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Forced Saving in a Keynesian Economy:

An Analysis of Demand-Pull Inflation

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A deterioration in the international situation necessitates a substantial increase in defense spending. An underdeveloped country attempts to finance economic growth by creating money, by "forced saving." What are the effects of such actions upon the price level, the rate of interest, private investment spending, and consumption? John Maynard Keynes concerned himself primarily with the tragedy of unemployment when he wrote the General Theory [13]; he did not concentrate attention on problems of inflation. I shall argue that the Keynesian tools constitute a powerful mechanism for the analysis of problems of inflation as well as unemployment. How to Pay For the War [14] was a pamphlet, not a treatise; the model of the General Theory implies much more about the inflationary process than Keynes presented in his brief discussion of the inflationary gap.

Post Keynesian studies of inflation have generally involved elaborations of the dynamics of the inflationary gap. But such studies as those of Tjalling Koopmans [16], James Duesenberry [3], Franklyn D. Holzman [11], and J. D. Pitchford [21] have involved the mobilization of only a part of the analytical machinery presented by Keynes in the General Theory. Both the

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* I am indebted to my undergraduate theory students at Yale University for emphasizing that if the Keynesian mechanism is to interest a generation untouched by the great depression of the thirties, it must be focused upon problems of post-war inflation as well as mass unemployment. At Reed College Seymour Fiekowsky first stimulated my interest in forced saving. Helpful criticism has been provided at a Yale Economics Faculty Seminar. Donald Hester, Arthur Okun, John Pitchford, Richard Porter, T. N. Srinivasan, and Bjørn Thalberg made useful comments on earlier drafts of this paper. Remaining errors are my responsibility, of course. Research time was made available through the generosity of the Cowles Foundation for Research in Economics at Yale University.

money supply and the interest rate, essential elements of the General Theory, are suppressed in these dynamic studies of inflation. A more complete set of weapons from the Keynesian arsenal is mobilized in the study of inflation presented in this paper. The explicit introduction of money into the analysis of the inflationary gap provides a framework more appropriate for the appraisal of monetary policy.

As a model of unemployment, the Keynesian system provided an explanation of the impact of taxation, changes in government spending, and adjustments in the money supply upon the level of economic activity. Why not modify the model in order to explain the impact of increases in government expenditure and adjustments in the money supply upon a fully-employed economy? As a first step in this venture it is necessary to spell out the equations of a Keynesian system. Then a concept of capacity for the economy is derived. This capacity concept suggests in turn a definition of demand-pull inflation, a construct which facilitates a simplification of the original model appropriate for the comparative statics analysis of monetary and fiscal policy in a fully-employed economy. The static argument is then utilized as a foundation in the construction of a dynamic model. Essential elements of the studies of the dynamics of the inflationary gap are combined with the monetary elements whose role is revealed in the static analysis. The dynamic model is utilized to appraise the implications of a forced saving policy of financing a maintained injection of real government spending by the continued creation of new money.

It may well be asked whether the Keynesian theory of employment, now twenty-five years old, is the most appropriate model for this task. Admittedly, the model of the General Theory has been extended in a number of directions during the quarter century that has elapsed since its appearance. One cost involved in utilizing the Keynesian model as the framework for the analysis of inflation is that certain complications of importance, such as those connected with a growing economy, are suppressed. At the price of additional complexity, much of the argument of this paper might be translated into the framework of any of a number of alternative, more modern, more complex macro-models of employment. As it stands, a good portion of the analysis that follows is little more than an elucidation of certain terse passages concerning inflation that appear in the writings of Keynes. In the General Theory Keynes states [13, p. 289]:

> "...as effective demand increases, employment increases, though at a real wage equal to or less than the existing one, until a point comes at which there is no surplus of labor available at the then existing real wage... After this point a unit of labour would require the inducement of the equivalent of an increased quantity of product, whereas the yield from applying a further unit would be a diminishing quantity of product. The conditions of strict equilibrium require, therefore, that wages and prices, and consequently profits also, should all rise in the same proportion as expenditure, the 'real' position, including the volume of output and employment, being left unchanged in all respects. We have reached, that is to say, a situation in which the crude quantity theory of money (interpreting 'velocity' to mean 'income-velocity') is fully satisfied; for output does not alter and prices rise in exact proportion to MV."

This line of argument is to be developed in detail in this paper.

At several points in the discussion, the Keynesian theory of inflation will be compared with the quantity theory of money. Under special conditions, it is demonstrated, the quantity theory of money will make correct predictions even in a world satisfying the postulates of the Keynesian system. Consideration of the problem of inflation within an essentially Keynesian, dynamic framework reveals conditions in which the income velocity of money will be invariant. But it also establishes that under the Keynesian assumptions the quantity theory would not be a useful tool for policy formulation, even under these special conditions. This means that empirical studies concerning the relative stability of either income or transactions velocity cannot in themselves suffice to establish that the quantity theory is an appropriate framework for analyzing questions of policy.

1. Feasible and Infeasible Solutions of a Keynesian System

As preparation for the development of a Keynesian analysis of demand-pull inflation we must first specify a notational system and enumerate those equations of the *General Theory* that are fundamental to our analysis. For ease of reference we define the relevant variables here:

Y	national income
I	investment
C	consumption
N_d	demand for labor
N_s	supply of labor
Y_f	full employment level of national income
Y_d	effective demand
Y_s	aggregate supply
K	stock of capital
r	the rate of interest
M	the nominal money supply
p	the price level
w	the money-wage rate
w^*	historically ruling money-wage rate

All but the last four variables are measured in real terms.

There are three sets of equations of the Keynesian model. First, there are the equations underlying effective demand:

(1.1)	$M/p = L(r, Y_d)$	liquidity preference*
(1.2)	$I = I(r)$	investment schedule
(1.3)	$C = C(Y_d)$	consumption function
(1.4)	$Y_d = I+C+G$	income defined

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* Keynes breaks the demand for money function into two components, but only as a "first approximation," [13, p. 199]; the distinction between "transactions" and "idle" balances, an artificial one useful for pedagogical purposes, may be ignored. Franco Modigliani assumes that in the Keynesian system the demand for money balances is determined by the rate of interest and money income, but is independent of the absolute price level [25, pp. 46 and 71]. This implies, contrary to the above equation, that if the price level were to double while real income and the rate of interest remained unchanged, the equilibrium quantity of money, measured in real terms, might change; it implies an element of money illusion. Don Patinkin has utilized the form of the equation to be followed here [19, p. 129, passim]. At one or two points below it is indicated that the assumption is at times a crucial one.

Second, there are the two equations that determine aggregate supply:

(1.5)	$Y_s = Y(N_d, K)$	production function
(1.6)	$\frac{w}{p} = \frac{\partial Y_s}{\partial N_d}$	labor demand equates the marginal productivity of labor with the real wage.

Two final equations determine the full-employment level of output for the economy:

(1.7)	$N_s = N_s(p, w, w^*)$	labor supply
(1.8)	$Y_f = Y(N_s, K)$	full employment output

This system of equations is most conveniently reviewed in terms of the familiar aggregate demand and supply diagrams of Keynesian analysis. We plot income on the abscissa and the price level on the ordinate. Let us consider each of the three sets of equations in turn.

The first set of four equations involves seven variables. Of these, the nominal money supply and the level of government expenditure may be regarded as parameters. This set of four equations in five unknowns suffices to determine the level of effective demand, Y_d , as a function of the price level and these two parameters, G and M .

$$(1.9) \qquad Y_d = Y_d(p; G, M) .$$

This familiar relation is plotted on Figure 1.1 for fixed values of M and G . Conventional assumptions concerning the properties of the first four equations imply that this aggregate demand schedule should have a negative slope.

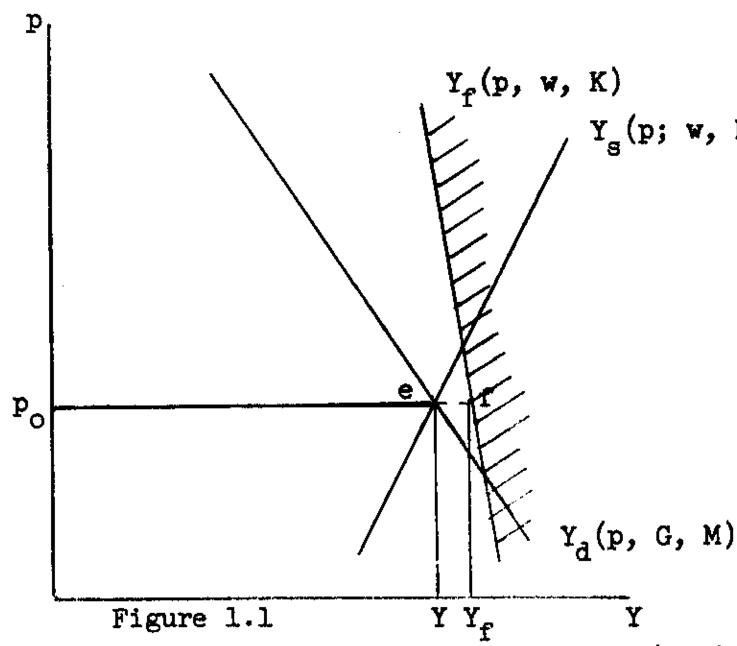

Figure 1.1

Next we derive the aggregate supply function from equations (1.5) and (1.6). We are armed with a production function involving both the available stock of capital and the quantity of labor employed. But the stock of capital, K , is regarded as fixed in Keynesian analysis. Effective supply, then, depends upon the level of employment, which in turn is given by the condition that workers are hired until their marginal productivity is equal to the real wage rate, a profit maximizing condition under pure competition. If we regard the level of money wages as a parameter, the volume of employment and, consequently, output is positively related to prices:

$$(1.10) \qquad Y_s = Y_s(p; K, w) .$$

This, the aggregate supply function, is also plotted on the diagram.

We specify an equilibrium condition. Aggregate supply must equal effective demand:

$$(1.11) \qquad Y_d = Y_s .$$

This serves to define equilibrium income, Y, and price level, the point e at which the aggregate demand and supply curves intersect on Figure 1.1.

There are two remaining equations, (1.7) and (1.8). These place a feasibility restraint upon our system, a condition that must be satisfied by any valid equilibrium solution of our model. Given the quantity of capital, enough labor must be supplied to produce the equilibrium level of income. A definition of full employment output is provided by substituting the labor supply equation, (1.7), into the production function:

$$(1.12) \qquad Y_f = Y[N_s(p,w,w^*);K] .$$

If money wages are set at the historically ruling level, the real wage will move reciprocally with the price level. If the supply of labor schedule is taken to have the customary positive slope, the supply of labor and hence the full employment level of output will be inversely related to the price level, given the money wage rate. The line Y_f on Figure 1.1 indicates this relation for given w.* Our feasibility constraint requires that the equilibrium

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* While we have arbitrarily drawn the aggregate demand curve more elastic than the full employment restraint, this is not essential for the analysis.

income be no greater than the full employment level which holds at the equilibrium level of prices, given the money wage rate:

(1.13) $$Y \leq Y_f .$$

Point e on Figure 1.1 satisfies this condition. The excess of full employment income Y_f over the equilibrium level Y is an index of the extent of unemployment, $N_s - N_d$.

