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Forced Saving in a Keynesian Economy:

An Analysis of Demand-Pull Inflation

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A deterioration in the international situation necessitates a substantial increase in defense spending. An underdeveloped country attempts to finance economic growth by creating money, by forced saving. What are the effects of such actions upon the price level, the rate of interest, private investment spending, and consumption? John Maynard Keynes concerned himself primarily with the tragedy of unemployment when he wrote the General Theory [13]; he did not concentrate attention on problems of inflation. I shall argue that the Keynesian tools constitute a powerful mechanism for the analysis of problems of inflation as well as unemployment. How to Pay for the War [14] was a pamphlet, not a treatise; the model of the General Theory implies much more about the inflationary process than Keynes presented in his brief discussion of the inflationary gap.

Post Keynesian studies of inflation have generally involved elaborations of the dynamics of the inflationary gap. But such studies as those of Tjalling Koopmans [16], James Duessenberry [3], Franklyn D. Holzman [11], and J. D. Pitchford [21] have involved the mobilization of only a part of the analytical machinery presented by Keynes in the General Theory. Both the money supply and the interest rate, essential elements of the General Theory, are neglected in these dynamic studies of inflation. The models attempt to explain the value of money; but money itself does not enter into

* I am indebted to my undergraduate theory students at Yale University for emphasizing that if the Keynesian mechanism is to interest a generation untouched by the great depression of the thirties, it must be focused upon problems of post-war inflation as well as mass unemployment. At Reed College Seymour Fiekowsky first stimulated my interest in forced saving. Helpful criticism has been provided at a Yale Economics Faculty Seminar and by John Pitchford of Australia, who read an earlier draft of this paper. Remaining errors are my responsibility, of course. Research time was made available through the generosity of the Cowles Foundation.

the equations! The dynamics of the inflationary gap in itself does not provide an appropriate framework for formulating monetary policy or appraising its effectiveness.

A more complete set of weapons from the Keynesian arsenal is mobilized in the study of inflation presented in this paper. As a model of unemployment, the Keynesian system provides an explanation of the impact of taxation, changes in government spending, and adjustments in the money supply upon the level of economic activity. Why not utilize the model to explain the impact of increases in government expenditure and adjustments in the money supply upon a fully-employed economy? As a first step in this venture it is necessary to spell out the equations of a Keynesian system. Then a concept of capacity for the economy is derived. This capacity concept suggests in turn a definition of demand-pull inflation which facilitates a simplification of the original model that proves appropriate for a comparative statics analysis of monetary and fiscal policy in a fully-employed economy. The static argument is then utilized as a foundation in the construction of a dynamic model suitable for the study of forced saving. This is achieved by combining the essential elements of the studies of the dynamics of the inflationary gap with the monetary elements whose role is revealed in the static analysis. The dynamic model is utilized to appraise the implications of financing a maintained injection of real government spending by the continued creation of new money.

It may well be asked whether the Keynesian theory of employment, now twenty-five years old, is the most appropriate model for this task. Admittedly, the model of the General Theory has been subjected to considerable modification in a number of divergent directions during the quarter century that has elapsed since its appearance. But a good portion of the analysis that follows

is little more than an elucidation of certain passages concerning inflation that appear in the writings of Keynes. Much of the argument of this paper could be readily translated into the framework of any of a number of alternative macro-models of employment.

At several points in the discussion, the Keynesian theory of inflation developed here is compared with the quantity theory of money. Under special conditions, it is demonstrated, the quantity theory of money will make correct predictions even in a world satisfying the postulates of the Keynesian system. Consideration of the problem of inflation within an essentially Keynesian, dynamic framework reveals conditions in which the income velocity of money will be invariant. But it also establishes that under the Keynesian assumptions the quantity theory would not be a useful tool for policy formulation, even under these special conditions. This means that empirical studies concerning the relative stability of either income or transactions velocity cannot in themselves suffice to establish that the quantity theory is an appropriate framework for analyzing questions of policy.

1. Feasible and Infeasible Solutions of a Keynesian System

As preparation for the development of a Keynesian analysis of demand-pull inflation we must first specify a notational system and enumerate those equations of the General Theory which are fundamental to our analysis.

For ease of reference we define the relevant variables here:

Y	national income
I	investment
C	consumption
N_d	demand for labor
N_s	supply of labor
Y_f	full employment level of national income

Y_d	aggregate demand
Y_s	aggregate supply
K	stock of capital
r	the rate of interest
M^n	the nominal money supply
p	the price level
w	the money wage rate

All but the last three variables are measured in real terms.

There are three sets of equations of the Keynesian model which we now enumerate. First there are the equations underlying effective demand:

(1.1)	$M^n/p=L(r,Y)$	liquidity preference*
(1.2)	$I=I(r,Y)$	investment schedule
(1.3)	$C=C(r,Y)$	consumption function
(1.4)	$Y=I+C+G$	income defined

* Keynes breaks the demand for money function into two components, but only as a "first approximation," [13, p. 199]; the distinction between "transactions" and "idle" balances, an artificial one useful for pedagogical purposes, may be ignored. Franco Modigliani assumes that in the Keynesian system the demand for money balances is determined by the rate of interest and money income, but is independent of the absolute price level [17, pp. 46 and 71]. This implies, contrary to the above equation, that if the price level were to double while real income and the rate of interest remained unchanged, the equilibrium quantity of money, measured in real terms, might change; it implies an element of money illusion. Don Patinkin has utilized the form of the equation to be followed here [19, p. 129, passim]. At one or two points below it is indicated that the assumption is at times a crucial one.

There are two equations determining effective supply:

(1.5)	$Y_s=Y(N_d,K)$	production function
(1.6)	$\frac{w}{p} = \frac{\partial Y}{\partial N_d}$	labor demand equates the marginal productivity of labor with the real wage.

Two additional equations serve to define the capacity of the economy:

(1.7) $N_s = N_s(p, w)$ labor supply

(1.8) $Y_f = Y(N_s, K)$ full employment output

This system of equations is most conveniently reviewed in terms of the familiar aggregate demand and supply diagrams of Keynesian analysis. We plot income on the abscissa and the price level on the ordinate. Let us consider each of the three sets of equations in turn.

The first four of our equations involve seven variables. Of these, the nominal money supply and the level of government expenditure may be regarded as parameters. This first set of four equations in five unknowns suffices to determine the level of effective demand, Y_d , as a function of the price level and these two parameters, G and M^n .

(1.9) $Y_d = Y_d(p; G, M^n)$.

This familiar relation is plotted on the aggregate supply diagram, Figure 1.1. Conventional assumptions concerning the properties of our first four equations imply that this aggregate demand schedule should have a negative slope.

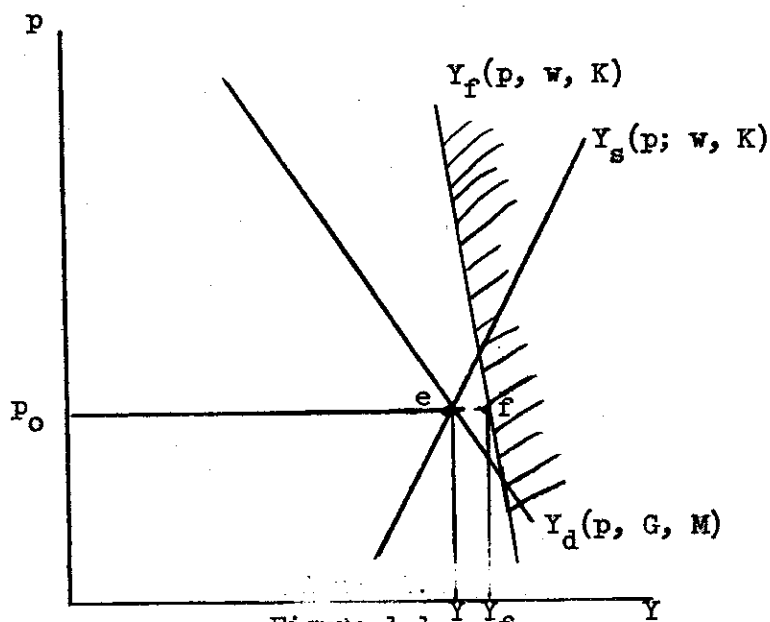


Figure 1.1

Next we derive the aggregate supply function from equations (1.5) and (1.6). We are armed with a production function involving both the available stock of capital and the quantity of labor employed. But the stock of capital, K , is regarded as fixed in Keynesian analysis. Effective supply, then, depends upon the level of employment, which in turn is given by the condition that workers are hired until their marginal productivity is equal to the real wage rate, a profit maximizing condition under pure competition. If we regard the level of money wages as a parameter, the volume of employment and, consequently, output is positively related to prices:

$$(1.10) \quad Y_s = Y_s(p; K, w).$$

This, the aggregate supply function, is also plotted on the diagram.

We specify an equilibrium condition. Aggregate supply must equal aggregate demand:

$$(1.11) \quad Y_d = Y_s .$$

This serves to determine the equilibrium income and the price level, the point e at which aggregate demand and supply curves intersect on our diagram.

There are two remaining equations of our system, (1.7) and (1.8). These equations place a feasibility restraint upon our system, a condition that must be satisfied by a valid equilibrium solution of our model. Given the quantity of capital, enough labor must be supplied to produce the equilibrium level of income. If money wages are set at the historically ruling level, the real wage will move reciprocally with the price level. Consequently, the supply of labor and hence the full employment level of income will also be inversely related to the price index. We have:

$$(1.12) \quad Y_f = Y [N_s(p, w); K] .$$

The line Y_f on figure 1.1 indicates this relation.* Our feasibility

* While we have arbitrarily made the aggregate demand curve more elastic than the full employment restraint, this is not essential for our analysis.

constraint requires that the equilibrium income be no greater than the full employment level which holds at the equilibrium level of prices, given the money wage rate:

$$(1.13) \quad Y \leq Y_f .$$

Point e on figure 1.1 satisfies this condition. The excess of full employment income Y_f over the equilibrium level Y is an index of the extent of unemployment, $N_s - N_d$.

This result is in a sense fortuitous. If G , K , M , and w are all regarded as fixed, there is no inherent reason why the level of income obtained by solving equations (1.1) through (1.6) should satisfy the feasibility condition that output be no greater than workers are willing to produce at the ruling real wage rate. After all, suppose that initially a feasible solution is obtained, as illustrated on Figure 1.1. Consider how this solution changes as one of our parameters, for instance government expenditure, is adjusted.** The aggregate demand curve shifts to the right

** Later we analyze the effects of changes in the nominal money supply, a second policy variable, as well as of G in greater detail.

as government expenditure increases.*** For a sufficiently large level of

*** The shift is not by the full amount of the multiplier obtained by taking the reciprocal of the marginal propensity to save. As income and the price level expand the rate of interest tends to rise curtailing the volume of private expenditure. This complication, also considered in greater detail later, may be seen from inspection of equations (1.1) through (1.4).

government expenditure, G_1 , we will find the situation illustrated on Figure 1.2. In this borderline case, all three curves intersect at a single point. The feasibility condition (1.13) is still satisfied. The economy operates at full employment with a price level somewhat higher than before.

