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FINANCIAL INTERMEDIARIES AND THE EFFECTIVENESS OF MONETARY CONTROLS

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Does the existence of uncontrolled financial intermediaries vitiate monetary control? What would be the consequences of subjecting these intermediaries to reserve requirements or to interest rate ceilings?

This paper is addressed to these questions, but it treats them theoretically and at a high level of abstraction. The method is to set up models of general equilibrium in financial and capital markets and to trace in these models the effects of monetary controls and of structural changes. Equilibrium in these models is an equilibrium of stocks and balance sheets -- a situation in which both the public and the financial institutions are content with their portfolios of assets and debts, and the demand to hold each asset is just equal to the stock supply. This approach has obvious limitations, among which the most important is probably that it has nothing to say about speeds of adjustment and other dynamic effects of crucial practical importance. On the other hand, monetary discussion has long suffered from being a set of *ad hoc*, partial, dynamic observations and assertions without solid foundation in any theory of general financial equilibrium. We feel that we can advance the discussion by outlining a systematic scheme for comparative static analysis of some of the questions at issue.[1]

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[1] This paper is based on work by both authors. Some of its topics were treated in a preliminary way in a Cowles Foundation Discussion Paper (No. 63, January 1958, mimeographed) of the same title, by James Tobin. The general approach of that paper was elaborated and extended in a systematic way by William Brainard in his Yale doctoral dissertation, Financial Intermediaries and a Theory of Monetary Control, submitted in 1962. Some of the results obtained in the dissertation are used in this paper.

The models discussed in the text are simple ones, designed to bring out the main points with few enough assets and interest rates so that graphical and verbal exposition can be used. The Appendix gives both the mathematical analysis of these simple models and their extension to models containing many financial intermediaries, assets, markets, and rates. The exposition in the text takes advantage of the fact that introducing non-bank financial intermediaries, uncontrolled or controlled, into a system in which banks are under effective monetary control presents essentially the same problems as introducing commercial banks as an intermediary, uncontrolled or controlled, into a system in which the government's essential control is the supply of its own currency. The analysis therefore centers on the more primitive question: the effects of financial intermediation by banks, the consequences of leaving their operation unregulated, and the effects of regulating them in various ways. The conclusions have some interest in themselves, in clarifying the functions of reserve and rate controls on commercial banks. By analogy they also bear on questions concerning the extension of such controls to other financial intermediaries.

The main conclusions can be briefly stated. The presence of banks, even if they were uncontrolled, does not mean that monetary control through the supply of currency has no effect on the economy. Nor does the presence of non-bank intermediaries mean that monetary control through commercial banks is an empty gesture. Even if increases in the assets and liabilities of uncontrolled intermediaries wholly offset enforced reductions in the supplies of controlled monetary assets, even if monetary expansion means equivalent contraction by uncontrolled intermediaries, monetary controls can still be effective. However, substitutions of this kind do diminish their effectiveness; for example, a billion dollar change in the supply of currency and bank reserves would have more effect on the economy if such substitutions were prevented.

Whether it is important that monetary controls be more effective in this sense is another question, to which this paper is not addressed. When a given remedial effect can be achieved either by a small dose of strong medicine or a large dose of weak medicine, it is not obvious that the small dose is preferable. Increasing the responsiveness of the system to instruments of control may also increase its sensitivity to random exogenous shocks.[2/] Furthermore, extension

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2/ The balancing of these considerations, and the desirability of finding structural changes which increase the first kind of responsiveness without increasing the second, are discussed in the Brainard dissertation cited above.

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of controls over financial intermediaries and markets involves considerations beyond those of economic stabilization; it raises also questions of equity, allocative efficiency, and the scope of governmental authority.

1. The nature of financial intermediaries.

The essential function of banks and other financial intermediaries is to satisfy simultaneously the portfolio preferences of two types of individuals or firms. On one side are borrowers, who wish to expand their holdings of real assets -- inventories, residential real estate, productive plant and equipment, etc. -- beyond the limits of their own net worth. On the other side are lenders, who wish to hold part or all of their net worth in assets of stable money value with negligible risk of default. The assets of financial intermediaries are obligations of the borrowers -- promissory notes, bonds, mortgages. The liabilities of financial intermediaries are the assets of the lenders -- bank deposits, savings and loan shares, insurance policies, pension rights.

Financial intermediaries assume liabilities of smaller default risk and greater predictability of value than their assets. The principal kinds of institutions take on liabilities of greater liquidity too; thus bank depositors can require payment on demand, while bank loans become due only on specified dates. The reasons that the intermediation of financial institutions can accomplish these transformations between the nature of the obligation of the borrower and the nature of the asset of the ultimate lender are these: (1) administrative economy and expertise in negotiating, accounting, appraising, and collecting; (2) reduction of risk per dollar of lending by the pooling of independent risks, with respect both to loan default and to deposit withdrawal; (3) governmental guarantees of the liabilities of the institutions and other provisions (bank examination, investment regulations, supervision of insurance companies, last-resort lending) designed to assure the solvency and liquidity of the institution. For these reasons, intermediation permits borrowers who wish to expand their investments in real assets to be accommodated at lower rates and easier terms than if they had to borrow directly from the lenders. If the creditors of financial intermediaries had to hold instead the kinds of obligations that private borrowers are capable of providing, they would certainly insist on higher rates and stricter terms. Therefore, any <u>autonomous</u> increase in the amount of financial intermediation in the economy can be expected to be, ceteris paribus, an expansionary influence. This is true whether the growth occurs in intermediaries with monetary liabilities -- i.e., commercial banks -- or in other intermediaries.

In the interests of concise terminology, _banks_ will refer to commercial banks, and _nonbanks_ to other financial institutions, including savings banks. Moreover, _intermediary_ will refer to an entire species, or industry, of financial institutions. Thus all commercial banks constitute one intermediary, all life insurance companies another, and so on. An _institution_ will mean an individual member of the species, an individual firm in the industry -- a bank, or a life insurance company, or a retirement program.

Financial institutions fall fairly easily into distinct categories, each industry offering a differentiated product to its customers, both lenders and borrowers. From the point of view of lenders, the obligations of the various intermediaries are more or less close, but not perfect, substitutes. For example, savings deposits share most of the attributes of demand deposits; but they are not means of payment, and the institution has the right, seldom exercised, to require notice of withdrawal. Similarly there is differentiation in the kinds of credit offered borrowers. Each intermediary has its specialty-- e.g., the commercial loan for banks, the real estate mortgage for the savings-and-loan association. But the borrowers' market is not completely compartmentalized. The same credit instruments are handled by more than one intermediary, and many borrowers have flexibility in the type of debt they incur. Thus there is some substitutability, in the demand for credit by borrowers, between the assets of the various intermediaries.

There is also product differentiation within intermediaries, between institutions, arising from location, advertising, and the other sources of monopolistic competition. But this is of a smaller order than the differentiation between intermediaries. For present purposes, the products offered by the institutions within a given intermediary can be regarded as homogeneous.

Among financial intermediaries commercial banks occupy a special position both in the attention of economic analysis and as the fulcrum of monetary control. The usual rationale for this special position is that, alone among intermediaries, banks "create" means of payment. This rationale is on its face far from convincing. It is more accurate to attribute the special place of banks among intermediaries to the quantitative restrictions to which banks alone are subjected than to attribute the quantitative restrictions to the special character of bank liabilities. The means-of-payment characteristic of demand deposits is indeed a feature differentiating bank liabilities from those of other intermediaries. Insurance against death is equally a feature differentiating life insurance policies from the obligations of other intermediaries, including banks. It is not obvious that one kind of differentiation should be singled out for special analytical and legal treatment. Like other differentia, the means-of-payment attribute has its price. Savings deposits, for example, are perfect substitutes for demand deposits in every respect except as a medium of exchange. This advantage of checking accounts does not give banks absolute immunity from the competition of savings banks, savings and loan associations, or even their own savings departments. It is a limited advantage that can be, at least in some part for many depositors, overcome by differences in yield.

<u>The substitution assumption</u>. These observations about the nature of financial intermediaries and the imperfect competition among them lead to a basic assumption of the analysis which follows, in the text and in the Appendix. The liabilities of each financial intermediary are considered homogeneous, and their appeal to owners of wealth is described by a single market rate of interest. The portfolios of wealth-owners are made up of currency, real capital, and the liabilities of the various intermediaries. These assets are assumed to be imperfect substitutes for each other in wealth-owners portfolios. That is, an increase in the rate of return on any one asset will lead to an increase in the fraction of wealth held in that asset, and to a decrease or at most no change in the fraction held in every other asset. Similarly, borrowers are assumed to regard loans from various intermediaries as imperfect substitutes. That is--given the profitability of the real investment for which borrowing is undertaken -- an increase in one intermediary lending rate will reduce borrowing from that intermediary and increase, or at least leave unchanged, borrowing from every other source.

2. <u>The criterion of effectiveness of monetary control</u>.

A monetary control can be considered <u>inflationary</u> if it <u>lowers</u> the rate of return on ownership of real capital that the community requires to induce it to hold a given stock of capital, and <u>deflationary</u> if it raises that rate of return. (The words inflationary and deflationary are used merely to indicate the direction of influence; the manner in which the influence is divided between price change and output change depends on aspects of the economic situation that are not relevant here.) The value of the rate of return referred to is a hypothetical one -- the level at which owners of wealth are content to absorb the given stock of capital into their portfolios or balance sheets along with other assets and debts. In full equilibrium, this critical rate of return must equal the expected marginal productivity of the capital stock, which depends technologically on the size of the stock relative to expected levels of output and employment. If a monetary action lowers the rate of return on capital that owners of wealth will accept, it becomes easier for the economy to accumulate capital. If a monetary action increases the rate of return on equity investments demanded by owners of wealth, then it discourages capital accumulation.

This paper concerns the financial sector alone, and we make no attempt here to describe the repercussions of a discrepancy between the rate of return on capital required for portfolio balance and the marginal productivity of capital. These repercussions occur in the market for goods and services and labor, and through them feed back to the financial sector itself. Let it suffice here to say that they are qualitatively of the same nature as the consequences of discrepancy between Wicksell's natural and market rates of interest.

We assume the value of the stock of capital to be given by its replacement cost, which depends not on events in the financial sphere but on prices prevailing for newly produced goods. We make this assumption because the strength of new real investment in the economy depends on the terms on which the community will hold capital goods valued at the prices of current production. Any discrepancy between these terms and the actual marginal productivity of capital can be expressed alternatively as a discrepancy between market valuation of old capital and its replacement cost. But the discrepancy has the same implications for new investment whichever way it is expressed.

This required rate of return on capital is the basic criterion of the effectiveness of a monetary action. To alter the terms on which the community will accumulate real capital -- that is what monetary policy is all about. The other criteria commonly discussed -- this or that interest rate, this or that concept of the money supply, this or that volume of lending -- are at best only instrumental and intermediate and at worst misleading goals.

3. Summary of regimes to be discussed.

The argument proceeds by analysis of a sequence of regimes. A regime is characterized by listing the assets, debts, financial intermediaries, and interest rates which play a part in it. In all the regimes to be discussed, net private wealth is the sum of two components: the fixed capital stock, valued at current replacement cost; and the non-interest-bearing debt of the government, taking the form either of currency publicly held or of the reserves of banks and other intermediaries. In the models of this paper, there is no government interest bearing debt.[3] Consequently there are no open market

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[3] This complication has been discussed in other works of the authors; in Brainard, op. cit., and in Tobin, An Essay in the Pure Theory of Debt Management, to be published in the research papers of the Commission on Money and Credit.

operations proper. Instead the standard monetary action analyzed is a change in the supply of non-interest-bearing debt relative to the value of the stock of capital. (Only the proportions between the two components of wealth matter, because it is assumed that all asset and debt demands are, at given interest rates, homogeneous with respect to the scale of wealth.)

The public is divided, somewhat artificially, into two parts, wealth-owners and borrowers. Wealth-owners command the total private wealth of the economy, and dispose it among the available assets, ranging from currency to direct ownership of capital. Borrowers use the loans they obtain from financial intermediaries to hold capital. This split should not be taken literally. A borrower may be, and usually is, a wealth-owner -- one who desires to hold more capital than his net worth permits.

A final simplification is to ignore the capital and nonfinancial accounts of intermediaries, on the ground that these are inessential to the purposes of the paper. Table 1 provides a summary of the regimes to be discussed.