This result is fortuitous. If G, K, M, and w are all regarded as fixed, there is no inherent reason why the level of income obtained by solving equations (1.1) through (1.6) must necessarily satisfy the feasibility condition that output be no greater than workers are willing to produce at the ruling money wage rate. After all, suppose that initially a feasible solution is obtained, as illustrated on Figure 1.1. Consider how this solution changes as one of the parameters, for instance government expenditure, is adjusted.*

* Later we analyze the effects of changes in the nominal money supply, a second policy variable, as well as of G in greater detail.

The aggregate demand curve shifts to the right as government expenditure increases.** For a sufficiently large level of government expenditure, G_1 ,

** The shift is not by the full amount of the multiplier obtained by taking the reciprocal of the marginal propensity to save. As income and the price level expand the rate of interest tends to rise curtailing the volume of private expenditure. This complication is apparent from an inspection of equations (1.1) through (1.4).

we will find the situation illustrated on Figure 1.2. In this borderline case, all three curves intersect at a single point. The feasibility condition (1.13) is still satisfied. The economy operates at full employment with a price level somewhat higher than before. Such a "full employment equilibrium" is

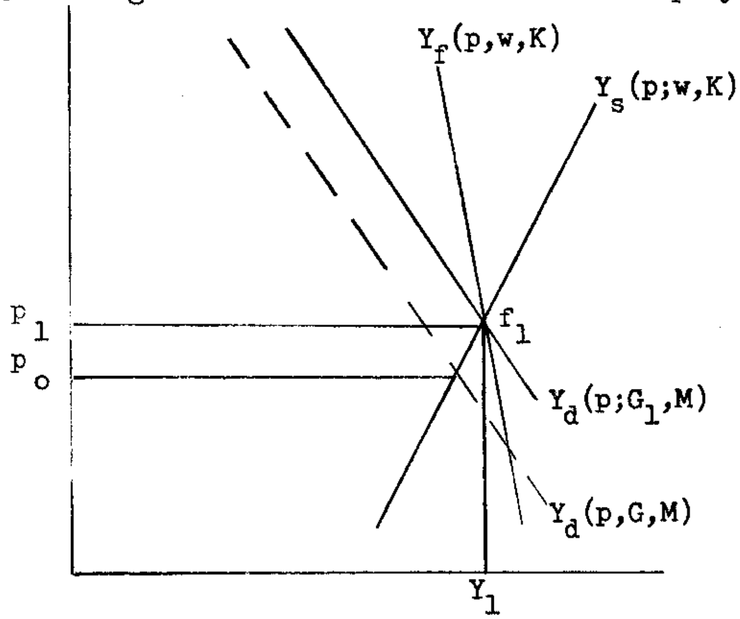

Figure 1.2

characterized by the condition:

$$(1.14) \qquad Y_d = Y_s = Y_f \ .$$

If government expenditure rises to a still higher level, G_2, we have the situation illustrated in Figure 1.3. Here the "equilibrium solution" (1.11), point e, is not feasible. The former full employment solution, f_1, is no longer valid at this higher level of government expenditure; it implies the existence of excess demand. Since point c implies that the real wage is not equal to the marginal productivity of labor, it cannot be regarded as a satisfactory solution.* Clearly, we have arrived at an impasse.

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* In the case illustrated, point c implies that the real wage is below the marginal productivity of labor. The direction of this inequality is reversed when the full-employment restraint is flatter than the aggregate demand schedule so that point c is southeast of f_1.

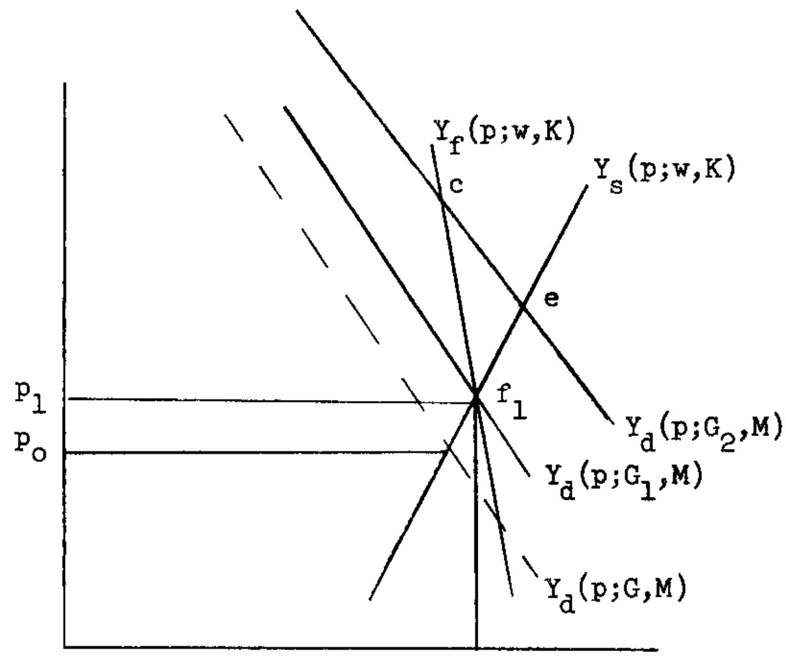

Figure 1.3

2. Capacity Output, Money Wage Rate Adjustments, and the Definition of Demand-Pull Inflation.

Given the level of government expenditure, the money supply, the capital stock, and the ruling money-wage rate, no feasible solution of the Keynesian system necessarily exists. Such an impasse cannot be circumvented unless at least one of the parameters of the system is allowed to adjust. Now the level of government expenditure and the nominal money supply, G and M, are policy variables; if we wish to analyze the consequences of government action we had better continue to regard these as fixed parameters. The capital stock parameter is a source of some embarrassment as the process of investment should lead to a change in its magnitude; but in a Keynesian analysis it is generally assumed that net-investment is sufficiently small in the short run relative to the size of the existing capital stock to permit the neglect of this complication. The money-wage rate is the only parameter remaining for consideration. If we allow money wages to vary we may find a feasible solution to our system. We shall look for a solution corresponding to full employment.

In a Keynesian economy the supply of labor is often taken to be an increasing function of the real wage rate, provided that money wages do not fall below the historically ruling level. It might appear, as a consequence, that the difficulties created by an expansion of government expenditure beyond the level required for full employment could be resolved, at least in part, by a favorable expansion in the level of output if only money wages were allowed to rise by a greater amount than any increase generated in the price level. But appearances are deceptive; this argument is not correct. Under the assumption of competition, workers are hired only until the marginal productivity of labor is equal to the real wage rate, condition (1.6). Assuming diminishing returns, the demand for labor is a decreasing function of real wages, given the supply of capital. If we have full employment, if $N_d = N_s$, the real wage rate and the number of workers employed are uniquely determined by the intersection of the demand for labor function and the supply of labor schedule. Substituting this full employment level of employment into the production function yields a "capacity level of output" Y_c . If there is no unemployment, if $N_d = N_s$, the economy must be operating at this capacity level of output, a level that depends on the existing quantity of capital and the production function, but is independent of the money wage rate, the level of government expenditure, and the money supply.

This digression has implications for the nature of shifts in the Y_f and Y_s schedules on the type of graph considered earlier. In terms of Figure 2.1 there exists a whole family of Y_f and Y_s schedules relating income to the price level, one for each level of the money-wage rate. But they all intersect at the capacity level of output, Y_c . Point f_1 represents

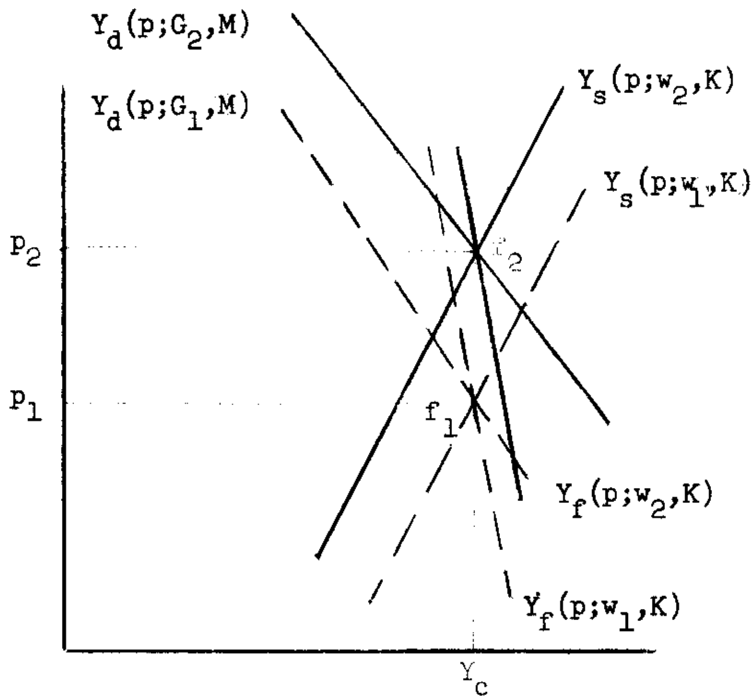

Figure 2.1

the old full-employment equilibrium corresponding to government expenditure G_1 , the equilibrium considered in the preceding section of this paper. When government expenditure increases to a higher level, G_2 , the aggregate demand schedule shifts upwards to $Y_d[p;\ G_2,\ M]$. Income is necessarily at Y_c when the economy is in full-employment equilibrium; consequently, a new full-employment equilibrium could only be obtained at point f_2 on the $Y_d[p;\ G_2\ ,\ M]$ schedule as it corresponds to income Y_c . Prices, therefore, must rise from p_1 to p_2 if the increase in government expenditure is to result in a full employment equilibrium. If the real wage remains unchanged, money wages climbing to the new level $w_2 = w_1\left(\frac{p_2}{p_1}\right)$, the Y_f and Y_s schedules will shift upwards so as to intersect at f_2 , as is required for the new full-employment equilibrium.

