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Entry, Gibrat's Law, Innovation, and the Growth of Firms

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I. Introduction

Because there have been so few econometric studies of the birth, growth and death of firms, we lack even crude answers to the following basic questions regarding the dynamic processes governing an industry's structure. What are the quantitative effects of various factors on the rates of entry and exit? How well can the growth of firms be represented by Gibrat's law of proportionate effect? What have been the effects of successful innovations on a firm's growth rate? What determines the amount of mobility within an industry's size structure? [1]

This paper provides some tentative answers to these questions. First, it constructs some simple models to estimate the effects of an industry's capital requirements, profitability, and other such factors on its entry and exit rates. In view of the significance of the rates of entry and exit, even rough estimates of the effects of these factors should be useful.

Second, the paper investigates how well Gibrat's law of proportionate effect can represent the growth of firms in each of the industries for which we have appropriate data. Although this law has played a prominent role in models designed to explain the size distribution of firms, it has been tested only a few times against data for very large firms.

Third, we estimate the difference in growth rate between firms that carried out significant innovations and other firms of comparable initial size. The results help to measure the importance of successful innovation as a cause of interfirm differences in growth rates, and they shed new light on the rewards for such innovations.

Fourth, the paper presents and tests a simple model to explain inter-industry and temporal differences in the extent to which firms change relative positions in the size distribution. Although economists have often noted the importance of the degree of mobility within an industry's size structure, they have not studied it thoroughly on either a theoretical or empirical level.

The paper is organized as follows. Sections 2-3 study the effects of several variables on the rates of entry and exit. Section 4 tests Gibrat's law, and Section 5 investigates the growth of the innovators. Section 6 studies the amount of mobility within an industry's size structure. Section 7 discusses some limitations of the results, and Section 8 concludes the paper. An Appendix presents the basic data on the birth, growth, and death of firms.

2. Entry, Profitability, and Capital Requirements

Although economists have traditionally been interested in the factors determining the rate of entry into an industry, there have been few attempts -- if any -- to estimate the quantitative importance of some of these determinants. In this section, we estimate the effects on the rate of entry of two factors that are generally regarded as important -- the profitability of the firms in the industry and the capital requirements involved.

Entry or exit can be defined as a net change in the number of firms in an industry. At the end of the following section, we present results based on this definition. In this section, we are interested in another concept of entry -- the extent to which new owners of productive facilities become established in an industry either through the construction of new plants or the purchase of existing firms. Of course, each concept has its own set of uses.[2/]

Perhaps the most obvious measure of the amount of entry into the i^{th} industry during the t^{th} period is E'_{it} -- the number of firms that entered during the period as a proportion of the number of firms in the industry at the beginning of the period. But the available data force us to use E_{it} -- the number of firms that entered during the period and survived until the end as a proportion of the original number of firms. Since E'_{it} and E_{it} should be highly correlated, this discrepancy is probably not too important for our purposes.[3/]

Letting C_{it} be the investment required to establish a firm of minimum efficient size in the i^{th} industry during the t^{th} period and letting Π_{it} be the average ratio of the rate of return in the i^{th} industry to that in all manufacturing during this period, we assume that

$$E_{it} = f(\Pi_{it}, C_{it}, \ldots). \tag{1}$$

Increases in Π_{it} -- because of their presumed effect on profit expectations -- make entry more attractive, and increases in C_{it} make it more difficult. Thus, E_{it} should be directly related to Π_{it} and inversely related to C_{it}.[4/]

More specifically, since the effects of these variables are likely to be multiplicative, we assume that

$$E_{it} = \alpha_0 \Pi_{it}^{\alpha_1} C_{it}^{-\alpha_2} Z_{it}, \qquad (2)$$

where Z_{it} is a random error term and the α's are presumed to be positive. To estimate the α's, data are needed on E_{it}, Π_{it}, and C_{it}. Table 1 shows the values of E_{it} during various periods in the history of the steel, petroleum refining, rubber tire, and automobile industries.[5] It also contains corresponding estimates of Π_{it} and C_{it}, the latter being based on Bain's figures [4] and the assumption that the ratio of the minimum efficient size to the average size of firm remained constant over time in each industry. Although these data are very rough they should be useful first approximations.[6]

Taking logarithms of both sides of eq. (2) and using these data to obtain least-squares estimates of the α's, we find that

$$\ln E_{it} = .49 + \underset{(.43)}{1.15} \ln \Pi_{it} - \underset{(.14)}{.27} \ln C_{it} \qquad (3)$$

where the quantities in parentheses are standard errors and $\ln Z_{it}$ is omitted. As one would expect, there is considerable variation about eq. (3), the coefficient of correlation (corrected for degrees of freedom) being about .70 (Figure 1). The residuals reflect the effects of differences in economies of scale, availability of raw materials, and other important factors that are omitted.[7]

Table 1 -- Values of Exogenous and Endogenous Variables in Eqs. (3),(7), and (16), Steel, Petroleum, Rubber Tire, and Automobile Industries, Selected Periods.

Industry and Time Period	E_{it} [a]	Π_{it} [b]	C_{it} [b]	R_{it} [c]	V_{it}^2	$\bar{S}_{it}/\hat{S}_{it}$ [d]	P_{it} [e]	A_{it} [f]	n_{it}
Steel:									
1916-26	.57	1.38	228	.20	18	1.15	.20	271	90
1926-35	.08	.38	214	.46	20	1.15	.17	281	122
1935-45	.20	.73	423	.16	12	1.15	.17	290	76
1945-54	.17	.77	465	.15	9	1.15	.26	300	81
Petroleum:									
1921-27	.66	.84	93	.59	11	.17	.36	62	314
1927-37	.46	.60	138	.65	13	.17	.42	68	335
1937-47	.78	.82	231	.42	15	.17	.35	78	269
1947-57	.25	1.01	238	.71	21	.17	.26	88	366
Tires:									
1937-45	.45	.84	11	.31	8	1.18	.30	41	49
1945-52	.68	.88	22	.46	10	1.18	.26	49	57
Autos:									
1939-49	.20	.94 [g]	316	.20	3	1.00	--	--	--
1949-59	.10	.36 [g]	575	.50	4	1.00	--	--	--

Source: See the Appendix and notes a-g below.

a/ For the definition and measurement of this variable, see Section 2 (notes 3 and 5 in particular) and the Appendix.

b/ For the assumptions underlying these estimates, see note 6. C_{it} is expressed in millions of dollars. It is the average of the upper and lower limits given by Bain [4] and includes initial losses in the case of the automobile industry.

c/ For the definition and measurement of this variable, see Section 3 and the Appendix.

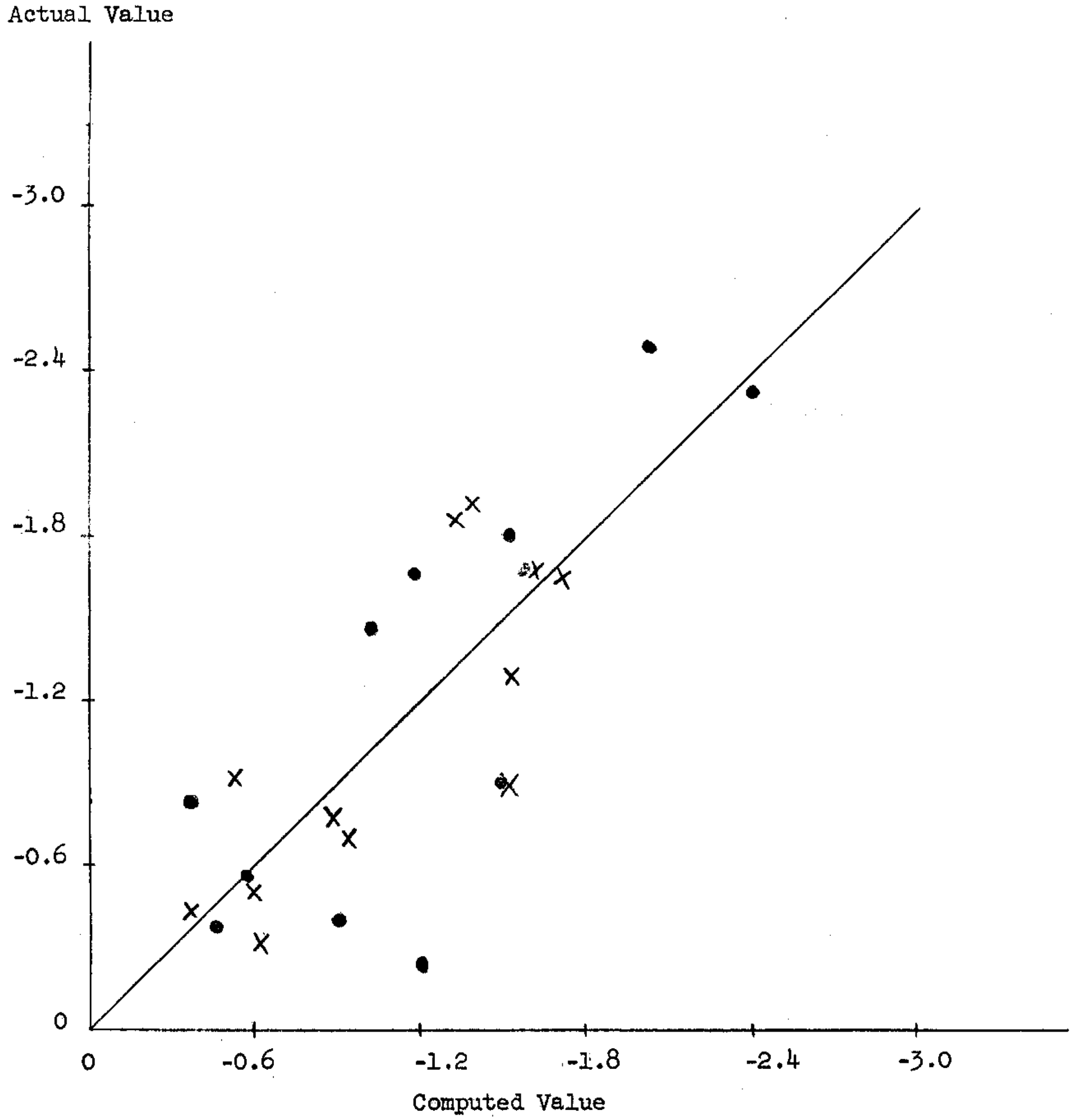
d/ See note 11 for the source of these estimates.

e/ See note 3 for a discussion of these estimates

f/ A_{it} is expressed in years. The derivation of these figures is discussed in note 32.

g/ See note 6 for some discussion of these figures.