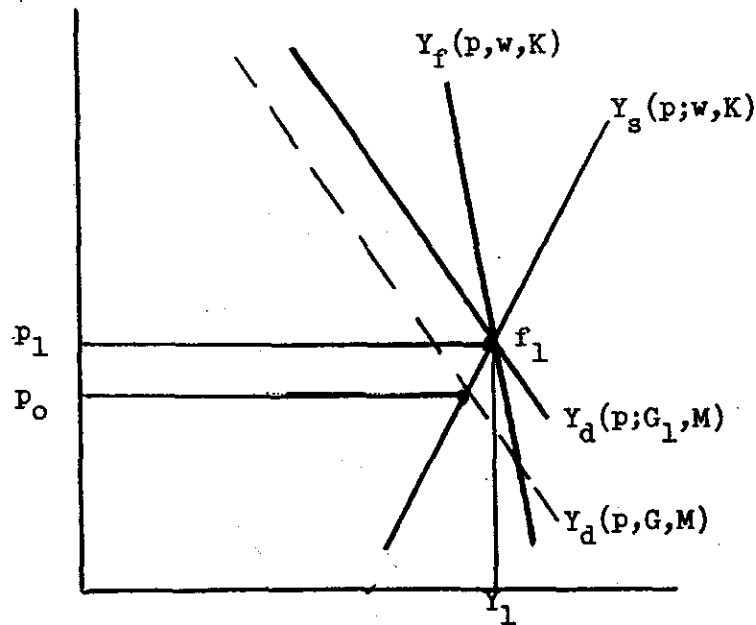


Figure 1.2

Such a "full employment equilibrium" is characterized by the condition:

$$(1.14) \quad Y_d = Y_s = Y_f .$$

If government expenditure rises to a still higher level, G_2 , we have the situation illustrated in Figure 1.3. Here the "equilibrium solution" (1.11), point e , is not feasible. The former full employment solution, f_1 , is no longer valid at this higher level of government expenditure; it implies the existence of excess demand. Since point c implies that the

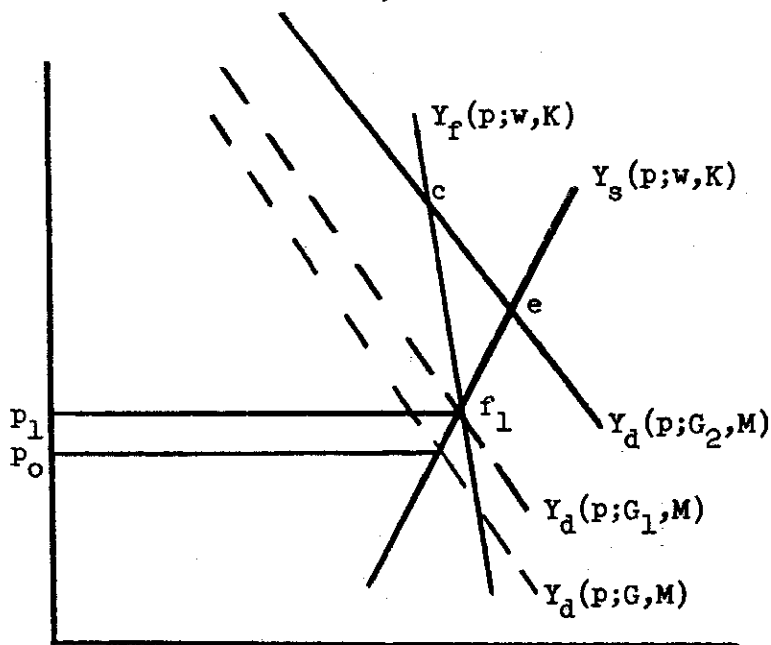


Figure 1.3

real wage is not equal to the marginal productivity of labor, it cannot be regarded as a satisfactory solution.* Clearly, we have arrived at an impasse.

* In the case illustrated, point c implies that the real wage is below the marginal productivity of labor. The direction of the inequality is reversed when the full-employment restraint is more elastic than the aggregate demand schedule.

2. Capacity Output, Money Wage Rate Adjustments, and the Definition of Demand-Pull Inflation.

Given the level of government expenditure, the money supply, the capital stock, and the ruling money-wage rate, no feasible solution of the Keynesian system necessarily exists. Such an impasse cannot be circumvented unless at least one of the parameters of the system is allowed to adjust. Now the level of government expenditure and the nominal money supply, G and M , are policy variables; if we wish to analyze the consequences of government action we had better continue to regard these as fixed parameters. The capital

stock parameter is a source of some embarrassment as the process of investment should lead to a change in its magnitude; but in a Keynesian analysis it is generally assumed that net-investment is sufficiently small in the short run relative to the size of the existing capital stock to permit the neglect of this complication. Only the money-wage rate remains for consideration. If we allow money wages to vary we may find a feasible solution to our system. We shall look for a solution corresponding to full employment. Before embarking on this quest, it will be useful to define the concept of capacity.

Given the capital stock, there exists an upper bound on the output of the economy, a capacity level of output that proves to be independent of government expenditure and the money supply. We shall see that this capacity level of output, Y_c , is necessarily attained by an economy in full-employment equilibrium. Now $Y_f(p; w, K) = Y_s(p; w, K)$ is a necessary condition for full-employment equilibrium, as defined by (1.14). Although aggregate supply and the full-employment level of income depend on money wages, at least in Keynesian analysis both may be assumed to be homogeneous of degree zero with respect to the real wage rate, w/p .^{*} The two functions

* To be precise, we have for $\lambda > 1$, $\lambda Y_f(p; w^*, K) = Y_f(\lambda p; \lambda w^*, K)$ and $Y_s(\lambda p; \lambda w, K) = \lambda Y_s(p; w, K)$, where w^* is the historically ruling wage rate.

are plotted on Figure 2.1 with income as a function of the real wage rate.

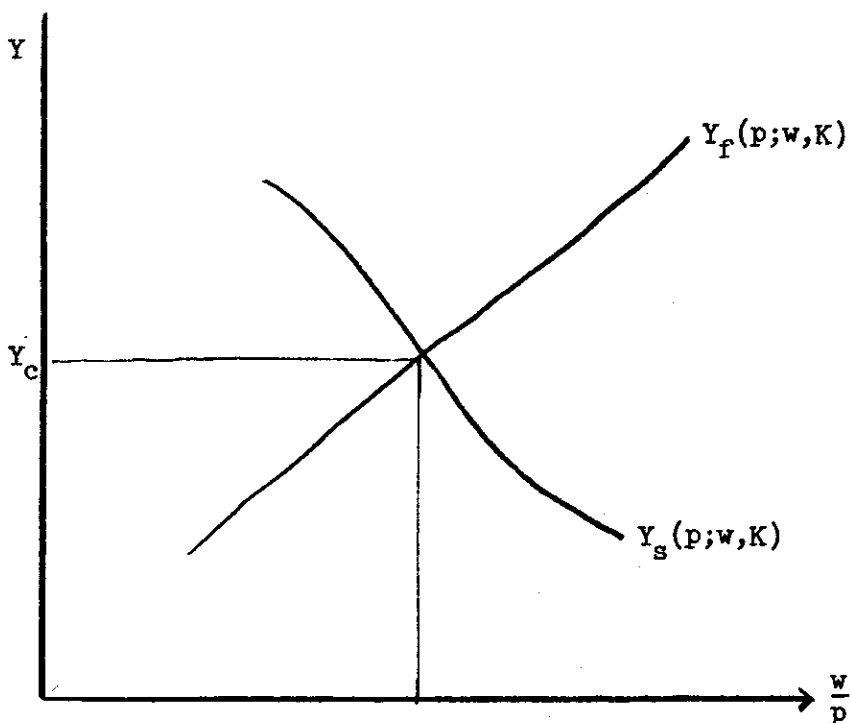


Figure 2.1

The output that can be produced with available labor is an increasing function of the real wage. Conversely, profit maximizing conditions imply that the higher the real wage rate the less entrepreneurs will be willing to produce. The intersection of these two curves determines the capacity output, Y_c . This equality condition, together with the homogeneity property, implies that full-employment equilibrium must be characterized by a particular level of income, a level that depends only on the capital parameter. In addition, a unique, full-employment equilibrium real wage is determined. Unless the real wage rate is equal to the marginal productivity of labor at capacity output, full-employment equilibrium cannot be achieved. Since government expenditure and the money supply appear only in the aggregate demand schedule, the definition of full-employment equilibrium implies that capacity income is independent of these two policy variables as well as the money-wage rate. If full-employment is to be achieved, it must be at output Y_c .

This discussion has implications for the nature of shifts in the Y_f and Y_s schedules on the type of graph we were considering earlier. In terms of Figure 2.2, there exists a whole family of Y_f and Y_s schedules relating income to the price level, one for each level of the money-wage rate. But they all intersect at the same level of income, Y_c . Point f_1 represents the old full-employment equilibrium corresponding to government expenditure G_1 , the equilibrium considered in the preceding section of this paper. Point f_2 on Figure 2.2 represents a new full-employment equilibrium corresponding to government expenditure G_2 . The level of output is the

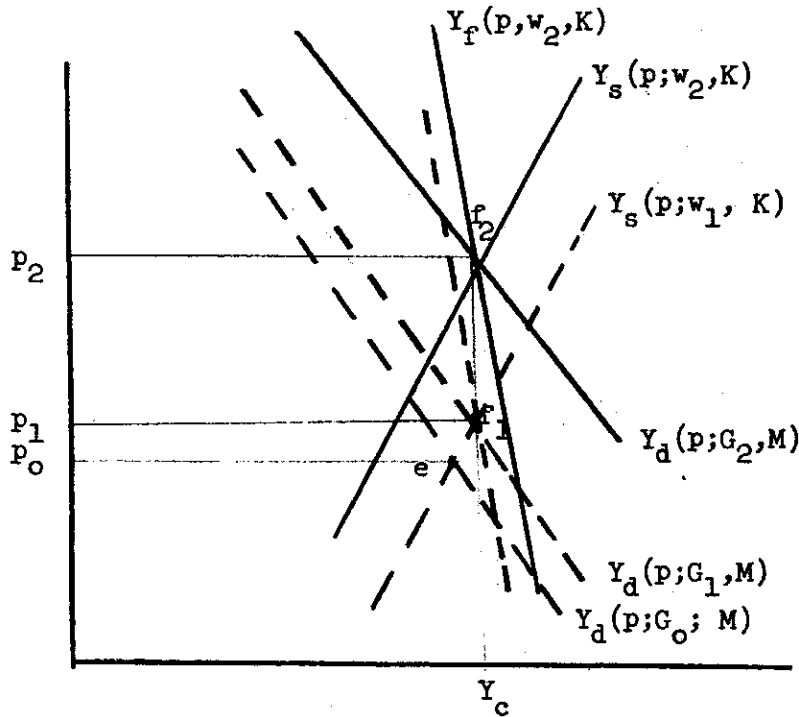


Figure 2.2

same as before. Prices have risen from p_1 to p_2 . Money-wages have risen to the level $w_2 = w_1 \left(\frac{p_2}{p_1} \right)$; i.e., the real wage rate does not change if an increase in government expenditure from G_1 to G_2 results in a new, full-employment equilibrium.

Let us consider other possible patterns of money-wage rate behavior. If money-wages had risen to a new level less than w_2 , the aggregate supply schedule and the full-employment restraint would have intersected below the aggregate demand curve and there would not have been a feasible equilibrium solution. Conversely, if money wages were pushed above w_2 , these two curves would intersect above the aggregate demand schedule; here we would have a feasible equilibrium at less than full-employment, an equilibrium similar to that illustrated on Figure 1.1. Observe too that if the new equilibrium were characterized by unemployment, money wages would have risen more than prices, and the price level in turn would have risen more than is required in order to obtain the full-employment equilibrium. Although output would be below capacity income, the real wage rate would be higher.

We are now in a position to summarize the argument by defining three essential phases of the inflationary process. First, there may be a quasi-inflationary phase that occurs when an upward shift in aggregate demand disturbs an unemployment equilibrium. Part of the increase in demand is met by expanding output, part by a rise in the general level of prices. In terms of Figure 2.2 this is represented by the increase in prices from p_0 to p_1 as a result of the upward shift in government expenditure to G_1 . Second, there is the demand-pull inflation generated when aggregate demand expands beyond the level required for full employment at the ruling money-wage rate. A new, feasible equilibrium can be achieved with government expenditure G_2 , but only if the money-wage rate moves upwards at least to the level

$w_2 = \left(\frac{p_2}{p_1} \right) w_1$. If money-wages rise by precisely this amount, we have pure demand-pull inflation. Output remains at the capacity level; the real wage rate is the same as before. Finally, an element of wage-push inflation may be present. If wages are pushed up above w_2 , prices will be shoved above p_2 .