Let us consider other possible patterns of money-wage rate behavior. If money-wages had risen to a new level less than w_2 , the aggregate supply schedule and the full-employment restraint would have intersected below the aggregate demand curve and there would not have been a feasible equilibrium solution. Conversely, if money wages were pushed above w_2, these two curves would intersect above the aggregate demand schedule; here we would have a feasible, less than full-employment equilibrium at the point where the higher aggregate supply curve intersects the new schedule of effective demand. Such an equilibrium must be characterized by a higher price level than that obtained when the new equilibrium is one of full-employment for it must be at a point to the left of f_2 on the same aggregate demand schedule, $Y_d[p; G_2, M]$. When a full-employment equilibrium is disturbed by an increase in government expenditure, unemployment may follow, but only if an increase in real wages occurs pushing prices above the level required for a new equilibrium at full-employment. On the other hand, full employment may be maintained if money wages rise *pari passu* with the price level. If cost-of-living escalator clauses generally prevail in wage contracts, the necessary upward adjustment in wages might be achieved automatically; alternatively, it might be achieved either by renegotiation of wage contracts or wage-drift.* But what is clear

* While common-sense might suggest that escalator wage contracts may contribute to unstable wage-price spirals, it will be argued in a later section of this paper that this is not necessarily the case.

is that the maintenance of full-employment requires that the equilibrium real wage be preserved.

Now we are in a position to define three essential phases of the inflationary process:

i. A "*quasi-inflationary*" phase occurs when an upward shift in aggregate demand disturbs an unemployment equilibrium. Part of the increase in demand is met by expanding output, part by a rise in the general level of prices. In terms of Figure 1.2, this is represented by an increase in prices from p_o to p_1 as a result of the upward shift in government expenditure to G_1 .

ii. "Demand-pull inflation" is generated when aggregate demand expands beyond the level required for full employment at the ruling money-wage rate. A new, feasible equilibrium can be achieved when government expenditure G_2 , but only if the money-wage rate moves upwards at least to the level $w_2 = \left(\frac{p_2}{p_1}\right) w_1$. If money-wages rise by precisely this amount, we have experienced a pure demand pull inflation. Output remains at the capacity level; the real wage rate is the same as before.

iii. An element of wage-push inflation may be present. If wages are pushed up above w_2 prices will be shoved above p_2 . Labor may achieve an increase in the real wage rate, but only at the expense of unemployment.*

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* Note that regardless of the type of inflation the assumption of competition implies that the real wage remains equal to the marginal productivity of labor. Consequently, wage push can be present even if real wages do not rise faster than productivity.

Admittedly, the three phases of the inflationary process just enumerated cannot be regarded as in any sense a historical sequence or a description of a chronological order in which events must necessarily unfold in practice. Keynes himself discussed the possibility of upward pressure on the money-wage rate before full-employment is achieved [13, p. 301]. All three elements of the process may be encountered in any inflation we have the misfortune to experience. But we shall find that for purposes of theoretical analysis the classification is not only legitimate, it facilitates concise analysis by pointing the way to appropriate simplifications.

Attention will be focused upon pure demand-pull aspects of the inflationary process in this paper. Aggregate demand and supply curve analysis has revealed two essential properties of this type of inflation:

(a) Pure demand-pull inflation is a full-employment phenomenon; real output remains at an unchanging, capacity level that is independent of government policy.

(b) Concentrating attention upon pure demand-pull inflation is equivalent to assuming that at full-employment a rise in prices must be precisely balanced by compensatory upward adjustments in the money-wage rate that leave real wages unchanged. Real wages remain equal to the marginal productivity of labor at capacity output.

These two properties permit a considerable reduction of the eight equation Keynesian system originally considered in Part 1.

3. Static Analysis of Demand-Pull Inflation:

A considerable simplification of the eight equation Keynesian system originally considered in Part 1 is possible when the area of investigation is restricted to the analysis of pure demand-pull inflation. As long as inflation is to be studied under full-employment conditions, output may be assumed to remain at Y_c ; the real wage rate is uniquely determined. If we substitute Y_c into the first four equations of our original model, we obtain a reduced system appropriate for the static analysis of demand-pull inflation:

$$(3.1) \qquad M/p = L(r, Y_c)$$

$$(3.2) \qquad I = I(r, Y_c)$$

$$(3.3) \qquad C = C(Y_c)$$

$$(3.4) \qquad Y_c = I + C + G$$

Admittedly, inflation may cause structural changes in the fundamental equations underlying Keynesian analysis. The very process of inflation might shift the consumption function, the marginal efficiency of capital schedule, and the demand for money equation. But it is appropriate to assume at the initial stage of the analysis that these relations are stable. Credence is added to this assumption by the necessity of allowing compensatory money wage increases to take place in order to preserve the constancy of real wages characteristic of pure demand-pull inflation; consequently, no wholesale redistribution of income need be involved in the inflationary process. We must also assume that investment has no significant effect upon either capacity or productivity in the short run. Secondary complications involving shifts in the fundamental relations may be readily introduced once the basic approach is outlined.

Let us suppose that an initial situation of full-employment equilibrium is disturbed by an increase in government spending to a new level G' . How will the price level, the rate of interest, consumption and private investment spending be affected?* Inspection of the reduced model reveals that the

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* The answer to this question depends, of course, upon how the increased government spending is financed. Purely in the interest of simplicity we assume in the text that a "neutral" means is utilized; such neutral financing might be achieved through change in the personal income tax structure involving a greater degree of progressivity so as to leave the consumption function unaffected while yielding the desired revenue. If, as is more likely in practice, taxes levied to finance the increased expenditure affect the consumption function and investment schedule, the shifts would have to be taken into account in deriving the full effects of the decision to increase government expenditure. The task of working out the various effects of alternative tax schemes can be safely left to the reader. The more interesting problem created when the increased expenditure is financed, at least in part, by the continued injection of new money will be analyzed at a later stage of the argument.

"inflationary gap" generated by the excess of spending over capacity output can be eliminated only if the increased government spending is offset by a corresponding reduction in private investment. Turning to the investment schedule, equation (3.2), we find a higher interest rate r' required to achieve the necessary reduction in private investment spending. The liquidity preference schedule, (3.1), may then be solved to obtain the reduction in the money supply required for this increase in the rate of interest. If the nominal money supply remains constant, the change in the price level required to achieve the new equilibrium is provided by the ratio:

$$(3.5) \qquad \frac{p'}{p} = \frac{L(r, Y_c)}{L(r', Y_c)} \quad .$$

It is clear that a ceiling exists to the change in the price level generated by an expansion of government expenditure.* Income and hence consumption in real terms will be the same after as before the inflation; but the rise in the rate of interest induced by inflation will curtail private investment spending so as to balance the increase in government expenditure.**

* It may be objected that if the marginal efficiency of capital schedule were sufficiently inelastic this might require an infinite rate of interest. Perhaps negative investment would be necessary. The argument that there exists a ceiling is strengthened below where a possible dependence of consumption upon the rate of interest and the money supply is considered.

** The term "forced saving" is sometimes reserved for situations in which increased government expenditure is financed, at least in part, by a curtailment of consumption spending, as would occur if consumption as well as investment were assumed to be negatively related to the rate of interest.

Several factors influence the magnitude of the price rise required to restore equilibrium. A more elastic marginal efficiency of capital schedule would mean that a smaller rise in the interest rate would curtail private investment by the amount required to offset the increased government spending.

Consequently, a less severe inflation would suffice to restore equilibrium. If consumption as well as investment were negatively related to the rate of interest, equilibrium could be obtained with a smaller rise in the rate of interest and the price level. Given the required change in the rate of interest, the extent of the inflation will be smaller the less the elasticity of the demand for money with respect to the rate of interest. If either the marginal efficiency of capital schedule is inelastic or the economy is in the liquidity trap, a much larger increase in the price level will be generated when an increase in government expenditure disturbs an economy initially at full-employment equilibrium.

Suppose that the factor disturbing the economy from its initial full-employment equilibrium is an increase in the nominal money supply rather than an expansion of government spending. If the nominal money supply increases from M to M', the effect may be immediately determined by inspection of equations (3.1) through (3.4). All that is required is for the price level to rise proportionately so that the real value of the money supply remains unaffected. While the rate of interest, consumption, and investment spending need not change, the price level must adjust to a new level $p' = p\left(\frac{M'}{M}\right)$. Only if the economy is in the liquidity trap or investment is completely independent of the rate of interest will this inflationary consequence of an injection of new money be avoided.

It is interesting to observe that the quantity theory has a certain validity even for an economy satisfying the postulates of the Keynesian theory. The quantity theory asserts that the behavior of prices is most appropriately analyzed with the aid of the equation:

$$Mv = pY .$$

Returning to (3.1) we may solve for v , the income velocity of money:

$$v = \frac{p}{M} \cdot Y = \frac{Y_c}{L(r, Y_c)} .$$

If a fully-employed Keynesian economy is disturbed by an increase in government spending, the nominal money supply remaining unchanged, the resultant rise in the interest rate means that velocity will have increased. But if it is an injection of new money that disturbs the fully-employed economy, the velocity of money will be stable. This suggests that a study of countries subjected to demand-pull inflation as a result of expansions in the money supply cannot provide an appropriate basis for comparing the validity of the two theories. When we focus attention on dynamic phenomena we shall find that the empirical problem is further confounded in that under certain other conditions the income velocity of money is also stable.

Certain qualifications of the elementary argument concerning the nature of demand-pull inflation remain to be considered. It is necessary to spell out factors that may cause shifts in the propensity to consume, the marginal efficiency of capital schedule, and the function describing the demand for money. At this stage, certain qualifications that arise from static considerations affecting the final equilibrium position will be appraised. This necessary step of the argument must be completed before more interesting, dynamic factors can be fully appreciated.

In an attempt to rally the classical forces against the Keynesian attack, Haberler [7, pp. 242, 491-503], Pigou [20] and others have asserted that consumption depends upon the real value of net assets held by individuals as well as the level of income. This complication, christened the "Pigou effect" by Patinkin [18], constitutes a crucial modification of the Keynesian argument concerning the ineffectiveness of flexible money wages and monetary policy as means for eliminating unemployment. How is the analysis of inflation altered by this complication of the Keynesian model? The process of inflation, by lowering the net purchasing power of assets whose value is expressed in money terms, would tend to curtail consumption spending, given the rate of interest and level of income. Since this would help eliminate the inflationary gap, a smaller rise in the price level would be required to restore a full employment equilibrium disturbed by an increase in government expenditure.* Since the market value of long-term government bonds will fall as a result of a rise in the rate of interest, the Pigou effect implies that consumption spending is sensitive, if indirectly, to the rate of interest.

A redistribution of income may affect the position of the consumption function. If lags in the adjustment of pensions, wages, and the size of dividend payments occur in inflationary conditions, the consumption function may shift temporarily. For the most part such effects may be regarded as transitory, dynamic phenomena not affecting the final equilibrium position. But redistribution effects cannot be neglected entirely, even at the comparative statics stage of the argument. For example, the real value of fixed interest payments will deteriorate as a result of inflation.