Figure 1 -- Plot of Actual Values of ln E_{it} and in R_{it} Against Those Computed from Eqs. (3) and (7), Steel, Petroleum, Rubber Tire, and Automobile Industries, Selected Periods.[1/]



Source: Table 1 and eqs. (3) and (7). The line is a 45° line through the origin.

1/ The dots represent ln E_{it} and the X's represent ln R_{it} .

The estimates of α_1 and α_2 have the expected signs and are both statistically significant (.05 level). Because of the small number of observations, they have fairly large standard errors, and because of errors in the exogenous variables and the probable effects of $\ln E_{it}$ on $\ln \Pi_{it}$ (cf. Section 7), they are probably biased somewhat toward zero. But despite these limitations, they shed new light on the effects of Π_{it} and C_{it} on E_{it} . For example, if the bias is in the expected direction, one can be reasonably sure that the average value of E_{it} would increase by at least 60 percent if Π_{it} doubled and that it would decrease at least 7 percent if C_{it} doubled. Lower bounds of this sort are obviously useful.[8/]

3. Exit Rates and Changes in the Number of Firms

This section estimates the effects of several factors on the rate at which firms leave an industry, and it takes up the effects of these, and related, variables on the amount of entry defined in terms of changes in the number of firms. We use R_{it} -- the proportion of firms in the i^{th} industry at the beginning of the t^{th} period that had left by the end -- as a measure of the exit rate. Both firms that scrapped their plant and those that sold out are counted as departures.

Letting $R_{it}(S)$ be the proportion of firms of size S (at the beginning of the period) that left during the period, we assume that

$$R_{it}(S) = g(S/\hat{S}_{it}, \prod_{it}, \dots), \tag{4}$$

where $\hat{S}_{it}$ is the minimum efficient size of firm at the beginning of the period. As a firm becomes smaller relative to the minimum efficient size, its chance of survival decreases; and as the industry becomes less profitable relative to others, firms become more likely to leave. Thus, $R_{it}(S)$ should be inversely related to both $S/\hat{S}_{it}$ and $\prod_{it}$.[9/]

Since their effects are likely to be multiplicative, we assume that

$$R_{it}(S) = \nu_0 (S/\hat{S}_{it})^{-\nu_1} \prod_{it}{}^{-\nu_2} Z'_{it} \tag{5}$$

where Z'_{it} is a random error term and the ν's are presumed to be positive. Letting $\rho_{it}(S)$ be the probability that a firm in the i^{th} industry at the beginning of the period is of size S and assuming that the distribution of firms by size is log-normal,[10/] we have

$$R_{it} = \int_0^\infty R_{it}(S)\, \rho_{it}(S) dS$$

$$= \nu_0 \left[\int_0^\infty (S/\hat{S}_{it})^{-\nu_1} \rho_{it}(S) dS \right] \prod_{it}{}^{-\nu_2} Z'_{it} ,$$

$$= \nu_0 \left[\bar{S}_{it}/\hat{S}_{it} \right]^{-\nu_1} (1 + V_{it}^2)^{\nu_1(\nu_1+1)/2} \prod_{it}{}^{-\nu_2} Z'_{it} \tag{6}$$

where $\bar{S}_{it}$ is the mean and V_{it} is the coefficient of variation of the distribution of firm sizes.

Table 1 contains rough estimates of R_{it} , $\bar{S}_{it}/\hat{S}_{it}$, and V^2_{it} .[11/] Taking logarithms of both sides of eq. (6), we can use these data to obtain least-squares estimates of the ν's . The results are

$$\ln R_{it} = \underset{}{-1.68} - \underset{(.14)}{.41} \ln (\bar{S}_{it}/\hat{S}_{it}) + \underset{(.25)}{.10} \ln (1 + V^2_{it}) - \underset{(.33)}{.60} \ln \Pi_{it}, \quad (7)$$

where the quantities in parentheses are standard errors and $\ln Z'_{it}$ is omitted. For simplicity, the coefficient of $\ln (1 + V^2_{it})$ was not constrained to equal $\hat{\nu}_1 (\hat{\nu}_1 + 1)/2$ -- although this constraint would have resulted in somewhat better estimates of the ν's. Fig.1 shows that there is considerable variation about eq. (7), the coefficient of correlation (corrected for degrees of freedom) being .70.[12/]

The estimates of ν_1 and ν_2 have the correct signs and are statistically significant. Although they contain fairly large sampling errors and are probably biased somewhat toward zero (because of errors in the exogenous variables and the probable effects of $\ln R_{it}$ on $\ln \Pi_{it}$), they provide useful information regarding the effects of $\bar{S}_{it}/\hat{S}_{it}$ and Π_{it} on R_{it} . For example, if the bias is in the expected direction, one can be reasonably sure that the average value of R_{it} would decrease by at least 15 percent if Π_{it} or $\bar{S}_{it}/\hat{S}_{it}$ doubled. Lower bounds of this sort can easily be computed for the effects of other percentage increases in Π_{it} or $\bar{S}_{it}/\hat{S}_{it}$.[13]

Note in passing that this model suggests a technique for estimating the minimum efficient size of firm in an industry. Suppose that $\hat{S}_{it}$, rather than R_{it} , were regarded as the "dependent" variable. If data regarding R_{it} , $\bar{S}_{it}$, Π_{it} , and V^2_{it} were obtained for some new industry or time period, they could then be used to estimate $\hat{S}_{it}$. Although this technique is likely to be rough, some work might be carried out to see how accurate it is and to sharpen it.[14]

Finally, we turn to the more familiar definition of entry. A reasonable measure in this case is D_{it} -- the change in the number of firms during the period as a proportion of the number at the beginning. Since the number of firms bought during the period must equal the number sold,

$$D_{it} = E_{it} - R_{it} , \tag{8}$$

and the effects of Π_{it} , C_{it} , $\bar{S}_{it}/\hat{S}_{it}$, and V^2_{it} on the average value of D_{it} can be estimated -- at least roughly -- from eqs. (3) and (7). Of course, one must bear in mind the limitations of eqs. (3) and (7) and the likelihood of considerable variation about this average value of D_{it}

But if they are used with caution, the resulting estimates should be useful first approximations.[15]

4. Gibrat's Law and the Growth of Firms

Gibrat's law is a proposition regarding the process of firm growth. According to this law, the probability of a given proportionate change in size during a specified period is the same for all firms -- regardless of their size at the beginning of the period. For example, a firm with sales of 100 million dollars is as likely to double in size during a given period as a firm with sales of 100 thousand dollars. Put differently, Gibrat's law states that

$$S_{ij}^{t+\Delta} = U_{ij}(t, \Delta) S_{ij}^{t} , \qquad (9)$$

where S_{ij}^{t} is the size of the j^{th} firm in the i^{th} industry at time t, $S_{ij}^{t+\Delta}$ is its size at time $t+\Delta$, and $U_{ij}(t, \Delta)$ is a random variable distributed independently of S_{ij}^{t}.

Since this law is a basic ingredient in most models designed to explain the shape of the size distribution of firms, some importance attaches to whether or not it holds. This section provides the first tests based on data for practically all firms -- large and small -- in individual industries. The results pertain to the steel, petroleum, and rubber tire industries. The automobile industry is omitted because, with only a handful of firms, it is unlikely to provide much evidence regarding a proposition of this sort.[16]

A simple way to test Gibrat's law is to classify firms by their initial size (S_{ij}^{t}), compute the frequency distribution of $S_{ij}^{t+\Delta}/S_{ij}^{t}$

within each of these classes, and use a χ^2 test to determine whether the frequency distributions are the same in each class. We rely heavily on this test, but supplement it with others described below. The basic data used in these tests are described and presented in the Appendix.[17]

Gibrat's law can be formulated in at least three ways, depending on the treatment of the death of firms and the comprehensiveness claimed for the law. First, one can postulate that it holds for all firms -- including those that leave the industry during the period. If we regard the size (at the end of the period) of each of these departing firms as zero (or approximately zero), this version can easily be tested. The results -- shown in Table 2 -- indicate that it generally fails to hold. In seven of the ten cases, the observed value of χ^2 exceeds the critical limit corresponding to the .05 significance level.[18]

Why does this version of the law fail to hold? Even a quick inspection of the transition matrices in the Appendix shows one principal reason. The probability that a firm will die is certainly not independent of its size. In every industry and time interval, the smaller firms were more likely than the larger ones to leave the industry. For this reason (and others indicated below), this version of the law seems to be incorrect.[19]

Second, one can postulate that the law holds for all firms other than those that leave the industry. Hart and Prais [12] seem to adopt this version. Omitting such firms, we ran another series of χ^2 tests, the results of which are shown in Table 2. In four of the ten cases, the evidence seems to contradict the hypothesis, the observed value of χ^2 exceeding the limit

Table 2 -- Observed Value of χ^2 Criterion, Estimated Slope of Regression of $\ln S_{ij}^{t+\Delta}$ on $\ln S_{ij}^{t}$, and Ratio of Variances of Growth Rates of Large and Small Firms, Steel, Petroleum, and Rubber Tire Industries, Selected Periods.