Table 1

Summary of Financial Regimes Discussed in Text

Regime	Structure of Assets and Debts: Holder	Assets (+), Debts (-)	Yields to be Determined on:
I	Private wealth-owners	+ Currency + Capital	Capital
II	Private wealth-owners	+ Currency, + Capital, + Intermediary Liabilities	Capital, Intermediary Liabilities and Loans
	Private borrowers	- Loans, + Capital	
	Intermediary	- Liabilities, + Loans, (+ Reserves)	
III	Private Wealth-owners	+ Currency, + Capital + Deposits	A) Capital, Loans, Deposits
	Private borrowers	- Loans, + Capital	B) Capital, Loans (Deposit rate fixed)
	Intermediary (Banks)	- Deposits, + Loans, + Reserves (Currency)	

4. Regime I: A Currency-Capital World

It is instructive to begin with a rudimentary financial world in which the only stores of value available are currency and real capital. There are no intermediaries, not even banks, and no credit markets. Private wealth is the sum of the stock of currency and the value of the stock of capital. The stock of currency is, in effect, the government debt, all in non-interest-bearing form. The required rate of return on capital R_o is simply the rate at which wealth-owners are content to hold the existing currency supply, neither more nor less, along with the existing capital stock, valued at replacement cost. The determination of R_o is shown in Figure 1. In Figure 1, the return on capital R_o is measured vertically. Total private wealth is measured by the horizontal length of the box OW, divided between the supply of currency OC and the replacement value of capital CW. Curve DD' is a portfolio choice curve, showing how wealth-owners wish to divide their wealth between currency and capital at various rates R_o. It is a kind of "liquidity preference" curve. The rate which equates currency supply and demand -- or, what amounts to the same thing, capital supply and demand -- is $\bar{R}_o$.

In this rudimentary world, the sole monetary instrument is a change in the supply of currency relative to the supply of capital. An increase in currency supply relative to the capital stock can be shown in Figure W simply by moving the vertical line CC' to the right. Clearly this will lower the required rate of return $\bar{R}_o$. Similarly, the monetary effect of a contraction of the currency supply can be represented by a leftward shift of the same vertical line.

5. Regime II: An Uncontrolled Intermediary

Now imagine that a financial intermediary arrives on the scene. The liabilities of the intermediary are a close but imperfect substitute for currency. Its assets are loans which enable private borrowers to hold capital in excess of their own net worth. How does the existence of this intermediary alter the effectiveness of monetary policy? That is, how does the intermediary affect the degree to which the government can change $\bar{R}_0$ by a given change in the supply of currency?

We will assume first that the intermediary is not required to hold reserves and does not hold any. Its sole assets are loans. To any institution the value of acquiring an additional dollar liability to the public is the interest at which it can be re-lent after allowance for administrative costs, default risk, and the like. Consequently, in unrestricted competition this rate will be paid to the intermediary's creditors. In equilibrium, the borrowers' demand for loans at the prevailing interest rate on loans will be the same as the public's supply of credits to the intermediary at the corresponding rate.

This regime is depicted in Figure 2. The axes represent the same variables as in Figure 1, and the supplies of currency and capital are shown in the same manner. But the demand for capital is now shown in two parts. The first, measured leftward from the right vertical axis to curve KK' , is the direct demand of wealth-owners. The second part, measured rightward from line CC' to curve LL' is the demand for capital by borrowers. This distance also measures the demand of borrowers for loan accommodation by the intermediary.

Curve DD' represents, as in Figure 1, the demand of wealth-owners for currency. The horizontal difference between DD' and KK' is their demand for the liabilities of the intermediary.

In this regime there is a second interest rate to be determined, the rate R_2 on intermediary liabilities. The rate on intermediary loans, r_2, is uniquely determined by R_2; competition among institutions keeps the margin between these rates equal to the cost of intermediation. The position of the three curves in Figure 2, and the demands which they depict, depend on this rate as well as on R_o. The three dashed curves represent a higher intermediary rate R_2 than the solid curves. However, only the solid curves represent an equilibrium combination of the two rates, at which (1) the demands for capital absorb the entire capital stock, (2) the loan assets of the intermediary equal its liabilities, and (3) the demand for currency is equal to the supply.

We may presume, of course, that the introduction of the intermediary lowers the required rate of return on capital $\bar{R}_o$. For wealth-owners, the intermediary's liabilities satisfy some of the same needs which would be met in regime I by an increase in the supply of currency. At the same time, some of the capital which wealth-owners formerly held can now be lodged with borrowers, at a lower rate of return. These borrowers were unable to obtain finance to hold capital in the regime I.

An autonomous growth of the intermediary can be formally represented by a reduction in the margin between the intermediary's lending and borrowing rates. As the intermediary becomes more efficient in administration, risk pooling, and in tailoring its liabilities and assets to the preferences of its customers on both sides, this margin will decline under the force of competition. It is shown in the Appendix, Part II Regime II, that a reduction in the margin always lowers the required rate of return on capital and increases the intermediary's assets and liabilities.

A reduction in the supply of currency will, in this regime, as in the first regime raise the required rate of return on capital. It will also raise the intermediary's rates. These results are illustrated in Figure 3. The solid curve indicates an initial equilibrium at $\bar{R}_o$. The effect of a reduction in the supply of currency (prior to any rate adjustments) is to shift leftward the curves CC' ~~cc'~~ and LL' to the position shown by the dashed curves cc' and ll'. At the initial deposit-loan rate, the rate on capital which would clear the capital market (R_o^k) is less than the rate on capital which would clear the currency market (R_o^c). If the rate on capital adjusts so as to clear the capital market, there will be an excess demand for currency, and an excess demand by borrowers for loans. The resulting increase in the deposit-loan rate: a) decreases the demand for currency (moves DD' to the left) b) decreases the direct demand for capital (moves KK' to the right), and c) decreases borrowers' demand for loans and capital (moves ll' to the left). The effect of (a) is to decrease the rate of return on capital which will clear the currency market. The effect of (b) and (c) is to increase the rate of return on capital which will clear the capital market. Equilibrium with the decreased supply of currency will be established when the deposit-loan rate increases sufficiently to clear the currency and capital markets at the same rate as capital. The small-dashed curves in Figure 3 indicate the new equilibrium; the rate of return on capital ($\bar{R}_o'$) is higher than initially.

The existence of the intermediary does not, therefore, mean that monetary control is ineffective. However, it normally means that the control is less effective, in the sense that a dollar reduction in the supply of currency brings about a smaller increase in $\bar{R}_o$ when it can be counteracted by an expansion of the intermediary. The possibility of substituting the intermediary's liabilities for currency offers a partial escape from the monetary restriction. But so long as the intermediary's liabilities are an imperfect substitute for currency, the escape is only partial.

6. Regime II: A controlled intermediary.

This proposition can be demonstrated by imagining that we can impose some quantitative restriction on the expansion of the intermediary. We can then compare the strength of monetary restriction in the Regime II, with and without this quantitative control.

Assume, therefore, that the government's non-interest-bearing debt is divided into two segregated parts: currency held by the public, and reserves held by the intermediary pursuant to a legal fractional reserve requirement. Assume further that this requirement is effective, i.e., that the aggregate size of intermediary liabilities permitted by the reserve requirement is smaller than the economic size which would result from unrestricted competition. When the reserve requirement is effective, the margin between the intermediary's lending and borrowing rates is greater than is needed to compensate for risk and administrative costs. Let the supply of currency to the public be reduced. In an uncontrolled Regime II, this will in certain circumstances lead to an expansion of intermediary assets and liabilities. In those circumstances, preventing such expansion by a reserve requirement will increase the effectiveness of monetary control. That is, a dollar reduction in the currency supply will raise $\bar{R}_o$ more if an expansionary response by the intermediary is prevented. There are also circumstances--probably less plausible--where monetary restriction would, in an uncontrolled regime, result in a contraction of the intermediary. In these cases, of course, control of the intermediary does not strengthen monetary control. These results are shown in the Appendix, Part II, Regime II.

The example just discussed is a simple and artificial one. But the point it makes is of quite general applicability. In the more complex real world, currency and commercial bank liabilities are together subject to control via monetary policy, while the scales of operations of other financial intermediaries are not. The freedom of these intermediaries offers an escape from monetary controls over commercial banks, but only a partial escape. Likewise, the effectiveness of monetary controls would be enhanced if each nonbank intermediary was subject to a specific reserve requirement which would keep it from expanding counter to policies which contract commercial banks. Again, see Appendix, Part III (B).

7. Regime III: Commercial Banking.

The reserve requirement introduced in Regime II was expressed in terms of a specific government debt instrument, available only for this purpose and only in amounts determined by the government. The more familiar situation is that the reserve asset is, for all purposes, currency itself. Currency, that is, can serve either as a means of payment in the hands of the public, or as reserves for the intermediary. The government determines the total size of its non-interest-bearing debt, but its allocation between currency and reserves is a matter of public choice. It is, of course, this kind of reserve and reserve requirement that we associate with commercial banks, the most prominent intermediary. Indeed the traditional business of banks is to accept deposit liabilities payable in <u>currency</u> on demand, and this obligation is the historical reason for banks' holding reserves in currency or its equivalent in "high-powered" money.

Let us consider, therefore, a third regime, in which the one intermediary is a commercial banking system required to hold as reserves in currency a certain fraction of its deposit liabilities. Total private wealth is, as in the first two regimes, the sum of currency supply and the capital stock. Wealth owners divide their holdings among currency, bank deposits, and direct ownership of capital. Banks dispose their deposits between reserves in currency and loans to borrowers, in proportions dictated by the legal reserve requirement. Borrowers hold that part of the capital stock not directly held by wealth-owners. So far as interest rates are concerned, there are two important variants:

(A) Interest on bank deposits is competitively determined, and stands in competitive relation to the interest rate on bank loans. This relationship will depend on, among other things, the reserve requirement, which compels a bank to place a fraction of any additional deposit in non-interest-bearing reserves.

(B) Interest on deposits is subject to an effective legal ceiling, and at the same time the reserve requirement normally restricts the banking industry to a scale at which the loan rate exceeds this fixed deposit interest rate by more than the competitive margin.

In this regime, there are two sources of demand for currency. One is the <u>direct</u> demand of the public. The other is the banks' reserve requirement; in effect, the public demand for deposits creates a fractional <u>indirect</u> demand for currency. This creates an interesting complication, as follows: The basic assumption about the portfolio behavior of wealth-owners is that assets are all substitutes for each other. Essentially the same assumption is applied to the behavior of borrowers. That is, a rise in the interest rate on any asset A induces, <u>ceteris paribus</u>, an increase in desired holdings of A and a decrease, or at most no change, in the desired holdings of every other asset. It is this assumption which enabled us to state unambiguously the direction of the effects of monetary actions in the regimes previously discussed. In the present regime the same substitution assumption still applies to the portfolio behavior of wealth-owners and borrowers. This means, among other things, that the public's <u>direct</u> demand for currency is assumed to decline, or at most not to rise, in response to an increase in the rate offered on bank deposits. But, of course, an increase in this rate also increases the demand for bank deposits.

Thus it indirectly increases the demand for currency to serve as bank reserves.

It is certainly conceivable, especially if the required reserve ratio is high, that the indirect effect of an increase in the deposit rate outweighs the direct effect. In that event currency and deposits are, taking account of public and banks together, complements rather than substitutes. This possibility is the simplest example of a very general phenomenon. Even though the substitution assumption applies to the portfolio choices of the public, and of every intermediary, taken separately, it is possible that assets will be complements in the system as a whole. This can happen whenever the public and intermediaries hold the same assets (currency, or government bonds, or other securities) or whenever one intermediary holds as assets the liabilities of another intermediary. Some of the implications of complementarity for both the stability of the system and its responses to various changes in parameters and in structure can be illustrated in the present regime.

The determination of equilibrium in regime III is illustrated in Figure 4. We shall consider first case (A) where the deposit rate is flexible and competitively determined. As before, the rate of return on capital is measured vertically. Total private wealth is indicated by the length of the horizontal axis, and the division of wealth between currency and capital is shown by the vertical line CC' . The currency supply is divided between public holdings and bank reserves by DD' , which expresses public demand for currency in relation to R_0 , for a given level of the deposit rate R_2 . For geometrical convenience, this curve is drawn vertical in the diagram: i.e., it is assumed that there is no relationship between direct currency demand

and R_o . This makes it possible to show on the diagram a unique volume of deposits DE at which banks will be using as reserves all the currency left over. The demand for deposits is measured as the difference between public currency holdings DD' and the curve KK' . This shows, again for a fixed deposit rate, that the demand for deposits is larger the lower is R_o . The remainder of public wealth, to the right of curve KK' , is the direct demand of wealth-owners for capital. Borrowers' demand for bank loans and for capital, an increasing function of R_o at a given bank loan rate, is shown by the horizontal distance between CC' and LL' .

When banks are in equilibrium, three conditions must be met: (1) deposits are equal to loans plus reserves, (2) deposits are at the level which just exhausts the supply of reserves, and (3) the loan rate is systematically related to the deposit rate, exceeding it by an amount which compensates banks both for costs of administration, etc., and for the requirement that a fraction of deposits must be invested in non-interest-bearing reserves. In Figure 4 the solid curves all assume a given level of the deposit rate, with the corresponding level of the loan rate. The dashed curves represent the same relationships with a higher deposit and loan rate. The solid curves depict an equilibrium situation; the dashed curves a disequilibrium.

