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Theory of an Efficient Several-Person Firm*

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by
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The firm of the classical theory is managed by a single person, the entrepreneur; he is certain of the future, and is efficient in the special sense that he chooses plans maximizing his profit. More recent work studies a firm that is led by one or more persons; that may operate under uncertainty; and that may or may not be efficient.

Efficiency. In one respect, my exposition will not be quite so general. For I shall confine myself to the efficient firm, although in a broader sense: the firm's behavior will be interpreted as if it consistently pursued a goal; but this goal need not be maximum profit.

Our chairman today, Richard Cyert, is one of those who have searched for significant theorems about inefficient behavior of firms. I submit that the analysis of efficient business firms, while less general and less closely portraying reality, is also useful, for two reasons:

1) it may well be that, on the whole, the efficiency assumption is not a bad approximation, especially for organizations that have survived over a long time; they have behaved as if they had pursued the goal of maximizing the chance of survival (not the profit);

2) we are often asked, not to describe how badly business people have solved their problems in the past, but to find the solution of a business problem.

Subjective probabilities and utilities. Strictly speaking, the manager of the classical theory, if he did not know the future with certainty, knew the probability distribution characterizing it; and he computed and maximized the

actuarial value (the mathematical expectation) of profit accordingly. When Frank Knight showed that relevant future events are, in the main, not repetitive, it was concluded that the theory of probability is of no interest to the theory of the firm. This made the analysis of decisions somewhat inarticulate. Yet in recent years the practical needs of business and military decision makers made it necessary to take a second look. Modern statisticians, asked to advise on action without being able to collect large samples, have approached the problem in the economist's spirit, as one of efficient behavior. Somewhat simplifying we can say that the "personalistic" view of L. J. Savage and others has revived the concepts and behavior postulates made in the 18th century by Thomas Bayes, a founder of the theory of probability, thus:

1) If, for whatever reason, you bet 9 shillings against 1 on one of two alternative events, you behave as if you were assigning at least .9 probability to one of them, and were choosing that action (viz., to bet rather than not to bet) which maximizes the actuarial value of your gain;

2) if the trials are repeated, you will achieve maximum actuarial gain by raising, after each trial, the odds for that event which has just occurred.

The definition of ("subjective") probabilities given in the first clause reminds the economist of the definition of the consumer's utilities: only if the decision maker is consistent do those numbers, or ranks, exist, i.e., can be ascertained from his behavior. And because of the second clause, the subjective probabilities will approach the relative frequencies, and hence approach the "objective" probabilities as repetitions become more numerous. Knight's "risk," the case when probabilities are known to the decision maker, is thus a limiting case.

The main step beyond Bayes made by modern thinkers (beginning with F. P. Ramsey) consists in replacing money by utility. This brings us, in fact, back to Marshall's view of gambling, but with probabilities re-interpreted subjectively. If business is gambling (as Mr. Baruch asserted against squeamish Mr. Morgan), it is less like roulette than like betting on horses. The efficient man behaves as if there existed two sets of numbers called, respectively, utilities (attached to the states of decision maker) and probabilities (attached to the states of the world), whose sum of products (the expected utility) he maximizes. This takes care of the manager who is cautious in the sense that he assigns an almost infinite negative utility to bankruptcy; or who is interested, not only in profits and losses, but also in power or status.

By incorporating subjective probabilities, economic theory of choice becomes a theory, not only of consistent tastes, but also of consistent beliefs. On this basis, a large number of business problems have been submitted to analysis: inventory control, production planning, portfolio selection, quality control. Let me point out some important concepts and relations fundamental to these studies and to the theory of the several-person firm, and not so clearly perceived in older theory.

Payoff function. The action of the decision maker, given the state of the outside world, determines his future state and thus also (depending on the particular criterion applied: profit, sales, status, survival, or what not; or any combination of these) the utility, or payoff, that the action will achieve. The table showing the payoff for each action at each state of the world is called payoff function. If, for example, a competitive firm is judged by the simple

criterion of profits, the payoff function coincides with the classical profit function-revenue minus cost, which essentially reflects the technology of production. The "actions" are, in this case, the inputs of raw materials, labor, etc.; and the product and input prices constitute the "state of the world."