A progressive income tax may also tend to eliminate the inflationary gap [24]. If a progressive income tax rate structure is not readjusted to allow for the impact of inflation, the size of the government's tax bite

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* In a revision of an article originally written in 1942, Milton Friedman adds that the Pigou effect alone leads to the elimination of the inflationary gap, the fall in the real value of liquid assets caused by inflation leading to a reduction in the volume of consumption that takes place at capacity income. He does not mention other factors, such as a rise in the interest rate, that may also serve to make the inflation itself a force tending to eliminate the gap through reducing private investment [4, pp. 253-7].

measured in real terms will depend on the price level. As prices rise, real disposable income and, consequently, consumption will be lower for each specified level of constant dollar GNP. Since a progressive income tax rate structure under inflation becomes equivalent to increasing tax rates with stable prices, reliance upon the progressivity of the income tax is a poor substitute for direct, anti-inflationary tax legislation. But it does serve to introduce a useful element of "money illusion" into the system.

A capital gains tax may also serve to curtail the rise in the price level generated by an injection of government spending. As the price level climbs, capital gains realized on assets whose values are not fixed in monetary terms will be partially siphoned off into the Treasury. The real value of the wealth held by stock market investors as well as those so timid as to invest only in bonds will be eaten away. If non-money assets as well as income and the rate of interest belong in the demand for money equation, this may act to curtail inflationary pressure. Given the level of income, the public will not be willing to hold as large a real stock of money at any given rate of interest as they were before the real value of their assets was reduced by inflation.

These qualifications contribute to the realism of the argument. Collectively they add further grounds for suggesting that the rise in the price level itself will help to curtail spending, combating the inflationary pressure. Additional complications are created when the method used to finance the increased government expenditure is not assumed to be neutral. It is a simple task to consider the effect of various taxation schemes for financing the increased expenditure. But when the higher level of spending is financed by a method that enlarges the money supply, further complications must be faced. The most interesting case concerns the long run effects of a "forced saving" policy.

An increase in government expenditure is financed by the continuous injection of new money into the system. Such a policy leads to dynamic complications that cannot be analyzed with static tools.

4. The Dynamics of Demand-Pull Inflation: A Synthesis

The preceding arguments have been exercises in comparative statics, an approach that can predict only the final outcome of an assumed change in the parameters. The argument has disclosed only the determinants of the magnitude of the ultimate rise in the price level generated by an inflationary gap. Static analysis cannot advise us as to the factors governing the rate at which prices will rise to the new equilibrium level or on more complicated issues of stability. Static analysis cannot reveal the consequences of a deliberate policy of continually expanding the money supply in order to finance government expenditure. Tjalling Koopmans [16], James Duesenberry [3], Franklyn D. Holzman [11], John D. Pitchford [21], and others have analyzed dynamic aspects of inflation. Here we shall show that the static arguments presented earlier in this paper provide a useful foundation for such dynamic superstructures. The dynamic model derived by this exercise in eclecticism will be used in the next section in a study of the forced saving process.

It was appropriate to suppress the time element in earlier sections where we were enumerating equations underlying the static analysis of inflation. But if we are to examine factors determining the speed of inflation, the rate at which prices rise, time must be introduced explicitly. When the Keynesian model is dynamized for purposes of business cycle analysis, any of a large number of alternative procedures for introducing time may be utilized. It is the same in the study of inflation. Any of an embarrassing variety of procedures might be utilized in the construction of a dynamic Keynesian model for the analysis of demand-pull inflation. Let us restrict attention to models that are Keynesian in the sense that the underlying equations have as static counterparts corresponding equations of traditional Keynesian analysis. In addition, let us introduce only a minimum of dynamic complications. Even with these restrictions, considerable scope remains for the arbitrary selection of dynamic elements in the construction of our dynamic models. While only one or two basic approaches will be outlined, it should be apparent that many alternative procedures for introducing time could be analyzed in essentially the same way.

We begin by adopting an assumption frequently invoked in dynamizing the Keynesian model. We assume that the current level of consumption spending is determined on the basis of yesterday's income. But in addition, we assume that the number of dollars allocated to this function is decided under the misapprehension that current prices as well as the other determinants will remain at yesterday's level. In other words, the number of dollars of yesterday's income allocated to consumption spending is given by the following modification of (1.3):

$$C_t^n = p_{t-1}\, C(Y_{t-1}) \ , \tag{4.1}$$

where the superscript n indicates that consumption is being measured in current dollars rather than deflated. Consequently, <u>real</u> consumption expenditure is given by the equation:

$$C_t = \frac{p_{t-1}}{p_t}\, C(Y_{t-1}) \ . \tag{4.2}$$

The same line of argument provides a similar expression for investment spending:

$$I_t = \frac{p_{t-1}}{p_t}\, I(r_{t-1}, Y_{t-1}) \ . \tag{4.3}$$

If the government also makes naive assumptions about the behavior of prices in appropriating funds, real government expenditure at time t is:

$$(4.4) \qquad G_t = \frac{p_{t-1}}{p_t} G \ .$$

It might be objected that these last three equations all embody an element of "money illusion." Of course, but it is not the type of money illusion usually considered in static analysis. Indeed, if the price level, the rate of interest, and real income are all unchanging, if static conditions are assumed, equations (4.2) and (4.3) reduce to their static, illusion free counterparts considered in Part 1. The illusion with which we are now concerned is the consequence of the rate at which prices change, not the current price level. Later we shall consider an alternative dynamic model of demand-pull inflation which is free of such elements of "inflation illusion."

As a first step in the study of the dynamic implications of the assumptions embodied in equations (4.2) through (4.4) concerning the way in which a changing price level affects real consumption and other components of total spending, let us explain what happens in the current period on the basis of known past events. It will be convenient to denote by D_t , effective demand at time t , the sum of actual consumption, investment, and government spending:

$$(4.5) \qquad D_t = C_t + I_t + G_t \ .$$

Since we are analyzing demand-pull inflation, the level of income may be regarded as fixed at Y_c . Consequently, equations (4.2), (4.3) and (4.4) imply that the level of effective demand depends only on lagged interest, the planned level of government expenditure, and the price ratio term:

$$(4.6) \qquad D_t = D\left(\frac{p_t}{p_{t-1}}, \ r_{t-1}, G\right) \ .$$

It is apparent that the assumptions made concerning the influence of rising prices on consumption, investment, and government expenditure imply that equation (4.6) is homogeneous of degree one with respect to the price ratio; we have the property:

$$D\left(\frac{p_t}{p_{t-1}}, \; r_{t-1}, \; G\right) = \frac{p_{t-1}}{p_t} \; D(1, r_{t-1}, G) \; .$$

Now we suppose that the price level must rise rapidly enough within each period to equate effective demand with capacity income. This supposition implies:

$$(4.7) \qquad Y_c = D\left(\frac{p_{t-1}}{p_t}, \; r_{t-1}, \; G\right) = \frac{p_{t-1}}{p_t} \quad D(1, r_{t-1}, G) \; .$$

Let us call $D(1, r_{t-1}, G) - Y_c$ the "savings-investment inflationary gap."*

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* This is the most frequently employed concept of the inflationary gap. Walter Salant has discussed this and various other concepts of the gap [22].

Equation (4.7) implies the following relation:

$$(4.8) \qquad \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{D(1, r_{t-1}, G) - Y_c}{Y_c} \; .$$

The speed of inflation is equal to the ratio of the savings-investment inflationary gap to capacity income.

This conclusion is, of course, sensitive to the particular assumptions made concerning the impact of inflation upon the various elements of effective demand. For example, if real government expenditure had been preserved at the level G rather than being reduced by the rising price level, prices would have risen more rapidly. If inflation had been partially anticipated, the rate of price increase would have been more rapid than equation (4.8) implies.

If a shortening occurs in the length of payment lags, the speed of inflation measured in terms of calender time will be more rapid. Tjalling Koopmans [16] has analyzed in considerable detail the speed of inflation under a variety of alternative assumptions concerning the effects of changing prices upon effective demand. In addition, he has provided a clever graphical technique useful for expository purposes. Measure income in real terms on the ordinate and the speed of inflation on the abscissa.

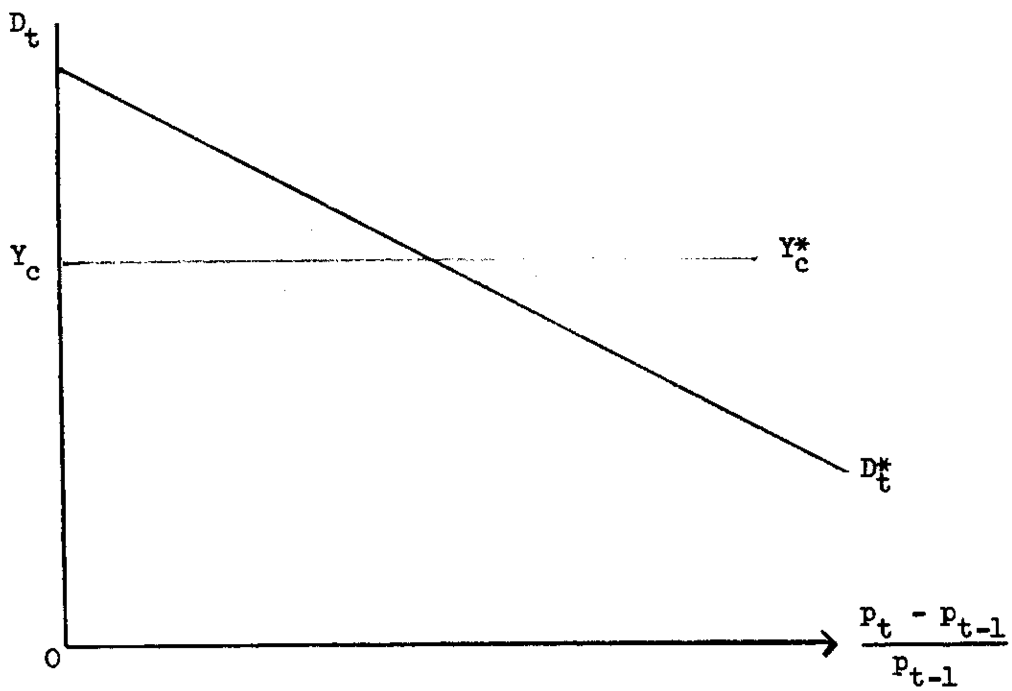

Figure 4.1

The curve $D_tD_t^*$ represents effective demand at time t as a function of the rate of price change. Its intersection with the capacity line $Y_cY_c^*$ reveals the speed of inflation at time t. Observe that the more sensitive effective demand to rising prices, the steeper the line $D_tD_t^*$, the less rapid is the inflation generated by any given savings-investment inflationary gap.

