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# Capital Formation and Technological Change*

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## 0. Introduction

The present study represents an attempt to apportion increases in output per man-hour between increases in capital employed per man-hour and a somewhat nebulous constellation of forces commonly referred to as "technological change." It is hoped that a quantitative estimate of the relative importance of these two factors in contributing to an increase in the average productivity of labor in the past will help policy-makers determine what proportion of our investment resources should be devoted to improving the technology, rather than to expanding existing types of capital equipment and structures.

My procedure is to examine the annual increases in output per man-hour of labor in the manufacturing sector of the United States economy between 1919 and 1955. I shall try to determine what proportion of these annual increases can be attributed to increases in capital input per man-hour, attributing the residual to technological change. The classification of causal forces is thus exhaustive, for technological change serves as a sort of catch-all category. The implications of this will be discussed in greater detail below.

## 1. Choice of Model

As a point of departure, we shall employ the model developed by Robert Solow,* which represents technological change (whether

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* "Technical Change and the Aggregate Production Function," The Review of Economics and Statistics, XXXIX, No. 3 (August, 1957), pp. 312-320.

neutral or non-neutral) as a shift in the aggregate production function. In its most general form, the production function can be written

(1) $Q = T(C, L; t)$,

where

$Q$ = output,

$C$ = capital input,

$L$ = labor input, and

$t$ = time.

Now, our task is to devise a measure of technological change, i.e., of the shift in the production function over time, which requires our making the fewest assumptions regarding the actual form of the function. Solow has shown (op. cit., pp. 312-313) that by assuming labor and capital to be paid according to their marginal products, we can write

(2) $$\frac{\dot{T}}{T} = \frac{\dot{P}}{P} - b\,\frac{\dot{K}}{K}$$

where

$P = Q/L$ (a dot denoting a time derivative),

$b$ = the ratio of net profits to net income,

$K = C/L$, and

$T$ = an index of technology.

An index of technology for each year can be derived from the expression*

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* See appendix A for the derivation of expressions (2) and (3). In fact, the assumption of competitive factor pricing together with exhaustive classification of inputs as capital or labor is equivalent to assuming a first degree homogeneous production function (see Solow, op. cit., p. 313).

$$(3) \qquad T(t+1) = T(t)\left(1 + \frac{\Delta T(t)}{T(t)}\right)$$

setting $T(19) = 1$. The index so derived should indicate the extent to which the production function has shifted since the preceding year. Three time series are required for computation of this index: output per man-hour, capital employed per man-hour, and capital's share of net output.

The assumption that factors are competitively priced frankly leaves me a little uneasy, especially during years characterized by either war or depression. It is awkward to speak of a factor receiving its marginal product when the economy is in a state of underemployment equilibrium (in the Keynesian sense), or when capital and labor rationing exist together with strict price controls. Nevertheless, the marginal productivity doctrine does not have to be taken too literally as an instantaneous equilibrating mechanism; rather, we can refer to a tendency for factors, in the long run, to be competitively priced, so that in most years, the rate of profit may serve as a first approximation to the marginal productivity of capital; and consequently capital's marginal product divided by its average product will be approximated by the ratio of net profits to net income originating in the manufacturing sector.

Our model, then, retains the full generality of expression (1), and neither restricts the form of the production function nor the character (as regards neutrality or biasedness) of shifts in the function. We are compelled to make only one restrictive assumption, at that one which is not uncommonly found in economic literature.

If, however, time can be factored out of equation (1), then technological change can be written as a multiplicative factor, so that we can write