Decision rule. Under uncertainty, the firm has to determine, not an optimal action but an optimal decision rule. The rule tells how to adjust action to information. To be sure, in the extreme case of uncertainty, when the man cannot learn anything beyond the probability distribution he believes in, he can do no **better** than fix some optimal routine, a constant action that maximizes expected payoff. For example, the expected profit of a firm may be maximized by fixing output so that, on the average, marginal revenue and cost are equal, the actual prices being unknown. In general, uncertainty is not that extreme. The firm can adjust its action to varying information even though information is not a complete or precise statement about the true state of the world (but may merely help to estimate it). An optimal decision rule calls for an action that maximizes the expected conditional payoff, given the information; for then the absolute (nonconditional) expected payoff will be, in the long run, higher than if information were ignored. In the previous example, the optimal decision rule would become: "given the information, choose an input level at which the conditionally expected marginal cost and revenue are equated."

A particularly important case of partial information was pointed out by Albert Hart, the economist. In general, the firm's profit depends upon a time-sequence of actions; but the firm's best plan is not a time-sequence of actions

but a time-sequence of decision rules, each relating the action at some given time to the partial information then available--viz., the information on what then will be the past and the present but not the future. In the theory of games a sequence of decision rules is called strategy (as distinct from a single action, or move). In statistics it is called sequential decision function. Its name in the current literature on operations research is: dynamic program. For brevity we shall use the term decision rule (in singular) to denote the whole sequence of such rules.

Information cost. Information is not costless. A firm which, not contented with a rough idea of averages, bases its decision on a very close continual study of various markets, has to pay for it. The closer is the information to the true state of the world, the better will be the best action chosen, in the sense of a higher expected utility of the outcome. That this gain may be offset by the cost of gathering information, was first emphasized in statistical decision theory: large samples are expensive. Similarly all research activities of a firm, and also its internal communications are costly, if only because they claim the manager's limited time.

Information rule. We shall call information rule the schedule that tells, for each state of the world, what the firm will know about it. Suppose the relevant aspect of the world is the set of prices of all raw materials and products of the firm and its competitors. Under one information rule, the firm learns all these prices daily, to the nearest half-cent. Under another information rule, the firm learns some of these prices weekly, others monthly and with less precision,

still others not at all. The former information rule may contribute more to the expected profit, but will also presumably cost more.

Decision cost. Cost is also attached to each decision rule. The processing of information into decision may be a difficult mental task; it is the more costly the larger its claims on the available capacity of the manager.

Organizational form. We shall define organizational form as a pair of two rules: the decision and the information rule. A schedule showing the cost of each organizational form may be called the organizational cost function.

If information and decision were costless, the efficient firm's problem would be to find a decision rule that bases action on all available information and maximizes expected utility. Since information and decision are not costless, the problem consists in finding simultaneously the decision rule and the information rule--i.e., in finding the organizational form--that will maximize expected utility net of organizational cost.

The solution of the problem--the optimal organizational form--will depend on the given circumstances. And what are these givens of the problem? They are, of course: the payoff function; the probability distribution of the states of the world; and the organizational cost function. For remember that each information rule translates states of the world into information, and each decision rule translates information into action. Hence each organizational form translates states of the world into action. But the action and the state of the world determine jointly the achieved utility, in a manner described by the payoff function. Therefore, if information and decision were costless, the utility achieved at a

given state of the world would depend on the payoff function and the organizational form. Therefore the expectation of utility, i.e., its average taken over all states of the world would depend on 1) their probabilities, 2) the payoff function, and 3) the organizational form. The net expected utility depends, in addition, on 4) the organizational cost function. This cost function, the payoff function, and the probability distribution are not under the firm's control. Given these three out of the four factors determining the net expected payoff, the firm can choose the fourth--the organizational form--so as to maximize the net expected payoff. (See Figure 1.)²⁾

Illustrations. To illustrate the effect of the probability distribution on the optimal organizational form: Suppose an external variable is subject only to small variations; then it may not pay to get information about them, and to adjust one's actions to this information. Or suppose two variables are strongly correlated; then it may suffice to get information about one of them.