Labor may achieve an increase in the real wage rate, but only at the expense of unemployment.*

* Note that regardless of the type of inflation the assumption of competition implies that the real wage remains equal to the marginal productivity of labor. Consequently, wage push can be present even if real wages do not rise faster than productivity.

Admittedly, the three phases of the inflationary process just enumerated cannot be regarded as in any sense a historical sequence or a description of the actual order in which events must necessarily occur in practice.

Keynes himself discussed the possibility of upward pressure on the money-wage rate before full-employment is achieved [13, p. 301]. All three elements of the process may be encountered in any inflation we have the misfortune to observe. But we shall find that for purposes of theoretical analysis the classification is not only legitimate, it facilitates concise analysis by pointing the way to appropriate simplifications.

Attention will be focused upon pure demand-pull aspects of the inflationary process in this paper. Aggregate demand and supply curve analysis has revealed certain properties of this type of inflation, properties that permit a considerable simplification of the analysis. We have demonstrated, in the first place, that the capacity level of output, Y_c , is unique.** Demand-pull inflation is a full-employment phenomenon; real

** Some writers, including Modigliani [17], introduce this as an independent assumption in their analysis, neglecting all complications relating to labor markets.

output remains at this unchanging, capacity level that is independent of government policy. In addition, concentrating attention upon pure

demand-pull inflation is equivalent to assuming that at full-employment a rise in prices must be precisely balanced by compensatory upward adjustments in the money-wage rate that leave real wages unchanged. Real wages remain equal to the marginal productivity of labor at capacity output. Constancy of the real wage rate may be regarded as a characteristic of pure demand-pull inflation. These properties will permit a considerable reduction of the eight equation Keynesian system originally considered in Part 2 above.

3. The IS-IM Framework for the Analysis of Demand-Pull Inflation

The IS-IM curve apparatus of John R. Hicks [10] provides a framework for the static analysis of demand-pull inflation. Income in real terms, Y , is measured along the horizontal axis. The rate of interest, r , appears on the ordinate.* It will be remembered that the equilibrium level of

* In static analysis where the price level is unchanging, the real and money rate of interest are identical.

income is determined by the intersection of two curves. The investment-saving or IS curve reveals the combinations of incomes and interest rates at which desired investment plus consumption would equal income; it is derived from equations (1.2) and (1.3). The liquidity-money or LM curve presents the combinations of interest rates and incomes at which the community would be willing to hold the existing nominal money supply. If the real money supply were to be kept constant, the LM curve could be derived directly from equation (1.1). But we wish the LM curve to specify the combinations of income and interest rates corresponding to equilibrium in the money market for given money wage rate and nominal money supply; equations (1.1), (1.5), and (1.6) all underlie this formulation of the IM curve construct.

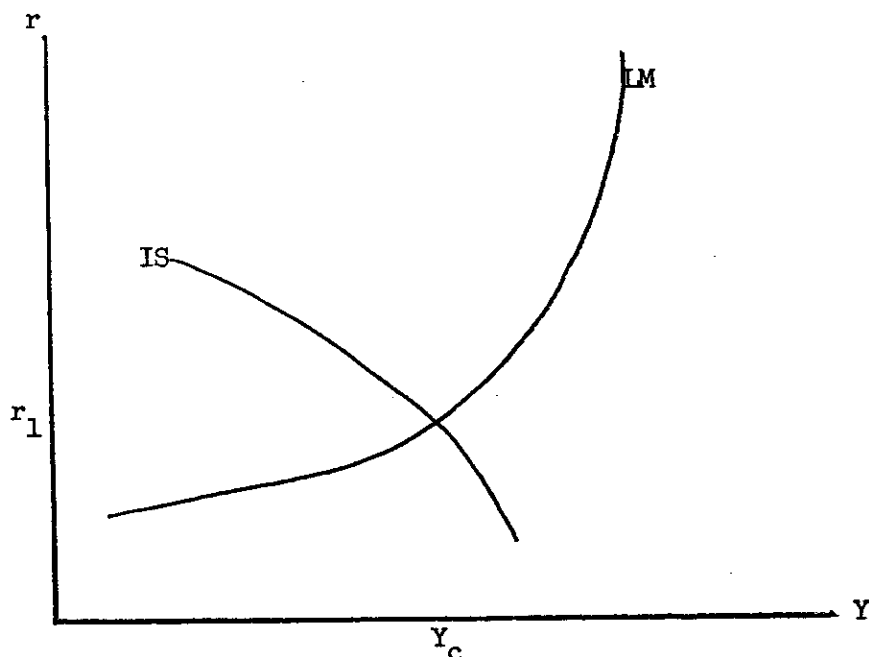


Figure 3.1

For purposes of analyzing demand-pull inflation it may be assumed that the initial equilibrium position is at the full-employment, capacity level of output, the level Y_c defined in Part 2.

Admittedly, inflation may cause structural changes in the fundamental equations underlying the Keynesian analysis. The very process of inflation might shift the consumption function, the marginal efficiency of capital schedule, and the demand for money equation. But it is appropriate to assume at the initial stage of the analysis that these relations are stable. Credence is added to this assumption by the necessity of allowing compensatory money wage increases to take place in order to preserve the constancy of real wages characteristic of pure demand-pull inflation; consequently, no wholesale redistribution of income need be involved in the inflationary process. We must also assume that investment has no significant effect upon either capacity of productivity in the short run. Secondary complications involving shifts in the fundamental relations may be readily introduced once the basic approach is outlined.

4. An Injection of Government Spending

An increase in government expenditure, ΔG , will disturb the initial full-employment equilibrium.* The IS_2 curve will shift horizontally to the

* For the time being it will be assumed that the increased government expenditure is financed by "neutral" means that do not change the money supply or affect any of the fundamental relationships. This is only an artifice adopted in order to allow things to be discussed one at a time.

right by the change in government expenditure times the multiplier, by

$\Delta G \left(\frac{1}{1-\alpha} \right)$, where α is the marginal propensity to consume. Let IS_2 be this new curve. If it were not for the capacity restraint, the level of income would expand to Y_2 (Figure 4.1); the equilibrium rate of interest would rise to r_2 , for an increased quantity of money would be desired for transactions purposes.

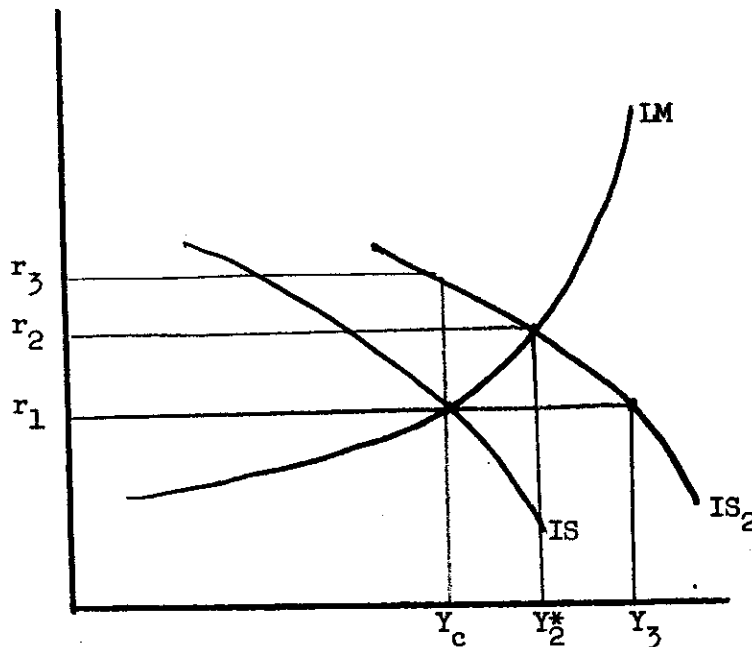


Figure 4.1

But the economy is already operating at capacity. The indicated expansion is a physical impossibility. Too much money is chasing too few goods.

If imports were to increase by ΔG , just matching the expansion in government expenditure, the shortage would be satisfied at the expense of a drain on the economy's foreign reserves. Alternatively, an expansion of capacity by $Y_2^* - Y_c$ would relieve the inflationary pressure; this required increase in capacity may be called the "inflationary gap," to utilize a phrase coined during World War II.* Unless the economy is in the liquidity

* In this section the term "inflationary gap" refers to what Walter Salant has called the "total income gap," the increase in capacity required to eliminate the inflationary pressure at the current price level, as opposed to the "saving-investment gap," the discrepancy when consumption is computed for the capacity level of output. Walter Salant presents a detailed discussion of alternative concepts of the inflationary gap [22, pp. 308-13].

trap or the marginal efficiency of capital schedule is completely inelastic, the gap is not as large as $Y_3 - Y_c$, the multiplier times the change in government expenditure; if the economy could expand, the interest rate would rise, curtailing private investment.

Suppose that the increased level of government expenditure is maintained. It might be thought that this would imply a continued expansion of the price level. Indeed, the argument of Keynes concerning the inflationary gap suggests that the inflation will continue indefinitely.** We shall examine

** Lawrence Klein also implies that the price level will rise without limit [15, Ch. 6]. Arthur Smithies has suggested a limit to the inflation generated by the gap [24]; his suggestion as well as other limits will be discussed below.

certain factors that limit the total change in the price level generated by the gap.

Observe that a rise in the rate of interest to r_3 would reduce private domestic investment just enough to offset the inflationary pressure generated by increased government expenditure. At this higher, "natural" rate of interest, consumption, private investment, and the enlarged level of government expenditure would be compatible with the full employment level of output, Y_c . But the very process of inflation will cause the interest rate to rise by the required amount. The LM curve is specified under the assumption that the real value of the money supply remains constant. With a stable price level, the LM curve will shift only with changes in the nominal money supply. With inflation, more currency is required to achieve a given physical volume of transactions; a rise in the price level tends to shift the LM curve to the left. As the LM curve shifts the rate of interest gradually rises and the size of the inflationary gap is reduced.

Prices will continue to rise until the gap is eliminated completely, until the economy is once more in equilibrium with the pre-inflation real income of Y_c , but with a rate of interest of r_3 . The process cannot stop short with a rate of interest below the natural rate; the inflationary gap would remain; further price increases would be generated. It is apparent that a ceiling exists to the change in the price level generated by a given expansion in government expenditure. Income and hence consumption in real terms will be the same after as before the inflation; but the rise in the rate of interest induced by inflation will curtail private investment spending so as to balance the increase in government expenditure.*

* It may be objected that if the marginal efficiency of capital schedule were sufficiently inelastic this might require an infinite rate of interest; perhaps negative investment would be necessary. The argument that there exists a ceiling is strengthened below where a possible dependence of consumption upon the rate of interest and the money supply is considered.

The change in the price level required to eliminate the inflationary gap cannot be read directly from the IS-IM diagram. Only the magnitude of the required interest rate change is revealed. But armed with this information, the extent of the inflation needed to restore equilibrium may be readily determined by stepping behind the IM curve. It will be assumed that the demand for real cash balances, M^r , is determined by the level of real income and the interest rate, that $M^r = L(Y,r)$. Excluding the absolute level of prices from the equation permits us to present it in the form made familiar by Alvin Hansen [8, p. 69].