An alternative way of depicting this regime is to consider separately the conditions of equilibrium in the "market" for currency, and in the market for loans. In Figure 5 the supply of currency is shown as the vertical line CC' . The demand for currency, in relation to the deposit rate R_2 , is the curve AA ' . This includes both the direct and the indirect demand for currency. As noted above, this relationship may be either downward-sloping, as in Figure 5.a or upward-sloping, as in Figure 5.b. The upward-sloping case is that of complementarity. In each case the position of the demand curve depends on the level of R_o ; the dashed curve represents a higher R_o , which tends to reduce the demand for currency. From the relationships involved in Figure 5 can be derived a locus of pairs of rates R_o and R_2 which equate demand and supply for currency. Such a relationship is shown in Figure 7 by the curve E_c . In the "substitutes case," corresponding to Figure 5.a, it is downward-sloping, as shown in Figure 7.a. In the "complements case," corresponding to Figure 5.b, it is upward-sloping, as shown in Figures 7.b and 7.c.

In Figure 6 the loan market is shown, the volume of loans on the horizontal axis and the deposit rate on the vertical axis. The supply of loans, BB' , is essentially the public demand for deposits, after allowance for the fractional reserve requirement. The demand for loans, LL' is the amount of loan accommodation borrowers wish at various deposit rates, taking account of the fact that the loan rate systematically exceeds the deposit rate for the reasons already mentioned. The positions of the loan supply and demand curves depend on R_o , the rate of return on capital. A higher R_o shifts both curves upward, as indicated by the dashed curves. From these relationships a second locus of the two rates R_o and R_2 can be derived, the pairs of rates which equilibrate the loan market. This is the upward-sloping relationship E_L , also shown in Figure 7.

Three possible cases for the system as a whole are shown in the three parts of Figure 7: the first, or "substitutes" case, in Figure 7.a, a moderate "complements" case in Figure 7.b, and an extreme "complements" case in Figure 7.c. Now if there is a reduction in the supply of currency, equilibrium in the currency market can be maintained only by an increase in the rate on capital R_o associated with any given deposit rate R_2. This can be seen by a shift left in the currency supply in Figure 5. Consequently the effect of a reduction in the currency supply can be shown in Figure 7 by a rightward shift in the curve E_c. In the first two cases, Figures 7.a and 7.b, this means an increase in both rates, as would be expected. However, in the "extreme complements" case, Figure 7.c, it indicates a decrease in both rates!

This implausible result arouses the suspicion that the solution indicated in Figure 7.c is an unstable equilibrium. In the Appendix, Part II, Regime III, stability is examined under the assumption that excess demand for capital leads to a fall in the rate of return on capital R_o, while excess borrowers' demand for loans, relative to the supply permitted by reserve requirements and depositor preferences, leads to a rise in loan -- deposit rate R_2. The case exhibited in Figure 7.c is indeed unstable, while the other two cases are stable.

Consider now alternative (B), in which the interest rate on deposits is subject to an effective legal ceiling. The competitive link between the deposit and loan rates is broken by this regulation. In Figure 5 in other words, there is only one applicable level of the deposit rate. Consequently, there is only one rate on capital which is consistent with equilibrium in the "market" for currency. Figure 8 exhibits this situation. The currency equilibrium curve of Figure 8 is simply a vertical line; although the loan rate r_2 measured on the vertical axis of Figure 8 can vary, its variation does not affect either the deposit rate or equilibrium in the currency "market."

The effect of an increase in the controlled deposit rate depends on whether currency and deposits are, when both indirect and direct demands are taken into account, substitutes or complements. _If they are substitutes_, an increase in the controlled deposit rate will reduce the net demand for currency. Therefore, the rate on capital R_o which balances the supply and demand for currency will be lower. In Figure 8, this means a movement to the left in the vertical line. In the new equilibrium both the rate on capital and the loan rate will be lower. An increase in the deposit rate is an expansionary monetary action.4/ _If, on the other hand, currency_

4/ It is perhaps not too fanciful to refer, as an example of this kind of effect, to the consequences of the increases in Regulation Q ceiling rates on time and savings deposits in commercial banks permitted in 1961. Contrary to many predictions, these increases led to lower, rather than higher, mortgage lending rates; they were expansionary. Given the low reserve requirement against these deposits, especially when compared to demand deposits, it is to be expected that time and savings deposits are strong substitutes for currency and reserves.

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and deposits are complements, the result of an increase in the controlled deposit rate is the reverse. Both the rate on capital and the loan rate are also increased. An increase in the deposit rate is a deflationary monetary action.

With a fixed deposit rate, a reduction in currency supply always is deflationary. This is true, of course, whether currency and deposits are substitutes or complements. But the question of real interest is whether monetary restriction is more deflationary -- i.e., produces a bigger increase in the return on capital -- when the deposit rate is flexible or when it is fixed. Monetary restriction will, in the flexible case, increase the deposit rate; in the other case the legal ceiling prevents this reaction. Now if currency and deposits are substitutes, an increase in the deposit rate is expansionary; it opposes and weakens the monetary contraction. But if they are complements, the reverse is true: flexibility in the deposit rate re-enforces and strengthens quantitative monetary control. These results are shown in Appendix, Part II Regime III.

Once there is a reserve requirement, variation in the required ratio is another instrument of monetary control. There are two questions to ask about such variation. The first concerns its effect upon the required rate of return on capital. The second concerns its effect on the strength of quantitative monetary control.

With a fixed deposit rate, an increase in the reserve requirement is always deflationary. With a given currency supply, the higher reserve requirement necessarily means that the public must curtail its holdings of currency or deposits or both. The only way they can be induced to do so is by an increase in R_o .

With a flexible deposit rate the same conclusion applies when currency and deposits are substitutes. However, it is conceivable, when they are complements, that an increase in the reserve requirement will be expansionary. This may be seen in the following way: The initial effect of an increase in the reserve requirement may be divided into two parts: (a) the increase in the demand for currency and decrease in the supply of loans which result from banks' attempts to meet the higher reserve requirement; and (b) the increased margin banks will require between their deposit and loan rates when they have to place a higher proportion of deposits in non-interest bearing reserves. The first of these effects is always contractionary. The second effect will also be contractionary in the "substitutes" case. In the "complements" case, however, the effect of increasing the margin between the rates is expansionary, and may even outweigh the first effect. At the same loan rate the deposit rate will be lower; as we have already noticed, lowering the deposit rate is expansionary in the "complements" case.

The answer to the second question, concerning the effect of increasing the reserve requirement on the strength of quantitative monetary control, similarly may depend on whether the deposit rate is fixed or flexible. With a fixed deposit rate an increase in the reserve requirement will decrease the response to changes in currency supply. This can be seen by imagining that regime II (A) is modified, first by fixing the deposit rate, and second by imposing a reserve requirement. With a fixed deposit rate and no reserve requirement, any reduction in the supply of currency is at the expense of the public's direct holdings of currency. The increase in the rate of return on capital necessary to reconcile the public to these reduced holdings of currency will also diminish their demand for bank deposits. But when banks hold no reserves, this cannot release any currency.

On the other hand, once banks are required to hold reserves, a contraction of bank deposits releases currency. Therefore direct holdings of currency do not have to absorb the full reduction in the currency supply. Consequently, the necessary increase in the rate of return on capital is smaller.

When the deposit rate is free to rise, we would expect an increase in the rate in the wake of currency contraction to increase the volume of bank deposits. This expansion tends to offset the reduction in the public's currency holdings. Substitution of deposits for currency moderates the increase in R_o necessary to reconcile the public to smaller holdings of currency. When banks hold no reserves, this substitution can proceed without any brake. But when banks are subject to a reserve requirement, it can proceed only to the extent that the public is induced to give up additional currency to serve as bank reserves. Therefore, a reserve requirement means that a larger increase in R_o , the rate of return on capital, is needed to make the public content with a larger reduction in its direct holdings of currency.

This is the essential reason why regulations preventing or limiting expansion of the intermediary strengthen monetary control. It may be observed that such regulations are of two kinds: either a rate ceiling which prevents the intermediary from bidding for funds, or a reserve requirement, or both. Once there is an effective rate ceiling, however, increasing the required reserve ratio -- though itself an effective instrument of control -- reduces the effectiveness of a given change in the supply of currency.

It is possible, even when the deposit rate is flexible, that bank deposits decline in response to a contraction of the currency supply. Then, just as in the case of a fixed deposit rate, increasing the reserve requirement diminishes the response of the system to such contraction. A reserve requirement is not necessary to prevent expansion of the intermediary from offering an escape from monetary control, because the intermediary would not expand anyway.[5]

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[5] This statement needs to be somewhat qualified to allow for the fact that a higher reserve ratio enlarges the competitively required margin between deposit and loan rates. This strengthens the contribution of a higher reserve requirement to the effectiveness of reduction in the currency supply, and makes it possible that this contribution is positive even when deposits do not expand.

The principal results for regime III are summarized in Table 2.

TABLE 2

Summary of Results for Regime III

Currency and Deposits:	SUBSTITUTES	COMPLEMENTS	EXTREME COMPLEMENTS
System:	Stable	Stable	Unstable
Increase in Deposit Rate:	Expansionary	Deflationary	
Variation of currency supply more effective when deposit rate is:	Fixed	Flexible	

These conclusions have been reached by adding one intermediary, banks, to a currency-capital model and then imposing rate and reserve regulations on banks. But they are illustrative of more general conclusions. In a many-intermediary world, similar propositions apply to the extension of nonbank intermediaries of the rate and reserve regulations to which banks are subject. The analogy is pursued in the Appendix, Part III, where nonbank intermediaries are required to hold reserves in currency in the same manner as banks.

APPENDIX

Part I. Notation and Assumptions

Suppose there are n types of financial assets that owners of wealth can hold; the first currency, the remaining $(n - 1)$ the liabilities of $(n - 1)$ financial intermediaries. An $n + 1$ asset, direct equity in capital is designated by subscript 0. Let $D_i \geqq 0$ be the proportion of the value of total wealth the public desires to hold in the i^{th} asset $(i = 0, 1, 2, \ldots, n)$. Let R_i be the rate of return offered owners of the i^{th} asset. Since the first financial asset is currency, R_1 is taken to be fixed at zero. Each D_i may be taken to be a function of all the R_i. We shall further assume that the demand for the various assets is homogeneous in wealth, i.e., the D_i do not depend on the level of wealth. There are n independent functions to distribute total wealth into $n + 1$ categories. Thus we may represent wealth-owners' desired allocation by:

$$(1) \qquad D_i = D_i(R_o, R_2, \ldots, R_n) \qquad\qquad (i = 1, 2, \ldots, n)$$

The assets are assumed to be imperfect substitutes, so that the effect of a reduction in the j^{th} interest rate, other rates remaining constant, is to diminish D_j and to increase or at least to leave unchanged the demand for each of the other assets, including D_0. Similarly, it is assumed that the effect of a reduction in R_o is to increase or leave unchanged every financial asset holding. Using the notation D_{ij} to represent the partial derivative of the function D_i with respect to the j^{th} rate, these assumptions are as follows:

(2)
$$D_{ij} > 0 \quad (i=j)$$
$$D_{ij} \leqq 0 \quad (i \neq j) \qquad (i,j = 1,2, \ldots, n)$$
$$\sum_{i=1}^{n} D_{ij} > 0$$
$$D_{i0} \leqq 0$$

Each of the financial intermediaries offers its own variety of loan to individuals who would like to hold capital beyond their net worth. The demands of borrowers for loans of each type depend jointly on the rate of return on capital, R_o, and on the $(n - 1)$ different borrowers' interest rates r_i:

(3)
$$L_i = L_i(R_o, r_2, \ldots, r_n) \qquad (i = 2, 3, \ldots, n)$$

Debts of different types are assumed to be gross substitutes, so that the effect of a reduction in the j^{th} borrowers' rate, other rates remaining constant, is to increase both total borrowing $\sum_{i=2}^{n} L_i$ and L_j specifically and to diminish or leave unchanged all other debts. The effect of a reduction in R_o is to diminish or leave unchanged borrowers' demands for each type of loan. These assumptions are as follows:

(4)
$$L_{ij} < 0 \quad (i=j)$$
$$L_{ij} \geqq 0 \quad (i \neq j) \qquad (i,j = 2, \ldots, n)$$
$$\sum_{i=1}^{n} L_{ij} < 0$$
$$L_{i0} \geqq 0$$

In addition to loans, an intermediary may hold other assets. The regimes considered in Part II (and their n - intermediary counterparts in Part III) make different assumptions about intermediaries' portfolios of loans, currency and other assets. The portfolio choices of intermediaries are constrained by the requirement that their assets equal their liabilities. (For the present purposes no harm is done by assuming shareholders' equity in intermediaries to be zero.) To simplify presentation, the balance sheet identity for each financial intermediary will be used to translate wealth-owners demand for each intermediary's liability into indirect demand for loans, currency, etc. Equilibrium in the various asset markets may then be represented by a system of equations of the following form:

$$
\begin{array}{llcl}
(5) \quad \text{Capital} & A_0\,(R_o, R_2, \ldots, R_n, r_2, \ldots, r_n) & = & S_o \\
\text{Currency} & A_1\,(R_o, R_2, \ldots, R_n, r_2, \ldots, r_n) & = & S_1 \qquad (\, i = 2, \ldots, n) \\
\text{Loans} & A_i\,(R_o, R_2, \ldots, R_n, r_2, \ldots, r_n) & = & 0 \\
\hline
& \sum_{o}^{n} A_i & = & \Sigma\, S_i \quad = 1
\end{array}
$$

The functions A_o, A_1, and A_i are derived from the demand functions D_i and L_i already discussed. A_o and A_1 represent the total private demand for capital and currency respectively. The supply of these assets, which comprise private wealth, are S_o and S_1. The A_i in the remaining $(n - 1)$ equations represent the excess demand functions for the various types of intermediary loans. When there are additional forms of government debt, this system is correspondingly augmented with equations like that relating to currency. The last equation indicates that the demands for the various assets always sum up to private wealth; hence one of the preceding demand equations is redundant. We follow the convention of omitting the first equation.

