	403
	C_2	4353	421	4723	435
	C_3	4335	417	4705	434
Consumption: Income Ratios	$C_1\|Y_1$	.978	.033	.978	.034
	$C_2\|Y_1$	1.025	.027	1.021	.030
	$C_3\|Y_2$	.992	.022	.989	023
Per-Capita Measures	$Y_1\|N$	1459	181	1419	172
	$Y_2\|N$	1500	182	1460	172
	$C_1\|N$	1423	163	1384	149
	$C_2\|N$	1492	175	1447	163
	$C_3\|N$	1486	175	1441	161

A parallel analysis was carried out for these two sets of observations. It is to be expected that the behavior of households with two or more persons will be more homogeneous because many of the smaller households are temporary or represent very old or very young families. These statistics show the quantitative consequences of the alternative definitions. Y_2 is $125 larger than Y_1 on the average and has about the same variability. C_2 is about $200 higher than C_1 and varies somewhat more. C_3 is slightly smaller and less variable than C_2. The C/Y ratios follow the same ranking as the consumption measures, but C_1/Y_1 is the most variable, C_3/Y_2 the least. Another finding from Table II is that while family income and consumption are larger in the sample of larger families, the per-capita figures are lower. Also the C/Y ratios are equal or insignificantly smaller for the larger families. Of immediate relevance to the remaining analysis is the limited range of variation of mean income among cities. Given such a small range the fits obtained by alternative functional forms will differ only slightly, any family of curves that has a variable slope and level will fit the data fairly well.

The Regression Results.

Several alternative functional forms were used, mostly for convenience in presentation and testing. In each case the city sample sizes were used as weights. First, simple linear equations were fitted relating consumption to income and family size, and per-capita consumption to per-capita income. The results are summarized in Table III. It should be noted that the R^2's and estimated slopes are both higher than for equation fitted to ungrouped household data. The former result suggests that the grouping served to reduce

TABLE III

Linear Consumption Functions

	$C = \alpha + \beta Y + \gamma^{N} + u$							
	α	(s_α)	β	(s_β)	γ	(s_γ)	s_u	R^2
All families								
$C_1;Y_1$	355	(452)	.832	(.061)	88	(125)	66	.903
$C_2;Y_1$	261	(390)	.922	(.053)	58	(108)	57	.939
$C_3;Y_2$	450	(305)	.908	(041)	-30	(86)	45	.961
2 or more persons								
$C_1;Y_1$	-753	(944)	.870	(.070)	380	(233)	66	.894
$C_2;Y_1$	-286	(902)	.932	(.067)	212	(222)	63	.917
$C_3;Y_2$	-320	(676)	.932	(.050)	179	(169)	49	.950

	$C \vert N = \delta + \lambda\ Y \vert N + u$					
	δ	(s_δ)	λ	(s_λ)	s_u	R^2
All Families						
$C_1;Y_1$	160	(76)	.866	(.052)	23	.930
$C_2;Y_1$	111	(65)	.947	(.044)	20	.956
$C_3;Y_2$	66	(56)	.947	(.037)	17	.969
2 or more persons						
$C_1;Y_1$	199	(75)	.835	(.053)	21	.923
$C_2;Y_1$	142	(71)	.919	(.065)	19	.943
$C_3;Y_2$	99	(57)	.920	(.039)	15	.964

the variability of residuals (mean transitory consumption) without eliminating all variability of the dependent variable. The latter suggests that either the transitory incomes do average out or else the upward bias from compensating differences is able to overcome the downward bias from transitory income. The $C_3;Y_2$ combination achieves the highest R^2's, but its slope estimates are very similar to the $C_2;Y_1$ pair. The large difference in family size coefficients between all families and large families is probably a freak result owing in part to the narrow range of mean family sizes observed.

Table IV displays the estimates for the log-linear, or constant elasticity consumption functions. These estimates correspond to Friedman's multiplicative formulation. The pattern of results is very similar to those for the linear model but the log-linear form has the advantage that there is a simple clear-cut null hypothesis to use for testing Friedman's "homogeneity hypothesis." If, by averaging out transitory income variation or by offsetting it with "compensatory income," P_Y has been made approximately equal to one then the income slopes in Table IV should provide unbiased estimates of the "Permanent income elasticity" of consumption. Equation one specifies unitary elasticity. Tests in columns A, B and C of Table V provide one-tail "t" tests of this null hypothesis against the alternative that the elasticity is less than one. In general the null hypothesis is rejected, usually with less than .05 probability of type I error. The last three columns of Table V show tests of a slightly different nature. Since equation (1) implies that C/Y should be independent of income, that specification can be tested by

TABLE IV

Constant Elasticity Consumption Functions

$$C = Y^{\beta} \cdot N^{\gamma} \cdot 10^{(\alpha+u)}$$

	α	(S_α)	β	(S_β)	γ	(S_γ)	S_u	R^2 for logs.
All families								
$C_1;Y_1$	.555	(.219)	.836	(.059)	.059	(.086)	.0067	.909
$C_2;Y_1$	.414	(.184)	.885	(.050)	.032	(.072)	.0057	.941
$C_3;Y_2$	.369	(.148)	.901	(.040)	.027	(.058)	.0045	.961
2 or more persons								
$C_1;Y_1$	.280	(.290)	.880	(.067)	.287	(.165)	0062	.904
$C_2;Y_1$	.298	(.267)	.900	(.062)	.148	(152)	0057	.924
$C_3;Y_2$	.173	(.211)	.933	(.049)	.131	(118)	0046	.952