Item	Steel				Petroleum				Tires	
	1916-1926	1926-1935	1935-1945	1945-1954	1921-1927	1927-1937	1937-1947	1947-1957	1937-1945	1945-1952
χ^2 criterion:[a]										
Including deaths	9.0	17.0*	22.5*	7.8	29.2*	44.9*	25.6*	42.7*	9.3	22.9*
Excluding deaths	7.1	3.3	9.5*	3.4	2.8	22.1*	17.7*	8.9	6.3	6.6*
Degrees of freedom (χ^2 tests):[b]										
Including deaths	6	6	6	6	6	6	6	6	6	4
Excluding deaths	4	4	4	4	4	4	4	4	4	2
Estimated slope:[c]										
Excluding deaths	.88*	.99	.92*	1.00	.94	.88*	.99	.94	.97	.97
Large firms only	.94	.96	1.00	.98	.99	.98	.93	1.10	1.07	.89
Standard error of slope:										
Excluding deaths	.05	.04	.03	.04	.05	.04	.03	.04	.05	.04
Large firms only	.16	.16	.07	.06	.24	.14	.07	.07	.10	.05
Number of firms:										
Excluding deaths	72	66	64	69	128	116	156	106	34	31
Large firms only	7	9	11	12	7	11	16	17	11	12
Variance of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ among small firms divided by variance of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ among large firms:[d]										
Excluding deaths	8.96*	.80	37.40*	5.06*	43.27*	19.25*	63.56*	147.1*	16.16*	.31
Large firms only	.63	161.00*	.90	8.50*	3.50	7.75*	4.00*	3.6*	39.25*	8.67

Source: See Section 4 and the Appendix.

[a] For the classification of firms by size and the classification of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ used in each industry, see notes 18 and 20.

[b] The number of degrees of freedom equals (a - 1) (b - 1) where a is the number of size classes and b is the number of classes of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ in the contingency table.

[c] The number of firms in each regression is shown in one of the last two rows of this table.

Table 2 (Continued)

d/ The firms regarded as "small" and "large" in the first row are as follows: In steel, small firms have 4,000-16,000 and large firms have 256,000-4,096,000 tone of capacity. In petroleum, small firms have 500-999 and large firms have 32,000-511,999 barrels of capacity. In tires, small firms have 80-159 and large firms have 640-5119 employees. The firms regarded as "small" and "large" in the second row are described in note 22.

* For X^2 criteria and ratios of variances, this means that the probability is less than .05 that a value would be this large (or larger) if Gibrat's law held. For estimated slopes, this means that they differ significantly from unity (.05 significance level).

corresponding to the .05 significance level.[20]

To see why this version must be rejected, note that eq. (9) implies that

$$\ln S_{ij}^{t+\Delta} = V_i(t, \Delta) + \ln S_{ij}^{t} + W_{ij}(t, \Delta), \tag{10}$$

where $V_i(t, \Delta)$ is the mean of $\ln U_{ij}(t, \Delta)$ and $W_{ij}(t, \Delta)$ is a homoscedastic random variable with zero mean. Thus, if $\ln S_{ij}^{t+\Delta}$ is plotted against $\ln S_{ij}^{t}$, the data should be scattered with constant variance about a line with slope of one. Table 2 contains the least-squares estimate of the slope of each of these lines. In half of the cases where the law was rejected the slope is significantly less than one.

In addition, the variance of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ tends to be inversely related to S_{ij}^{t}. Taking in each case a group of small firms and dividing the variance of their values of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ by the variance among a group of large firms, we obtain the results shown in Table 2. In eight of the ten cases the variances differed significantly. Thus, contrary to this version of the law smaller firms often tend to have higher and more variable growth rates than larger firms.[21]

Third, one can postulate that the law holds only for firms exceeding the minimum efficient size in the industry -- the size (assuming the long-run average cost curve is J-shaped) below which unit costs rise sharply and above which they vary only slightly. This is the version put forth by Simon and Bonini [24]. One is faced once again with the problem of whether or not to include firms that die. We excluded them, but the major results would almost certainly have been the same if they had been included.

This version was tested in two ways. First, we estimated the slope of the regression of $\ln S_{ij}^{t+\Delta}$ on $\ln S_{ij}^{t}$, but included only those firms that were larger than Bain's [4] estimate of the minimum efficient size. The results are quite consistent with Gibrat's law (the slopes never differing significantly from one). Second, we used F tests to determine whether the variance of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ was constant among these firms. Contrary to Gibrat's law, the variance of $S_{ij}^{t+\Delta}/S_{ij}^{t}$ tends to be inversely related to S_{ij}^{t} in six of the ten cases.[22]

Thus, regardless of which version one chooses, Gibrat's law fails to hold in more than one-half of these cases. What sort of mechanism produced the observed departures from this law? The reasons for the inverse relationships between a firm's chance of death and its initial size seem fairly obvious, but why should the data for the survivors show that the smaller firms tend to have higher and more variable growth rates than the larger ones?[23]

One model that might help to account for this is as follows. Consider the distribution of growth rates of firms of size S_{ij}^{t} that would have resulted if none had left the industry. It is not unreasonable to suppose that above some minimum value of S_{ij}^{t} the average of this distribution would be about the same in each size class. Moreover, one might also suppose that it would exceed the average for those that left the industry by about the same amount in each size class.

In addition, because larger firms are to some extent a collection of somewhat independent smaller firms, the variance of this distribution would be expected to be inversely related to S_{ij}^{t} .[24/] And under these conditions, if the actual growth rate of each survivor is proportional to what it would have been if all firms had survived and if the death rate is inversely related to S_{ij}^{t} , one would expect to encounter very frequently the sorts of departures from Gibrat's law that were found above.[25/]

Research should be conducted to determine the conditions under which models of this sort lead to a log-normal or Yule distribution of firm sizes. Although Gibrat's law is very convenient from an analytical point of view, it does not seem to hold up very well empirically. It seems to be a rather unreliable base on which to rest theories of the size distribution of firms.

5. Successful Innovation and the Growth of Firms

How much of an impact does a successful innovation have on a firm's growth rate? In another study [16], I presented a list of the firms that were first to introduce the significant new processes and products that emerged since World War I in the steel and petroleum refining industries. A comparison of their growth rates -- during the period in which the innovation occurred -- with those of other comparable firms should help to indicate how great the pay-off is (in terms of growth) for a successful innovation.

For each period for which we have data, Table 3 estimates the average annual growth rate of (1) firms that carried out significant innovations during the period, and (2) other firms that were equal in size to the successful innovators at the beginning of the period. There is a marked difference between the two groups. In every time interval and in both industries, the successful innovators grew more rapidly than the others; and in many cases, their average rate of growth was more than twice that of the others.[26/]

Taking each innovator separately, the difference between its growth rate and the average growth rate of other comparable firms seems to have been inversely related to its size. As one would expect, a successful innovation had a much greater impact on a small firm's growth rate than on a large firm's. The fact that fewer of the successful innovators in more recent periods were small firms probably accounts in part for the decrease over time in the average difference (in Table 3) between the two groups.[27/]

Each growth rate in Table 3 pertains to the entire period indicated in the caption -- whereas the innovations occurred sometime within the period. Consider the period from time t to time $t + \Delta$. Suppose that the j^{th} successful innovator in this period introduced its innovation at time t_j, that its average annual growth rate from time t to time t_j exceeded that of other comparable firms by e_j, and that its average annual growth rate from time t_j to time $t + \Delta$ exceeded that of other comparable firms by $e_j + d_j$. What were the average values of e_j and d_j?

Letting $S_j^{t+\Delta}$ be the size (i.e., capacity) at time $t + \Delta$ of the

Table 3 -- Average Annual Growth Rates of Successful Innovators and Other Forms (of comparable initial size), Computed Values of $\bar{e}$ and $\bar{d}$, and Regressions (excluding innovators) of $\ln S_{ij}^{t+\Delta}$ on $\ln S_{ij}^{t}$, Steel and Petroleum Refining Industries, Selected Periods.

Item	Steel				Petroleum			
	1916-1926	1926-1935	1935-1945	1945-1954	1921-1927	1927-1937	1937-1947	1947-1957
Average annual growth rate (percent):								
Innovators	13.7	6.5	3.4	3.2	13.1	7.9	3.6	6.7
Other Firms[a/]	3.7	3.3	2.0	2.4	6.6	4.1	3.6	4.2
Computed value of:[b/]								
$\bar{e}$ (percent)	--	0.7	0.7	--	--	4.2	-2.5	-2.8
$\bar{d}$ (percent)	--	3.9	5.2	--	--	5.7	3.6	13.4
Regression of $\ln S_{ij}^{t+\Delta}$ on $\ln S_{ij}^{t}$:[c/]								
Intercept (a_i)	1.68	.55	1.34	.18	1.10	1.68	.41	1.27
Slope (b_i)	.88	.97	.90	1.01	.93	.84	.98	.90

Source: See Section 4 and Mansfield [16].

a/ See note 26 for the source of these estimates.

b/ No figures are computed in cases where there were only a few innovators. See note 28 for a discussion of the derivation of these figures.

c/ Note 26 describes how this regression is used to estimate the average annual growth rate of the "other firms" that were of the same initial size as the innovators

j^{th} innovator and $Q_j^{t+\Delta}$ be the average logarithm of the sizes at time $t + \Delta$ of the other firms that were equal in size to the j^{th} innovator at time t, one can show that

$$(\ln S_j^{t+\Delta} - Q_j^{t+\Delta})/\Delta = e_j + [1 - (t_j - t)/\Delta]d_j \tag{11}$$

Letting $\bar{e}$ and $\bar{d}$ be the average values of e_j and d_j and assuming that $(e_j - \bar{e})$ and $(d_j - \bar{d})$ are statistically independent of $(t_j - t)/\Delta$, we have

$$(\ln S_j^{t+\Delta} - Q_j^{t+\Delta})/\Delta = \bar{e} + [1 - (t_j - t)/\Delta]\bar{d} + W_j, \tag{12}$$

where W_j can be treated as a random error term. Using eq. (12) we can apply least-squares to obtain $\bar{e}$ and $\bar{d}$.[28]

The results (in Table 3) indicate that $\bar{d}$ was always positive, but that the sign of $\bar{e}$ varied. This means two things. First, in the period immediately before they introduced the innovations, there was no persistent tendency for the successful innovators to grow more rapidly than other comparable firms. In some cases they grew more rapidly, but in others they did not. Thus, their higher growth rate cannot be attributed to their pre-innovation behavior. Second, in the period after they introduced the innovations their mean growth rates consistently exceeded that of other comparable firms by more than it had before their introduction -- which is what one would expect.