The n independent equations (5) contain $2n - 1$ rates: R_o the yield of capital, $n - 1$ intermediary lending rate, and $n - 1$ intermediary borrowing rates. Consequently, $n - 1$ additional equations are needed. For example, lending and borrowing rates may be assumed equal for all intermediaries in a competitive regime where intermediaries hold no assets but loans. Then

$$(6) \qquad R_i = r_i \qquad (i = 2, \ldots, n)$$

This need not be literally interpreted to mean that competition among financial institutions within a given intermediary brings the rates into equality. The assumption could be relaxed to permit a premium to compensate for administrative costs and risks of default and illiquidity, without essential difference so long as the premium is a constant or increasing function of the total volume of assets and liabilities of the intermediary.

When the deposit rates are regulated, or competition is ineffective, a different set of $n - 1$ conditions will apply to the rates.

Use of equations (6) or their counterpart will enable us to eliminate all but n variables from equation system (5). The effects of changes in parameters on the rates, and in particular on R_o, may then be found by differentiating this system. The results depend crucially upon the partial derivatives of the demand equations A_i, which differ from regime to regime.

Part II. Analysis of Regimes Discussed in Text

REGIME I

(I.1) $$D_1\ (R_o) = S_1 \qquad \text{Currency}$$

$$\frac{\partial R_o}{\partial S_1} = \frac{1}{D_{10}} < 0$$

REGIME II

(A) Uncontrolled intermediary.

(II.1)
$$\begin{aligned} D_1\ (R_o, R_2) &= S_1 && \text{Currency} \\ D_2\ (R_o, R_2) - L_2(R_o, r_2) &= 0 && \text{Intermediary} \\ r_2 - R_2 &= a && \text{Relation between rates} \end{aligned}$$

(i) Effect of reduction in margin between rates:

(II.2)
$$\begin{bmatrix} D_{10} & D_{12} \\ D_{20} - L_{20} & D_{22} - L_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial a} \\ \frac{\partial R_2}{\partial a} \end{bmatrix} = \begin{bmatrix} 0 \\ L_{22} \end{bmatrix}$$

The restrictions on D_i and L_i assumed in Part I assure that the Jacobian is negative, that $\frac{\partial R_o}{\partial a} \geq 0$, that $\frac{\partial R_2}{\partial a} \leq 0$, and that $\frac{\partial D_2}{\partial a} = D_{20} \frac{\partial R_o}{\partial a} + D_{22} \frac{\partial R_2}{\partial a} \leqq 0$. That is, an increase in the efficiency of intermediation lowers the required return on capital, raises the deposit rate, and expands the liabilities and assets of the intermediary.

(ii) Effect of change in currency supply.

$$\text{(II.3)} \qquad J' \begin{bmatrix} \dfrac{\partial R_o}{\partial S_1} \\ \\ \dfrac{\partial R_2}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix}$$

where J' is the Jacobian of (II.2). It follows that $\frac{\partial R_o}{\partial S_1}$, $\frac{\partial R_2}{\partial S_1}$, $\frac{\partial r_2}{\partial S_1} \leqq 0$. $\frac{\partial D_2}{\partial S_1} = D_{20} \frac{\partial R_o}{\partial S_1} + D_{22} \frac{\partial R_2}{\partial S_1}$ may have either sign.

(B) Controlled intermediary.

$$\text{(II.4)} \qquad \begin{aligned} D_1 (R_o, R_2) &= S_1 && \text{Currency} \\ e\, D_2 (R_o, R_2) &= S_2 && \text{Specific Reserve} \\ (1-e)\, D_2 (R_o, R_2) - L_2(R_o, r_2) &= 0 && \text{Intermediary} \end{aligned}$$

Here S_2 is the supply of government debt in the form of the reserve asset, expressed as a proportion of total wealth. The required ratio is $0 \leq e \leq 1$. There are three equations in the three rates R_o, R_2, r_2 ; the third equation of (II.1) drops out. (However, the inequality $r_2 - R_2 \geqq a$ must hold; otherwise equations II.4 are supplanted by II.1.)

(i) Effect of change in currency supply.

$$\text{(II.5)} \quad \begin{bmatrix} D_{10} & D_{12} & 0 \\ D_{20} & D_{22} & 0 \\ (1-e)D_{20}-L_{20} & (1-e)D_{22} & -L_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The first two equations can be solved separately, and it is easily seen that $\frac{\partial R_o}{\partial S_1}, \frac{\partial R_2}{\partial S_1} \leqq 0$. Since $D_{20} \frac{\partial R_o}{\partial S_1} = - D_{22} \frac{\partial R_2}{\partial S_1}$ from the second equation, the third equation reduces to

$$- L_{20} \frac{\partial R_o}{\partial S_1} = L_{22} \frac{\partial r_2}{\partial S_1} .$$

Therefore $\frac{\partial r_2}{\partial S_1}$ is also non-positive.

(ii) Comparison with uncontrolled regime.

The effect of restraining the expansion of the intermediary can be found by subtracting equation system II.3 from the first two equations of II.5.

(II.6)

$$\begin{bmatrix} D_{10} & D_{12} \\ D_{20} & D_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \end{bmatrix}_5 - \begin{bmatrix} D_{10} & D_{12} \\ D_{20}-L_{20} & D_{22}-L_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \end{bmatrix}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} D_{10} & D_{12} \\ D_{20} & D_{22} \end{bmatrix} \left[\begin{pmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \end{pmatrix}_5 - \begin{pmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \end{pmatrix}_3 \right] = \begin{bmatrix} 0 & 0 \\ -L_{20} & -L_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \end{bmatrix}_3$$

$$= \begin{bmatrix} 0 \\ -\left(\frac{\partial D_2}{\partial S_1}\right)_3 \end{bmatrix}$$

It follows that

$$\operatorname{sign} \left(\frac{\partial R_o}{\partial S_1}\right)_5 - \left(\frac{\partial R_o}{\partial S_1}\right)_3 = \operatorname{sign} \left(\frac{\partial D_2}{\partial S_1}\right)_3$$

$$\operatorname{sign} \left(\frac{\partial R_2}{\partial S_1}\right)_5 - \left(\frac{\partial R_2}{\partial S_1}\right)_3 = \operatorname{sign} -\left(\frac{\partial D_2}{\partial S_1}\right)_3$$

If reduction of currency supply would lead to an expansion of the intermediary when it is uncontrolled, then preventing this expansion will enhance the effectiveness of the currency restriction.

REGIME III

(A) Deposit rate flexible.

$$\text{(III.1)} \qquad \begin{aligned} D_1(R_o, R_2) + c\, D_2(R_o, R_2) &= S_1 && \text{Currency} \\ (1-c) D_2(R_o, R_2) - L_2(R_o, r_2) &= 0 && \text{Intermediary (Banks)} \\ \frac{R_2}{1-c} - r_2 &= 0 && \text{Relation between rates} \end{aligned}$$

Here the required reserve is in currency, and the required ratio is $0 \leq c \leq 1$. The three equations determine R_o, R_2, r_2.

(B) Deposit rate fixed.

$$\text{(III.2)} \qquad \begin{aligned} D_1(R_o, \bar{R}_2) + c\, D_2(R_o, \bar{R}_2) &= S_1 && \text{Currency} \\ (1-c) D_2(R_o, \bar{R}_2) - L_2(R_o, r_2) &= 0 && \text{Intermediary (Banks)} \end{aligned}$$

Here the deposit rate is fixed at $\bar{R}_2$, and the two equations determine R_o, r_2. In order for this regime to apply, the following inequality must hold:

$$\frac{\bar{R}_2}{1-c} \leqq r_2$$

(i) Change in fixed deposit rate in (E) .

$$(\text{III.3}) \quad \begin{bmatrix} D_{10} + c\,D_{20} & 0 \\ (1-c)\,D_{20} - L_{20} & -L_{22} \end{bmatrix} \begin{bmatrix} \dfrac{\partial R_o}{\partial \bar{R}_2} \\ \dfrac{\partial r_2}{\partial \bar{R}_2} \end{bmatrix} = -\begin{bmatrix} D_{12} + c\,D_{22} \\ (1-c)\,D_{22} \end{bmatrix}$$

The Jacobian is negative. Therefore

$$(\text{III.4}) \qquad \text{sign}\ \frac{\partial R_o}{\partial \bar{R}_2} = \text{sign}\ (D_{12} + c\,D_{22})$$

Currency and deposits are defined to be <u>substitutes</u> if $(D_{12} + c\,D_{22}) < 0$, and <u>complements</u> if $(D_{12} + c\,D_{22}) > 0$. Thus (III.4) says that raising the fixed deposit rate is <u>expansionary</u> if currency and deposits are substitutes, and <u>deflationary</u> if they are complements.

In either case, $\dfrac{\partial r_2}{\partial \bar{R}_2} > 0$.

(ii) Complementarity and stability.

The sign of $D_{12} + c\,D_{22}$ is also important in determining the stability of the equilibrium represented by the solution of III.1. Let the Jacobian of III.1 be:

$$J' = \begin{bmatrix} D_{10} + c\,D_{20} & D_{12} + c\,D_{22} \\ (1-c)\,D_{20} - L_{20} & (1-c)\,D_{22} - L_{22} \cdot \dfrac{1}{1-c} \end{bmatrix}$$

The solution is stable if $|J'| < 0$ and unstable if $|J'| > 0$. If $D_{12} + c\,D_{22} < 0$, $|J'| < 0$. That is, a sufficient condition for stability is that currency and deposits be substitutes. This is not a necessary condition. But "extreme" complementarity is associated with instability.

The proof that stability depends on the sign of $|J'|$ is as follows:

Assume that excess demand for capital leads to a fall in the rate of return on capital and that an excess of borrowers' demand for loans over banks' supply of loans leads to a rise in the deposit rate R_2 and accordingly in the loan rate r_2.[1/]

(III.5) $$\dot{R}_o = -K_o A_o (R_o, R_2)$$

$$\dot{R}_2 = -K_2 A_2 (R_o, R_2)$$

where K_o, $K_2 > 0$ are speeds of adjustment, which by choice of units may be taken as unity.

Here, $$A_o = 1 - D_1 - D_2 + L_2 - S_o$$
$$= S_1 - D_1 - D_2 + L_2$$

and $$A_2 = (1 - c)\,D_2 - L_2$$

1/ We assume that the relation $\frac{R_2}{(1-c)} - r_2 = \dot{0}$ always holds. Our results would not be altered if we allowed a "profit" margin in disequilibrium and assumed that it tends toward zero.

By Taylor's theorem we can approximate A_i in the neighborhood of equilibrium by the linear expression:

(III.6) $$A_i = \sum_j a_{ij} (R_j - \bar{R}_j) \qquad i = 0, 2$$

where a_{ij} is the partial derivative of excess demand for the i^{th} asset with respect to the j^{th} rate and $\bar{R}_j$ is the equilibrium R_j .

Substituting (III.6) in (III.5) and using the relationships given in (III.1) we obtain:

(III.7)
$$\begin{bmatrix} \dot{R}_o \\ \dot{R}_2 \end{bmatrix} = \begin{bmatrix} D_{10} + D_{20} - L_{20} & D_{12} + D_{22} - \frac{L_{22}}{(1-c)} \\ -(1-c)D_{20} + L_{20} & -(1-c)D_{22} + \frac{L_{22}}{(1-c)} \end{bmatrix} \begin{bmatrix} R_o - \bar{R}_o \\ R_2 - \bar{R}_2 \end{bmatrix} = A \begin{bmatrix} R_i - \bar{R}_i \end{bmatrix}$$

This system is stable if and only if the real parts of the characteristic roots of the Jacobian A are all negative. A 2 x 2 matrix with negative diagonal elements (always the case in our system) has the real parts of its characteristic roots negative if and only if the determinant of the matrix is positive. That is, stability under our assumptions requires $|A| > 0$.

By adding the last row of the determinant of A to the first row it can be seen that:

$$| J' | = - | A |$$

hence $|J'| < 0$ is a necessary and sufficient condition for stability under our assumptions.