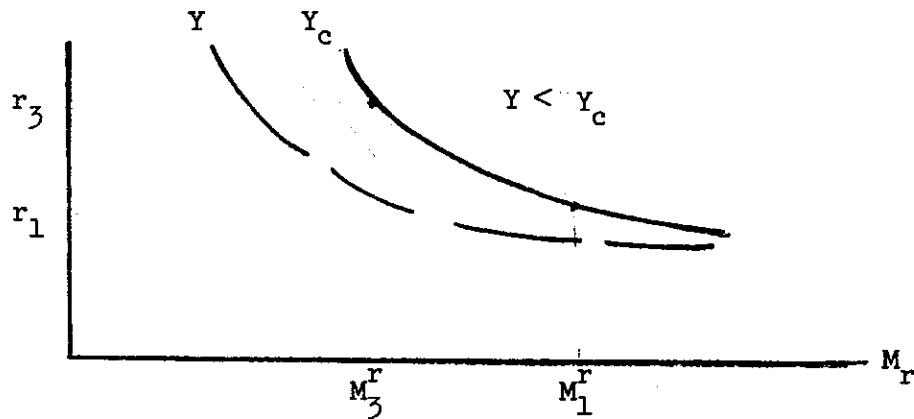


Figure 4.2

If the rate of interest is to rise from r_1 to r_3 , income remaining at Y_c , the real value of the money supply must contract from M_1^r to M_3^r ; i.e., $\frac{p_1}{p_3} M_1^r = M_3^r$. The nominal money supply is constant; this means that $M_1/M_3^r = p_3/p_1$ is the required adjustment of the price level.

It is apparent that several factors influence the magnitude of the price rise required to restore equilibrium. A more elastic marginal efficiency of capital schedule or a greater marginal propensity to consume would flatten the IS_2 curve; consequently, a smaller increase in the rate of interest, a less severe rise in the price level, would be required in order to restore equilibrium. Given the required change in the rate of interest, the extent of the inflation will be smaller the larger the elasticity of demand for money with respect to money income and the less its elasticity with respect to the rate of interest. If either the marginal efficiency of capital schedule is inelastic or the economy is in the liquidity trap, a much larger increase in the price level will be generated when an increase in government expenditure disturbs an economy initially at full-employment equilibrium.

5. Some Essential Modifications of the Static Argument:

It is not, of course, asserted that the IS-IM analysis takes into account all the complications of demand-pull inflation. It is necessary to spell out certain factors that may cause shifts in the propensity to consume, the marginal efficiency of capital schedule, and the function describing the demand for money. At this stage, certain qualifications that arise from static considerations affecting the final equilibrium position will be appraised. This necessary step of the argument must be completed before more interesting, dynamic factors can be fully appreciated.

In Hicks' original exposition of the IS-IM curve apparatus, he explained how a slight generalization of the Keynesian model was readily achieved [10, pp. 472-5]. Investment may depend upon the level of income as well as upon the rate of interest. Savings may depend upon the rate of interest as well as income. These qualifications simply affect the shape of the IS and IM curves. If private investment depends directly upon

income, the multiplier will be larger than is implied by the value of the marginal propensity to consume; consequently, a given increase of government expenditure at full employment will generate a greater expansion in the price level. To the extent that saving is positively related to the rate of interest, the IS curve is flatter than the factors considered in elementary Keynesian analysis indicate; consequently, a smaller rise in the interest rate and hence less inflation suffices to eliminate the inflationary gap. In addition, the end result under full employment of an expansion in government expenditure is a reduction in consumption spending as well as in private investment.

In an attempt to rally the classical forces against the Keynesian attack, Haberler [7, pp. 242, 491-503], Pigou [20] and others have asserted that consumption depends upon the real value of net assets held by individuals as well as the level of income. This complication, christened the "Pigou effect" by Patinkin [18], constitutes a crucial modification of the Keynesian argument concerning the ineffectiveness of flexible money wages and monetary policy as means for eliminating unemployment. How is the analysis of inflation altered by this complication of the Keynesian model? The process of inflation, by lowering the net purchasing power of assets whose value is expressed in money terms, will tend to push the IS curve to the left. Since this will help eliminate the inflationary gap, a smaller rise in the price level will be required to restore a full employment equilibrium disturbed by an increase in government expenditure.* Since the market value

* In a revision of an article originally written in 1942, Milton Friedman adds that the Pigou effect alone leads to the elimination of the inflationary gap, the fall in the real value of liquid assets caused by inflation leading to a reduction in the volume of consumption that takes place at capacity income. He fails to mention other factors, such as a rise in the interest rate, that may also serve to make the inflation itself a force tending to eliminate the gap through reducing private investment [4, pp. 253-7].

of long-term government bonds will fall as a result of a rise in the rate of interest, the Pigou effect implies that consumption spending is sensitive to the rate of interest.

A redistribution of income will affect the position of the consumption function. If lags in the adjustment of pensions, wages, and the size of dividend payments did occur as a result of the inflationary process, the consumption function, and consequently the IS curve, would shift temporarily. For the most part such effects are transitory, dynamic phenomena not affecting the final equilibrium position. But redistribution effects cannot be neglected entirely, even at the comparative statics stage of the argument; for example, the real value of dividend payments and the market value of outstanding government bonds will deteriorate as a result of inflation, affecting consumption, if the Pigou effect is operative. Such factors are not difficult to incorporate into the static analysis. Whether they all add up to a substantial effect is an empirical issue, a question that cannot be explored here.

There is another way in which the distribution of income might affect the analysis, a point made by Arthur Smithies [24, p. 119]. He has suggested that a progressive income tax might tend to eliminate the inflationary gap. Suppose progressive income tax rates are not readjusted to allow for the impact of inflation. Then the government will levy a larger and larger bite at each level of money income. As the price level rises, real disposable income and, consequently, consumption will be lower for each specified level of constant dollar GNP. Since a progressive income tax under inflation becomes equivalent to a larger tax bite, reliance upon the progressivity of the income tax is a poor substitute for direct anti-inflationary tax legislation. But it does serve to introduce a useful element of money illusion into the system.

A capital gains tax may also serve to curtail the rise in the price level generated by an injection of government spending. As the price level climbs, capital gains realized on assets whose values are not fixed in monetary terms will be partially siphoned off into the Treasury. The real value of the wealth held by stock market investors as well as those so timid as to invest only in bonds will be eaten away. If non-money assets as well as income and the rate of interest belong in the demand for money function, this may well cause the IM curve to shift to the left. Given the level of income, the public will not be willing to hold as large a real stock of money at any given rate of interest as they were before the real value of their assets was reduced by inflation.

These qualifications contribute to the realism of the argument. Collectively they add further grounds for suggesting that the rise in the price level itself will help to curtail spending, combating the inflationary pressure. Additional complications are created when the method used to finance the increased government expenditure is not assumed to be neutral. It is a simple task to consider the effect of various taxation schemes for financing the increased expenditure. But when the higher level of spending is financed by a method that enlarges the money supply, further complications must be faced. As a first step in analyzing these complications, the effect of an increase in the money supply will be considered in abstraction from changes in the level of government expenditure.

6. An Increase in the Money Supply: The Quantity Theory

Keynes stated in the General Theory that the quantity theory is indeed valid in certain special circumstances [13, P. 289].

...as effective demand increases, employment increases, though at a real wage equal to or less than the existing one, until a point comes at which there is no surplus of labor available at the then existing real wage...We have reached...a situation in which the crude quantity theory of money (interpreting 'velocity' to mean 'income-velocity') is fully satisfied; for output does not alter and prices rise in exact proportion to MV.

Now the argument of the preceding section demonstrates that the quantity theory, if defined in terms of stability of income velocity, does not necessarily hold beyond full-employment. If an increase in effective demand beyond capacity generated by an expansion of government expenditure is to be offset by an inflationary process that reduces the real value of the money supply, income velocity must rise in strict proportion to the change in the price level. But under certain other conditions the quantity theory fares better. Here we shall demonstrate that if the inflationary gap is generated by an increase in the money supply, the equilibrium income velocity of money is unaffected. But the question of whether Keynes correctly defined the conditions in which velocity would be unchanged will not be discussed here.*

* Don Patinkin has presented much of the argument of this section, but with a slightly different mechanism; he draws a very different moral, citing chapter and verse in an attempt to establish that Keynes delimited too narrow a set of conditions in which the quantity theory holds [19].

Suppose the factor disturbing the economy from its initial full employment equilibrium, Y_c , is an increase in the nominal money supply from M to $M + \Delta M$. Such an expansion will cause the LM curve to shift outwards to the right, to LM_2 , the magnitude of the shift being $V \cdot \frac{\Delta M}{P_t}$ if $V = \frac{\partial L}{\partial Y}$, the marginal income velocity, is constant.** Inspection of Figure 6.1 reveals

** Constant here means that $\frac{\partial^2 L}{\partial Y^2} = \frac{\partial^2 L}{\partial Y \partial M} = 0$.

that this will generate an inflationary gap, except under most exceptional circumstances. Unless the economy is in the liquidity trap or the IS curve is a vertical line, the marginal efficiency of capital schedule being

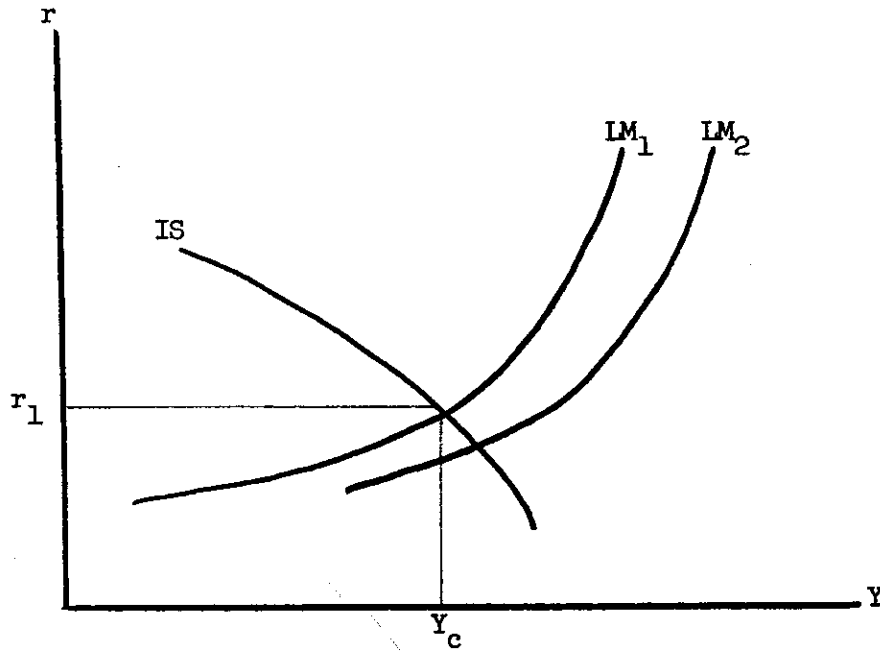


Figure 6.1

completely inelastic, inflationary pressure will be generated. The rising level of prices generated by the inflationary gap will tend to shift the LM curve to the left. In order to eliminate the inflationary pressure entirely, it must shift all the way back to IM_1 . This requires that the real money supply shrink to its former value; consequently, at the new equilibrium we will have $\frac{M}{P_1} = \frac{M + \Delta M}{P_2}$ or $\frac{P_2}{P_1} = \frac{M + \Delta M}{M}$. The change in the price level is proportional to the expansion of the nominal money supply.

The real value of the money supply is unaffected; the income velocity of money is stable. The quantity theory of money indeed holds as a special case! Even in a hypothetical economy satisfying the postulates of the Keynesian model, the quantity theory of money holds for increases in the money supply that disturb a full employment equilibrium. This suggests that

a study of countries subjected to demand-pull inflation as a result of expansions in the money supply cannot provide an appropriate basis for comparing the validity of the two theories.