If one makes the crude assumption that the pre-innovation difference in average growth rate between successful innovators and other firms would have been maintained from time t to time $t + \Delta$ if the innovations had not been introduced, $\bar{d}$ measures the average effect of these successful innovations on a firm's growth rate during the relevant period. Based on this assumption, their average effect was to raise a firm's growth rate by 4-13 percentage points, depending on the particular time interval and industry. In view of the widespread interest in measures of the pay-off from successful innovation, these estimates, despite their crudeness, should be useful.[29/]

6. Mobility Within an Industry's Size Structure

Economists have become increasingly aware of the importance of the amount of mobility in an industry -- i.e., the extent to which firms change their relative positions in the size distribution [12, 19, 24, 25]. This section measures the amount of mobility in several industries and constructs a simple model to help explain the observed variation. The results shed additional light on the process of firm growth, since mobility is obviously related to the amount of interfirm variation in growth rates.[30/]

To measure the amount of mobility in the i^{th} industry during the t^{th} period, suppose that we have a list of all firms that were in existence at both the beginning and end of the period. Suppose that a firm is chosen at random from this list. Then suppose that another firm is chosen at random from those that were 60-70 percent as large as the first firm at the beginning of the period. The probability that the second (initially smaller) firm will be bigger than the first (initially larger) firm at the end of the period is a rough measure of the amount of mobility. Let this probability be P_{it} .[31/]

Table 1 contains estimates of P_{it} for various periods in the history of the steel, petroleum refining, and rubber tire industries. Because of the small number of firms, it was impossible to obtain meaningful estimates for the automobile industry. To help explain the considerable variation in P_{it} , we assume that

$$P_{it}(S) = h\,(S/n_{it}\,\bar{S}_{it}\,,\ A_{it}\,,\ S^{*}_{it}/\tilde{S}_{it}\,,\ n_{it}\,,\ \ldots\ )\,, \tag{13}$$

where $P_{it}(S)$ is the probability that a firm of size S at the beginning of the period will be smaller at the end of the period than a firm originally of size .6S - .7S, n_{it} is the number of firms in the industry at the beginning of the period, $\bar{S}_{it}$ is their mean size, $\tilde{S}_{it}$ is their median size, S^{*}_{it} is the size of firm such that firms exceeding it accounted for one-half of the market, and A_{it} is the age of the industry.

What are the effects of these variables? First, the smaller firm's chance of overtaking the larger one will be inversely related to the initial difference between their market shares -- which is proportional to $S/n_{it}\ \bar{S}_{it}$. Second, as an industry grows older, stronger ties are established between firms and their customers, the technology becomes more settled, and the industry's structure tends to become more rigid. Thus, A_{it} -- which is a proxy variable for these factors -- is likely to be important.[32] Third, $P_{it}(S)$ is likely to be inversely related to the amount of concentration in the industry. Thus, n_{it} and $S^*_{it}/\tilde{S}_{it}$ (a convenient measure of the amount of inequality among firm sizes) are included.[33]

Since the effects of these variables are likely to be multiplicative, we assume that

$$P_{it}(S) = \beta_0(S/n_{it}\ \bar{S}_{it})^{-\beta_1} A_{it}^{-\beta_2} (S^*_{it}/\tilde{S}_{it})^{-\beta_3} n_{it}^{\beta_4} Z''_{it} \tag{14}$$

where Z''_{it} is a random error term and the β's are presumed to be positive. Assuming again that the distribution of firms by size is log-normal, we have

$$P_{it} = \int_0^\infty P_{it}(S)\, \rho_{it}(S)dS$$

$$= \beta_0[\int_0^\infty S^{-\beta_1} \rho_{it}(S)dS]\bar{S}_{it}^{\beta_1} A_{it}^{-\beta_2} (S^*_{it}/\tilde{S}_{it})^{-\beta_3} n_{it}^{\beta_4+\beta_1} Z''_{it}$$

$$= \beta_0(1 + V^2_{it})^{\beta_1(1+\beta_1)/2 - \beta_3} A_{it}^{-\beta_2} n_{it}^{\beta_4+\beta_1} Z''_{it} . \tag{15}$$

To see how well this model can represent the data, we take logarithms of both sides of eq. (15), and using the data in Table 1, we obtain least-squares estimates of the coefficients. The results are

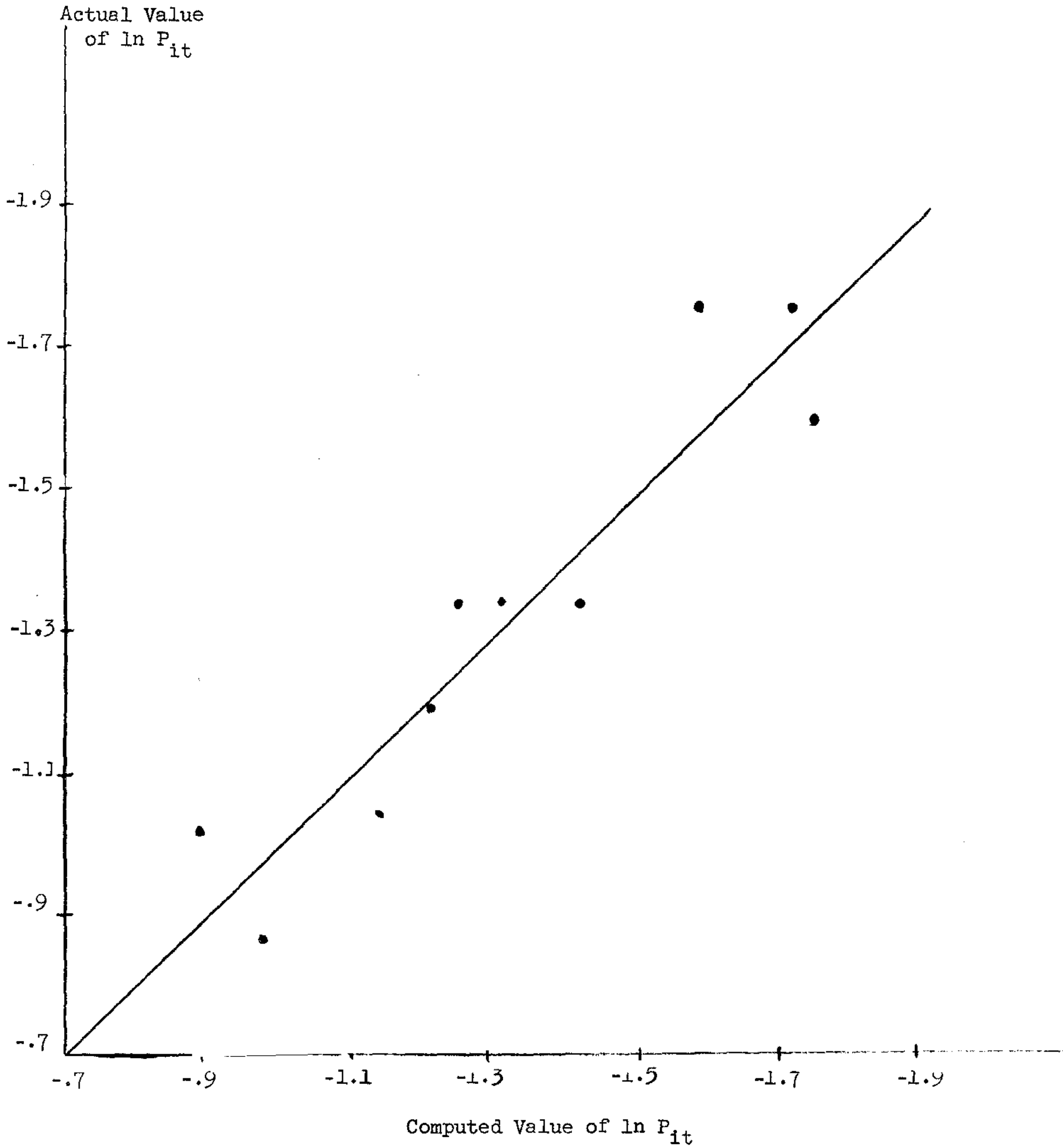
$$\ln P_{it} = -.55 - \underset{(.20)}{.57} \ln (1 + V_{it}^2) - \underset{(.07)}{.15} \ln A_{it} + \underset{(.08)}{.29} \ln n_{it} , \qquad (16)$$

where the quantities in parentheses are standard errors and $\ln Z_{it}''$ is omitted. Figure 2 shows that eq. (16) can explain much of the variation in $\ln P_{it}$, the coefficient of correlation (corrected for degrees of freedom) being about .90. All of the regression coefficients have the expected signs and are statistically significant at the .05 level. Thus, what evidence we have seems to be quite consistent with the model. Indeed, it fits the data surprisingly well.[34]

7. Limitations

Before concluding, we discuss briefly some of the limitations of this study. First, the empirical results in Sections 2, 3 and 6 are based on relatively few observations. Of course, considerable work was required to obtain even this small number because each observation is based on a large amount of relatively inaccessible data. But regardless of the reasons, the smallness of the samples results in fairly substantial sampling errors and obvious dangers of bias.

Figure 2 - Plot of Actual Values of ln P_{it} Against Those Computed from Eq. (16), Steel, Petroleum, and Rubber Tire Industries, Selected Periods.



Source: Table 1 and eq. (16).