$$(4) \qquad Q = T(t)f(C, L).$$

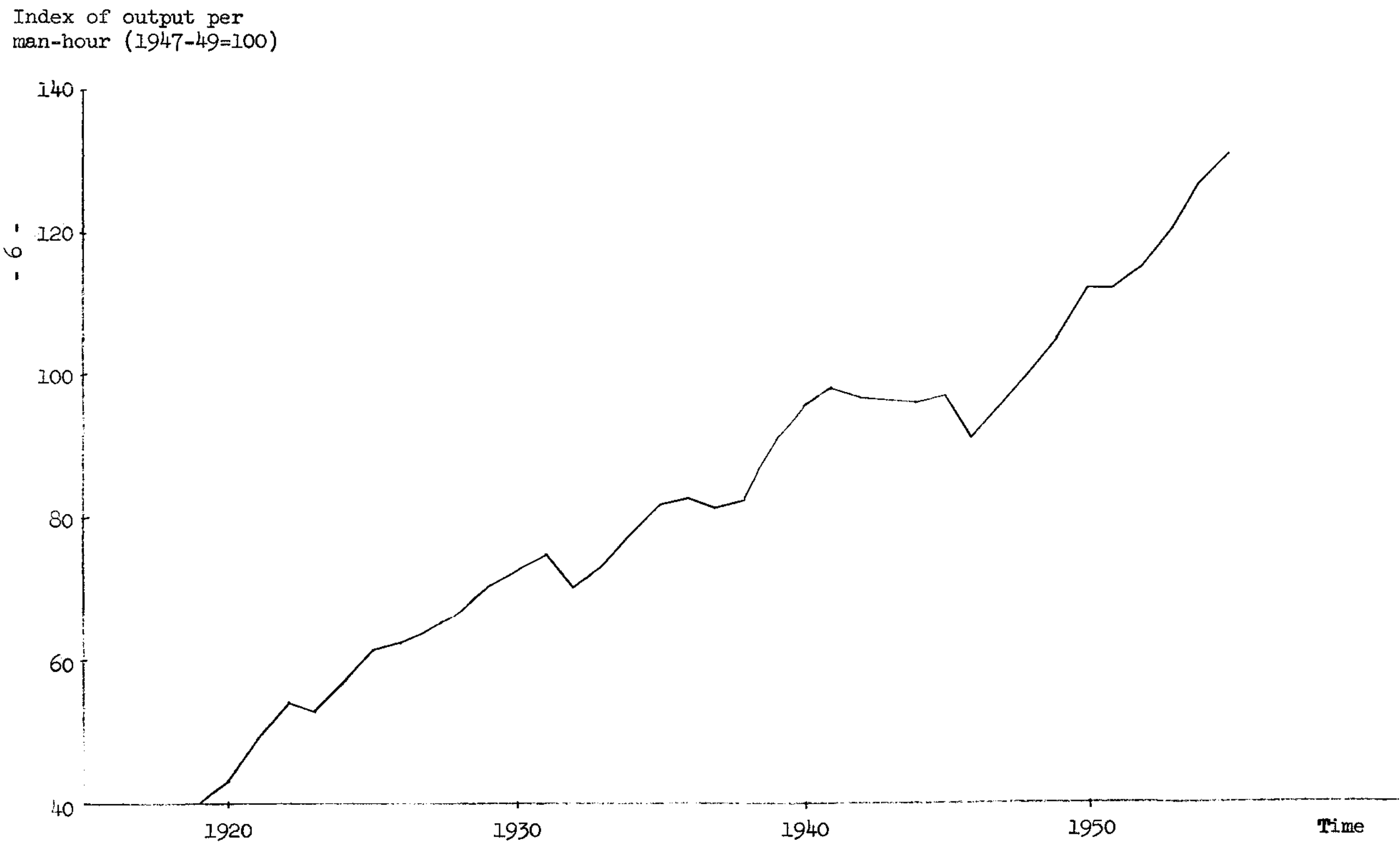
The function can be written in this form only if technological change is neutral, i.e., such as to leave the marginal rate of substitution between capital and labor unaltered at given amounts of both factors. A test will later be introduced to determine whether changes in the technology were such as to enable us to employ equation (4) rather than (1). The advantage offered by equation (4) consists chiefly in its intuitive appeal. A parallel shift in the function will permit technological change to be measured in a way which is independent of the quantities of the factors employed; while in the more general case, the extent of the shift in the function will itself be a function of the position on the aggregate production isoquant at which the economy is operating. If $\frac{\dot{T}}{T}$ does in fact prove to have been neutral, we have the best of both worlds, for without the necessity of making further assumptions which limit the generality of the conclusions, we are nevertheless enabled to use equation (4).