If inflation contributes to an inefficient allocation of resources, the total output curve as well as the total demand schedule will slope downwards to the right, increasing the speed of inflation. When the total output curve was horizontal, when capacity was independent of the speed of inflation, rising prices might have had undesired effects upon the distribution of wealth and income. Conceivably, inflation might distort the choice between current consumption versus investment for future use, lowering the rate of economic expansion. But when the total output curve is downward sloping, inflation causes an actual reduction in physical output. There is a diminution in the size of today's pie as well as a change in the relative magnitudes of individual servings.*

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* It is conceivable for the capacity level of output to be an increasing function of the speed of inflation. If workers are misled by rising prices into thinking that real wages are higher than is actually the case, they may be induced to provide more labor services than they otherwise would at any given real wage. Again, if employers are misled by errors of accounting connected with inflation into confusing paper windfall profits with the actual gains of business, workers may in general be paid more than their marginal productivity. This was recognized by Keynes [13, p. 290]. The assumption that Y_c is a function of the rate of price change is perfectly compatible with the conclusion of Part 2 that it is stable under static conditions.

This inflationary gap analysis is not complete. It must be recognized that the equilibrium determined by this procedure holds only for the current time period. The assumptions advanced concerning the inflationary process do not imply that the events of tomorrow are a simple repetition of today's. Does not the speed of inflation determined by condition (4.7) depend upon last period's rate of interest? The rising price level will fail to reduce the real value of liquid assets only if the government takes steps to expand the nominal money supply. Unless the economy is in the liquidity trap, any decline in the real value of the money supply will raise the interest rate. This will curtail private investment and possibly consumption spending as well. Only if both consumption and investment spending are independent of the rate of interest will the higher interest rate not cause a contraction in the size of the inflationary gap and a consequent reduction in the speed of inflation.

In order to understand the details of this process it is necessary to return to a further consideration of the liquidity preference schedule, equation (1.1). We shall not bother to insert lags into the equation, but shall assume simply:

$$(4.9) \qquad \frac{M_t^n}{p_t} = L(r_t, Y_t) .$$

Now under conditions of demand-pull inflation, real income remains constant at a capacity ceiling Y_c , a level which may, as a first approximation at least, be assumed constant even under dynamic conditions. As a consequence, there is a precise relation between the rate of interest and the real value of the money supply; let us write it as:

$$(4.10) \qquad r_t^n = R_c\left(\frac{M_t^n}{p_t}\right) .$$

It is the opportunity costs of holding money that must be equated with the marginal gains from liquidity. The rate of interest determined by liquidity preference is the nominal, not the real rate. Investors, on the other hand, are concerned with the real rather than the nominal rate of interest.*

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* If one pays $130 back on a loan of $100, the nominal rate of interest is 30 per cent. But if the market value of physical assets rises by 25 per cent during the period of the loan, a five per cent real rate of return on investment would be sufficient to justify borrowing at this high nominal rate. Keynes discusses the tax of inflation upon holders of liquid assets in the Tract [12, pp. 48-9].

The ex ante real rate of interest, the nominal rate of interest minus the anticipated speed of inflation, belongs in the investment function. Of course, the real and nominal rates of interest are identical in comparative statics.

The distinction can be neglected in dynamic analysis if naive anticipations are assumed, if it is assumed that investors in real assets fail to recognize that prices are rising.

Let us substitute equation (4.10) directly into our expression for investment, (4.8), in order to obtain a modification of the equation explaining the speed of inflation. This is equivalent to assuming that investors in real assets fail to appreciate that prices are rising. This simplifying assumption is certainly not essential, but the argument is not seriously modified when it is assumed that investors correctly anticipate the speed of inflation. We have:

$$\frac{p_t - p_{t-1}}{p_{t-1}} = \frac{C(Y_c) + I\left[R_c\left(\frac{M^n_{t-1}}{p_{t-1}}\right), Y_c\right] + G - Y_c}{Y_c} \tag{4.11}$$

If the nominal money supply is constant, a rising price level, by reducing the real value of the money supply, tends to raise the rate of interest; this in turn curtails investment spending. Given M^n_t, the inflationary gap is a decreasing function of the absolute level of prices. Equation (4.11) reveals the dependence of the speed of inflation upon the price level. In terms of the diagram developed by Koopmans, the D_t line shifts downwards through time as prices rise to successively higher levels, approaching the equilibrium whose properties were derived in the static analysis of Part 3.

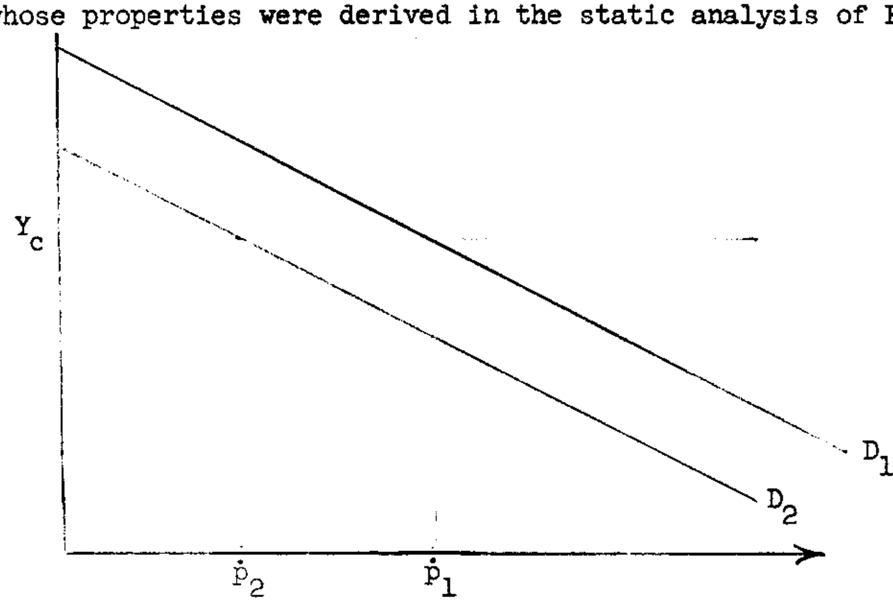

Figure 4.2

The very process of inflation tends to cure the disease of rising prices.

Inspection of equation (4.11) reveals that the speed of inflation is a non-decreasing function of the real money supply. Differentiation yields:

$$(4.12) \qquad \frac{\partial \dot{p}_t}{\partial M^r_{t-1}} = \frac{\frac{\partial I}{\partial r^n} \frac{\partial r^n}{\partial M^r_{t-1}}}{Y_c} \quad \text{and} \quad \frac{\partial \dot{p}_t}{\partial G} = \frac{1}{Y_c} \text{ , where } \dot{p}_t = \frac{p_t - p_{t-1}}{p_{t-1}} \text{ .}$$

These properties will be found to have important implications when we turn to analyze forced saving phenomena.

The positive relationship between the real money supply and the speed of inflation is only the consequence of the particular procedures that have been utilized in adding dynamic elements to the static Keynesian model. To see this, we shall consider a second, alternative set of dynamic complications. An element of illusion on the part of spenders concerning the future course of the price level has been involved in all the dynamic complications so far considered. Spenders have been assumed to act under the supposition that the price level is not going to change. In addition, investors are alleged to confuse the nominal with the real rate of interest. The dynamic Keynesian model that will now be developed is free of such elements of inflation illusion. A comparison of the properties of this second model with those of equation (4.11) will reveal certain implications for the inflationary process of naive errors of judgment made by spenders in anticipating the rate of price change. Under certain conditions, examination of the second model reveals that a larger real money supply may lead to an actual reduction in the speed of inflation.

The dynamic complication now to be considered involves the consequence of the capital-loss inflicted upon the holders of money assets by inflation. When the price level doubles, an individual who holds $1,000 cash suffers a capital loss of $500. This is the difference between yesterday's and today's purchasing power of the money assets carried over from yesterday. It is the amount that would have to be set aside in order to restore the real value of liquid assets to their former level. The size of the capital loss measured in real terms, L_t , is obtained by subtracting from the real value of yesterday's money supply its value in terms of today's price level.*

$$(4.13) \qquad L_t = \frac{M_{t-1}}{p_{t-1}} - \frac{M_{t-1}}{p_t} = \frac{p_t - p_{t-1}}{p_t \, p_{t-1}} \; M_{t-1} = \frac{p_t - p_{t-1}}{p_t} \quad M^r_{t-1} \quad .$$

* For simplicity we neglect the capital loss suffered on government bonds as a result of inflation. The complication could be readily introduced if it were assumed that the volume of net publicly held assets of fixed nominal value is linearly related to the money supply.

It seems reasonable to suppose that any capital loss inflicted by inflation is taken into account in planning current expenditure. Let us suppose that actual effective demand is overstated by the sum of the consumption, investment, and government spending as defined by the static equations of Part 1, the overstatement in spending being proportionate to the size of the capital loss experienced. The symbol γ will represent the proportion of the capital loss that is replaced by reductions in current expenditure:

$$(4.14) \qquad D_t = C(Y_t) + I(r^r) + G - \gamma \left(\frac{p_t - p_{t-1}}{p_t} \right) \quad M^r_{t-1} \quad .$$

A consumer wishing to restore his liquid assets to their former level might well curtail his consumption spending by the full amount of the loss; for such an individual, $\gamma = 1$. Alternatively, the capital-loss may be regarded as a reduction in income, expenditures being curtailed by the marginal propensity to consume times the size of the capital loss. Such adjustments in expenditure do not necessarily involve any error of judgment or mistake in anticipating the speed of inflation in that the capital loss involved may have been correctly anticipated and added to the real interest cost in determining the appropriate size of cash balances to be held for liquidity purposes. But the size of the coefficient relating consumption to current capital losses may well be smaller if inflation illusion is present; if additional price rises are not anticipated, if the capital loss is regarded as a one-shot affair, the individual may plan to gradually replenish his liquid assets over several future periods. Contrariwise, if the price index is expected to continue its climb there will be additional capital losses to worry about in the future; as a consequence, such replenishment as is to be made as a result of the current capital loss had better take place immediately.*

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* In terms of the language of the Modigliani-Brumberg and Friedman hypothesis, it is a question of whether a reduction in transient or permanent income is involved.

If prices are to rise sufficiently fast in each period to equate effective demand with capacity output we must have:

$$(4.15) \qquad Y_c = D_t = C(Y_t) + I\left[R\left(\frac{M_{t-1}}{p_{t-1}}\right) - \dot{P}_t\right] + G - \gamma\, P_t\, M^r_{t-1} ,$$

$$\text{where} \quad \dot{P}_t = \frac{p_t - p_{t-1}}{p_t} = \frac{p_{t-1}}{p_t} \cdot \mathring{p}_t \quad .$$

The inclusion of the real rate of interest in the investment equation, $r^r = R\left(\frac{M_{t-1}}{p_{t-1}}\right) - \dot{P}_t$, serves to eliminate the last vestige of the inflation illusion that characterized the earlier model.**

** Investment is assumed to depend upon <u>last</u> period's money supply in order to simplify the analysis. If this is not done, the dynamic models of Part 5 become of a higher order that proves more difficult to manipulate.