Second, the basic data in Table 1 are often very rough. Difficulties in the estimates of the amount of entry and exit are discussed in the Appendix. In addition, the estimates of C_{it}, Π_{it}, A_{it}, and $\bar{S}_{it}/\hat{S}_{it}$ are based on the rather crude assumptions described in notes 6, 11, and 32. Unfortunately, no better data could be found. To the extent that they are distributed randomly, the errors of measurement in the exogenous variables tend to bias the estimates of the coefficients in eqs. (3), (7), and (16) toward zero.[35/]

Third, the models in Sections 2, 3, and 6 are obviously oversimplified. The small number of observations, as well as measurement problems and lack of data, limited the number of explanatory variables that could be included. In passing, it might be noted that another independent variable -- the length of the time period -- was used initially in eqs. (3), (7), and (16), but its effect turned out to be non-significant.[36/]

Fourth, the estimating procedures are sometimes rough. In Section 5, the computed values of $\bar{e}$ and $\bar{d}$ are based on a rather bold assumption. In Sections 2-3, there is probably some least-squares bias toward zero in the estimates of the α's and ν's because Π_{it} is inversely related to E_{it} and directly related to R_{it}. But in the case of α_2 and ν_2; this bias should not be very large, and considering the quality of the basic data, it did not seem worthwhile to use more complicated estimating procedures.[37/]

8. Summary and Conclusions

This paper constructs some econometric models to estimate the effects of various factors on the rates of birth, growth, and death of firms. Unfortunately, the processes of firm formation, growth, and decline, although they are fundamental parts of micro-economics, have received little attention from econometricians, and existing theories regarding these processes -- to the extent that they exist at all -- have little empirical content. Our results, despite their limitations, should help to fill this gap.

First, we estimated the effects of an industry's profitability, capital requirements, and minimum efficient size of firm on its rates of entry and exit. Because of the roughness and sparseness of the basic data, the results must be treated with caution. But they provide some idea of how rapidly firms appear and disappear in response to the incentives and penalties of the profit system. Moreover, they help to gauge the importance of large capital requirements as a bar to entry.

Second, we tested several variants of Gibrat's law of proportionate effect. This law -- which states that a firm's growth rate is independent of its initial size -- is an important part of most theories designed to explain the size distribution of firms. Contrary to the law, smaller firms have relatively high death rates and those that survive tend to have higher and more variable growth rates than larger firms. An alternative theory which may help to account for these deviations from Gibrat's law is sketched.

Third, evidence is presented showing for the first time the effects of successful innovations on a firm's rate of growth. The results indicate that on the average the successful innovators in these industries grew about twice as rapidly as other comparable firms during the relevant period.[38/] In terms of short-term growth, the rewards for successful innovation seem to have been substantial, particularly for smaller firms.

Fourth, a simple model is presented to help explain variation among industries and over time in the extent to which firms change their relative positions in the size distribution. Of course, this aspect of an industry's structure is closely related to the amount of interfirm variation in growth rates. Our tentative results indicate that the amount of mobility in an industry depends significantly on its age and its market structure.

These results should be useful elements in building a richer theory of the dynamic aspects of industrial structure. In recent years, economists have begun to study in a systematic way the changes over time in an industry's composition and structure, but because so little econometric work has been carried out, they have had relatively little to go on in constructing models to represent the relevant dynamic processes. For this reason (and others), they have not proceeded far beyond the simplest sorts of stochastic models -- Markov processes with constant transition probabilities based on Gibrat's law, constant entry rates, etc.[39/] By providing some of the necessary econometric results, this paper should contribute to the development of more useful theories in this area.

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Appendix: Transition Matrices

This Appendix presents the basic data regarding the birth, growth, and death of firms in the steel, petroleum refining, and rubber tire industries. (The data for the automobile industry pertain to only a small number of firms and are readily available in Automobile Industries.) These data can be summarized most easily in the form of transition matrices -- shown in Tables A, B, C, D, and E. If all firms are classified into n size classes, the ij^{th} element of the transition matrix for a particular period $(i, j = 1, \ldots, n)$ is the number of firms in the i^{th} class at the beginning of the period that were in the j^{th} class at the end. The general usefulness of such matrices seems obvious. But despite the pioneering work of Hart and Prais [12] and Adelman [1], few have been constructed.

Tables A and B contain transition matrices for the steel industry for 1916-26, 1926-35, 1935-45, and 1945-54. Tables C and D contain matrices for petroleum refining for 1921-27, 1927-37, 1937-47, and 1947-57. Table E contains matrices for the rubber tire industry for 1937-45 and 1945-52. Some of these periods were dictated by the availability of data; others were chosen rather arbitrarily. A firm's size is measured in terms of gross tons of ingot capacity (steel), daily crude capacity (petroleum), or employment (tires). In steel and petroleum, all firms with ingot capacity or crude capacity are included. In rubber tires, all firms cited in the Rubber Red Book as manufacturers of rubber tires are included.

The basic data were derived from the Directory of the American Iron and Steel Institute [3] Bureau of Mines bulletins [6], the Petroleum Refiner, the Rubber Red Book, Moody's Industrials, and correspondence with particular firms. To construct each matrix, we obtained from these sources complete lists of the firms in the industry at the beginning and end of the period and the size of each firm at both points in time. With this information at hand, it was a simple matter to construct each matrix.

Three points should be noted regarding these data. First, when a firm's name appeared on a list for the first time, we assumed that it entered the industry during the preceding period. Similarly a firm is regarded as having left the industry when its name disappeared from the lists. Although we tried to keep track of mere changes in company names, (where changes in ownership were not involved), some were undoubtedly missed and hence the entry and exit rates may be inflated. But on the other hand, they may also be underestimated because some firms may have kept the same names despite a change in ownership. (Of course, for large corporations, changes in ownership occur to some extent all the time and are not very important unless changes in the control of the firm are involved.) Unfortunately, the available data force us to use a firm's name as an indicator of its ownership. But one would certainly expect the resulting rates of entry and exit to be closely related to the actual ones. Of course, if they are proportional on the average, there is no problem.

Second, when mergers occurred, they were treated as if the largest firm involved in the merger bought the others. That is, the resulting firm was regarded as a continuation of the largest of its components, and the other parties to the merger were treated as if they went out of business. This procedure is arbitrary, but no other seems clearly preferable. Fortunately, it should not affect the results very substantially. Third, some members of the industry did not provide data regarding their size at some of these points in time, and there was no choice but to omit them during the relevant periods. This should be of little consequence because only a few such cases were encountered. Note that this accounts for the fact that the number entering a particular size class sometimes differs from the number in that size class at the beginning of the next period.

Table A -- Transition Matrices for the Steel Industry, 1916-26, and 1926-35.[a]

Capacity (tons) at beginning of the period	Total	Ingot capacity (tons) at end of the period: Disappearances	Under 4,000	4,000-15,999	16,000-63,999	64,000-255,999	256,000-1,023,999	1,024,000-4,095,999	Over 4,095,999
					[Number of firms]				
					1916-1926				
Entrants	51	--	10	10	15	12	4	0	0
Under 4,000	10	2	4	3	1	0	0	0	0
4,000-15,999	17	5	1	6	4	1	0	0	0
16,000-63,999	14	1	0	1	11	1	0	0	0
64,000-255,999	31	6	0	0	0	16	8	1	0
256,000-1,023,999	10	2	0	0	0	0	7	1	0
1,024,000-4,095,999	7	2	0	0	0	0	0	4	1
Over 4,095,999	1	0	0	0	0	0	0	0	1
					1926-1935				
Entrants	10	--	0	4	2	2	2	0	0
Under 4,000	14	11	3	0	0	0	0	0	0
4,000-15,999	20	11	1	7	1	0	0	0	0
16,000-63,999	30	17	0	1	10	1	0	1	0
64,000-255,999	31	12	0	0	1	13	5	0	0
256,000-1,023,999	19	5	0	0	0	0	11	3	0
1,024,000-4,095,999	6	0	0	0	0	0	1	4	1
Over 4,095,999	2	0	0	0	0	0	0	0	2

Source: See the Appendix.

[a] The first column (labeled "total") contains the number of firms in each size class at the beginning of the period. For each beginning-of-period size class (i.e., each row), the remaining columns show the end-of-period size distribution.

Table B -- Transition Matrices for the Steel Industry, 1935-45, and 1945-54.[a]

Capacity (tons) at beginning of the period	Total	Disap-pearances	Under 4,000	4,000 15,999	16,000-63,999	64,000-255,999	256,000-1,023,999	1,024,000-4,095,999	Over 4,095,999
			Ingot capacity (tons) at end of the period						
			[Number of firms]						
			1935-1945						
Entrants	15	--	0	2	7	3	3	0	0
Under 4,000	4	4	0	0	0	0	0	0	0
4,000-15,999	12	0	0	5	6	1	0	0	0
16,000-63,999	14	4	0	1	6	3	0	0	0
64,000-255,999	16	0	0	0	0	14	2	0	0
256,000-1,023,999	19	3	0	0	0	0	14	2	0
1,024,000-4,095,999	8	1	0	0	0	0	1	5	1
Over 4,095,999	3	0	0	0	0	0	0	0	3
			1945-1954						
Entrants	14	--	1	1	6	2	3	1	0
Under 4,000	0	0	0	0	0	0	0	0	0
4,000-15,999	9	2	0	4	3	0	0	0	0
16,000-63,999	20	1	0	0	14	4	1	0	0
64,000-255,999	21	6	0	0	1	12	2	0	0
256,000-1,023,999	20	3	0	0	0	0	12	5	0
1,024,000-4,095,999	7	0	0	0	0	0	0	3	4
Over 4,095,999	4	0	0	0	0	0	0	0	4

Source: See the Appendix.

[a] See note a, Table A.