$$C|N.1000 = \left(\frac{Y}{N.1000}\right)^{\gamma} \cdot 10^{\delta+u}$$

	δ	(S_δ)	λ	(S_λ)	S_u	R^2 for logs.
All families						
$C_1;Y_1$	.0106	(.0082)	.871	(.049)	.0068	.938
$C_2;Y_1$	.0245	(.0069)	.912	(.041)	.0057	.960
$C_3;Y_2$	.0063	(.0065)	.942	(.036)	.0048	.971
2 or more persons						
$C_1;Y_1$	.0132	(.0078)	.843	(.049)	.0062	.933
$C_2;Y_1$	.0253	(.0071)	.890	(.045)	.0056	.969
$C_3;Y_2$	.0080	(.0063)	.919	(.037)	.0045	.967

TABLE V

"t" Test Coefficients
for Friedman's "Homogeneity" Hypothesis

Test	A	B	C	D	E	F
All families						
$C_1;Y_1$	-2.80	-2.87	-2.63	-2.72	-2.69	-2.58
$C_2;Y_1$	-2.36	-2.30	-2.15	-2.31	-2.27	-2.13
$C_3;Y_2$	-2.50	-2.48	-1.61	-2.52	-2.31	-1.52
Two or more persons						
$C_1;Y_1$	-2.61	-1.79	-3.21	-2.62	-1.78	-3.13
$C_2;Y_1$	-2.21	-1.63	-2.45	-2.20	-1.59	-2.43
$C_3;Y_2$	-1.86	-1.37	-2.19	-1.89	-1.31	-2.09
Critical values						
.10	-1.32	-1.33	-1.32	-1.32	-1.33	-1.32
.05	-1.72	-1.73	-1.72	-1.72	-1.73	-1.72
.025	-2.08	-2.09	-2.08	-2.08	-2.09	-2.08

Test	equation	Null Hypothesis	Alternative Hypothesis
A	$\log C = a + b \log Y$	$b \geqq 1$	$b < 1$
B	$\log C = a + b \log Y + c \log N$	$b \geqq 1$	$b < 1$
C	$\log C/N = a + b \log Y/N$	$b \geqq 1$	$b < 1$
D	$C/Y = a + b Y$	$b \geqq 0$	$b < 0$
E	$C/Y = a + bY + cN$	$b \geqq 0$	$b < 0$
F	$C/Y = a + b \ Y/N$	$b \geqq 0$	$b < 0$

estimating the relation between the ratio and income. In this case the null hypothesis is that the income slope is zero but the "t" test coefficients have almost the same pattern and magnitude as those for the log-linear case. While there is clearly very little independence among the 36 test coefficients in Table V it is of some interest to note that they are not very sensitive to the changes in formulation and definition that have been tried. The only exception is due to the erratic behavior of the estimated family size coefficients.

One additional feature of the results should be noted. The estimated marginal propensities to consume shown in Table III are almost as large, and in several cases larger than the corresponding elasticities in Table IV. On Friedman's theory the former should be equal to $k \cdot P_y$, the product of his "true" marginal propensity to consume permanent income and the proportion of the variation of income accounted for by permanent income. Similarly, as explained earlier, the log-linear slope should estimate P_y, this time, however, the proportion has to do with variation in the log scale. Since the range of variation of income is fairly small the logarithmic transformation is nearly linear and this should not markedly affect the value of P_y. Now, even if the averaging of transitory incomes did not make P_y equal to 1, the ratio of the linear to the log-linear slope should provide an estimate of k. The relative magnitudes of these slopes indicate a value of 1 or larger for k, a larger value than most students of the problem would assign _a priori_. One is thus faced with a choice of accepting so large a value for k or modifying some of the other specifications of equation (1).

It may be that the "true" elasticity of consumption is less than one and/or the propensity to consume transitory income may be larger than zero. This latter explanation of the phenomena can be valid only if there is some remaining inter-city variation in transitory income, i.e., $P_y < 1$. All things considered, the most acceptable interpretation of the findings is that the income elasticity of consumption is something less than unity. More simply, permanent consumption does not by this evidence, seem to be a linear homogeneous function of permanent income.

Concluding Remarks.

The question of the form of the relation between the permanent components of income and consumption is quite independent of the other features of Friedman's theory. The remaining features of primary importance are: a) The household consumption is scaled to a long-run, normal, or "permanent" level of income and b) income over short periods can be usefully separated into two uncorrelated components, a permanent component which determines consumption up to a random factor and a transitory component which is totally independent of consumption. These propositions are also mostly out of range of the evidence presented above. There is perhaps some support from the estimates of the M.P.C. and income elasticity. These are quite high relative to estimates from typical cross-section of households. The effect is clearly consistent with Friedman's model. Given the peculiarities of the set of data.. non random sample of cities, use of city means, etc., it would be hazardous to infer anything from the magnitude of the numerical estimates.

The consequences of abandoning the homogeneity hypothesis are a loss of simplicity in the theory and gain of identification difficulties in empirical analysis. Explanation for the near constancy of the aggregate consumption: income ratio in the U.S. over several decades must be found outside the basic micro-theory, but given the growth of population and per capita incomes, changing taste and products, etc., a proportional micro-equation for consumption is not of great importance in any case. While the evidence reviewed in the paper supports the Keynesian proposition that, Ceteris Paribus, people do not increase consumption in proportion with income, it does not warrant any simple generalization about the behavior of aggregates over time.

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