In the text we defined "extreme" complementarity as the case where the currency equilibrium curve E_c cut the loan equilibrium curve E_L above from the left as in Figure 7.c. Let $E_L(R_o)$ be the value of R_2 which clears the loan market, and $E_c(R_o)$ the value of R_2 which clears the currency "market."

Then $\quad \frac{\partial E_L}{\partial R_o} > \frac{\partial E_c}{\partial R_o} > 0 \quad$ for $\quad (D_{12} + c\,D_{22}) > 0$

since $\quad \frac{\partial E_L}{\partial R_o} = \frac{-\left[(1-c)\,D_{20} - L_{20}\right]}{\left[(1-c)D_{22} - \frac{L_{22}}{(1-c)}\right]} \quad$ and $\quad \frac{\partial E_c}{\partial R_o} = \frac{-\left[D_{10} + c\,D_{20}\right]}{\left[D_{12} + c\,D_{22}\right]}$

and $\quad |J'| = [D_{10} + cD_{20}]\,[(1-c)D_{22} - \frac{L_{22}}{(1-c)}] - [(1-c)D_{10} - L_{20}]\,[D_{12} + cD_{22}]$

it is clear that in the complements case:

$$\frac{E_L}{\partial R_o} \gtrless \frac{E_c}{\partial R_o} \qquad \text{implies} \qquad |J'| \gtrless 0$$

The "extreme" complements case is unstable; the "moderate" complements case is stable.

(iii) Effect of change in currency supply

(A) Deposit rate flexible

Differentiating III.1 gives:

$$\text{(III.8)} \qquad \begin{bmatrix} \\ J' \\ \\ \end{bmatrix} \begin{bmatrix} \dfrac{\partial R_o}{\partial S_1} \\[2ex] \dfrac{\partial R_2}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\[2ex] 0 \end{bmatrix}$$

Assuming stability, $|J'| < 0$, and $\dfrac{\partial R_o}{\partial S_1}$, $\dfrac{\partial R_2}{\partial S_1} \leqq 0$.

(B) Deposit rate fixed:

Differentiating III.2, and letting J^2 be the Jacobian of this system, written out in III.3, we have:

$$\text{(III.9)} \qquad \begin{bmatrix} \\ J^2 \\ \\ \end{bmatrix} \begin{bmatrix} \dfrac{\partial R_o}{\partial S_1} \\[2ex] \dfrac{\partial r_2}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\[2ex] 0 \end{bmatrix}$$

Again, $\dfrac{\partial R_o}{\partial S_1}$, $\dfrac{\partial r_2}{\partial S_1} \leqq 0$.

Comparison of responses in (A) and (B).

We can compare the responses in the two cases by assuming a restriction of currency supply beginning at the same equilibrium. In one case, (B), the deposit rate is prevented from rising, but the loan rate may rise. In the other case, (A), the two rates remain in the equilibrium relationship given by the third equation of III.1. However, in order to make the comparison, we must use that equation to eliminate R_2, rather than r_2, from the first two equations of III.1.

$$\text{(III.10)}\quad \begin{bmatrix} D_{10} + c\,D_{20} & (D_{12} + c\,D_{22})\,(1-c) \\ (1-c)D_{20} - L_{20} & (1-c)^2\,D_{22} - L_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \end{bmatrix}_1 - \begin{bmatrix} D_{10} + c\,D_{20} & 0 \\ (1-c)\,D_{20} - L_{20} & -L_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore

$$\text{(III.11)}\quad \begin{bmatrix} D_{10}+cD_{20} & (D_{12}+cD_{22})(1-c) \\ (1-c)D_{20}-L_{20} & (1-c)^2\,D_{22}-L_{22} \end{bmatrix} \begin{bmatrix} \left(\frac{\partial R_o}{\partial S_1}\right)_1 - \left(\frac{\partial R_o}{\partial S_1}\right)_2 \\ \left(\frac{\partial r_2}{\partial S_1}\right)_1 - \left(\frac{\partial r_2}{\partial S_1}\right)_2 \end{bmatrix}$$

$$= -\left(\frac{\partial r_2}{\partial S_1}\right)_2 \begin{bmatrix} (D_{12}+cD_{22})(1-c) \\ (1-c)^2\,D_{22} \end{bmatrix}$$

The determinant of the Jacobian in III.11 is $(1-c)\ |\ J^2\ |$, and is therefore negative whenever the system is stable. We know also that $\left(\frac{\partial r_2}{\partial S_1}\right)_2$ is negative.

Therefore:

$$\left(\frac{\partial R_o}{\partial S_1}\right)_1 - \left(\frac{\partial R_o}{\partial S_1}\right)_2 = \frac{-\left(\frac{\partial r_2}{\partial S_1}\right)_2}{|\ J^2\ |} \cdot \begin{vmatrix} D_{12} + c\ D_{22} & 0 \\ (1-c)^2\ D_{22} & -L_{22} \end{vmatrix}$$

$$\text{Therefore} \quad \text{sign} \left(\frac{\partial R_o}{\partial S_1}\right)_1 - \left(\frac{\partial R_o}{\partial S_1}\right)_2 = \text{sign} \ -(D_{12} + c\ D_{22}),$$

$$\text{or} \quad \text{sign} \left|\left(\frac{\partial R_o}{\partial S_1}\right)_1\right| - \left|\left(\frac{\partial R_o}{\partial S_1}\right)_2\right| = \text{sign} \ D_{12} + c\ D_{22}$$

That is, changes in currency supply are more (less) effective with a <u>fixed</u> deposit rate if currency and deposits are substitutes (complements).

(iv) Effect of change in reserve requirement.

(A) Deposit rate flexible.

Differentiating III.1 gives:

$$\text{(III.12)} \qquad \begin{bmatrix} & \\ & J' & \\ & \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial c} \\[2ex] \frac{\partial R_2}{\partial c} \end{bmatrix} = \begin{bmatrix} -\ D_2 \\[2ex] D_2 + \frac{L_{22}\ R_2}{(1-c)^2} \end{bmatrix}$$

Since $|J'|$ is negative for stable solutions,

$$\operatorname{sign} \frac{\partial R_o}{\partial c} = \operatorname{sign} \begin{vmatrix} D_2 & D_{12} + c\, D_{22} \\ -D_2 - \dfrac{L_{22} R_2}{(1-c)^2} & (1-c)\, D_{22} - \dfrac{L_{22}}{1-c} \end{vmatrix}$$

$$= \operatorname{sign} \begin{vmatrix} D_2 & D_{12} + c\, D_{22} \\ -\dfrac{L_{22} R_2}{(1-c)^2} & D_{12} + D_{22} - \dfrac{L_{22}}{1-c} \end{vmatrix}$$

Therefore, if $D_{12} + c\, D_{22} < 0$, then $\dfrac{\partial R_o}{\partial c} > 0$. (Substitutes case). But if $D_{12} + c\, D_{22} > 0$ (complement case), $\dfrac{\partial R_o}{\partial c}$ may be negative. For example, let $D_2 = 0$.

(B) Deposit rate fixed.

Differentiating III.2 gives

$$\text{(III.13)} \qquad \begin{bmatrix} & \\ & J^2 & \\ & \end{bmatrix} \begin{bmatrix} \dfrac{\partial R_o}{\partial c} \\ \dfrac{\partial r_2}{\partial c} \end{bmatrix} = \begin{bmatrix} -D_2 \\ D_2 \end{bmatrix}$$

Therefore, $\dfrac{\partial R_o}{\partial c} = \dfrac{D_2\, L_{22}}{|J^2|} > 0$.

(v) Effect of change in reserve requirement on response of system to change in currency supply.

(A) Deposit rate flexible.

Differentiating (III.8) with respect to c gives:

$$\begin{bmatrix} & \\ J' & \\ & \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial c}\left(\frac{\partial R_o}{\partial S_1}\right) \\ \frac{\partial}{\partial c}\left(\frac{\partial R_2}{\partial S_1}\right) \end{bmatrix} = - \begin{bmatrix} D_{20} & D_{22} \\ -D_{20} & -D_{22} - \frac{L_{22}}{(1-c)^2} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \end{bmatrix}$$

$$= - \begin{bmatrix} \frac{\partial D_2}{\partial S_1} \\ -\frac{\partial D_2}{\partial S_1} - \frac{L_{22}}{(1-c)^2}\frac{\partial R_2}{\partial S_1} \end{bmatrix}$$

$$\text{sign}\ \frac{\partial}{\partial c}\left(\frac{\partial R_o}{\partial S_1}\right) = \text{sign} \begin{vmatrix} \frac{\partial D_2}{\partial S_1} & D_{12} + c\,D_{22} \\ -\frac{\partial D_2}{\partial S_1} - \frac{L_{22}}{(1-c)^2}\frac{\partial R_2}{\partial S_1} & (1-c)\,D_{22} - \frac{L_{22}}{1-c} \end{vmatrix}$$

$$= \text{sign} \begin{vmatrix} \frac{\partial D_2}{\partial S_1} & D_{12} + c\,D_{22} \\ -\frac{L_{22}}{(1-c)^2}\frac{\partial R_2}{\partial S_1} & D_{12} + D_{22} - \frac{L_{22}}{1-c} \end{vmatrix}$$

If $D_{12} + c\, D_{22} < 0$ (substitutes case) and $\frac{\partial D_2}{\partial S_1} < 0$, then $\frac{\partial}{\partial c}\left(\frac{\partial R_o}{\partial S_1}\right) < 0$, i.e., an increase in c increases the response. In particular $D_{12} + c\, D_{22} < 0$ when $c = 0$. If $D_{12} + c\, D_{22} > 0$ (complements case) and $\frac{\partial D_2}{\partial S_1} > 0$, then $\frac{\partial}{\partial c}\left(\frac{\partial R_o}{\partial S_1}\right) > 0$.

(B) Deposit rate fixed.

Differentiating III.9 with respect to c gives:

$$\begin{bmatrix} & \\ & J^2 & \\ & \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial c}\left(\frac{\partial R_o}{\partial S_1}\right) \\ \frac{\partial}{\partial c}\left(\frac{\partial r_2}{\partial S_1}\right) \end{bmatrix} = \begin{bmatrix} -D_{20}\,\frac{\partial R_o}{\partial S_1} \\ D_{20}\,\frac{\partial R_o}{\partial S_1} \end{bmatrix}$$

$$\text{Therefore } \frac{\partial}{\partial c}\left(\frac{\partial R_o}{\partial S_1}\right) = \frac{D_{20}\, L_{22}\, \frac{\partial R_o}{\partial S_1}}{|J^2|} > 0 \quad .$$

That is, increasing the required reserve ratio always diminishes the response.

PART III. Extension to Many Intermediaries.

The discussion of regimes below parallels Part II.

Regime II

(A) Uncontrolled Intermediaries.

$$(\text{II}.1) \quad D_1(R_o, R_2, \ldots, R_n) = S_1 \quad \text{Currency}$$

$$D_i(R_o, R_2, \ldots, R_n) - L_i(R_o, r_2, \ldots, r_n) = 0 \quad \text{Intermediaries}$$

$$r_i - R_i = a_i \quad \text{Relation between the rates}$$

$$i = 2, \ldots, n$$

(i) Effect of change in currency supply.

Using the $n - 1$ rate relations to eliminate the r_i and differentiating with respect to S_1 :

$$(\text{II}.2) \quad \begin{bmatrix} D_{10} & D_{12} & \cdots & D_{1n} \\ D_{20}-L_{22} & D_{22}-L_{22} & \cdots & D_{2n}-L_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ D_{n0}-L_{n0} & D_{n2}-L_{n2} & \cdots & D_{nn}-L_{nn} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \\ \vdots \\ \frac{\partial R_n}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

By assumptions (2) and (4) in Part I the first column of the $n \times n$ matrix in (II.2) is composed entirely of non-positive elements. In the remaining columns, the diagonal elements are positive and the rest non-positive. Moreover, the sum of the elements in every column but the first is positive. It is shown in Part IV that the determinant of such a matrix is negative and that all the cofactors of the first row are positive. Hence all the derivatives that solve (II.2) are negative. An increase in the supply of currency will lower rates at all intermediaries and will lower the acceptable return on direct equity in capital.

The change in the volume of each intermediary $\frac{\partial D_i}{\partial S_1} = \sum_j D_{ij} \frac{\partial R_j}{\partial S_1}$,

$(i = 2, \ldots, n)$, and the corresponding derivative for aggregate intermediary liabilities $\sum_{i=2}^{n} \frac{\partial D_i}{\partial S_1} = \sum_i \sum_j D_{ij} \frac{\partial R_j}{\partial S_1}$ may have either sign.

(B) Controlled intermediaries.

$$
\text{(II.3)} \qquad
\begin{aligned}
D_1 (R_o, R_2, \ldots, R_n) &= S_1 && \text{Currency} \\
e_i D_i (R_o, R_2, \ldots, R_n) &= S_i && \text{Specific Reserves} \\
(1-e_i) D_i (R_o, R_2, \ldots, R_n) - L_i (R_o, r_2, \ldots, r_n) &= 0 && \text{Intermediaries} \\
& && i = 2, \ldots, n
\end{aligned}
$$

Here S_i is the supply of the reserve asset specific to the i^{th} intermediary expressed as a proportion of total wealth. The required ratio for the i^{th} intermediary is $0 \leq e_i \leq 1$. There are $2n - 1$ equations in the $2n - 1$ rates R_o, R_i, r_i ; the last $n - 1$ equations in (II.1) drop out. (However, the inequalities $r_i - R_i \geq a_i$ must hold.)