Table C -- Transition Matrices for the Petroleum Refining Industry, 1921-27, and 1927-37.[a]

Daily capacity (bbls.) at beginning of the period	Total	Disap- pearances	Under 1,000	1,000-3,999	4,000-15,999	16,000-63,999	64,000-255,999	Over 255,999
			Daily capacity (bbls.) at end of the period					
			[Number of firms]					
			1921-1927					
Entrants	207	--	75	92	36	4	0	0
Under 1,000	58	34	13	6	4	1	0	0
1,000-3,999	173	119	3	34	15	2	0	0
4,000-15,999	61	29	1	5	17	9	0	0
16,000-63,999	15	4	0	0	0	7	4	0
64,000-255,999	6	0	0	0	0	0	6	0
Over 255,999	1	0	0	0	0	0	0	1
			1927-1937					
Entrants	153	--	44	62	43	3	1	0
Under 1,000	92	74	8	8	2	0	0	0
1,000-3,999	137	94	1	27	14	0	1	0
4,000-15,999	72	40	1	6	19	6	0	0
16,000-63,999	23	11	0	1	2	5	4	0
64,000-255,999	10	0	0	0	0	0	6	4
Over 255,999	1	0	0	0	0	0	0	1

Source: See the Appendix.

[a] See note a, Table A. Both domestic and foreign capacity owned by firms are included.

Table D -- Transition Matrices for the Petroleum Refining Industry, 1937-47, and 1947-57.[a]

Daily capacity (bbls.) at beginning of the period	Total	Disappearances	Daily capacity (bbls.) at end of the period: Under 1,000	1,000-3,999	4,000-15,999	16,000-63,999	64,000-255,999	Over 255,999
			[Number of firms]					
			1937-1947					
Entrants	210	--	83	75	43	9	0	0
Under 1,000	54	26	21	6	1	0	0	0
1,000-3,999	104	53	4	34	13	0	0	0
4,000-15,999	80	30	1	7	27	15	0	0
16,000-63,999	14	3	0	0	2	8	1	0
64,000-255,999	12	1	0	0	0	0	8	3
Over 255,999	5	0	0	0	0	0	0	5
			1947-1957					
Entrants	90	--	7	33	31	16	3	0
Under 1,000	109	98	4	5	2	0	0	0
1,000-3,999	122	95	0	16	10	1	0	0
4,000-15,999	86	54	0	3	22	7	0	0
16,000-63,999	32	13	0	0	2	12	5	0
64,000-255,999	9	0	0	0	0	0	5	4
Over 255,999	8	0	0	0	0	0	0	8

Source: See the Appendix.

[a] See note a, Table A. Both domestic and foreign capacity owned by firms are included.

FOOTNOTES

* A preliminary version of this paper was read at the August 1961 meetings of the Econometric Society. The work on which it is based is part of a larger project on industrial research and technical change supported by the Office of Special Studies of the National Science Foundation, by a Ford Foundation Faculty Research Fellowship, and by the Cowles Foundation for Research in Economics. The paper has benefitted from discussions with D. Fink, J. Muth, and N. Seeber. My thanks also go to G. Haines and D. Remington for their assistance and to various companies for data.

1/ With regard to the effects of various factors on the rates of entry and exit, there has been considerable theorizing [4, 13, 18, 27] and a few relevant empirical studies [4, 7], but there has been no systematic attempt to estimate the quantitative effect of various factors. With regard to the growth of firms, there have been several studies of Gibrat's law [10, 12, 14, 24], one of which dealt in part with the determinants of the amount of mobility, but these studies are based only on the largest firms. There are no previous studies (that I know of) of the effects of innovation on a firm's growth rate.

2/ The first definition of entry is useful in analyzing problems regarding market structure and industrial concentration. The number of firms in an industry is an important factor in such problems, and it is important to understand how a change in this number -- i.e., entry or exit -- takes place.

The second definition of entry (i.e., the one considered in this section) is useful in measuring the ease with which new entrepreneurs can become established in an industry and the extent to which they do so. For this purpose, it would be misleading to ignore those that entered by purchasing existing concerns. (Of course, one problem is that changes in ownership may not mean changes in control and vice versa.)

Finally, it is possible that a third definition of entry would be useful for some purposes. This would measure the number of firms that entered with new plant, regardless of the number of firms that scrapped their plant during the period. That is, it would be a gross measure of entry. Bain's discussion [4], p. 4 ff. generally runs in these terms. The available data do not permit us to measure this gross concept of entry. See the Appendix.

3/ Of course, other measures might be used. E.G., the absolute number of entrants is a possible alternative. But the establishment of two new firms would seem to mean one thing if there previously were two firms and something else if there previously were 100. Moreover, one would expect that ease of entry would be directly related to the number of firms in the industry [20]. Although it is somewhat arbitrary, it seems sensible to follow the Department of Commerce's procedure [7] and to normalize for the original number of firms.

The size of the entrants -- as well as their number -- might be very important for some problems. Although we ignore this aspect of the problem, it could be included fairly easily. Note too that, in comparisons of the values of E_{it}, differences in length of the period might be important. Although we tried to obtain periods of equal length, this was not always feasible. However, when introduced into eqs. (3), (7), and (16), this factor has no significant effect on E_{it}, R_{it}, or P_{it}.

Having only the transition matrices in the Appendix, we had no choice but to use E_{it}. So long as the survival rate for new firms is relatively independent of E'_{it} or positively correlated with it, E_{it} should be a reasonably good surrogate. Moreover, if one believes that we should only be concerned with entrants that survive for some specified length of time, E_{it} may be closer to what we want than E'_{it}. Finally, E_{it} has the advantage that it equals $D_{it} + R_{it}$. See Section 3.

4/ As a first approximation, it may not be too unreasonable to assume that the profit expectation of potential entrants during the period is a function of Π_{it}. But many other factors are obviously of importance -- the variability of the industry's profits during the period, the absolute level of profits, the probability that new processes or related new products will be developed, the outlook with regard to factor prices, etc.

5/ The Appendix describes the data on entry for the steel, petroleum refining, and rubber tire industries, and it points out the difficulties in them. Probably the most important difficulty is that lists based wholly on company names are used to identify entrants. The data would permit no other procedure. For further details, see the Appendix.

Note that the steel data pertain to all firms with ingot capacity (open-hearth, bessemer, or electric) and that the petroleum data pertain to all firms with crude capacity (operating or shut-down). For some purposes, it might have been preferable to have excluded electric furnaces and shut-down capacity. The data on entry (and also those on exit used in the following section) for the automobile industry were derived from the annual statistical issues of Automotive Industries.

Finally, note that eq. (2) may be much more effective in explaining changes in the number of new firms with new plant than changes in the number of firms that are bought. But, since the former are a large percentage of the total number of entrants included in E_{it}, eq. (2) seems sensible. (In recent years, the former account for about 2/3 of the total, according to [7].) However, one might argue that the error term is additive.

6/ To estimate Π_{it}, we needed figures on profits after taxes as a percentage of net worth in each industry. For rubber tires, the data came from the Statistics of Income. For petroleum, they came from Epstein [9], De Chazeau and Kahn [8], and the Statistics of Income. For steel, they came from Schroeder [23], but some adjustment was made for differences in concept. The data for automobiles came from Moody's and pertained to the largest five firms. The 1925-57 data for all manufacturing came from the First National City Bank of N. Y. (as reported in the 1959 Petroleum Facts and Figures). The earlier data for all manufacturing came from Epstein [9].

In many respects, the data are rough. An unweighted average of the profit rates of firms above the minimum efficient size would seem appropriate here. But judging by Bain's figures [4], there is little correlation between size and profit rate in steel, petroleum, and tires; and firms above the minimum efficient size account for almost all of the assets. Thus the weighted averages that we use should be fairly good approximations. In autos, there seems to be some correlation of this sort and consequently we use an unweighted average. This results in a much lower figure for autos than the weighted average that is generally published. For 1949-59, the figure seems much too low, but it could be appreciably higher without affecting the results substantially.

To obtain C_{it} in each industry, we multiplied Bain's estimate [4] of the required investment by the ratio of the average size of firm at the beginning of the t^{th} period (measured in terms of capacity in steel and petroleum, production in automobiles, and employment in tires) to the average size of firm in 1945 (steel), 1947 (petroleum), 1949 (autos), or 1945 (tires). See note b, Table 1 for further comments on C_{it}. If the ratio of the average size to the minimum efficient size of firm remained constant over time in each industry and if the necessary investment varied in proportion to the minimum efficient size, this would be all right. This is probably as sensible as any of the simple, operational assumptions we could make, but its crudeness should be obvious.

Finally, note that C_{it} -- even if it were accurately measured -- would not necessarily be the minimum investment for an entrant because the typical entrant was below the minimum efficient size. For the same reason, the typical entrant could not expect to earn profits of $\overline{\Pi}_{it}$. But it seems reasonable that the expected profitability of the typical entrant would be closely related to $\overline{\Pi}_{it}$. And since the average size of an entrant is a relatively constant proportion of the minimum efficient size in these cases, it is pretty certain that the average capital requirements would be closely related to C_{it}.

7/ For an elementary account of some of the factors omitted here, see [21]. For a discussion of the automobile industry, see [28]. Of course, the influence of World War II (with controls of various sorts) should not be overlooked either.

8/ The tests described above are one-tailed tests -- which are appropriate here. Of course, our primary purpose is to estimate the effects of $\overline{\Pi}_{it}$ and C_{it} on E_{it}, rather than to see if they have any effect. They almost certainly do have an effect but research to date provides little or no clue regarding its magnitude.

For a discussion of the biases due to measurement errors and least-squares, see Section 7 and note 37 in particular. If there were no bias, the likelihood that the lower bounds in the test would be exceeded would equal .85. Given the probable bias, it should be much higher.

The percentage change in the average value of E_{it}, given a doubling of $\overline{\Pi}_{it}$, is $2^{\alpha_1} - 1$. The effect of doubling C_{it} is given by substituting $-\alpha_2$ for α_1 . Of course, one could get lower bounds for the effects of a 10, 20,...percent change in Π_{it} and C_{it} in exactly the same way.

9/ Of course, the sale of a firm need not mean that it was a failure. Eq. (4) is likely to represent the scrappage or abandonment rate better than the rate at which firms are sold. But the former is likely to be a large part of the total and hence eq. (4) is likely to represent R_{it} fairly well. The data are such that firms that scrapped their plant cannot be separated from those that sold out. See the Appendix.