The predictions of the two theories are identical only as a special case; the policy implications are not necessarily the same. The condition of repressed inflation that threatened the stability of numerous countries at the close of World War II provides an important illustration. Price controls, rationing, and other restrictions served to suppress inflationary pressure. While controls may repress inflation by forcing the IS curve downwards to the left, Keynes had argued in How to Pay for the War that moral sanctions against price increases as well as the application of direct controls to repress inflation contribute to an inefficient allocation of resources that is worse than the disease of inflation itself [14, pp. v,vi]. A drastic reduction in the nominal money supply attempted by numerous European countries hastened post World War II recovery; by shifting the LM curve to the left, such reforms can serve to reduce or eliminate the condition of repressed inflation without creating open inflation or necessitating wholesale reductions in government spending. Suppose that the monetary authorities estimate the percentage reduction in the money supply required to eliminate the inflationary gap by application of the quantity theory under the assumption that the income velocity of money will return to its prewar level. If the postulates of the Keynesian system are satisfied, the quantity theory of money would reveal the size of the cut in the money supply required to eliminate inflationary pressure only under certain conditions. Suppose the prewar situation was one of full employment equilibrium without inflationary pressure; suppose that government spending, the consumption and investment functions, the liquidity preference schedule, and capacity are

all unchanged, the inflationary gap being generated by a swollen money supply; then clearly, the quantity theory would correctly prescribe a cut in the real money supply to its prewar value. But except in such special circumstances, a fortuitous balance of forces would be required in order for the quantity theory to prescribe the required reduction in the money supply.*

* V. A. DeRidder discusses the utilization of the quantity theory of money in planning the Belgium monetary reform [2, p. 57]. For a discussion of European monetary reforms see John G. Gurley [6].

The basic argument concerning the effects of an increase in the money supply upon an economy initially at full employment equilibrium is readily modified in order to take into account such qualifications as those mentioned in the earlier analysis. At first glance it might appear that the introduction of the Pigou effect would necessitate modification of the assertion that the predictions of the quantity theory will be correct; the inflationary gap would be larger initially, for the increase in the money supply would cause the IS as well as the IM curve to shift. But the process of inflation will push both curves back towards their original position simultaneously until equilibrium is restored with the real value of the money supply at its former level. The argument must be qualified only if an element of "money illusion" is present, only if the position of either the IS or IM curves is influenced directly by the level of prices, the real value of the money supply being unchanged. Professor Smithies' progressive income tax argument makes the IS curve's position depend upon the absolute level of prices. The IM curve will depend upon the absolute price level if the demand for real speculative balances is not independent of the

price level.* Only such secondary complications lead to a discrepancy between

* Cf. p. 4 above for a discussion of such possible dependence.

the predictions of the quantity theory and the Keynesian mechanism in the particular circumstances spelled out here.

Before turning to the dynamic aspects of the inflationary process, it will be advisable to present an additional conclusion that follows immediately at this stage of the analysis. Suppose that a single, one period injection of government expenditure occurs. Suppose that this expenditure is financed at least in part by an increase in the money supply. What will be the consequence of this operation if the economy is initially at full employment? Here we must combine the results of two separate pieces of analysis. But observe that the expenditure is temporary, so that only the change in the money supply has a prolonged effect. With the exception of arguments involving money illusion, such as the progressive income tax case, nothing prevents the quantity theory from correctly prescribing the consequences of this action in the long run after the effects of the spending have worn off. While this argument is almost trivial, it does eliminate one case from later consideration. A more interesting case, which can be analyzed only with dynamic tools, concerns the long run effect of a maintained injection of government expenditure financed by continuously expanding the money supply.

7. The Dynamics of Demand-Pull Inflation: A Synthesis

The preceding arguments have been exercises in comparative statics, an approach that can predict only the final outcome of an assumed change in the parameters. The argument has informed us only of the determinants of the magnitude of the ultimate rise in the price level generated by an inflationary

gap. Static analysis cannot advise us as to the factors governing the rate at which prices will rise to the new equilibrium level or on more complicated issues of stability. Tjalling Koopmans [16], James Duessenberry [3], Franklyn D. Holzman [11], John D. Pitchford [21], and others have analyzed dynamic aspects of inflation. Here we shall show that the static arguments of earlier sections provide a useful foundation for such dynamic superstructures. The model derived by this exercise in eclecticism will be used in the next section in a study of the forced saving process.

It was appropriate to suppress the time element in Part 1 where we were enumerating equations underlying the static analysis of inflation. But if we wish to examine factors determining the speed of inflation, time must be introduced explicitly. Only one of a large number of alternative procedures which could be utilized in dynamizing the static Keynesian model for the study of inflation will be considered. Once the basic approach is outlined, it will be apparent that alternative procedures for introducing time could be analyzed in essentially the same way.

We begin by adopting an assumption frequently invoked in dynamizing the Keynesian model. We assume that current levels of consumption and investment spending are determined on the basis of yesterday's interest rate and income. But in addition, we assume that the number of dollars allocated to these functions is decided on the assumption that current prices as well as the other determinants will remain at yesterday's level. In other words, the number of dollars allocated to consumption spending is given by the following modification of (1.3):

$$(7.1) \quad C_t^m = p_{t-1} C(r_{t-1}, Y_{t-1}),$$

where the superscript m indicates that consumption is being measured in

current dollars rather than deflated. Consequently, real consumption expenditure is given by the equation:

$$(7.2) \quad C_t = \frac{P_{t-1}}{P_t} C(r_{t-1}, Y_{t-1}) .$$

We may derive in the same way a similar expression for investment spending:

$$(7.3) \quad I_t = \frac{P_{t-1}}{P_t} I(r_{t-1}, Y_{t-1}) .$$

If we wish to assume that the government also makes naive assumptions about the behavior of prices in appropriating funds, we have for real government expenditure at time t :

$$(7.4) \quad G_t = \frac{P_{t-1}}{P_t} G .$$

It might be objected that these last three equations all embody an element of "money illusion." Of course, but it is not the type of money illusion usually considered in static analysis. Indeed, if the price level, the rate of interest, and income are all unchanging, if static conditions are assumed, equations (7.2) and (7.3) reduce to their static, illusion free counterparts considered in Part 1. The illusion with which we are concerned is the consequence of the rate at which prices change, not the current price level. If the existence of such illusion is denied, a lagless system with prices always in the equilibrium defined in earlier sections is implied. If the lags did not exist, dynamic analysis would not be required.

As a first step in the study of the dynamic implications of the assumptions embodied in equations (7.2) through (7.4) concerning the way in which a changing price level affects real consumption and other components of total spending, let us explain what happens in the current period on the basis of known past events. It will be convenient to denote by D_t , effective

demand at time t , the sum of actual consumption, investment, and government spending:

$$(7.5) \quad D_t = C_t + I_t + G_t .$$

Since we are analyzing demand-pull inflation, the level of income may be regarded as fixed at Y_c . Consequently, equations (7.2), (7.3) and (7.4) imply that the level of effective demand depends only on lagged interest, the level of government expenditure, and the price ratio term:

$$(7.6) \quad D_t = D\left(\frac{p_t}{p_{t-1}}, r_{t-1}, G\right) .$$

It is apparent that the assumptions made concerning the influence of rising prices on consumption, investment, and government expenditure imply that equation (7.6) is homogeneous of degree one with respect to the price ratio; we have the property:

$$(7.7) \quad D\left(\frac{p_t}{p_{t-1}}, r_{t-1}, G\right) = \frac{p_{t-1}}{p_t} D(0, r_{t-1}, G) .$$

Now we suppose that the price level must rise rapidly enough within each period to equate effective demand with capacity income. This supposition implies:

$$Y_c = D\left(\frac{p_{t-1}}{p_t}, r_{t-1}, G\right) = \frac{p_{t-1}}{p_t} D(0, r_{t-1}, G) .$$

Let us call $D(0, r_{t-1}, G) - Y_c$ the "savings-investment inflationary gap."*

* This, the most frequently employed concept of the inflationary gap, is not to be confused with the total income gap discussed in Part 4 of this paper. Salant has discussed these and other concepts of the gap [22].

Equation (7.7) implies the following relation:

$$(7.8) \quad \frac{P_t - P_{t-1}}{P_t} = \frac{D(0, r_{t-1}, G) - Y_c}{Y_c} .$$

The speed of inflation is equal to the ratio of the savings-investment inflationary gap to capacity income.

This conclusion is, of course, sensitive to the particular assumptions made concerning the impact of inflation upon the various elements of effective demand. For example, if real government expenditure had been preserved at the level G rather than being reduced by the rising price level, prices would have risen more rapidly. If the price rise had been partially anticipated, the rate of price increase would have been more rapid than equation (7.8) implies. Tjalling Koopmans [16] has analyzed in considerable detail the speed of inflation under a variety of alternative assumptions concerning the effects of changing prices upon effective demand. In addition, he has provided a clever graphical technique useful for expository purposes. We measure income in real terms on the ordinate and the speed of inflation on the abscissa.

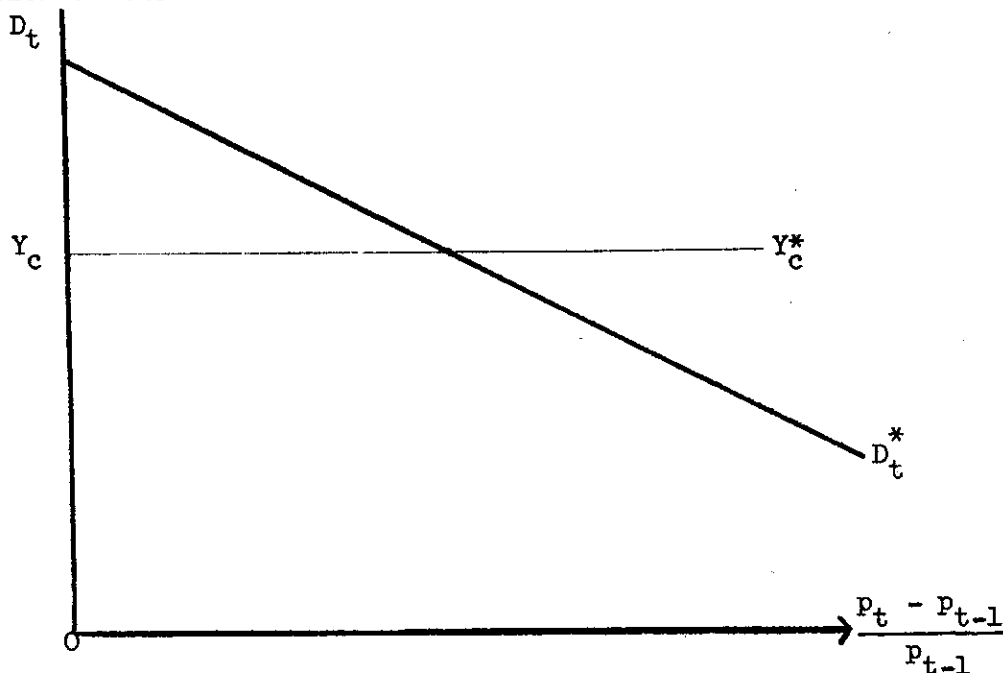


Figure 7.1

The curve $D_t D_t^*$ represents effective demand at time t as a function of the price level. Its intersection with the capacity line $Y_c Y_c^*$ reveals the speed of inflation at time t . Observe that the more sensitive effective demand is to rising prices, the steeper the line $D_t D_t^*$, the less rapid the inflation generated by any given savings-investment inflationary gap. The speed of inflation is seen to depend upon the length of a number of lags.