## 2. Selection and Adjustment of Data

Where Solow's analysis was concerned with the non-farm private sector of the economy, he suggested (op. cit., p. 312n) that a more appropriate study would be one dealing with a narrowly defined production function, one in which inputs and outputs would be specifically enumerated. While this has not been attempted here, a step has been made in this direction in considering only the manufacturing sector. While it is obvious that the inputs and product outputs of the aggregate production function of this sector do not entirely comply with Solow's specifications, it is advantageous, we believe, to limit the study to a sector producing physical goods only, and that increased homogeneity of output is thereby achieved.

The output series (see chart 1) refers to "real" output of the manufacturing sector of the U. S. economy, from 1919 to 1955. While a more desirable measure of output might be "valued-added" by manufacturing, which would be net of depreciation and would eliminate double-counting in aggregation, the available data relating to this magnitude are of dubious value, largely due to the difficulties involved in the measurement of depreciation, which depends to a much too great extent on the conventions pursued by accountants, and on prevailing tax laws. While there is still some question as to how "real" is real output as used here, there is no series which is clearly preferable to this one. The series, which divides the annual output figures by the corresponding figure for man-hours worked, was taken from <u>Productivity, Prices, and Incomes</u>, a Joint Economic Committee publication, p. 148. It is assumed, in using gross output rather than some measure akin to value-added, that the latter changes proportionately to the former, both secularly and cyclically; and to the extent that this assumption is not borne out by the data, there will be an unknown bias in the results.

Chart 1. Output per Man-hour in U.S. Manufacturing, 1919-1955



Other writers on technical change and/or capital formation* have

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* See, for example, W. Duane Evans, "Indexes of Labor Productivity as a Partial Measure of Technological Change," Netherlands Economic Institute (ed.), Input Output Relations, Proceedings of a Conference on Inter-Industrial Relations Held at Dreibergen, Holland; and E. F. Denison, "Theoretical Aspects of Quality Change, Capital Consumption, and Net Capital Formation," (with the comments which follow), Problems of Capital Formation, National Bureau of Economic Research, Studies in Income and Wealth, Volume 19, pp. 215-284.

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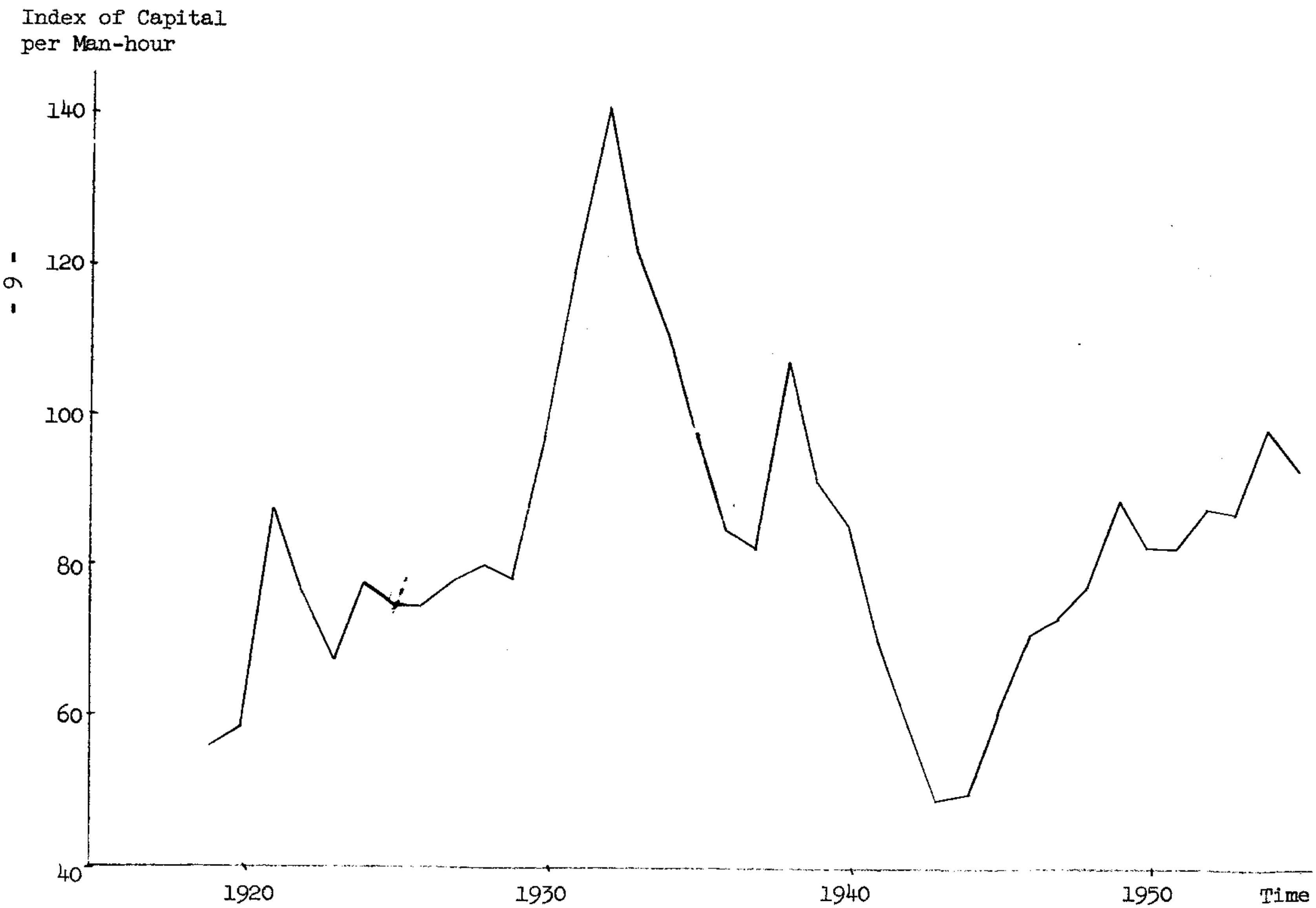
had much to say about the difficulties introduced by quality changes in inputs and in output. We feel that their discussion of this problem is sufficiently applicable to this paper, and is known to most readers, so that this need not be taken up here.