10/ The log-normal distribution seems to provide a reasonably good (but by no means perfect) fit to the distribution of firms by size. See [12]. The Appendix gives the units in which a firm's size is measured.

11/ The sources of the data on R_{it} for the steel, petroleum and tire industries are discussed in the Appendix. For the automobile industry, they came from Automotive Industries. The estimates of V_{it} were obtained from the frequency distributions in the Appendix and from Moody's figures on assets of automobile firms.

To estimate the ratio of the average size of firm to the minimum efficient size, we divided Bain's estimate [4] of the minimum efficient size in each industry into the average size of firm in 1947 (petroleum), 1945 (steel and tires), or 1949 (autos). The estimates of the minimum efficient size were 1,000,000 net tons of capacity (steel), 120,000 barrels of capacity (petroleum), 1-1/2 percent of total employment (tires), and 10 percent of total production (autos). The average size of firm in each case came from the Appendix and Automotive Industries. Then we assumed that this ratio was constant over time. The crudeness of these estimates should be obvious. Note too that the minimum efficient size for the production of specialty items and in certain locations may be less than this. For further comments, see note 22.

12/ The residuals reflect the effects of various important variables that are omitted -- the extent to which the plants in the industry can be adapted for other uses, the adaptability and mobility of the management and the work force, the liquidity of the firms, the durability of their equipment, the rate at which costs rise when firms are less than minimum efficient size, etc.

13/ The procedure used to obtain those figures is just like that described in note 8. Again, our primary purpose is to estimate the effects of the exogenous variables, not to test whether they have any effect. Almost certainly, they have some effect on R_{it} .

14/ Of course, this presumes that the long-run average cost curve is J-shaped and consequently that a minimum efficient size of firm exists in the new industry. For another technique that is somewhat similar in spirit, see Stigler [26].

15/ It would be preferable to combine eqs. (2) and (6), obtain D_{it} as a function of Π_{it} , C_{it} , etc., and estimate the α's and ν's all at once. But the difference between eqs. (2) and (6) is awkward to work with. Perhaps some other form of eqs. (2) and (6) that is more convenient in this respect will fit as well.

Although the results help to indicate the effects of changes in Π_{it} , C_{it} , etc. on the average value of D_{it} , the combination of eqs. (3) and (7) may be only moderately useful in forecasting D_{it} -- because of the effects of the variables cited in notes 7 and 12 and because of the difficulty in forecasting Π_{it} . In the twelve cases for which we have data the correlation between the actual and forecasted values of D_{it} is .60.

16/ For a discussion of the use of Gibrat's law in explaining the size distribution of firms, see [11, 12, 24]. For previous tests, see also [10, 14].

17/ Any of the standard statistical texts describes tests of this sort under the heading of "contingency tables." The assumptions involved can be found there.

18/ The following size classes were used in these tests. In steel, we classified firms by their value of S_{ij}^{t} into four classes: 4,000-15, 999 tons, 16,000-63, 999 tons, 64,000-255, 999 tons, and 256,000-4,096,000 tons. In tires, we used four classes: 20-79 men, 80-159 men, 160-639 men, and 640-5119 men. And in petroleum, there were four classes: 500-999 barrels, 2,000-3,999 barrels, 8,000-15,999 barrels, and 32,000-511,999 barrels. To cut down the computations involved, only firms in these classes were included. Thus, some of the largest and smallest firms were omitted in steel and tires, and some small, medium-sized, and large firms were excluded in petroleum. But had all firms been included, the results would almost certainly have been much the same.

In all cases, the firms in a size class were divided into three groups -- those where $S_{ij}^{t+\Delta}/S_{ij}^{t}$ was less than .50, between .50 and 1.50, and 1.50 or more. Those classes were chosen so that the expected number of firms in each cell of the contingency table would be five or more. (According to a well-known rule of thumb, the expected number in each cell should be this large.) This did not always turn out to be the case, but further work showed that the results would stand up if cells were combined.

19/ The way mergers are handled here (see the Appendix) may help to produce an inverse relationship between a firm's size and its probability of death. But this alone cannot account for this result. Such a relationship has often been noted before. E.g., see [1, 20].

20/ With the following exceptions, the classifications in note 18 were used in these tests too. In steel and tires, the two smallest size classes were combined. In some cases, firms were classified into groups where $S_{ij}^{t+\Delta}/S_{ij}^{t}$ was less than

1.00, between 1.00 and 2.00, and 2.00 or more. These changes were made to meet the rule of thumb in note 18. Despite these changes, the expected number of firms in some cells was not quite five, but the results would not be affected if some cells were combined.

21/ These results differ in part from those of [12, 24, 10, 14]. The latter conclude that there was no tendency for the smaller firms to grow more rapidly than the large ones. But this was due to the fact that they included only very large firms. With regard to the larger variation in growth rates among smaller firms, our findings agree with those in [10, 14], but differ from those in [24]. There is no treatment of this in [12].

All firms that survived during the period are included in these regressions. Note that all crude capacity -- domestic and foreign -- is included for each firm in the petroleum industry. The data on foreign capacity had to be obtained from the individual firms.

In one case in steel, the slope is significantly less than one but this does not show up in the χ^2 test -- largely because of the incomplete coverage in the latter. See note 18. One-tailed F tests are used to determine whether the variances differ. In several cases, the variances differ significantly, but it does not show up in the χ^2 tests.

22/ The χ^2 tests had to be abandoned here because of the small number of firms. Firms with more than 64,000 barrels of capacity (petroleum), 1,000,000 net tons of capacity (steel) or 0.8 percent of total employment (tires) were included in the regression. The number included in each case is shown in the last row of Table 2. The fact that none of the slopes differs significantly from one indicates that there is no evidence that among these firms the average growth rate depended on a firm's initial size.

In the variance ratio tests we divided these firms into two size (S_{ij}^t) groups, the dividing line being 150,000 barrels of capacity (petroleum), 3,000,000 tons of capacity (steel), and 30,000 employment (tires). Then F tests were used to determine whether the variances of $S_{ij}^{t+\Delta}/S_{ij}$ differed. This test is not too robust with regard to departures from normality, but it should perform reasonably well here.

Note that in petroleum and tires we include firms that are more than one-half of the minimum sizes given in note 11. According to Bain [4], the cost curve is quite flat back to one-half of those sizes. Thus, it seemed acceptable to include the additional firms and to increase the power of the tests in this way.

23/ Note that the inverse relationship between S_{ij}^t and the average growth rate shows up only when all firms are included. There is no evidence of this among firms exceeding the minimum efficient size. The inverse relationship between S_{ij}^t and the variability of growth rates shows up in both cases.

24/ The growth rate of a large firm can be viewed as the mean of the growth rates of its smaller components. This point has also been made in [14].

Note too, that in the last sentence of the previous paragraph "the average for those that left the industry" is the average growth rate they would have experienced if they had not left.

25/ If the conditions set forth in this paragraph and the preceding one hold, one can show that

$$\sigma_S^2(S) = \mu^2 \left\{ \sigma_t^2(S) - \frac{P(S)\ K^2}{[1 - P(S)]^2} \right\}, \qquad (1')$$

where $\sigma_S^2(S)$ is the variance of the growth rates of the survivors (originally of size S_{ij}^t), $\sigma_t^2(S)$ is the variance of the growth rates of all firms (originally of size S_{ij}^t), that would have resulted if all had survived, K is the difference between the average growth rate of all firms (if none had left) and the average growth rate that would have been experienced by those leaving the industry, $P(S)$ is the probability of death for firms initially of size S_{ij}^t and μ is the ratio of a survivor's actual growth rate to what it would have been if all had survived. This assumes that if they had not left, the firms originally of size S_{ij}^t that left the industry would have had growth rates with a variance of $\sigma_t^2(S)$.

In addition, one can show that the average growth rate of the survivors originally of size S_{ij}^t equals

$$\overline{S}(S) = M \left\{ \overline{t} + \frac{P(S)\ K}{1 - P(S)} \right\}, \qquad (2')$$

where $\overline{t}$ is the average growth rate for all firms of size S_{ij}^t if none had left the industry. Note that we, in the same spirit as [24], are stipulating only that $\overline{t}$ will be constant above some minimum size.

According to the reasoning in the text, $P(S)$ and $\sigma_t^2(S)$ are inversely related to S_{ij}^t . Thus, from eq. (1'), it follows that $\sigma_s^2(S)$ will be inversely related to S_{ij}^t so long as $P(S) > 1/3$ or $d\sigma_t^2(S)/dS_{ij}^t$ is large in absolute terms relative to $\frac{d\ P(S)}{dS_{ij}^t}$. And from eq. (2') it follows that $\bar{S}(S)$ will always be inversely related to S_{ij}^t . Thus, as stated in the text, one would often expect the observed departures from Gibrat's law to occur. Of course, there may be many other explanations for this too.

26/ Note four things. (1) We are not comparing innovators with non-innovators since some of the "other firms" may have been unsuccessful innovators. Because we can only include successful innovation (the data being what they are), it is not surprising that they have higher growth rates, and we are much more interested in the size of the difference than in its existence. (2) Some of the innovators introduced more than one innovation during the period. Thus, the difference in growth rates is not due entirely to a single innovation. But in the subsequent analysis (involving $\bar{d}$) only cases involving a single innovation are included. (3) It would be interesting to see how an innovation's effects depended on its character, but we have too little data to attempt this.