Evaluation of the implications for the inflationary process of equation (4.15) is greatly simplified if we are permitted to assume that the investment equation is linear. If $I_t = I^o - \beta r_t^r$, $\beta \geq 0$, equation (4.15) is easily manipulated in order to obtain as the expression for the speed of inflation:

$$(4.16) \qquad \dot{P}_t = \frac{p_t - p_{t-1}}{p_t} = \frac{C(Y_t) + I^o - \beta R(M_{t-1}^r) + G - Y_c}{\gamma M_{t-1}^r - \beta} .$$

The numerator of this expression is the inflationary gap.* The model is

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* $\beta \dot{p}$ does not have to be subtracted from the numerator of (4.16) in order to obtain the gap as it is again defined as the excess of spending that would take place if the speed of inflation were zero over actual capacity.

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of interest only when the denominator $\gamma M_{t-1}^r - \beta \geqslant 0$; otherwise, either the inflationary gap or the speed of inflation is negative. The speed of inflation for this model is equal to the size of the gap divided by the sum $\gamma M_{t-1}^r - \beta$. In contrast, equation (4.11) revealed that for the earlier model involving inflation illusion the speed of inflation was simply the gap divided by the capacity level of output.

The effect of an increase in government expenditure upon the speed of inflation, given the money supply, is easily obtained by differentiation of equation (4.16):**

$$(4.17) \qquad \frac{\partial \dot{P}_t}{\partial G} = \frac{1}{\gamma M_{t-1}^r - \beta} > 0$$

** We again assume that the increase in government spending is financed by neutral means that do not affect the behavior of the system. See p.16 , fn. above.

We also have:

$$(4.18) \qquad \frac{\partial \dot{P}_t}{\partial G \partial M^r_{t-1}} = \frac{-\gamma}{(\gamma M^r_{t-1} - \beta)^2} < 0 \ .$$

Differentiating with respect to the real money supply yields:

$$(4.19) \qquad \frac{\partial \dot{P}_t}{\partial M^r_{t-1}} = \frac{-\beta R'(M^r_{t-1})}{\gamma M^r_{t-1} - \beta} - \frac{\gamma[C+G+I^o-\beta R(M^r_{t-1})-Y_c]}{(\gamma M^r_{t-1} - \beta)^2} = \frac{-\beta R'(M^r_{t-1})-\gamma \dot{P}_t}{\gamma M^r_{t-1} - \beta} \ .$$

The first of the two terms in this derivative arises from the tendency for a larger money supply to reduce the rate of interest, encourage investment, and thus contribute to more rapidly rising prices. The second term, on the other hand, reflects the tendency for a larger money supply to be associated with larger capital losses. The relationship between the real stock of money and the speed of inflation will be positive in sign only if the first of these effects outweighs the second. Two special cases definitely imply a negative relation. If the volume of investment is insensitive to fluctuations in the interest rate, $\beta = 0$, the first term of (4.19) is zero, and $\frac{\partial \dot{P}_t}{\partial M^r_{t-1}} \leq 0$. In fact, the elasticity of the response of the speed of inflation to changes in the money supply, $\frac{\partial \dot{P}_t}{\partial M^r_{t-1}} \frac{M^r_{t-1}}{\dot{P}_t}$, is minus one. As a second special case, suppose the economy were in the liquidity trap: $R'(M^r_t) = 0$ would again imply a negative impact upon the speed of inflation of a larger money supply, but with an elasticity that is less than minus one.

The two models considered in this section constitute only a rough sampling from the host of dynamic complications that might be appropriately introduced into a static Keynesian framework for the purpose of analyzing demand-pull inflation. No attempt at an exhaustive coverage is attempted. It is clear that the elements that have been considered could have been combined in many alternative arrangements. The inflation illusion and the capital-loss models could be united into still a third model. But it hardly seems necessary to work out the details of such analysis. The important thing to observe is that it is conceivable for the speed of inflation to be either positively or negatively related to the quantity of money. In the next section we spell out the implications of the direction of this dependence for the process of inflation and the policy of forced saving. The actual direction of dependence and the dynamic elements of real importance could only be determined by detailed empirical investigation of the relation in past inflations* or a

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* The only empirical analysis related to this question is a brilliant study by Philip Cagan [2] suggesting that the size of real money balances in situations of hyper-inflation is negatively related to an exponentially moving average of price changes. The distributed lag nature of the relationship may result, as Cagan himself admits, from lags involved in the adjustment of payment procedures to changing costs of liquidity as the speed of inflation changes [1, pp. 74-5]. Cagan's evidence is far from conclusive because his study neglected the effects of government spending, because of a number of qualifications that he himself mentions [1, pp. 51-64], and because his empirical results may actually reflect the liquidity preference relation rather than the nature of the dependence of the speed of inflation upon the real money supply.

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study of the dependence of the consumption, investment, and government spending components of total spending upon the rate of price change.**

** Arthur Okun's investigation [17, Ch. 3] casts serious doubt upon the empirical validity of the lagged adjustment type of behavior underlying equation (4.2). I know of no empirical study of the capital loss adjustment effects of inflation upon consumption behavior, although the vast literature on the Modigliani-Brumberg and Friedman hypothesis is, at least tangentially, related to the problem.

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5. Forced Saving Equilibrium:

A richer if more complicated analysis of the impact of fiscal and monetary policy upon a fully employed economy is permitted with the dynamic apparatus developed in Part 4. Static analysis sufficed to reveal the existence of a new equilibrium price level when a continued injection of government expenditure that disturbs a full-employment equilibrium is financed in a neutral fashion that does not alter the money supply. The case of a one shot injection of new money could also be handled with static tools. But suppose government expenditure is not only maintained at the higher level; suppose it is financed, at least in part, by a continuous expansion of the nominal money supply. This, the most interesting case of "forced saving," is to be examined. Forced saving is an intriguing doctrine of long history. Hayek [9] traces the concept to John Stuart Mill, even Bentham. But we shall analyze the phenomenon within the framework of a dynamic, Keynesian model of inflation. It will be shown that in a certain sense the quantity theory of money still holds as a description of equilibrium, but not in a way that is useful for policy formulation.

Suppose that a fully employed economy is disturbed by an increase in the real value of government expenditure to a higher level G. In addition, suppose that the government spending is to be financed, at least in part, by an expansion of the money supply. Let δ be the coefficient relating the expansion of the money supply to the level of government spending. Either directly, through the printing of new money, or indirectly, through an enlargement of the reserves of the banking system, $p_t \delta G$ new dollars are to be pumped into the money supply each period. We wish to examine the dependence of the rate of price change upon δ and G, the two control variables that characterize a policy of forced saving.

Under a forced savings policy, the real value of the money supply is influenced by the extent to which the injection of new money is offset in each period by depreciation of the existing money stock as a result of inflation. If M^r_{t-1} was the real value of the money supply in period $t-1$, its nominal value must have been $p_{t-1}M^r_{t-1}$. Consequently, the purchasing power at time t of currency already in circulation will be $\left(\frac{p_{t-1}}{p_t}\right) M^r_{t-1}$. Adding to this the new injection, we obtain an expression for the total real money supply at time period t :

$$(5.1) \qquad M^r_t = \left(\frac{p_{t-1}}{p_t}\right) M^r_{t-1} + \delta G .$$

Subtracting M^r_{t-1} from both sides of the equation reveals the change in the real value of the money supply:

$$(5.2) \qquad \Delta M^r_t = M^r_t - M^r_{t-1} = \delta G - \dot{P}_t \, M^r_{t-1} \text{ , where } \dot{P}_t = \frac{p_t - p_{t-1}}{p_t} .$$

Time may be measured in continuous as well as discrete units. In order to develop the differential equation counterpart of (5.2) observe that $M(t) = p(t)\, M^r(t)$ by definition as well as $\frac{\partial M(t)}{\partial t} = p(t)\, \delta\, G$ by assumption. Differentiate the first expression and equate it with the second to obtain: $\frac{\partial M(t)}{\partial t} = \frac{\partial p}{\partial t} \cdot M^r(t) + p(t) \frac{\partial M^r(t)}{\partial t} = p(t)\, \delta\, G$. This implies:

$$(5.2^*) \qquad \frac{\partial M^r(t)}{dt} = \delta G - \dot{p}(t)\, M^r(t) \text{ , where } \dot{p}(t) = \frac{dp}{dt}\Big/ p(t) .$$

Depending upon the rate of price change and the extent of the injection of new money, the real value of the money supply may either rise or fall through time. It is conceivable that the injection of new money may be precisely offset by inflation, leaving the real value of the money supply unchanged. This indeed proves to be characteristic of forced saving equilibrium.

Equation (5.2) does not suffice to determine the behavior of the real value of the money supply. The path of the price index must also be specified. In Part 4 we analyzed the dynamics of price behavior. Equation (4.11) explained the current rate of price change in terms of the real money supply, the level of government expenditure, and certain parameters characterizing inflation illusion behavior. Equation (4.16) was derived with an alternative argument involving a capital-loss adjustment factor but free of elements of inflation illusion. If either of these two equations is combined with function (5.2) describing the behavior of the money supply under a forced saving policy the path of the price level is determined for specified initial conditions and policy parameters G and δ . Although the equation system is non-linear, it is, fortunately, of the first order; consequently, certain general characteristics of its solution may be easily specified. Only the differential equation version of the argument will be considered; time will be assumed to be continuous.

Consider once more the relationship between the rate of price change and the real money supply. From equation (5.2*) we can determine for any given level of the policy variables δG a relationship that must be satisfied if the real value of the money supply is to remain stable. If $\frac{\partial M_1^r}{\partial t} = 0$, we must have:

$$(5.3) \qquad \dot{p}(t)\, M^r(t) = \delta G \ .$$

This relation between $\dot{p}(t)$ and $M^r(t)$ is represented by the rectangular hyperbola on Figure 5.1. Combinations of $M^r(t)$ and $\dot{p}(t)$ below the hyperbola imply an expanding money supply. Conversely, if today's money supply and speed of inflation are represented by a point above the hyperbola, the money supply is contracting. The curve summarizes, then, the implications