(i) Effect of change in currency supply.

$$
\text{(II.4)} \quad \left[\begin{array}{cccc|ccc}
D_{10} & \cdots & & D_{1n} & 0 & \cdots & 0 \\
e_2 D_{20} & \cdots & & e_2 D_{2n} & & & \\
\vdots & & & \vdots & \vdots & & \vdots \\
e_n D_{n0} & \cdots & & e_n D_{nn} & 0 & \cdots & 0 \\
\hline
(1-e_2)D_{20}-L_{20} & (1-e_2)D_{22} & \cdots & (1-e_2)D_{20} & -L_{22} & \cdots & -L_{2n} \\
\vdots & \vdots & & & \vdots & & \vdots \\
(1-e_n)D_{n0}-L_{n0} & (1-e_n)D_{n2} & \cdots & (1-e_n)D_{n0} & -L_{n2} & \cdots & -L_{nn}
\end{array} \right]
\left[\begin{array}{c}
\frac{\partial R_o}{\partial S_1} \\
\frac{\partial R_2}{\partial S_1} \\
\vdots \\
\frac{\partial R_n}{\partial S_1} \\
\hline
\frac{\partial r_2}{\partial S_1} \\
\vdots \\
\frac{\partial r_n}{\partial S_1}
\end{array} \right]
=
\left[\begin{array}{c}
1 \\
0 \\
\vdots \\
\\
0
\end{array} \right]
$$

The first n equations can be solved separately from the last $(n-1)$ equations for the n derivatives $\frac{\partial R_o}{\partial S_1}$, $\frac{\partial R_i}{\partial S_1}$. Since this subsystem has the same features as system (II.2) it is easily seen that $\frac{\partial R_o}{\partial S_1}$,

$\frac{\partial R_i}{\partial S_1} \leq 0 \quad (i = 2, \ldots, n)$.

The remaining $(n - 1)$ equations may be solved for the $\frac{\partial r_i}{\partial S_1}$. Since the first n equations are satisfied, $\sum_j e_i D_{ij} \frac{\partial R_j}{\partial S_1} = 0$, this system reduces to:

$$(\text{II.5}) \quad \begin{bmatrix} -L_{22} & \cdots & -L_{2n} \\ \vdots & & \\ -L_{n2} & \cdots & -L_{nn} \end{bmatrix} \begin{bmatrix} \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix} = \begin{bmatrix} L_{20} \\ \vdots \\ L_{n0} \end{bmatrix} \frac{\partial R_o}{\partial S_1}$$

The diagonal elements of the matrix in (II.5) are all positive, the off diagonal element non-positive, and the column sums positive. In Part IV it is shown that the determinant of such a matrix is positive and that the cofactors of each element are positive. Hence

$$\frac{\partial r_i}{\partial S_1} \leq 0 , \quad i = 2, \ldots, n .$$

(ii) Comparison with uncontrolled regime.

The effect of restraining the expansion of each intermediary can be found by subtracting equation system (II.2) from the first n equations of II.4.

(II.6)

$$J_4 \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \vdots \\ \frac{\partial R_n}{\partial S_1} \end{bmatrix}_4 - J_2 \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \vdots \\ \frac{\partial R_n}{\partial S_1} \end{bmatrix}_2 = 0$$

where J_4 is the sub-Jacobian in II.4 and J_2 the Jacobian in (II.2)

Given

$$J_2 = J_4 + \begin{bmatrix} 0 & \cdots & 0 \\ -L_{20} & \cdots & -L_{2n} \\ \vdots & & \\ -L_{n0} & \cdots & -L_{nn} \end{bmatrix} = J_4 + \Delta$$

(II.7)

$$J_4 \left[\begin{bmatrix} \frac{\partial R_i}{\partial S_1} \end{bmatrix}_4 - \begin{bmatrix} \frac{\partial R_i}{\partial S_1} \end{bmatrix}_2 \right] = \Delta \begin{bmatrix} \frac{\partial R_i}{\partial S_1} \end{bmatrix}_2 = - \begin{bmatrix} 0 \\ \frac{\partial D_2}{\partial S_1} \\ \vdots \\ \frac{\partial D_n}{\partial S_1} \end{bmatrix}_2$$

Given $|J_4| < 0$ it follows that

$$(\text{II.8}) \qquad \text{sign}\left(\frac{\partial R_o}{\partial S_1}\right)_4 - \left(\frac{\partial R_o}{\partial S_1}\right)_2 = \text{sign}\begin{vmatrix} 0 & D_{12} & \cdots & D_{12} \\ \left(\frac{\partial D_2}{\partial S_1}\right)_2 & D_{22} & \cdots & D_{2n} \\ \vdots & \vdots & & \vdots \\ \left(\frac{\partial D_n}{\partial S_1}\right)_2 & D_{n2} & \cdots & D_{nn} \end{vmatrix}$$

If all the $\left(\frac{\partial D_2}{\partial S_1}\right)_2$ are negative the determinant at the right will be negative; this is a sufficient, not a necessary, condition.

If reduction of the currency supply would lead to an expansion of all the intermediaries when they are uncontrolled, then preventing this expansion will enhance the effectiveness of currency restriction.

REGIME III

(A) Deposit Rates flexible.

$$(\text{III.1}) \qquad D_1(R_o, R_2, \ldots, R_n) + c \sum_2^n D_i(R_o, R_2, \ldots, R_n) = S_1 \qquad \text{Currency}$$

$$(1-c)D_i(R_o, R_2, \ldots, R_n) - L_i(R_o, r_2, \ldots, r_n) = 0 \qquad \text{Intermediaries}$$

$$\frac{R_i}{(1-c)} - r_i = 0 \qquad \text{Relation between rates}$$

$$(i = 2, \ldots, n)$$

Here the required reserve for each intermediary is in currency, and the required ratio is $0 \leq c \leq 1$. For convenience we assume the required ratio for all intermediaries is equal. The $2n - 1$ equations determine the $2n - 1$ variables R_o, R_i, r_i $(i = 2, \ldots, n)$.

(B) Deposit rates fixed.

$$\text{(III.2)} \qquad D_1(R_o, \bar{R}_2, \ldots, \bar{R}_n) + c \sum_2^n D_i(R_o, \bar{R}_i) = S_1 \qquad \text{Currency}$$

$$(1-c)D_i(R_o, \bar{R}_2, \ldots, \bar{R}_n) - L_i(R_o, r_2, \ldots, r_n) = 0 \qquad \text{Intermediaries}$$

$$(i = 2, \ldots, n)$$

Here the deposit rate at the i^{th} intermediary is fixed at $\bar{R}_i$, and the n equations determine the n variables R_o, r_i $i = 2, \ldots, n$. In order for the regime to apply, the following inequality must hold:

$$\frac{\bar{R}_i}{(1-c)} \leq r_i$$

(i) Effect of change in currency supply.

(A) Deposit rates flexible.

Differentiating (III.1) gives:

(III.3)

$$\begin{bmatrix} D_{10}+c\Sigma D_{i0} & D_{12}+c\Sigma D_{i2} & \cdots & D_{1n}+c\Sigma D_{in} \\ (1-c)D_{20}-L_{20} & (1-c)D_{22}-\frac{L_{22}}{(1-c)} & \cdots & (1-c)D_{2n}-\frac{L_{2n}}{(1-c)} \\ \vdots & \vdots & & \vdots \\ (1-c)D_{n0}-L_{n0} & (1-c)D_{n2}-\frac{L_{n2}}{(1-c)} & \cdots & (1-c)D_{nn}-\frac{L_{nn}}{(1-c)} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \\ \vdots \\ \frac{\partial R_n}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Since the cofactors of the first row are all positive the derivatives that solve (III.3) are all of the same sign:

$$\text{sign} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial R_2}{\partial S_1} \\ \vdots \\ \frac{\partial R_n}{\partial S_1} \end{bmatrix} = \text{sign} \; |J^3| \qquad \text{where}$$

J^3 is the Jacobian in (III.3).

A sufficient, but not necessary condition that $|J^3| < 0$ is that all the deposits be "substitutes" with currency:

$$(D_{1j} + c\sum_{i=2}^{n} D_{ij}) \leq 0 \qquad j = 2, \ldots, n$$

This condition is analogous to the substitutes case discussed in the text and Part II of the Appendix. In this case however an increase in the i^{th} rate causes a decrease in the demand for currency reserves by the other intermediaries as well as an increase in the demand for reserves by the i^{th} intermediary.

(B) Deposit rate fixed.

Differentiating (III.2) gives:

(III.4)

$$\begin{bmatrix} D_{10} + c\Sigma D_{i0} & 0 & \cdots & 0 \\ (1-c)D_{20} - L_{20} & -L_{22} & \cdots & -L_{2n} \\ \vdots & \vdots & & \vdots \\ (1-c)D_{n0} - L_{n0} & -L_{n2} & \cdots & -L_{nn} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The Jacobian in this system of equations meets the same sign conditions as the Jacobian in system (II.2), hence $\frac{\partial R_o}{\partial S_1}, \frac{\partial r_i}{\partial S_1} \leq 0 \quad (i = 2, \ldots, n)$.

Comparison of Response in (A) and (B)

Following the procedure discussed in Part II Regime III (iii) we may compare the responses in the two cases by subtracting system (III.3) (but using the rate relations to eliminate the deposit rather than loan rates) from system (III.4):

(III.5)

$$J^4 \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix}_4 - J^{3'} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix}_{3'} = \begin{bmatrix} 0 \\ \\ \vdots \\ 0 \end{bmatrix}$$

where J^4 is the Jacobian in (III.4) and $J^{3'}$ is the Jacobian of the system (III.3) when the deposit rates rather than the loan rates are eliminated.

(III.6)

$$J^4 \left[\begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix}_4 - \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix}_{3'} \right] = - \begin{bmatrix} J^4 - J^{3'} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix}_{3'}$$

$$= (1-c) \begin{bmatrix} 0 & (D_{12}+c \sum_{2}^{n} D_{i2}) & \cdots & (D_{1n}+c \sum_{2}^{n} D_{in}) \\ \vdots & (1-c)\, D_{22} & \cdots & (1-c)\, D_{22} \\ & \vdots & & \vdots \\ 0 & (1-c)\, D_{n2} & \cdots & (1-c)\, D_{n2} \end{bmatrix} \begin{bmatrix} \frac{\partial R_o}{\partial S_1} \\ \frac{\partial r_2}{\partial S_1} \\ \vdots \\ \frac{\partial r_n}{\partial S_1} \end{bmatrix}_{3'}$$

Since $|J^4| < 0$ and the cofactor of the first element in the first row of J^4 is positive:

$$\text{sign}\left(\frac{\partial R_o}{\partial S_1}\right)_4 - \left(\frac{\partial R_o}{\partial S_1}\right)_{3'} = \text{sign} - \sum_{j=2}^{n} \left(D_{1j} + c\sum_{i=2}^{n} D_{ij}\right)\left(\frac{\partial r_j}{\partial S_1}\right)_{3'}$$

When $|J^3| < 0$, $\left(\frac{\partial r_j}{\partial S_1}\right)_{3'} \leq 0$ for all j.

Changes in currency supply are more (less) effective with fixed deposit rates if currency and each variety of "deposits" are substitutes (complements).

(ii) Effect of change in reserve requirement on response of system to change in currency supply.

(A) Deposit rate flexible.

Differentiating (III.3) with respect to c gives:

(III.7)

$$J^3 \begin{bmatrix} \dfrac{\partial\left(\dfrac{\partial R_i}{\partial S_1}\right)}{\partial c} \end{bmatrix} = - \begin{bmatrix} \sum_2^n D_{i0} & \sum_2^n D_{i2} & \cdots & \sum_2^n D_{in} \\ -D_{20} & -D_{22} - \dfrac{L_{22}}{(1-c)^2} & \cdots & -D_{2n} - \dfrac{L_{22}}{(1-c)^2} \\ \vdots & \vdots & \cdots & \vdots \\ -D_{n0} & -D_{n2} - \dfrac{L_{n2}}{(1-c)^2} & \cdots & -D_{nn} - \dfrac{L_{nn}}{(1-c)^2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial R_i}{\partial S_1} \end{bmatrix}$$

where J^3 is the Jacobian in (III.3).