(4) The average annual growth rate of the "other firms" in Table 3 was computed as follows. If the innovator was smaller than the sizes given in the second sentence in note 22, we used the following technique to estimate the average annual growth rate of the other firms of its initial size. We assumed that, for the j^{th} "other firm" in the i^{th} industry, $\ln S_{ij}^{t+\Delta} = a_i + b_i \ln S_{ij}^t + Z_{ij}''$, where Z_{ij}'' is a random error term. An equation of this form fits the data for the smaller firms quite well. We then obtained least-squares estimates (shown in Table 3) of a_i and b_i ; and taking each innovator, we used this regression to estimate the average value of $\ln S_{ij}^{t+\Delta}$ for the "other firms" corresponding to its value of S_{ij}^t . Deducting its value of $\ln S_{ij}^t$ from this computed average value and dividing the result by Δ , we obtain an estimate of the average annual growth rate of "other firms" of the same original size as this innovator.

If the innovator was larger than the sizes given in note 22, we had to use another method because the regressions do not always fit the larger firms very well. In these cases, we used the average annual growth rate of the "other firms" larger than the sizes given in note 22. Finally, to obtain the figures in the second row of Table 3, we took the resulting average growth rate for the "other firms" corresponding to each innovator during the period (whether or not it was above the sizes in note 22) and averaged them.

27/ If the innovators in steel are divided into two groups -- those above 1,000,000 tons and those less than (or equal to) 1,000,000 tons at the beginning of the period -- the average difference between their growth rates and the growth rates of other comparable firms differs considerably between the groups. Among the larger firms, the average difference is generally about 0.5 points whereas it is 3-10 points among the smaller ones. Similarly, if the innovators in petroleum are divided into two groups -- those above 32,000 barrels and those less than (or equal to) 32,000 barrels at the beginning of the period -- the average difference is practically zero among the larger firms but 6-24 points among the smaller ones.

28/ Consider the k^{th} "other firm" of the same size as the j^{th} innovator at time t. If r_{1k} is its average rate of growth between time t and time $t+j$, r_{2k} is its average rate of growth between time t_j and time $t+\Delta$, and $S_{jk}^{t+\Delta}$ is its size at time $t+\Delta$,

$$\ln S_{jk}^{t+\Delta} = \ln S_j^t + r_{1k}(t_j - t) + r_{2k}(t + \Delta - t_j).$$

Thus, if r_1 is the average value of r_{1k} and r_2 is the average value of r_{2k},

$$Q_j^{t+\Delta} = \ln S_j^t + r_1(t_j - t) + r_2(t + \Delta - t_j) .$$

But by the definitions of e_j and d_j,

$$\ln S_j^{t+\Delta} = \ln S_j^t + (r_1 + e_j)(t_j - t) + (r_2 + e_j + d_j)(t + \Delta - t_j) .$$

Thus,

$$\ln S_j^{t+\Delta} - Q_j^{t+\Delta} = e_j\Delta + d_j(t + \Delta - t_j) ,$$

and eq. (11) follows.

To estimate $Q_j^{t+\Delta}$, we use the procedure described in note 26. In computing $\bar{e}$ and $\bar{d}$, innovators that introduced more than one innovation had to be excluded (except in a few cases where the innovations were all introduced at the same time). These relatively few omissions are ignored, and we act as if we had the entire population of innovators in the analysis.

Of course, the assumption that $(e_j - \bar{e})$ and $(d_j - \bar{d})$ are statistically independent of $(t_j - t)/\Delta$ is rather bold. Some bias may result if d_j is higher immediately after the introduction of an innovation. If so, $(d_j - \bar{d})$ and $1 - (t_j - t)/\Delta$ may be negatively correlated, and we would probably overestimate $\bar{e}$ and underestimate $\bar{d}$.

Where there were only a few innovators, this assumption (and the one in the previous paragraph) seemed particularly risky and we did not compute values of $\bar{e}$ and $\bar{d}$. But some preliminary work suggested that, had we done so, the results would have been much the same.

29/ For example, Alan Waterman of the National Science Foundation, in testimony before the Joint Economic Committee in 1959, cited the need for information regarding the effect of a firm's research activities on its growth rate. Of course, it would be even more useful to know the effects of successful innovation and research on a firm's profits, but this lies outside the scope of this paper. Note that the successful innovators tend to be the large spenders on research. I shall present evidence on this score in a forthcoming paper.

Note that if we had complete, year-by-year data on each firm's size, we could compute $\bar{e}$ and $\bar{d}$ without making the assumption discussed in note 28. Unfortunately, we do not have such data. Note too, that the differences in growth rates shown in Table 3 are averages over periods of 1-10 years after an innovation was introduced. Obviously, the effects of an innovation damp out as time goes on.

Finally, for the reason cited in note 28, the estimates of $\bar{d}$ may be biased downward. On the other hand, in the petroleum industry in 1947-57, $\bar{d}$ may be unduly affected by one firm and is probably too high. Note too, that the observed differences in growth rate may be due in part to other factors that are associated with a firm's willingness to innovate.

30/ If Gibrat's law held and if $W_{ij}(t, \Delta)$ were normally distributed, the amount of mobility would be solely a function of the latter's variance. For this reason, its variance has sometimes been suggested as a measure of the amount of mobility. But, since Gibrat's law generally does not hold and the normal distribution is only an approximation, the measure discussed below seems preferable.

31/ Of course, the choice of 60-70 percent is arbitrary. We could have experimented with alternative ranges, but the amount of clerical work involved would have been prohibitive. Note that this measure is based on all firms and that, for some purposes, one might be interested in the amount of mobility among the larger firms only. If so, one could easily modify the measure by including only such firms.

Note too, that this measure is based solely on firms that survived until the end of the period. If we included all firms -- regardless of whether or not they survived -- the results would depend heavily on the death rate. Because the latter was already taken up in Section 3, it seemed preferable to use this measure. If one is interested in results that include deaths, it is relatively easy to combine the findings in this Section with those in Section 3.

Let $P'_{it}(S)$ be the probability that a firm initially of size S will be smaller at the end of the period than -- or as small as -- a firm that initially was 60-70 percent of its size. If a firm dies, let its size be zero. Then

$$P'_{it}(S) = R_{it}(S) + [1 - R_{it}(S)]\ [1 - R_{it}(S')]\ P_{it}(S) \quad ,$$

where $R_{it}(S')$ is the probability that a firm of initial size, $.6S$-$.7S$ will die during the period. Since Section 3 takes up $R_{it}(S)$ -- and $R_{it}(S')$ -- and this section analyzes $P_{it}(S)$, together they provide information regarding $P'_{it}(S)$.

32/ Of course, the age of an industry is a somewhat slippery concept. In 1896, Goodyear made the first American tires for commercial vehicles. In 1859, oil production began in the United States. The production of iron began here in 1645. Using these years as estimates of the dates of birth of the industries, A_{it} was derived by subtracting them from the initial year of the t^{th} period. Although the results seem reasonable, their crudeness should be obvious.

33/ Since we assume that the distribution of firms by size is log-normal, the obvious measure of their inequality is the variance of the logarithms of the firm sizes. It can be shown that the latter is equal to $\ln\,(S^*_{it}/\bar{S}_{it})$. Thus, our measure -- which is convenient because it allows some terms in eq. (15) to be collected -- is a monotonic increasing function of the variance.

The chief reason for using a measure of inequality of firm sizes plus the number of firms as a measure of concentration is convenience. For some disadvantages in the use of such measures, see [2]. For some evidence that increases in concentration are associated with decreases in mobility, see [14]. Of course, one would expect this variable to be important because where markets are highly concentrated it is more likely that discipline can and will be maintained to see to it that firms remain in about the same relative positions.

34/ The expression in eq. (15) is not a very obvious one. E.g., it is not obvious (at least to me) that $\ln P_{it}$ should be a linear function of $\ln(1 + V^2_{it})$. Consequently, it is all the more satisfying that this function turns out to be such a good representation of the data.

There is some evidence that the variability of firm growth rates increases with the industry's over-all rate of growth [14]. Of course, A_{it} and the industry's growth rate are liable to be related in general. But in the cases used here, there is no correlation between the two variables. Consequently, the observed effects of A_{it} are not mere reflections of the effects of the industry's growth rate.

Another factor that may be important is the extent to which the smaller firms tend to be the innovators. In addition, Hart and Prais [12] provide some evidence that there is more mobility during depressions, but this may be due to the differences among industries in the extent to which sales fall during a recession. Our data -- which do not lump all industries together -- do not show any obvious signs of such a tendency.

Note too, that in deriving eq. (15), we presume that $P_{it}(S)$ exists for all S. But in some cases there are no firms 60-70 percent as large as another firm. Thus, strictly speaking, we should use the size distribution of firms where it exists.

35/ Note too that the residuals in these equations -- e.g., Z_{it} -- may not be entirely independent because of the effects of factors that persist in a given industry.

36/ Of course, the exclusion of important factors can create biases of various sorts. E.g., it is possible that C_{it} is correlated with other barriers to entry and that consequently its effects on E_{it} are overstated.

37/ For an elementary discussion of least-squares bias of this sort, see [5]. To evaluate the bias, we first formed a complete system of equations by adding to eqs. (3) and (7) a third equation in which $\ln \Pi_{it}$ is represented as a linear function of $\ln E_{it}$, $\ln R_{it}$, two unspecified exogenous variables, and an error term.

(Of course, the use of two exogenous varibles is arbitrary.) As noted in the text, the coefficient of $\ln E_{it}$ in the third equation is assumed to be negative and the coefficient of $\ln R_{it}$ is assumed to be positive. Next, we assumed that the residuals in the equations were uncorrelated (which is consistent with the data we have), and we assumed for simplicity that the covariances of the exogenous variables were all zero.

Under these conditions we found that the asymptotic bias in the estimates of the α's and ν's was always towards zero and that it was likely to be small (percentage-wise) for the estimates of α_2 and ν_2. Of course, this only holds in the limit and our assumptions are obviously rough. When better and more extensive data become available, it should be worthwhile to use limited information or two-stage least squares estimates. It did not seem worthwhile here.

38/ The average growth rate in the first row of Table 3 is approximately double the average growth rate in the second row.

39/ Judging by our results and the transition matrices in the Appendix, the assumption -- sometimes made -- that the transition probabilities are constant over time is likely to be a poor one. E.g., the extent of "mixing" [24] seems to decrease with time.