Labor input is measured in man-hours worked in the manufacturing sector (from Joint Economic Committee, op. cit., p. 148). This represents an attempt to measure all labor time, that of self-employed persons as well as employees. This corresponds best with the economist's notion of labor as a factor of production, rather than as a social class, or according to some other distinction. No attempt is made to allow for different grades of labor, for changes in the intensity with which the labor is applied, for changes in the composition of the labor force, nor for the increased importance of skills and education in more recent years. We have taken labor in terms of man-hours of some constant, or average quality. This surely results in an understatement of the increase in man-hours worked because it is beyond doubt that today's worker is better equipped intellectually, and possibly physically (due to better diets) than his temporal predescessor. It is debatable, though, whether this increase properly should be included as an increase in labor input, or whether it is a form of technical change, which thus allows a man to produce more with a given amount of effort, or time. Again, we can consider the skills and knowledge of which a man is possessed as a form of stored-up capital; just as today's worker is better equipped with physical capital, so is he better supplied with mental capital as a result of the dual process of capital formation and technical progress.

Capital input refers to structures, equipment, and inventory actually in use, rather than merely in existence. Although ideally the capital series would be one referring to "physical" capital, corresponding to the measure of output, we have had to use a value measure, in terms of constant dollars of 1947 purchasing power. The problem of converting capital in existence to capital employed is somewhat complex, so this will now be discussed in some detail.

Appendix B contains the details relating to our adjustment of the raw data to obtain a series of capital-in-existence. The raw data, for the years from 1929 to 1955, are from Donald G. Wooden and Robert C. Wasson, "Manufacturing Investment Since 1929 in Relation to Employment, Output, and Income," Survey of Current Business, XXXVI, No. 11 (November, 1956). For the years prior to 1929, little data relating to the capital stock in manufacturing were available, so a series was pieced together from a number of sources, as indicated in the appendix. The series for the entire period, giving the ratio of capital to man-hours worked, appears in chart 2; this series gives evidence of a strong upward trend, but is nevertheless marked by sharp cyclical instability. While the secular rise in the capital-labor ratio so measured probably represents a "real" phenomenon, in the sense that it signifies a changing composition

Chart 2. Ratio of Capital Assets to Man-hours of Labor in U.S. Manufacturing, 1919-1955



of inputs in the aggregate production function, the cyclical variation does not necessarily signify such a change. There is good reason to believe that the cyclical variation represents for the most part idle capacity, which is included in the measurement of capital. While we can, as a rule, get a reasonably good approximation to actual man-hours worked, for years in which there is considerable unemployment as well as in "full" employment years, corresponding figures for the capital stock are not available. To what extent the figures for capital in existence represent idle structures and equipment rather than employed inputs, comparable to the labor figures, is a matter of speculation.