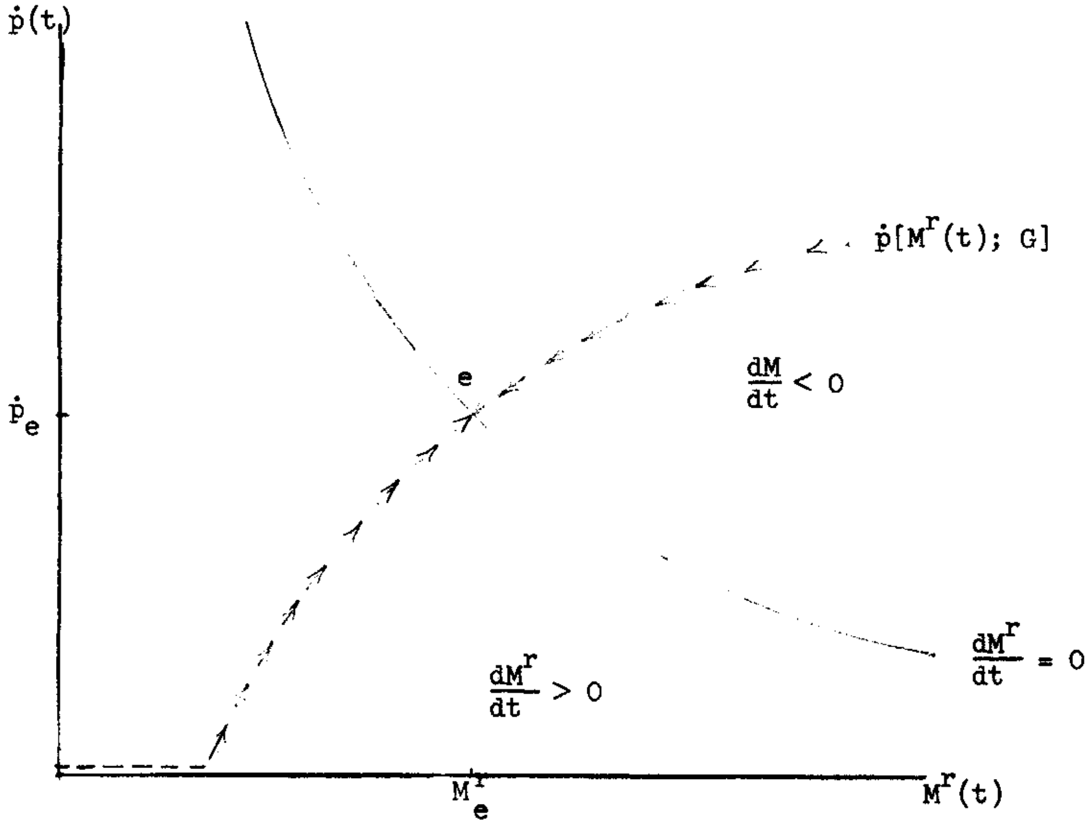

Figure 5.1

for the money supply of the specified forced saving policy. For higher values of δG the hyperbola would be shifted outwards in the northeast direction.*

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* If $\delta = 0$, the case analyzed in Part 4, $\dot{p} > 0$ implies a contracting money supply.

Let us first suppose that the speed of inflation is an increasing function of the money supply. This is necessarily the case for the inflation illusion behavior underlying equation (4.11). It is a possibility for the capital-loss adjustment type of behavior explained in the derivation of equation (4.16). The positively sloped line on Figure 5.1 labeled $\dot{p}[M(t); G]$ represents the relationship.** An increase in the level of government expenditure causes

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** Needless to say, the speed of inflation depends upon the level of taxes and their structure as well as the money supply and volume of government spending. But for purposes of exposition it will suffice to present the argument for the case of neutral tax effects; the complications of changes in tax structure concomitant with adjustments in government spending can be safely left to the reader.

an upward shift of this curve. Point e on the graph represents a forced saving equilibrium, but it is a moving equilibrium characterized by a constant speed of inflation rather than a stable price level. The behavioral relationship, $\dot{p}[M(t); G]$, advises us that if the money supply is M_e^r, the speed of inflation will be $\dot{p}_e$. Since point e is also on the rectangular hyperbola, the injection of new money will just cancel the loss from inflation; consequently, the real value of the money supply remains unchanged. This equilibrium necessarily exists and is unique as it is determined by the intersection of a rectangular hyperbola with a non-decreasing function. Under quite general assumptions -- of which equations (4.2), (4.3) and (4.9) constitute a special case -- not only the level of income but also its allocation between consumption, investment and government spending will be uniquely determined in this condition of forced savings equilibrium for given values of δ and G.

The situation is quite different when the speed of inflation is negatively related to the money supply, as with the capital-loss adjustment model summarized by equation (4.16). No equilibrium necessarily exists; multiple equilibria are a possibility. Figure 5.2 reveals certain possibilities. The rectangular hyperbola again represents those points corresponding to an unchanging real money supply. If the speed of inflation function is of the form represented by the dotted curve $\dot{p}_\alpha$, the money supply will always contract; the arrows indicate the direction of movement of prices and the money supply through time; for this curve, forced savings equilibrium is not a possibility. Speed of inflation curves $\dot{p}_\beta$ and $\dot{p}_\delta$ each yield a single forced saving equilibrium point. Curve $\dot{p}_\gamma$ provides us with two forced saving equilibria, points u and s.

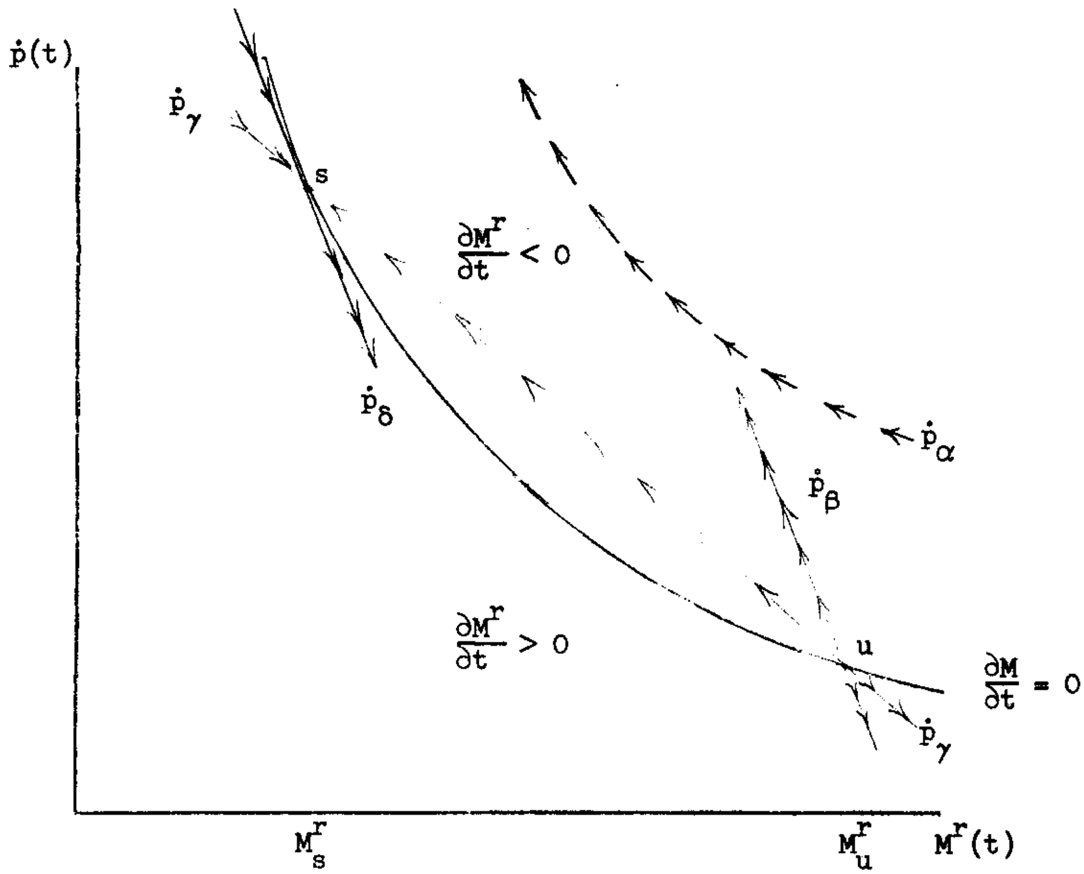

Figure 5.2

For any of the speed of inflation functions graphed on Figure 5.2 the direction of movement of the money supply and the speed of inflation through time can be readily determined. When the speed of inflation curve is above the rectangular hyperbola, the money supply is contracting, and movement is to the left. Conversely, points below the hyperbola correspond to an expanding money supply and movement is to the right. The arrows on the various speed of inflation functions on Figure 5.2 indicate the direction of movement. They can be read in order to determine which of the forced saving equilibria are stable. All equilibria at point u are unstable. On the other hand, curve $\dot{p}_\gamma$ has a stable equilibrium at s . Observe that for curve $\dot{p}_\delta$, the equilibrium at s is stable from the left but not from the right. The conditions for stability can be formalized if it is remembered that the phase line of a first order linear system must have a negative slope at a stable equilibrium.* Differentiating (5.2*) yields as the condition for stability:

$$(5.4) \qquad \frac{\partial^2 M^r(t)}{\partial t \partial M_r(t)} = - \frac{\partial \dot{p}(t)}{\partial M^r(t)} M^r(t) - \dot{p}(t) < 0 \ .$$

But this is equivalent to:

$$(5.5) \qquad \frac{\partial \dot{p}(t)\ M^r(t)}{\partial M^r(t)\ \dot{p}(t)} > -1$$

Stability of forced saving equilibrium requires that the elasticity of the speed of inflation function be greater than minus one.** This condition is

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* Cf. [1, pp. 314-7].

** Cagan's analysis demonstrates that the conditions for stability are weakened when a distributed lag complication is introduced [2, pp. 64-73].

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necessarily satisfied only for the inflation illusion model, not for the model incorporating the capital loss adjustment factor. In particular, the elasticity is precisely minus one if investment is insensitive to changes in the interest rate. If the economy were in the liquidity trap the elasticity would be less than minus one, implying instability.***

*** See p. 34 above for the derivation of these interesting if improbable cases.

A forced saving equilibrium is characterized by a stable real value of the money supply if rising price level. Since real output remains at Y_c, this means that the income velocity of money is constant when an economy is in forced saving equilibrium. Clearly, a stable income velocity of money is consistent with the Keynesian model under these dynamic conditions. But in actual fact, hyperinflations have generally been characterized by an acceleration in the speed of inflation accompanied by a gradual fall in the real value of the money supply and a concomitant increase in the velocity of circulation [2, pp. 25-6]. This type of data might be generated by a Keynesian model under conditions of forced saving even with the positively sloped speed of inflation curve that necessarily exists when only inflation illusion is involved. Even if an expanding money supply implies an increasing rate of price climb, given the level of government expenditure, the opposite relation might be observed as a result of upward shifts in government expenditure to successively higher levels. If the injection of new money were constant in real terms, δ falling as G increased, we might observe a succession of points close to the rectangular hyperbola of Figure 5.1. But if there is a negative relationship between the speed of inflation and the money supply, the declining real value of money characteristic of hyperinflations is more readily explained. It is not necessary to invoke changes in the policy variable to explain the observed behavior if the relationship between the money supply and the rate of price rise is given by curve $\dot{p}_\alpha$ of Figure 5.2. The essential characteristics of hyperinflation would again be observed if curve $\dot{p}_\beta$ represented the speed of inflation function but the money supply were less than M_u, or in the central portion of curve $\dot{p}_\gamma$. While the accelerating speed of inflation and declining real money supply generally experienced in hyperinflation are compatible with either a positive or negative relationship between the speed of inflation and the money supply, the data is most easily explained by the negative relationship that can occur only with capital loss adjustment behavior.

