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Testing for Neutrality of Technological Change

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What follows is a brief comment upon this passage in Robert Solow's "Technical Change and the Aggregate Production Function":

"It can be shown, by integrating a partial differential equation, that if \dot{F}/F is independent of K and L (actually under constant returns only K/L matters) then ... shifts in the production function are neutral." (Review of Economics and Statistics, August 1957, p. 313)

Solow is employing a production function $F(K, L; t)$, assumed to be linear homogeneous in K and L for all values of the parameter t , which stands for all influences tending to shift the function. He shows that the relation

$$Q = F(K, L; t)$$

plus the marginal productivity theory of distribution yields the formula

$$\frac{\dot{q}}{q} = \frac{1}{F} \frac{\partial F}{\partial t} + W_K \frac{\dot{k}}{k}$$

where $q = \frac{Q}{L}$, $k = \frac{K}{L}$, W_K denotes the share of capital, and dots indicate time derivatives. Since time series on the profit share and on the growth of output per man hour and capital per man hour are available, this relation permits $\frac{1}{F} \frac{\partial F}{\partial t}$ to be estimated.

The partial differential equation to which Solow refers is presumably

$$\frac{1}{F} \frac{\partial F}{\partial t} = g(t),$$

so that the right-hand side is not a function of K or L . This may be integrated:

$$\begin{aligned}\frac{\dot{F}}{F} &= g(t)\partial t \\ \log F &= \int g(t)\partial t + \varphi(K,L) \\ F &= e^{\int g(t)\partial t} e^{\varphi(K,L)}\end{aligned}$$

Equivalently, set

$$\begin{aligned}\int g(t)\partial t &= \log A(t) \\ \text{and} \\ \varphi(K,L) &= \log f(K,L)\end{aligned}$$

The result is

$$F = A(t)f(K,L) ,$$

which is Solow's "neutral change" form of F . The parenthetical comment on the constant returns case can be interpreted as follows: It can be shown that the requirement that F be linear homogeneous in K and L for all t implies that $\frac{\dot{F}}{F}$ is likewise linear homogeneous in K and L for all t . Consequently, $\frac{\dot{F}}{F}$ is homogeneous of degree zero in K and L , and depends on the ratio K/L if it depends on K and L at all. Therefore, a showing that $\frac{\dot{F}}{F}$ does not depend on K/L is a showing that it does not depend on K and L , and neutrality is assured.

The discussion above tends to confirm the statement quoted. However, it does not validate the test for neutrality made by Solow (and by B. Massell*)

* "Capital Formation and Technological Change," CFDP 58

in his empirical work. This test was a scatter diagram of the historically observed values of \dot{F}/F against $k = K/L$, which showed little evidence of a relationship. Both authors concluded that neutrality is a satisfactory assumption. Such a test actually affords no positive evidence for neutrality.

What is required is a showing that for each value of t , different values of k do not result in different values of \dot{F}/F . This implies that each shift moves a particular isoquant of F toward the origin in a uniform way: in the same proportion along all rays through the origin. While the constant returns assumption guarantees that all points on a particular ray are shifted in the same proportion, it does not assure that this proportion is the same on all rays. Consequently, without knowledge of what a given shift does along several rays, it is not possible to draw conclusions about neutrality.

This argument can be based upon the partial differential equation. Let us assume, provisionally, that it has the form

$$\dot{F}/F = g\left(\frac{K}{L}, t\right)$$

and that we have a series of observations on \dot{F}/F , K/L , and t , no value of t being repeated. Suppose \dot{F}/F shows no tendency to change with K/L ; for simplicity, assume \dot{F}/F is observed to be constant. Does this indicate that g does not depend on K/L ? Not at all, it merely indicates that the combinations $K/L, t$ which we have observed lie on a contour line of g . Only when the constancy of \dot{F}/F is observed for a fixed t is the conclusion $\frac{\partial g}{\partial k} = 0$ justified. An example is instructive.

Let

$$Q = F(K, L; t) = K^{1-\frac{\alpha}{t^\epsilon}} L^{\frac{\alpha}{t^\epsilon}}$$

where $\epsilon > 0$

This is a familiar Cobb-Douglas function modified by labor saving technological change. We must assume that (in the units we happen to employ) we are operating in the region $K > L$, so that the change is an improvement -- something else is going on outside this region. We assume K is growing faster than L . Now,

$$\frac{\partial F}{\partial t} = K^{1-\frac{\alpha}{t^\epsilon}} L^{1-\frac{\alpha}{t^\epsilon}} \alpha \epsilon t^{-\epsilon-1} \log\left(\frac{K}{L}\right)$$

Thus

$$\frac{1}{F} \frac{\partial F}{\partial t} = \log\left(\frac{K}{L}\right) \alpha \epsilon t^{-\epsilon-1}$$

Suppose $\frac{K}{L}$ is observed to increase over time in a manner described by

$$\frac{K}{L} = e^{\frac{St^{1+\epsilon}}{\alpha\epsilon}}$$

For small values of ϵ , $\frac{K}{L}$ is increasing slightly faster than exponentially. Now

$$\log\left(\frac{K}{L}\right) = \frac{St^{1+\epsilon}}{\alpha\epsilon}$$

or

$$\log\left(\frac{K}{L}\right) \alpha \epsilon t^{-\epsilon-1} = S$$

Thus, $\frac{\dot{F}}{F}$ will be observed to be a constant, S , and Solow's test will lead one to conclude that technological change is neutral!

The conclusion seems to be that the Solow-Massell empirical work is to be interpreted as an investigation of the implications of an untested assumption that technological change is neutral. We seem to have returned to that unfortunate situation in which the effects of technological change and capital accumulation cannot be separated without more knowledge of the production function at each point of time.