From the standpoint of the production function, our interest centers not on the ratio of capital in existence to labor, but on the ratio of capital in use to labor. The latter corresponds more closely to the notion of factor proportions in economic theory, while the former, insofar as it has any meaningful interpretation at all, denotes a sort of historical accident. To convert the raw capital data to what we are after, some measure of idle capacity must be devised. Although some attention has been devoted to the concept of capacity and its utilization in the post World War II years, no attempt was made, for earlier periods, to gauge the extent to which capital in the economy was actually employed, so that no data are available relating to this issue. Nevertheless, an attempt must be made to adjust the available data to remove the assymetry caused by our labor figures referring to employed labor, and our capital data relating to existing capital.

One means toward this end would be to express both labor and capital inputs not in terms of actually employed factors, but in terms of existing resources. In years when both labor and capital are fully employed, the ratio obtained by this procedure would precisely correspond to that of employed capital to employed labor. When either factor is underemployed, however, the figures would deviate from those which are relevant to this study. In fact, use of this procedure is equivalent to making the assumption that labor and capital will suffer underemployment to the same extent, i.e., that the percentage of the labor force employed is equal to the percentage of the capital stock in use. This assumption is in fact made by Solow; his procedure is to multiply the capital stock, for each year, by the proportion of the labor force employed in that year. This, of course, leads to the same results as though he had merely used the ratio of existing capital to existing labor, though his "adjustment" may lend an air of sophistication to the method. While admitting that this adjustment is not ideal, Solow expresses the belief that the results which are based on it are probably better than had no adjustment for idle capacity been made (op. cit., p. 314). He seems to consider, that is, that the assumption that capital and labor are laid off in constant proportions during a recession is valid as a first approximation.

One must remember, however, that Solow's figures for the capital stock in existence refer not to the economy as a whole, but to the non-farm private sector. And there is some question whether one can speak of the percentage of the labor force employed, when referring to a sector of the economy. If the employment figures relate to the economy as a whole, then one must assume that the ratio of employed workers to total workers in the particular sector under consideration is identical with this ratio for the economy. Specifically, Solow must assume that the percentage of the non-farm private labor force (whatever that means) that is employed is the same as the percentageoof the overall labor force which is employed; and if hardly seems likely that cyclical changes in employment will fall evenly on all sectors. Hence, there is some question whether one is justified in applying unemployment figures for the economy as a whole to a particular sector within the economy.

If, instead, a measure of unemployment for the sector in question is attempted, a conceptual difficulty is encountered. To refer to a non-farm private labor force is in effect to assume absence of inter-sector labor mobility, an assumption which is contradicted by empirical evidence on cyclical variations in employment.

Let us grant, however, that this problem can be overcome, and that one is able to obtain a measure of the fraction of the total non-farm private labor force which is employed, for each year. To postulate that this ratio corresponds to that of employed capacity to total capacity will lead to an <u>over</u>-correction for idle capacity. This is most evident in the early thirties, when net disinvestment was occurring, so that the capital stock was diminishing in size. During this period, however, the "labor force" (in the non-farm private sector) was increasing, both due to a secular influence and to the fact that during a depression, more members of the household are often compelled to join the labor force. In the early years of the depression, then, a given number of employed workers constitutes a smaller percentage of labor force employment than during the late twenties, when the labor force was smaller. This tends to overstate the amount of idle capacity. Due to net disinvestment, the capital stock itself was decreased during the years following 1929, tending to further overstate the amount of idle capacity. The net result is that for some early depression years, the ratio of employed capital to man-hours worked, according to Solow's paper, is actually lower than in 1929, rather than higher, as one would expect. Most economists would assert that there is a tendency to substitute capital for labor during a depression. At most, one might assume, as Solow undoubtedly intended, that there are constant factor proportions in the short run, so that capital and labor will be laid off proportionately. But in this case the proper adjustment for idle capacity is to multiply the ratio of the number employed in the year in question to the labor force during the last "full" employment year, by the capital stock existing before net disinvestment occurred. This in effect will hold the capital-labor ratio constant throughout the period of underemployment. Ideally, one would like to permit the capital-labor ratio to increase, in the case of a prolonged period of underemployment, as in the thirties; i.e., in the initial stages of the depression, the constant proportions assumption might serve as a first approximation to actual conditions, but after a year or two, it appears likely that some substitution of capital for labor will occur. This is to be expected because of the inflexibility of wage rates downward, as well as because of the limit set to the rate of net disinvestment. Possibly, after a still longer period of time, when capital in use is again equal to capital in existence, the capital-labor ratio might begin again to decline. We might expect this result in the late thirties (as well as during the war years).