In implementing a policy of forced saving as a means of financing government expenditure, the government is presumedly interested in curtailing both the tax burden and the speed of inflation. Given these preferences, the appropriate policy in determining the values of the two policy variables δ and G depends upon whether the speed of inflation curve has a positive or negative slope. We will first evaluate the problem of selecting the appropriate value of δ for given government expenditure G when a positive relationship exists between the money supply and the speed of inflation and then turn to the more complicated case in which the speed of inflation function has a negative slope.

Examination of Figure 5.1 will quickly reveal the relevant issues involved when the speed of inflation function has a positive slope. Given the level of government spending, the larger the proportion of government spending financed by issuing currency, the larger the δ coefficient, the higher the rectangular hyperbola representing points of unchanging money supply. Consequently, an increase in δ means a higher equilibrium speed of inflation, an equilibrium that is stable. The rational policy maker selects the appropriate value of δ for given G on the basis of a comparison of the disutilities of inflation versus the political complications of harsher tax laws.

Figure 5.3 is of assistance in evaluating the problem faced by the policy maker in determining the appropriate injection of real dollars to be made each period in financing government expenditure when the rate of price rise is negatively related to the money supply. The line $\dot{p}_\alpha$ representing the speed of inflation function is drawn with an elasticity greater than minus one throughout its length. Point s represents a unique, stable equilibrium. But this is a non-optimal policy, for if a larger proportion of government spending were financed by printing money, say $\delta' > \delta$, the rectangular hyperbola corresponding to a stable money supply would shift outwards to the northeast, leading to a new equilibrium at point s'. Increasing the proportion

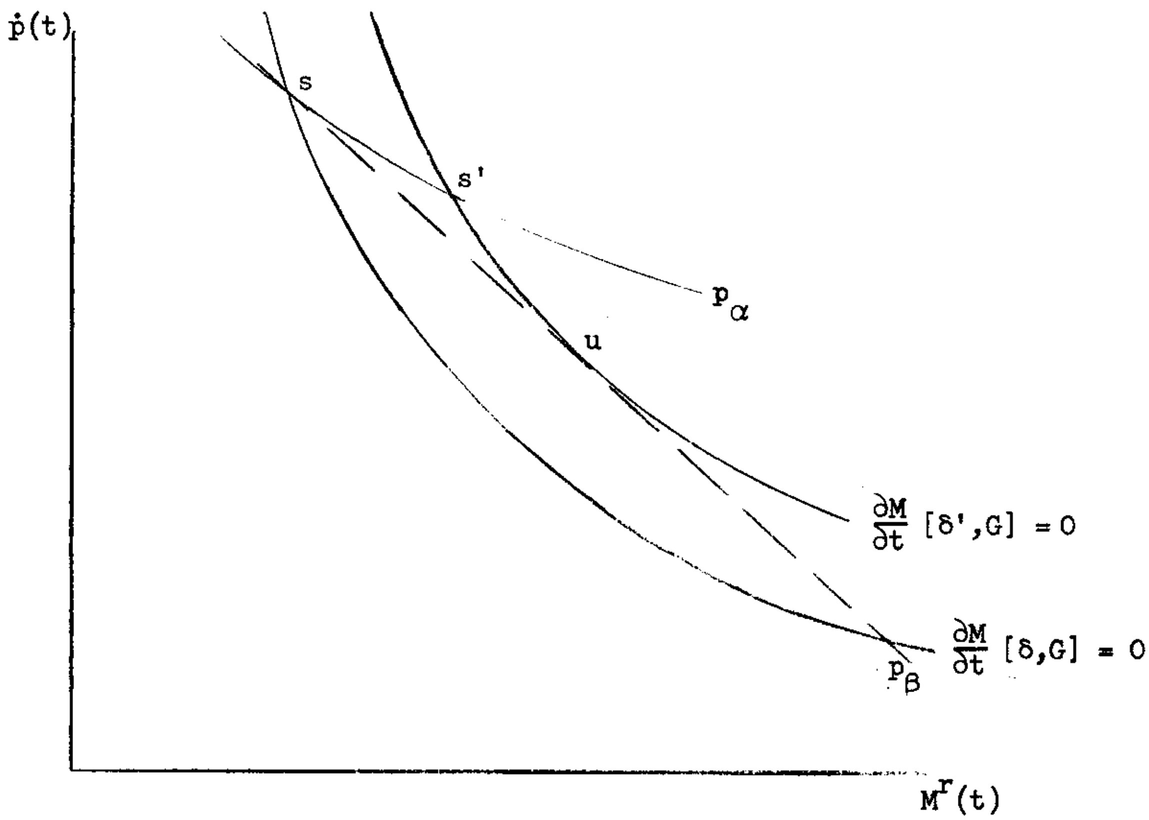
$\dot{p}(t)$
s
s'
p_α
u
$\frac{\partial M}{\partial t}$ [δ',G] = 0
$\frac{\partial M}{\partial t}$ [δ,G] = 0
p_β
$M^r(t)$

Figure 5.3

of government spending financed by printing money has the unexpected boon of reducing the speed of inflation. Suppose the relationship between the rate of price rise and the money supply is more nearly vertical, as with dotted line p_β . If the economy is initially at point s corresponding to a stable equilibrium, the increase in δ to δ' leads to a new equilibrium, stable from the left at u . This point corresponds to the maximum injection of new money that is compatible with a forced saving equilibrium. But again this is not an optimal point.* A further increase in the proportion of government

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* Cagan concluded [2, pp. 80-1] that if attention is restricted to situations in which the nominal money supply expands at a constant rate, the revenue collected from a forced saving policy is maximized when the elasticity of the function relating the speed of inflation to the real money supply has elasticity of minus one, as at point u . The optimal policy, in fact, need not be characterized by a stable real money supply.

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spending financed by injecting money would lead to a continuing fall in the rate of price rise. We are led to the conclusion that if there is a negative relationship between the speed of inflation and the money supply, the appropriate policy for reducing the rate of price climb is one that contributes to a continuously expanding money supply. The larger γ , the greater the injection of new money into the economy, the more rapid the deceleration of the inflation process.

Theoretical analysis of the dynamics of demand-pull inflation has not provided any simple rule of thumb that may be applied in determining the appropriate monetary policy to be followed in order to minimize the inflationary pressure generated under conditions of forced saving. The consequences of a given forced saving policy depend in a crucial fashion upon whether there is a positive relation between the speed of inflation and the money supply, as is necessarily the case under inflation illusion behavior, or the negative one possible with the capital-loss adjustment model. By its nature, theoretical analysis can only suggest conceivable modes of behavior.* The theory only

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* We have not spelt out in the text richer behavior patterns conceivable when the argument is formulated in terms of difference rather than differential equations. When the discussion is developed in terms of a first order non-linear difference equation, stable oscillations of varying period and explosive cycles become theoretical possibilities. A perverse example demonstrates the possibility of oscillations. Suppose that $G = 10$ billion and $\delta = 0.4$. In addition, suppose that $M_{t-1} = \$20$ billion yields in terms of equation (4.11) a speed of inflation of 0.05. Equation (5.2) then yields $M_t = \$23$ billion. Suppose that equation (4.11) then yields $\dot{p}_{t+1} = 7/23 = 0.3$. Equation (5.2) now yields a reduction in the money supply of \$3 billion. The money supply returns to \$20 billion. This limit-cycle repeats itself again and again; the system never converges to equilibrium. Baumol describes a graphical technique for the analysis of non-linear first order difference equations [1, pp. 258-65].

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reveals what empirical issues are of importance. The appropriate monetary policy could only be determined on the basis of a detailed, empirical investigation.

6. Summary: Forced Saving and the Quantity Theory

The nature of demand-pull inflation in an economy satisfying essentially Keynesian assumptions may be conveniently summarized by contrasting the implications of the preceding analysis with the predictions of the quantity theory. A leading advocate of the quantity theory, Professor Milton Friedman of the Chicago School, has stated [5, p. 20]:

> ...there is perhaps no other empirical relation in economics that has been observed to recur so uniformly under so wide a variety of circumstances as the relation between substantial changes over short periods in the stock of money and in prices; the one is invariably linked with the other and is in the same direction; this uniformity is, I suspect, of the same order as many of the uniformities that form the basis of the physical sciences. And the uniformity is in more than direction. There is an extraordinary empirical stability and regularity to such magnitudes as income velocity that cannot but impress anyone who works extensively with monetary data.

Under certain conditions, the income velocity of money may be stable even in a Keynesian environment. No change in the income velocity of money would be revealed by a before and after examination of the effects of an inflationary gap generated by a one shot injection of new money at full employment. Of course, a fall in income velocity would be revealed by a comparative statics investigation of the effect of the injection of a continued stream of government expenditure into a fully employed economy financed by some neutral means. But if a policy of forced saving is being followed, if a continued injection of real government spending is financed at least in part by creating money, a new equilibrium defined by fixed levels of consumption and investment spending would be characterized by an unchanging income velocity of money but a constantly rising price level.

It must be stressed that no empirical evidence concerning the validity of the quantity theory underlies the argument. Only the difficulty of such empirical investigation is insisted upon. Even if a stable income velocity of money were observed, this would not in itself refute the Keynesian analysis of unemployment and inflation. It would not guarantee that the quantity theory would be an appropriate guide in the planning of monetary reform or in appraising the consequences of a forced saving policy.

In a recent study of inflation, Richard T. Selden considered the appropriateness of a Keynesian rather than a quantity theory framework for his analysis. He concluded that "for an *ex post* study...the equation of exchange is much the more valuable of the two" on the basis of the following argument [23, p. 5]:

> Since, in the Keynesian system, the division of GNP among consumption, investment, government and net foreign purchases *usually* is not considered a function of the price level, the system provides no way of determining the source of an inflation.

The argument of this paper reveals that if traditional behavioral equations of the Keynesian system are utilized in the static analysis of inflation, a most important role must be assigned to the price level.

Analysis of the dynamics of demand-pull inflation revealed that monetary complications are important, even in a Keynesian environment. The very process of inflation, by reducing the real value of the money supply, affects the size of the inflationary gap. It has been demonstrated that in a Keynesian economy the all too common practice of financing a portion of budget deficits by creating money may, under certain conditions, lead to a forced saving equilibrium characterized by a constant speed of inflation and a stable real money supply. But this does not imply that monetary factors may be suppressed in the analysis of forced savings. It does not serve to vindicate those post-Keynesian arguments that neglect monetary complications in the analysis of the dynamics of the inflationary gap. Under a forced saving policy, two policy variables govern the speed of inflation. In analyzing forced saving policy, it is necessary to consider the extent to which the government's budgetary demands are met by expanding the money supply as well as the actual level of government expenditure. The real value of the money supply, as well as the speed of inflation, depends upon these two policy variables.

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