To illustrate the effect of the payoff function: Suppose two inputs are mutually strong complements, i.e., the marginal payoff due to one of them strongly depends on the amount of the other. Then the firm benefits from knowing the variations in the prices of both; it will increase one input not only whenever the price of that input falls but also whenever the price of its complement falls (or that of its substitute rises).

To illustrate the effect of organizational cost: If it's large, the firm may prefer to pursue "routines" policy, and not to be kept informed about the variations of any of the external variables.

Several-person firm. How is the problem modified if the firm consists of several--say, n --decision makers (executives)? Each of them decides about different things, and on the basis of different information. Our concept of organizational form has to be generalized. There are now n information rules, each translating the true state of the world into the information available to a different executive; and n decision rules, each translating the information of a given executive into his action. The set of n information rules--"who learns what?"--is generated by a communication network and the rules of operating it: "who talks to whom and when?" Again, decision and information cost will mainly consist of claims on the executives' time. It is mostly a fixed cost inasmuch as these persons are usually on long-term contracts.

The problem of the optimal network of communication and optimal rules to operate it may require difficult analysis in any particular case. Economic theory can do no more than establish some general results. Even these are, so far, fragmentary rather than systematic.

Suppose an outsider wants to organize or reorganize a firm, according to his own criteria and beliefs. How will his choice of organizational form depend on his views of the payoff function, the probability distribution and the schedule of organizational costs? How do these factors affect the need for more or less communication between executives?

Properties of payoff function: Complementarity. The allotted roles of two executives may be such that, with communication, they can increase each other's effectiveness; without communication, they may step on each other's toes. At first sight, there seems to be more complementarity, and hence more need for

communication, when the different operations must be performed in succession-- e.g., along a conveyor, or at successive stops of an airline (studied by M. Beckmann)--than in the case of "parallel coupling" as among branch managers of a hotel chain. However, simultaneous operations may also imply high complementarity, if the branches have to compete for a limited capacity of some central facility: as with the salesmen of a bakery (studied by C. B. McGuire). Finally, a special case (emphasized in the theory of non-constant-sum games) occurs when the payoff function has several maxima: e.g., two or more timetables are often equally good, but some "coordinator" has to choose one.

Person-by-person maximization. In the case of some payoff functions, the maximizing decision rules can be found by step-wise approximations, person by person. This is true of smooth payoff functions beloved of classical economic theory: the summit of a smooth hill can be reached step-wise by moving due North and stopping at the highest point on that route; moving due East from there, and finding the highest point on that route; moving North from there, etc. Yet the hill representing the payoff function may have a "ridge," as when, e.g., the sales of a nonstorable product are proportional to production or to demand, whichever is smaller; alternate adjustments by the manager of production and that of promotion will lead to one of the many points where production equals demand, but this need not be the highest one.

Properties of the probability distribution. We have already mentioned, for the single-person case, the effect of variances and of correlations. The extension is obvious. As another example, suppose the branch manager has power to decide on his own, except in emergencies: e.g., a bank branch manager can grant a

loan only below a certain limit. Clearly, the optimal limit will depend both on the relative cost of the central and local officers' time, and on the probabilities of applications for loans of various amounts.