Solow's results were probably affected by another fact, namely, the inappropriate measurement of depreciation which enters into his capital stock values. Goldsmith's estimates of capital, which were used by Solow, are based upon an accounting concept of depreciation, which though an adequate reflection of actual capital consumption in the long run, may deviate sharply from the latter during the cycle. It is clearly capital consumption, rather than the accounting notion of depreciation charges, which should be subtracted from gross investment to yield net investment, or net additions to the capital stock. While in a "normal" year, i.e., a peace-time, full-employment year, depreciation charges may give a fair approximation to capital consumption, in a "troubled" year, such as 1930, when output has fallen considerably below the expected, or normal, value, depreciation in the accounting sense may greatly exceed the actual decline in the value (in real terms) of the capital stock. During the '30's, straight line depreciation was predominantly used, so that the allowance for deterioration (and obsolescence) in a given year was based on the original cost of the asset and its expected life. However, when output is cut back, unexpectedly, by a significant amount, the asset is likely to last longer than expected, so that actual deterioration will be less than the amount allowed on the books. This fact leads one to surmise that the figures for the capital stock, for the period from 1930 to 1937, for example, are likely to be too low. If the ratio of net to gross investment during this period was greater than business records indicate, then the capital stock must have been larger than appears from the data.

The rationale behind our criticism, in other words, is that while in some cases depreciation is a function of time only, in other cases it is a function of both time and output. While a straight line depreciation allowance may serve as a good approximation to actual capital consumption when actual output equals expected output, it is a poorer approximation when output either greatly exceeds or greatly falls short of its expected value. Furthermore, in periods of depression or of war, the rate of technological change, and hence the rate at which capital goods acquire obsolescence, is likely to be less than the rate assumed when computing the depreciation allowance.

Several other ways of converting capital in existence to capital in use suggest themselves. One way which, though as arbitrary as any other, is perhaps more theoretically satisfying, is to separate the capital-labor ratio into a secular and a cyclical component, by means of a regression technique. The cyclical component, once determined, can be eliminated, leaving a capital-labor ratio which is influenced by secular forces only. This method, unfortunately, is not as satisfactory as at first sight appears. First, we do not really wish to eliminate the entire cyclical component but only that part of it which represents a spurious increase in the capital-labor ratio due to the presence of idle capacity when there is a sharp decline in man-hours worked during a recession. The real substitution of capital for labor, brought about by maximizing behavior on the part of entrepreneurs in the presence of fixed plant and equipment with virtually zero opportunity cost (which induces decision makers to attempt to reduce current, i.e., labor, costs and to utilize more fully fixed capital which would otherwise merely accumulate obsolescence and deterioration) should be included in the analysis. The change to be eliminated is that due to the limitation on the adjustment (through disinvestment) of the capital stock to the decline in man-hours worked. The multiple regression technique will not achieve this end.