If inflation contributes to an inefficient allocation of resources, the total supply curve as well as the total demand schedule will slope downwards to the right, increasing the speed of inflation. When the total supply curve was horizontal, when output was independent of the speed of inflation, rising prices might have had undesired effects upon the distribution of wealth and income; inflation might have distorted the choice between current consumption versus investment for future use, lowering the rate of economic expansion. But when the supply curve is downward sloping, inflation causes an actual reduction in physical output. There is a diminution in the size of today's pie as well as a change in the relative magnitudes of individual servings.*

* It is conceivable for the capacity level of output to be an increasing function of the speed of inflation. If workers are misled by rising prices into thinking that real wages are higher than is actually the case, they may be induced to provide more labor services than they otherwise would at any given real wage. Again, if employers are misled by errors of accounting connected with inflation into confusing paper windfall profits with the actual gains of business, workers may in general be paid more than their marginal productivity.

This inflationary gap analysis is not complete. It must be recognized that the equilibrium determined by this procedure holds only for the current time period. The assumptions advanced concerning the inflationary process do not imply that the events of tomorrow are a simple repetition of today's.

Does not the speed of inflation determined by condition (7.7) depend upon last period's rate of interest? Unless the government takes steps to expand the nominal money supply, the rising price level will reduce the real value of liquid assets, thus tending to raise the rate of interest. This will curtail private investment and possibly consumption spending as well. But this means a contraction in the size of the inflationary gap and a consequent reduction in the speed of inflation.

In order to understand the details of this process it is necessary to return to a further consideration of the liquidity preference schedule, equation (1.1). We shall not bother to insert lags into the equation, but shall assume simply:

$$(7.9) \quad \frac{M_t^n}{P_t} = L(r_t, Y_t).$$

Now under conditions of demand-pull inflation, real income remains constant at a capacity ceiling Y_c , a level which may, as a first approximation at least, be assumed constant even under dynamic conditions. In terms of Figure 4.2, we are interested only in the Y_c contour line. We have a precise relation between the rate of interest and the real value of the money supply;*

* The rate of interest determined by liquidity preference is the nominal, not the real rate. If one can receive \$130 back on a loan of \$100, the nominal rate of interest is 30 per cent. This is the opportunity cost of holding money that must be equated with the marginal gains from liquidity. Keynes discusses the tax of inflation upon the holders of liquid assets in the Tract [12, pp. 48-9]. Investors, on the other hand, are concerned with the real rate of interest; a 5 per cent real return on investment may be sufficient to induce investment even if the nominal rate of interest is 30 per cent, provided that one expects the market value of physical assets to rise by 25 per cent. The ex ante real rate of interest, the nominal rate of interest minus the anticipated speed of inflation, belongs in the investment function. The rates are identical in comparative statics; the distinction can be neglected in dynamic analysis if static anticipations are assumed, if it is assumed that investors in real assets fail to recognize that prices are rising. In the exposition of our argument, it is assumed that investors in real assets fail to appreciate that prices are rising. But this is not essential, and the argument is not seriously modified even if it is assumed that investors correctly anticipate the speed of inflation.

let us write it as;

$$(7.10) \quad r_t = R_c \left(\frac{M_t^n}{P_t} \right).$$

We are now prepared to substitute expression (7.10) into our investment and consumption equations in order to obtain a modification of the expression explaining the speed of inflation:

$$(7.11) \quad \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{C \left[R_c \left(\frac{M_{t-1}^n}{P_{t-1}} \right), Y_c \right] + I \left[R_c \left(\frac{M_{t-1}^n}{P_{t-1}} \right), Y_c \right] + G - Y_c}{Y_c}$$

A rising price level, by reducing the real value of the money supply, $\frac{M_t^n}{P_t}$, tends to raise the rate of interest; this in turn curtails the sum of investment plus consumption spending. The inflationary gap is a decreasing function of the absolute level of prices. Equation (7.11) reveals the dependence of the speed of inflation upon the level of prices. In terms of the diagram developed by Koopmans, the $D_t - D_t^*$ line shifts downwards through time as prices rise to successively higher levels.

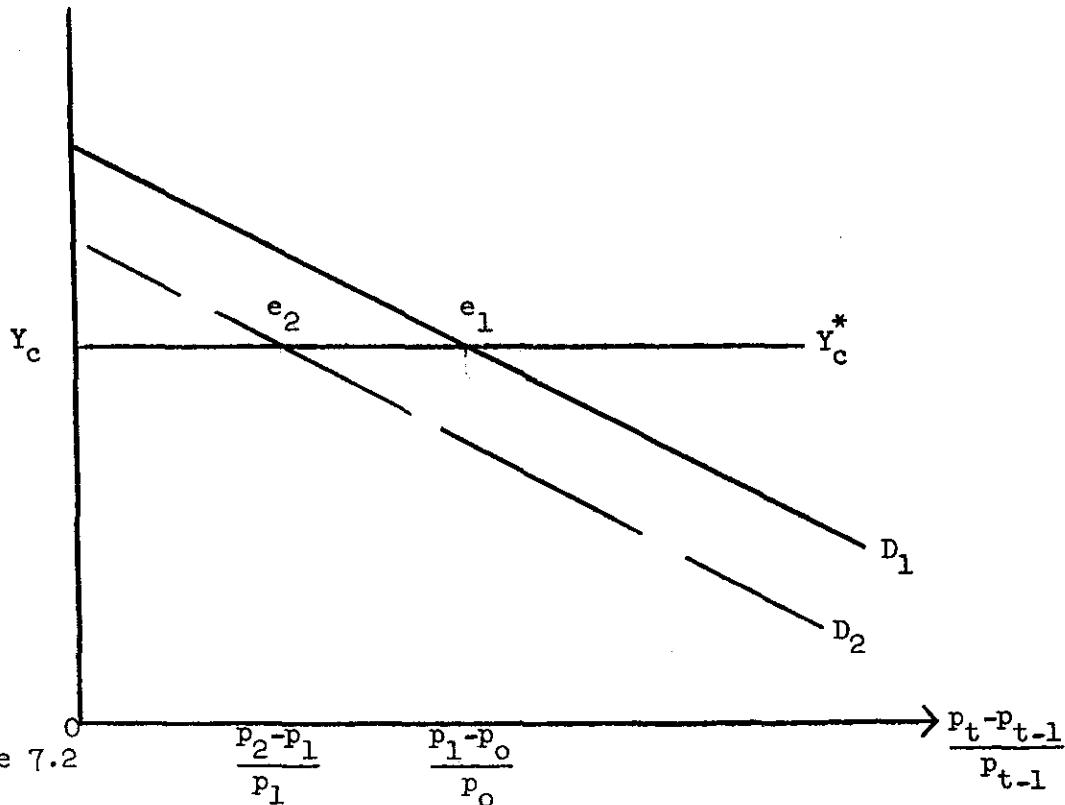


Figure 7.2

The very process of inflation tends to cure the disease of rising prices.

Only under special conditions may these monetary complications be avoided. If the economy were to remain in the liquidity trap, of course, the fall in the real value of the money supply would not affect the rate of interest. Again, if investment and consumption spending were both independent of the rate of interest, the speed of inflation would be unabated. Finally, if government actions preserved the real value of the money supply, the rate of interest and the speed of inflation would again be stabilized and prices would rise indefinitely. The last possibility, the exception of greatest interest, will be explored in the next section of this paper.

8. Forced Saving Equilibrium:

A richer if more complicated analysis is permitted by the apparatus that has been developed. There exists a new equilibrium price level, we have seen, if a continued injection of government expenditure that disturbs a stable full-employment situation is financed in a "neutral" fashion that does not alter the money supply. The case of a temporary injection financed by an increase in the money supply has also been discussed. But suppose that the injection of government expenditure is not only made each period; suppose it is financed, at least in part, by a continuous expansion of the nominal money supply. This, the most interesting case of "forced saving," is to be examined. Forced saving is an intriguing doctrine of long history. Hayek [9] traces the concept to John Stuart Mill, even Bentham. But we shall analyze the phenomenon within the framework of a dynamic, Keynesian model of inflation. It will be shown that in a certain sense the quantity theory of money still holds as a description of equilibrium, but not in a way that is useful for policy formulation.

We shall suppose that a fully employed economy is disturbed by an increase in the real value of government expenditure to a new level G . In addition, we suppose that government expenditure is to be financed, at least in part, by an expansion of the money supply. Either directly, through the printing of money, or indirectly, through an enlargement of the reserves of the banking system, $p_t \gamma G$ new dollars are pumped into the money supply each period.

With this forced saving policy, the real value of the money supply is influenced by the extent to which the injection of new money is offset in each period by depreciation of the existing money stock as a result of inflation. If M_{t-1}^r was the real value of the money supply in period $t-1$, its nominal value must have been $p_{t-1} M_{t-1}^r$. Consequently, the purchasing power at time t of currency already in circulation will be $\left(\frac{p_{t-1}}{p_t}\right) M_{t-1}^r$. Adding to this the new injection, we obtain an expression for the total real money supply at time period t :

$$(8.1) \quad M_t^r = \left(\frac{p_{t-1}}{p_t}\right) M_{t-1}^r + \gamma G .$$

Subtracting M_{t-1}^r from both sides of the equation reveals the change in the real value of the money supply:

$$(8.2) \quad \Delta M_t^r = M_t^r - M_{t-1}^r = \gamma G - \dot{p}_t M_{t-1}^r , \text{ where } \dot{p}_t = \frac{p_t - p_{t-1}}{p_t} .$$

Depending upon the rate of price change and the extent of the injection of new money made each period, this expression may be either positive or negative.*

* If we wish to measure time in continuous rather than discrete units, we have $M(t) = p(t) M^r(t)$ by definition as well as $\frac{\partial M(t)}{\partial t} = p(t) \gamma G$ by assumption. If we differentiate the first expression and equate it with the second we obtain: $\frac{\partial M(t)}{\partial t} = \frac{\partial p}{\partial t} \cdot M^r(t) + p(t) \frac{\partial M^r(t)}{\partial t} = p(t) \gamma G$. This implies: $\frac{\partial M^r(t)}{\partial t} = \gamma G - \dot{p}(t) M^r(t)$, where $\dot{p}(t) = \frac{dp}{dt} / p(t)$.

It is conceivable that the injection of new money may be precisely offset by inflation, leaving the real value of the money supply unchanged. We shall see that this is indeed a characteristic of equilibrium.

Equation (8.2) does not in itself suffice to determine the behavior of the real value of the money supply. The path of the price index must also be specified. In the preceding section we analyzed the dynamics of price behavior. Equation (7.11) summarized that argument. But there is no need to restrict ourselves to that particular formulation. For the discussion that follows we need only assume:

$$(8.3) \quad \dot{p}_t = S(M_{t-1}^r, G)$$

$$\text{where } \frac{\partial S}{\partial M_{t-1}^r} \geq 0 \text{ and } \frac{\partial S}{\partial G} > 0 .$$

Equation (7.11) is but a particular example of this more general assumption relating the speed of inflation to last period's real money supply and the level of government spending.*

* If time is continuous rather than discrete, only the details rather than the essential points of the analysis that follows are affected.

Equations (8.2) and (8.3) together with knowledge of initial conditions and policy parameters G and γ suffice to determine the path of prices and the money supply through time. This might be carried out in an iterative fashion, utilizing today's value of the real money supply and the speed of inflation to determine the events of tomorrow. But it is possible to specify certain general characteristics of solutions that would be obtained in this way, even though the difference equations are non-linear. Let us proceed to analyze the behavior of prices under our conditions of forced saving.