This reduces to:

$$(\text{III.8}) \qquad J^3 \begin{bmatrix} \dfrac{\partial\left(\dfrac{\partial R_i}{\partial S_1}\right)}{\partial c} \end{bmatrix} = - \begin{bmatrix} \dfrac{\partial \Sigma D_i}{\partial S_1} \\ -\dfrac{\partial D_2}{\partial S_1} - \sum_2^n \dfrac{L_{2i}}{(1-c)^2} \dfrac{\partial R_i}{\partial S_1} \\ \vdots \\ -\dfrac{\partial D_n}{\partial S_1} - \sum_2^n \dfrac{L_{ni}}{(1-c)^2} \dfrac{\partial R_i}{\partial S_1} \end{bmatrix}$$

Assuming $|J^3| < 0$: If $\dfrac{\partial D_i}{\partial S_1} < 0 \; (i = 2, \ldots, n)$,

$(D_{ij} + c \sum_{i=2}^{n} D_{ij}) < 0 \quad (j = 2, \ldots, n)$ (all substitutes case), and

$\sum_{i=2}^{n} \dfrac{L_{ji}}{(1-c)^2} \left(\dfrac{\partial R_i}{\partial S_1}\right) > 0$ then $\dfrac{\partial}{\partial c}\left(\dfrac{\partial R_o}{\partial S_1}\right) < 0$, i.e., an increase in c increases the response of the rate on capital.

(B) Deposit rates fixed.

Differentiating (III.4) with respect to c gives:

$$J^4 \begin{bmatrix} \dfrac{\partial\left(\dfrac{\partial R_i}{\partial S_1}\right)}{\partial c} \end{bmatrix} = - \begin{bmatrix} \sum_{i=2}^{n} D_{i0} \\ -D_{20} \\ \vdots \\ -D_{n0} \end{bmatrix} \dfrac{\partial R_o}{\partial S_1}$$

Therefore,

$$\frac{\partial\left(\frac{\partial R_o}{\partial S_1}\right)}{\partial c} = \frac{-\frac{\partial R_o}{\partial S_1}}{|J^4|} \begin{vmatrix} \sum_{i=2}^{n} D_{io} & 0 & \cdots & 0 \\ -D_{20} & -L_{22} & & -L_{2n} \\ \vdots & \vdots & & \vdots \\ -D_{20} & -L_{22} & \cdots & -L_{nn} \end{vmatrix} > 0$$

That is, increasing the required reserve ratio always diminishes the response of the rate on capital to changes in the supply of currency.

Part IV.

1. Let A be a non-singular square matrix with non-positive off-diagonal elements, positive diagonal elements, column sums $\sum_i a_{ij}$ positive. To prove that det(A) is positive.

Consider the matrix B where $b_{ij} = -a_{ij}/a_{jj}$ for $i \neq j$ and $b_{jj} = 0$.
B is then a matrix of non-negative elements with column sums $\sum_i b_{ij} < 1$.
det(A) will be positive if det(I-B) is positive, for $\det(I-B) = (1/\prod_j a_{jj})\det(A)$.

Proof that $\det(I-B) > 0$:-/

-/ For this proof, I am much indebted to Martin Beckmann.

Suppose $\det(I-B) \leqq 0$. Since for sufficiently large λ, $\det(\lambda I-B) > 0$, there must exist a root $\lambda_o \geqq 1$ with $\det(\lambda_o I-B) = 0$. The equation system $[\lambda_o I-B]'x = 0$ has a solution vector $x \neq 0$. Let x_j be the element of largest absolute value $|x_j| > 0$. $|x_j| \leqq |\lambda_o x_j|$. By the j^{th} equation of the system, $|\lambda_o x_j| = |\sum_i b_{ij} x_i|$. $|\sum_i b_{ij} x_i| \leqq \sum_i b_{ij} |x_i| \leqq \sum_i b_{ij} |x_j| < |x_j|$.
This contradicts that $\lambda_o \geqq 1$. Hence $\det(I-B) > 0$.

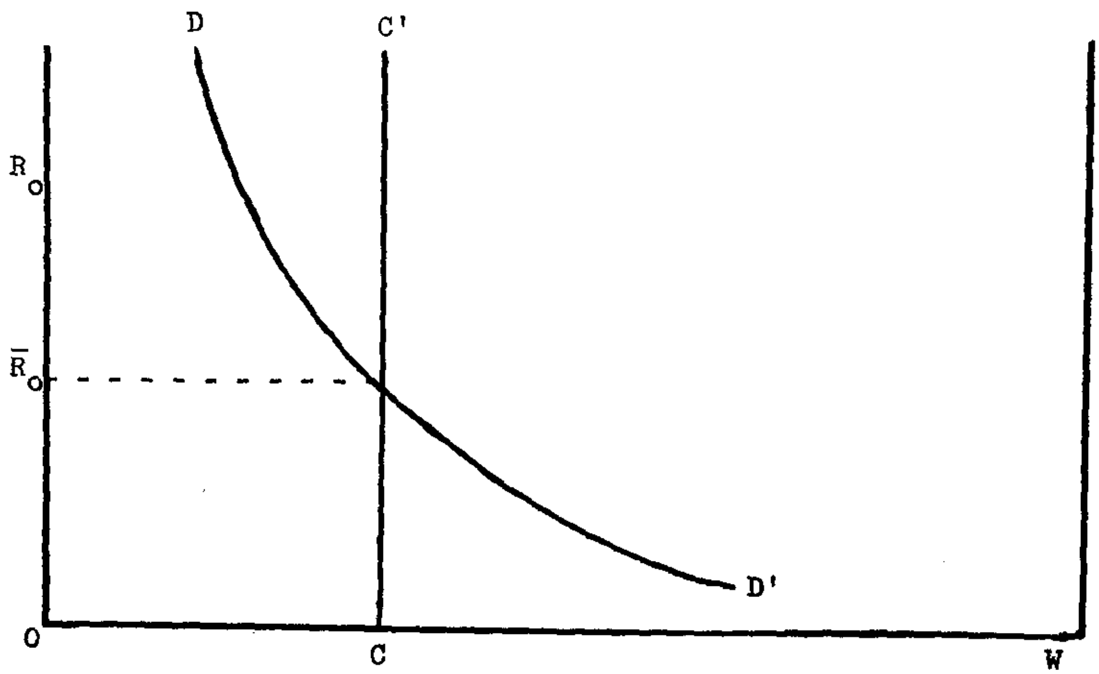
2. Consider a non-singular matrix A_1 formed by substituting for the first column of A a vector of non-positive elements. The proposition is that $\det(A_1)$ is negative. Proof by induction:

If the proposition is true for $n \times n$ square matrices, then it is true for $n+1 \times n+1$ square matrices. Add to $A(n)$ a new first row and first column so that the resulting matrix is $A_1(n+1)$. Expand A_1 by the first row. The first cofactor is $\det(A(n))$, which is positive according to note 1 above. The cofactors of the remaining elements of the first row all involve $n \times n$ minors of which the first column consists entirely of non-positive elements, while the remaining columns come from $A(n)$. The minor of the second element is $A_1(n)$ and by assumption negative; thus the second cofactor is positive. The minor of the third element can be made into an $A_1(n)$ by placing the third row at the top of the minor. This interchange alters the sign; hence the minor and cofactor are both positive. In general, the minor of the i^{th} element can be made into an $A_1(n)$ by the $i-2$ interchanges necessary to place the i^{th} row at the top of the minor. The minors will be positive for i odd and negative for i even; therefore all the cofactors are positive. Since all the elements of the first row are non-positive, $A_1(n+1)$ is negative.

The proposition is true for $n = 2$. $A_1(2) = \begin{vmatrix} - & - \\ - & + \end{vmatrix}$.

If the first and last rows of the matrix of coefficients in (II.2) are interchanged, the resulting matrix is $A_1(n+1)$. Hence the matrix in (II.2) has a positive determinant, with negative cofactors for the last row.

Figure 1



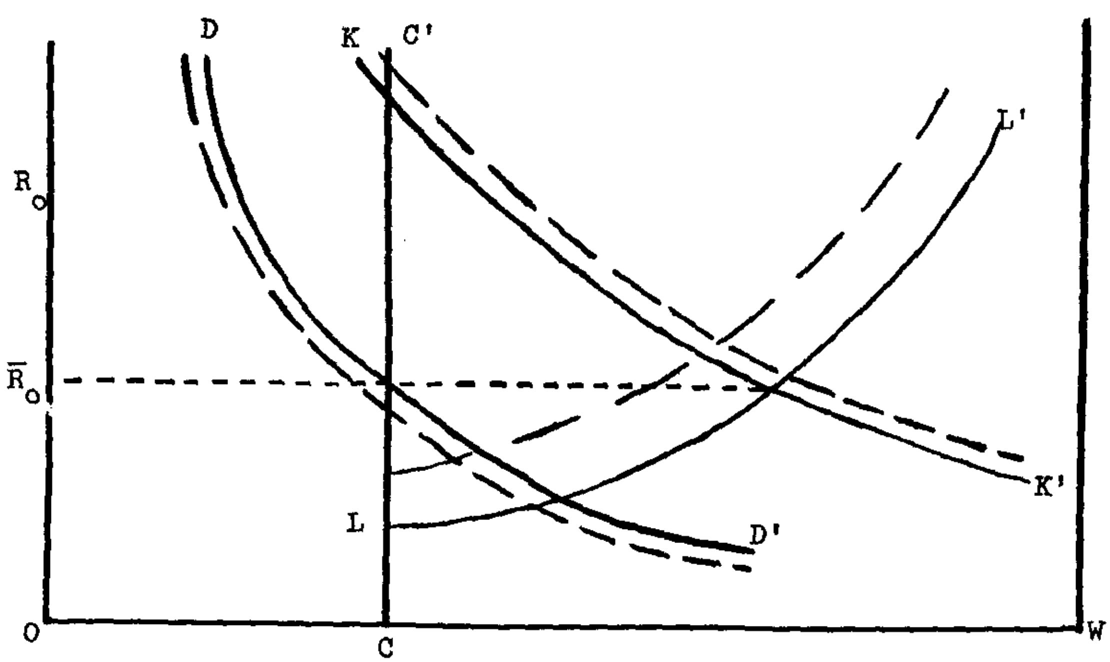
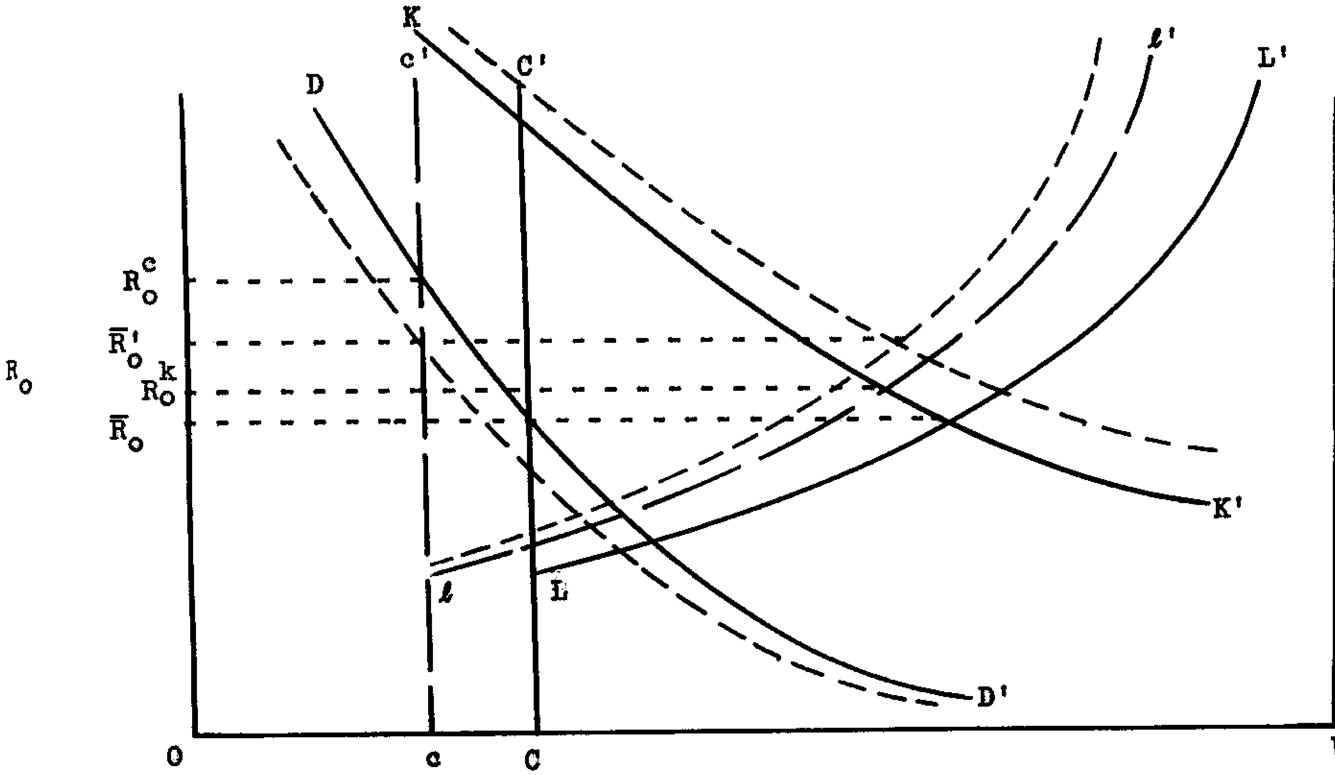