As an alternative procedure consider a Cobb-Douglas function as an approximation to the aggregate production function. If the assumptions underlying this approximation are met, and if in addition technological change can be factored out, as in equation (4), then we may write

$$(5) \qquad Q = T(t)C^{b}L^{1-b}.$$

If, in addition, we can represent technological change by an exponential function of time, (5) can be rewritten

$$(6) \qquad Q = ae^{rt}C^{b}L^{1-b} \text{ , or}$$

$$(7) \ \ln(Q) = \ln(a) + rt + b \ln(C) + (1-b) \ln(L).$$

The three parameters, a, b, and r can be estimated by least squares, taking observations only from years which have been (arbitrarily) designated as full employment; and these values can then be substituted in the production function for underemployment years to yield the values of C for these years. The values for capital in use thus found may be used in the original model to yield a better approximation to technical change. This procedure relieves us of the need to make ad hoc adjustments to allow for idle capacity; instead, we arrive at the desired figures directly by substituting in the production function. This method thus leads to substitution of capital for labor during a recession, as one would expect. Moreover, capital's elasticity of production can be estimated without having to employ data relating to profits and net income, concepts which are unreliable due to their dependence on accounting conventions and on tax laws. We can thus employ the profits and income data to check the reasonableness of our assumption about competitive pricing of capital and labor.

Other difficulties are present, however. One major disadvantage is that we are forced to assume that the rate of technological change is constant over time, and even though this assumption is relaxed eventually, any results which are based on it are of doubtful validity. This is especially so because the parameters are estimated on the basis of full employment years, while inference is made as to their value when the economy experiences some degree of unemployment. There is also the problem of multicollinearity, sometimes associated with the results obtained from fitting a Douglas function.*

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* See, for example, Horst Mendershausen, "On the Significance of Professor Douglas' Production Function," Econometrica VI, No. 1 (1938), pp. 143-53, for an excellent discussion of this issue.

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The procedure which was employed in this paper is to think of the ratio of capital in use to labor in use as the product of two ratios: the capital-output ratio, and the output-labor ratio. (Of course, the relationship holds, by identity.) Changes in the capital-output ratio can be divided into two classes: those which have as their origin long run forces in the economy, such as to lead to a secular change in the average productivity of capital (the reciprocal of the capital-output ratio), and those which consist of cyclical fluctuations in this ratio. It is our belief that while the latter may exist, they are of small enough magnitude to be safely ignored. By assuming that cyclical changes in the ratio of employed capital to output are non-existent, we are in effect maintaining that any deviation from the trend in the observed ratio of capital to output is spurious. Thus, a trend line was fitted (see chart 3) by the method of least squares to the capital-output ratio figures, using full-employment years only. It was postulated that deviations from this trend represented, in the large, fluctuations in employment rather than real changes in the factor proportions used in manufacturing. This seems reasonable, for the types of factors which determine the ratio of capital to output in the production function are forces which do not vary much from year to year, and hence can be considered negligible compared to the apparent changes in this ratio.

Having thus computed a trend for the capital-output ratio, we multiplied the observed changes in the output-labor ratio, which we accepted as being representative of the "true" changes, by the corresponding trend values of the capital-output ratio. The result was a cyclically adjusted series for the "real" capital-labor ratio, as appears in chart 4. The decision to use trend values of the capital-output ratio rather than fitting a trend line directly to the capital-labor ratio observations was made on empirical grounds. The capital-output trend fitted the data better than did a capital-labor trend. The data suggested that deviations from the former trend were largely a result of the capital figures' reflecting idle capacity. The observations which were omitted were either years with substantial unemployment of labor or war years; the former were substantially above the trend line, while the latter fell below the line. The observations which were actually employed represented a compromise in the sense that while some less-than-normal years were included, we felt that fewer observations would have made the least squares line less meaningful. It should be pointed out that use of this method has its disadvantages. First, as can be seen from chart 4, the curve dips in 1932-33, which runs counter to our earlier remarks concerning behavior of the capital-labor ratio in a depression. Second, use of this procedure is equivalent to postulating (though empirically rather than on a priori grounds) a constant percentage rate of growth in the capital-labor ratio, so that in effect annual fluctuations in output-per-man-hour are all attributed to technical change. We believe that this assumption regarding capital is justified as a first approximation, for the period under consideration.