Consider once more the relationship between the rate of price change and the real money supply. From equation (8.2) we can determine for any given level of the policy variables γG a relationship that must be satisfied if the real value of the money supply is to remain stable. If $\Delta M_t^r = 0$, we have:

$$(8.4) \quad \dot{p}_t M_{t-1}^r = \gamma G .$$

This relation between \dot{p}_t and M_{t-1}^r is represented by the rectangular hyperbola on Figure 8.1. Combinations of M_{t-1}^r and \dot{p}_t below the rectangular hyperbola imply an expanding money supply. Conversely, if yesterday's money supply and today's speed of inflation are represented by a point above the hyperbola, the money supply is contracting. The curve summarizes, then, the implications for the money supply of the specified forced saving policy. For higher values of γG the hyperbola would be shifted outwards in the northeast direction.

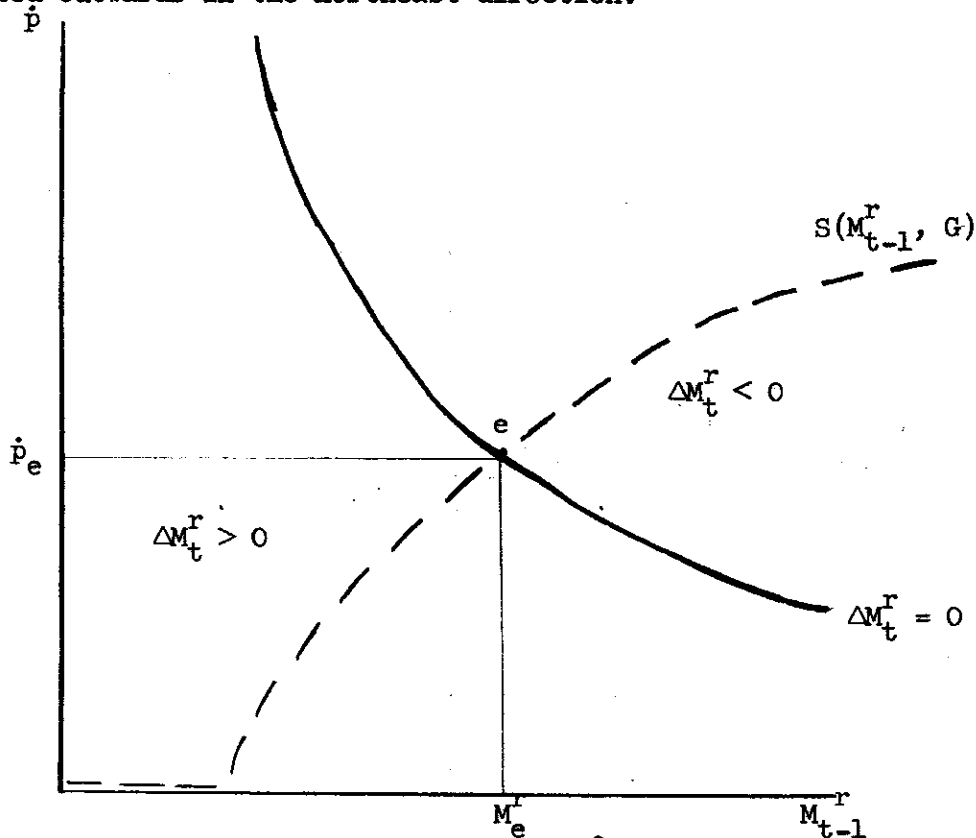


Figure 8.1

The dotted line on the graph represents the second equation of our system, function (8.3). This behavioral relation specified that the speed of inflation is a non-decreasing function of yesterday's money supply, given the level of government expenditure. The function is drawn to approach a maximum, as would be implied by a liquidity trap. An increase in the level of government expenditure causes an upward shift of this curve.

Point e on the graph represents an equilibrium, but it is a moving equilibrium characterized by a constant speed of inflation rather than a stable price level. The behavioral relationship, equation (8.3), advises us that if yesterday's money supply is M_e^r , the speed of inflation will be \dot{p}_e . Since point e is on the rectangular hyperbola, the injection of new money will just cancel the loss from inflation; consequently, the real value of the money supply remains unchanged. This equilibrium necessarily exists and is unique as it is determined by the intersection of a rectangular hyperbola with a non-decreasing function. Under quite general assumptions--of which equations (7.2), (7.3) and (7.9) constitute a special case--not only the level of income but also its allocation between consumption, investment and government spending will be uniquely determined in this condition of forced savings equilibrium for given values of γ and G .

Returning to Figure 8.1, one can immediately ascertain the impact upon the economy of adjustments of the two policy variables. If a larger proportion of government expenditure is to be financed by creating money, if the γ coefficient increases, both the equilibrium speed of inflation and the equilibrium real money supply will increase as the rectangular hyperbola corresponding to a stable real money supply will move in a northeast direction. If there is an increase in government spending, the other policy variable, the speed of inflation will be accelerated as the dotted

line will be shifted upwards. If the increase in the level of government spending is financed in part by issuing additional money each period, the hyperbola will shift upwards as well, causing a further acceleration of the speed of inflation.

This type of diagram serves to reveal certain dynamic aspects of the adjustment process as well as the effects of changes in the policy variables upon the forced saving equilibrium. In Figure 8.2 the rectangular hyperbola has been assumed to shift outwards as a result of an increase in the fraction of government spending financed by creating money. Government spending and,

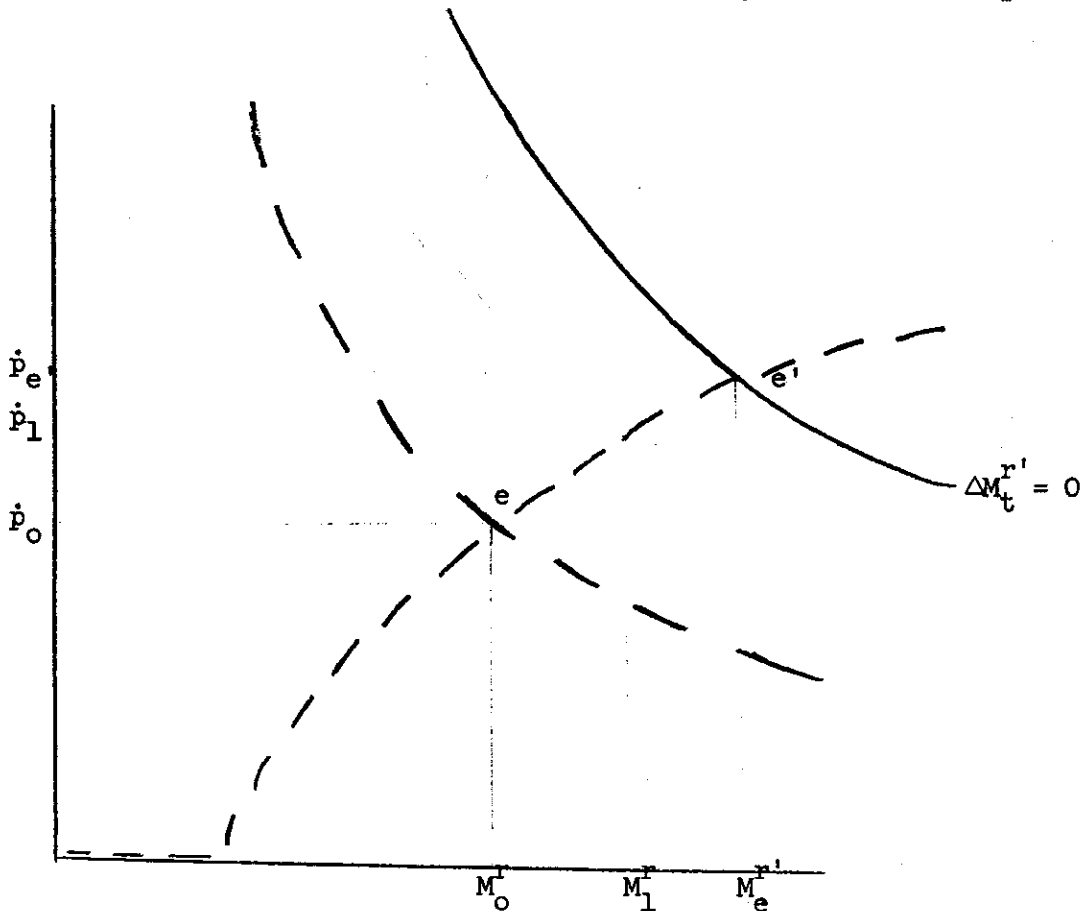


Figure 8.2

consequently, the speed of inflation function are unchanged. Point e' represents the new equilibrium, as contrasted with the old, point e , at which we had an initial money supply M_0^r and speed of inflation \dot{p}_0 .

In the first period after the change in policy, prices increase by the same amount as before. But point e is now below the relevant rectangular hyperbola. Consequently, the speed of inflation is not sufficient to offset the injection of new money; the real value of the money supply increases by the amount specified by equation (8.2). Let us suppose that the new value of the money supply is M_1^r . We see that this implies a speed of inflation \dot{p}_1 . The money supply will continue to expand in this way as long as $\dot{p}_t < \dot{p}_e'$, as long as the money supply is below the equilibrium level M_e^r . Clearly, $M_t^r < M_e^r$ implies $\Delta M_t^r > 0$. The converse also holds. If the money supply is above the equilibrium level, prices rise at a sufficiently rapid clip to more than offset the injection of new dollars; the real value of the money supply falls.

It might appear at first sight that a forced saving equilibrium is stable as well as unique. But appearances are to some extent misleading. As long as the argument is formulated in terms of difference equations, monotonic convergence to the new equilibrium, stable oscillations, and even explosive cycles are all theoretical possibilities. One perverse example serves to convince us of the possible varieties of behavior. Suppose that $G = 10$ billion and $\gamma = 0.4$. In addition, suppose that the initial money supply of \$20 billion yields in terms of equation (8.3) a speed of inflation of 0.05. From equation (8.2) it is seen that this implies that the money supply increases by \$3 billion to the level \$23 billion in the next period. Now suppose that equation (8.3) yields a speed of inflation of $7/23 \doteq 0.3$ for this larger money supply. Returning to equation (8.2) we find that this implies a reduction in the money supply of \$3 billion. The money supply returns to \$20 billion. This limit-cycle repeats itself again and again, the money supply and prices oscillating in a two period cycle of unchanging amplitude. Clearly, further investigations of the

dynamics of forced saving is required.*

* If time is regarded as continuous rather than discrete, the differential equation form of the argument is relevant. This system is necessarily stable and incapable of oscillation. Baumol has presented a concise elementary account of the solution of non-linear first order differential equation systems [1, pp. 315-6]. The phase line of the system has a negative slope throughout, implying monotonic convergence to equilibrium.

In order to appraise the stability of forced saving equilibrium, first observe that equation (8.3) may be substituted into (8.1) to provide for given γ and G a first order non-linear difference equation in terms of the money supply alone:

$$(8.5) \quad M_t^r = \gamma G + (1-p_t)M_{t-1}^r = \gamma G + [1-S(M_{t-1}^r)] M_{t-1}^r .$$

It will be helpful to analyze this function graphically.** On the abscissa

** This graphical technique for solving non-linear first-order difference equations is presented in [1, pp. 258-65].

of Figure 8.3 we have yesterday's real money supply. If the money supply is unchanging, the situation would be represented by a point on the forty-five degree line labeled M_{t-1} . The second line, $M_{t-1} + \gamma G$, indicates the contribution of the injection of new money to the existing money supply. If the price level were stable, a point on this line would indicate the relation between yesterday's and today's money supply. The curve M_t^r takes into account the fact that today's money supply is partially affected by the depreciation of the existing money stock.