Figure 2

Figure 3



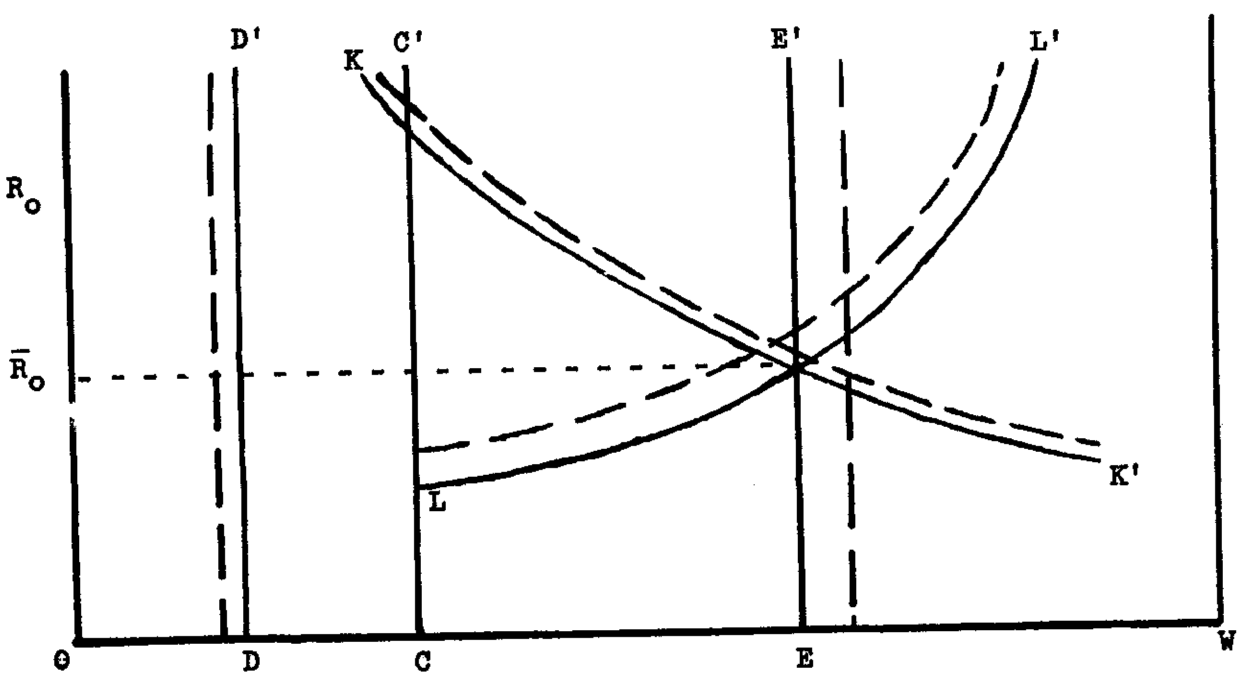
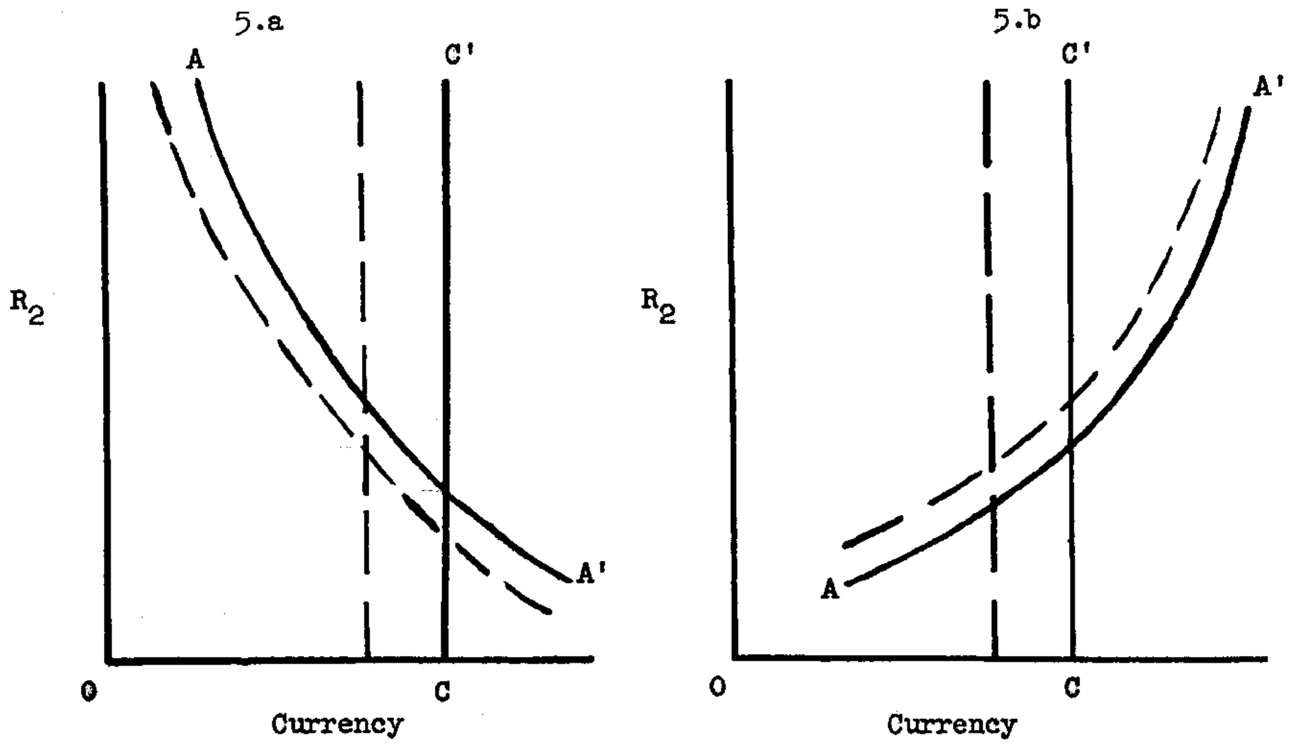


Figure 4

Figure 5



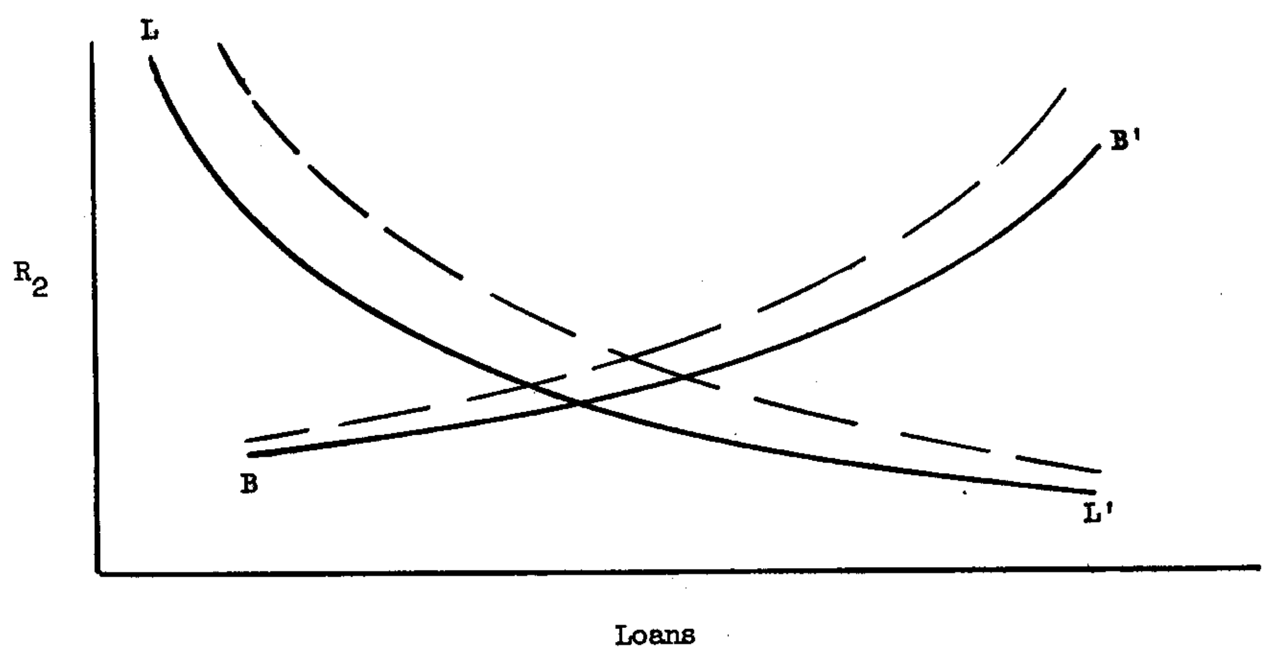


Figure 6

Figure 7

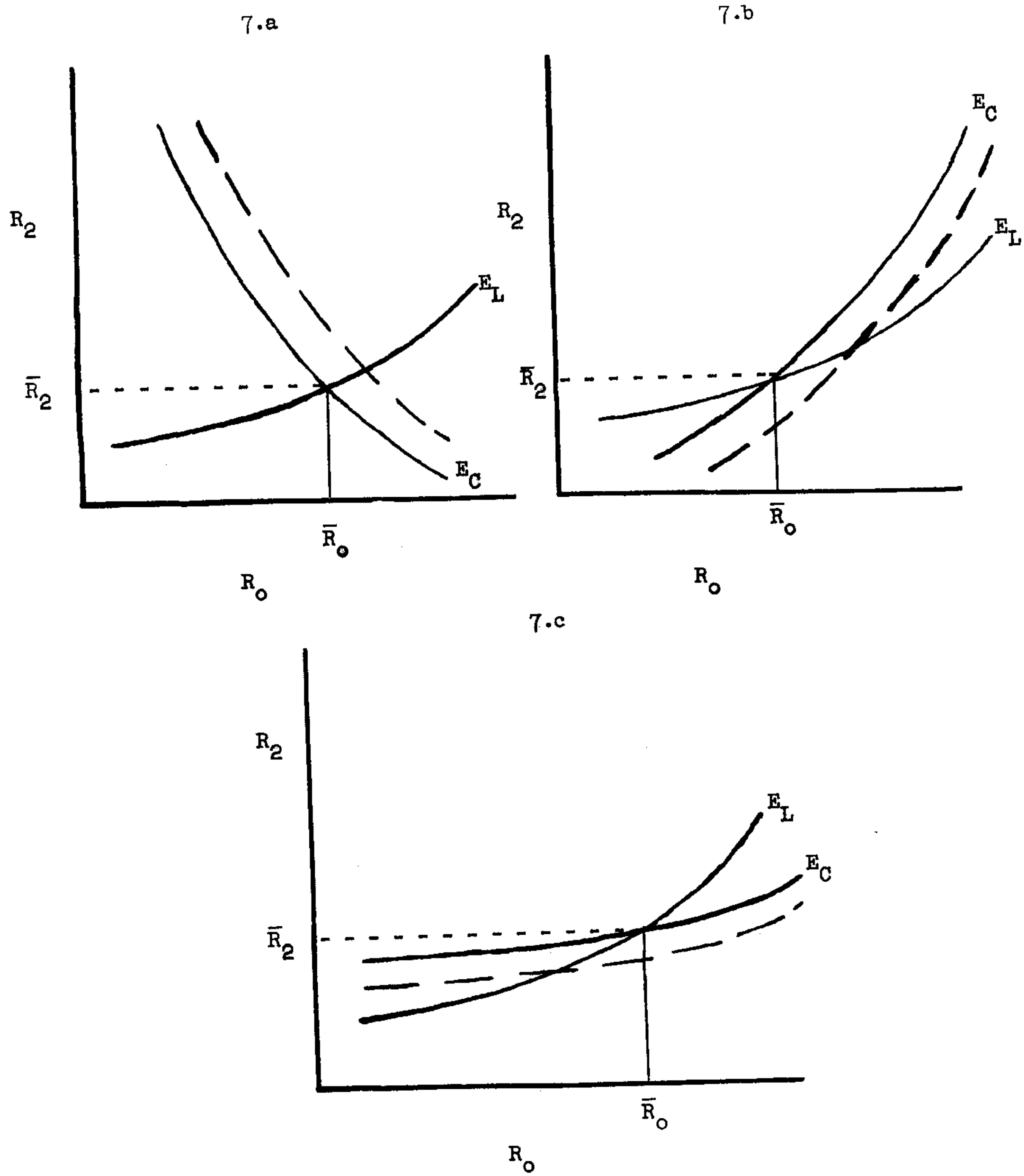

7.a
R_2
$\bar{R}_2$
E_L
E_C
$\bar{R}_o$
R_o
7.b
R_2
$\bar{R}_2$
E_C
E_L
$\bar{R}_o$
R_o
7.c
R_2
$\bar{R}_2$
E_L
E_C
$\bar{R}_o$
R_o

Figure 8