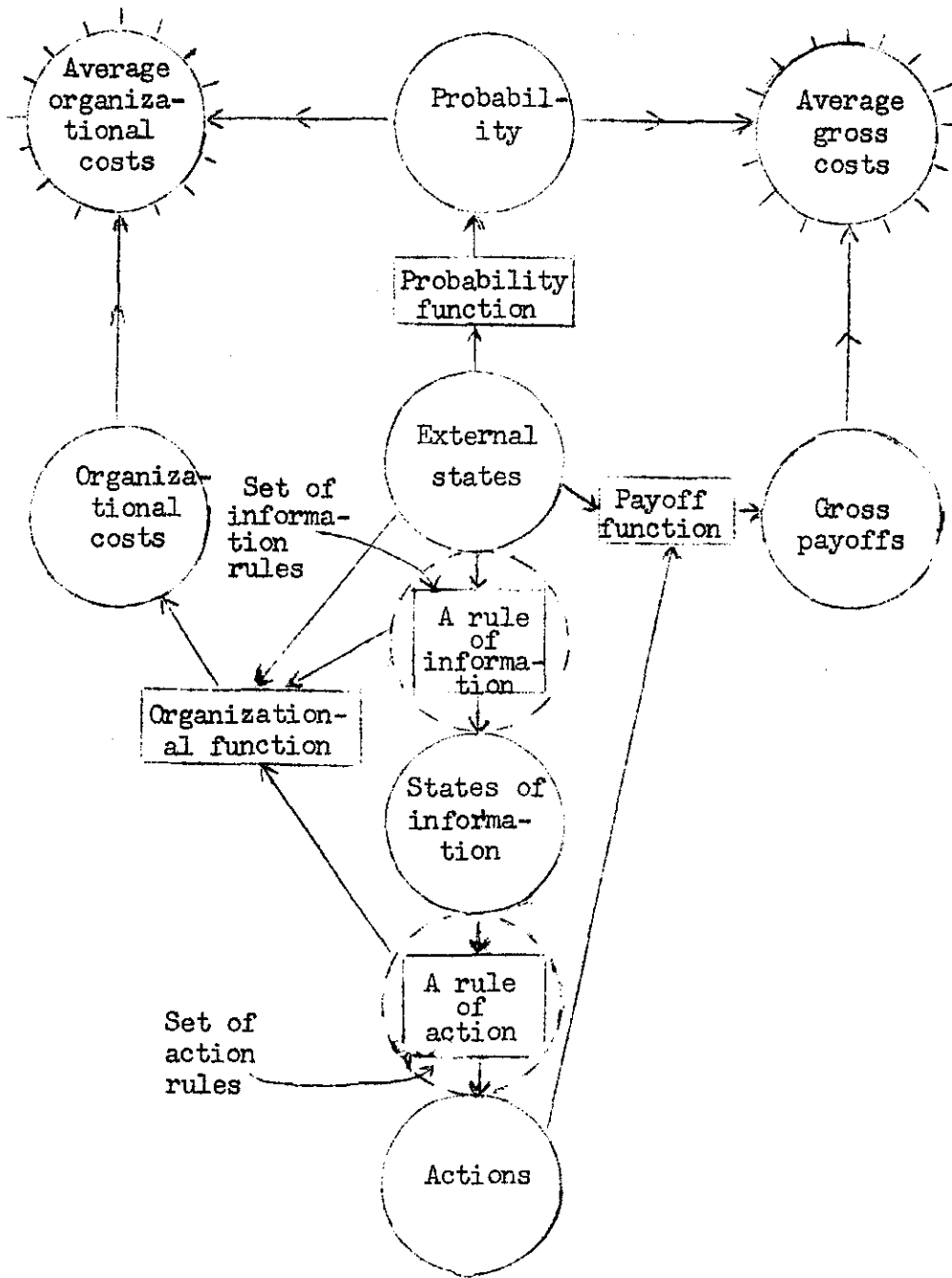
Incentives and leadership. So far, we discussed the goodness of alternative organizational forms chosen on the basis of the goals and beliefs of some outsider: an "organizer," a management consultant. The goals and the beliefs of the several executives themselves will, in general, differ. E.g., the goal of the owner-manager is not that of his officers. Goal divergence is diminished by appropriate incentives--positive such as a bonus, or negative such as the threat of dismissal. A decision-rule takes the form: "upon receiving information proceed so as to maximize the actuarial value of utility to yourself, using as much additional information as you possess or can gather on your own." But even a complete identification of goals would not make beliefs identical. And an action optimal when the actuarial values are computed on the basis of one set of probabilities is not optimal under another such set. A leader is that member of the organization who imposes his goals and beliefs on the choice of the organizational form; he does so by setting incentives and thus controlling the actions of other members. (A whole team of leaders is, of course, thinkable.)

Let me conclude with this very sketchy challenge to a future economics of leadership, a necessary part of economics of the several-person firm, and more generally, of the economics of organizational forms.

Footnotes

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2) Taken from the author's "Efficient and Viable Organizational Forms," Modern Organization Theory, ed. Mason Haire, Wiley, 1959, pages 307-320.



Circles are sets (variables)
 Boxes are operators (functions)
 Dotted circles, () are sets of controlled
 operators which can be chosen so as to
 maximize the net expected payoff, i.e.,
 radiant circles ☼

Figure 1. Determination of average gross payoff and average organizational costs