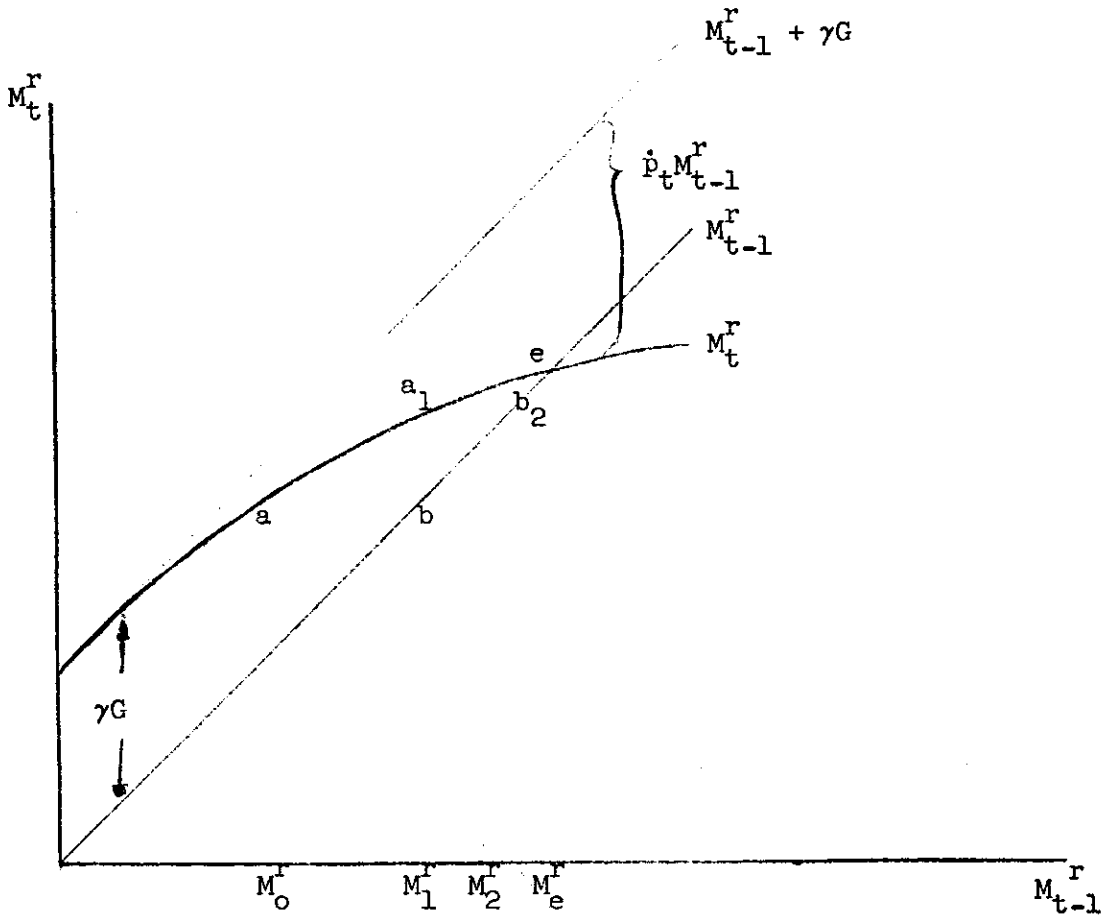


Figure 8.3

Let us see how this graph helps us to understand the way in which events unfold through time. If we start with yesterday's money supply M_0^r , today's money supply is determined by point a on the M_t^r schedule; it is the vertical distance $M_0^r - a$. This distance may be recorded on the abscissa by moving horizontally to the forty-five degree line at point b , and then dropping to record as M_1^r today's money supply. To determine the money supply in period 2, one moves vertically from M_1^r to the M_t^r schedule, moves horizontally to b_2 , and records the money supply as M_2^r on the abscissa. Since the M_t^r schedule is drawn with a positive slope we will converge monotonically to point e , the equilibrium point with money supply M_e^r .

Other possible modes of behavior are illustrated on Figure 8.4. If we start with money supply M_e nothing tends to change; the system is in equilibrium. But with money supply M_a , one moves in the next period to M_b and then back again to M_a in the type of limit cycle described earlier. If the initial money supply is between M_a and M_b , but not the equilibrium value M_e , the system moves in explosive oscillations towards the limit cycle path. Again, if one starts with an initial money supply below M_a or above M_b the system oscillates but in cycles of gradually decreasing amplitude, converging towards the limit cycle.

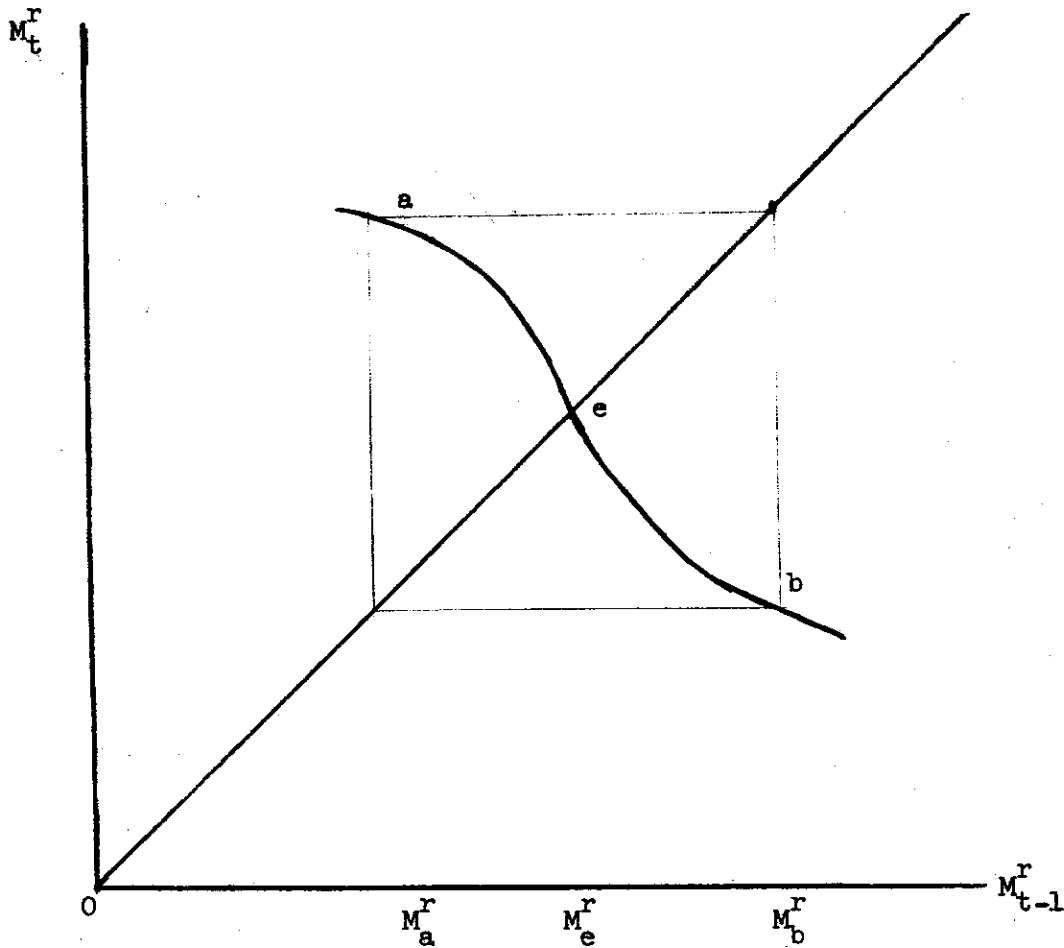


Figure 8.4

Consideration of Figures 8.3 and 8.4 suggests that the stability of forced saving equilibrium is related in a crucial fashion to the slope of the M_t^r schedule, the derivative:

$$(8.6) \quad \frac{\partial M_t^r}{\partial M_{t-1}^r} = 1 - \dot{p}_t - \frac{\partial \dot{p}_t}{\partial M_{t-1}^r} M_{t-1}^r .$$

If this expression is non-negative, we have the situation exhibited in Figure 8.3. Regardless of the initial money supply, the system converges to the forced savings equilibrium without oscillation. Now $\frac{\partial \dot{p}_t}{\partial M_{t-1}^r} \geq 0$ by assumption; therefore, the possibility of oscillations cannot be excluded for all initial conditions if $\dot{p}_t > 1$ for some M_{t-1}^r . If this condition is not satisfied, the system might still converge monotonically to the new equilibrium, but oscillations are a conceivable outcome for appropriate initial conditions. Explosive oscillations involve a stronger condition. Explosive oscillations can occur, for some set of initial conditions, if $\frac{\partial M_t^r}{\partial M_{t-1}^r} < -1$. Finally, for a stable system, the speed of convergence to equilibrium is more rapid the flatter the M_t^r schedule.

9. Forced Saving and the Quantity Theory

The nature of demand-pull inflation in an economy satisfying essentially Keynesian assumptions may be conveniently summarized by contrasting the implications of the preceding analysis with the predictions of the quantity theory. The quantity theorist asserts that the behavior of prices is most appropriately analyzed with the aid of the equation:

$$M v = p Y^F .$$

A leading advocate of the quantity theory, Professor Milton Friedman of the Chicago School, has stated [5, p. 20]:

...there is perhaps no other empirical relation in economics that has been observed to recur so uniformly under so wide a variety of circumstances as the relation between substantial changes over short periods in the stock of money and in prices; the one is invariably linked with the other and is in the same direction; this uniformity is, I suspect, of the same order as many of the uniformities that form the basis of the physical sciences. And the uniformity is in more than direction. There is an extraordinary empirical stability and regularity to such magnitudes as income velocity that cannot but impress anyone who works extensively with monetary data.

Under certain conditions, the income velocity of money may be stable even in a Keynesian environment. No change in the income velocity of money would be revealed by a before and after examination of the effects of an inflationary gap generated by a one shot injection of new money at full employment. Of course, a fall in income velocity would be revealed by a comparative statics investigation of the effect of the injection of a continued stream of government expenditure into a fully employed economy financed by some neutral means. But if a policy of forced saving is being followed, if a continued injection of real government spending is financed at least in part by creating money, a new equilibrium defined by fixed levels of consumption and investment spending would be characterized by an unchanging income velocity of money but a constantly rising price level.

It must be stressed that no empirical evidence concerning the validity of the quantity theory underlies the argument. Only the difficulty of such empirical investigation is insisted upon. Even if a stable income velocity of money were observed, this would not in itself refute the Keynesian analysis of unemployment and inflation. It would not guarantee that the quantity theory would be an appropriate guide in the planning of monetary reform or in appraising the consequences of a forced saving policy.

In a recent study of inflation, Richard T. Selden considered the appropriateness of a Keynesian rather than a quantity theory framework for his analysis. He concluded that "for an ex post study...the equation of exchange is much the more valuable of the two" on the basis of the following argument [23, p. 5]:

Since, in the Keynesian system, the division of GNP among consumption, investment, government and net foreign purchases usually is not considered a function of the price level, the system provides no way of determining the source of an inflation.

The argument of this paper reveals that if traditional behavioral equations of the Keynesian system are utilized in the analysis of inflation, a most important role must be assigned to the price level.

Analysis of the dynamics of demand-pull inflation revealed that monetary complications are important, even in a Keynesian environment. The very process of inflation, by reducing the real value of the money supply, tends to eliminate the inflationary gap. It has been demonstrated that in a Keynesian economy the all too common practice of financing a portion of budget deficits by creating money may lead to a forced saving equilibrium characterized by a constant speed of inflation and a stable real money supply. But this does not imply that monetary factors may be suppressed in the analysis of forced savings. It does not serve to vindicate those post-Keynesian arguments that neglect monetary complications in the analysis of the dynamics of the inflationary gap. Under a forced saving policy, two policy variables govern the speed of inflation. In analyzing forced saving policy, it is necessary to consider the extent to which the government's budgetary demands are met by expanding the money supply as well as the actual level of government expenditure. The real value of the money supply, as well as the speed of inflation, depends upon these two policy variables.

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