The last series, property's share of net income originating in manufacturing, was obtained from two distinct series, one relating to labor income, the other to total net income originating in the sector. The ratio of labor income to total income, for each year, was subtracted from unity to obtain capital's share. It is interesting to note that in two years, viz., 1932 and 1933, labor income exceeded total net income, indicating an apparent net loss for capital. There is some doubt, however, whether capital's share was in fact negative in these years; an alternative explanation would focus on the difference between actual depreciation of fixed capital and apparent depreciation, as represented in business records. (The income data have been adjusted

Chart 3. Capital-output Ratio in Manufacturing, full-employment peacetime years 1919-55

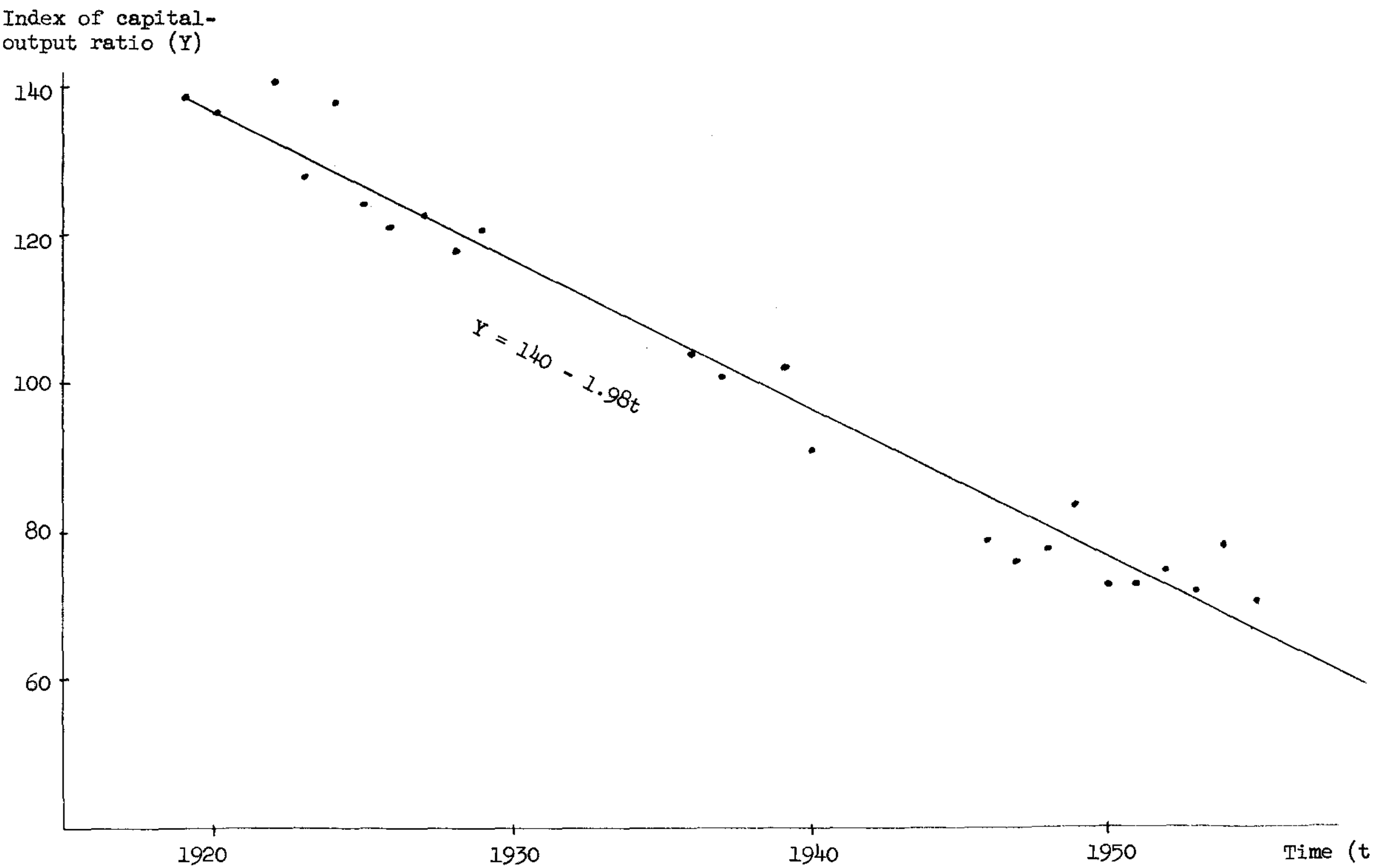
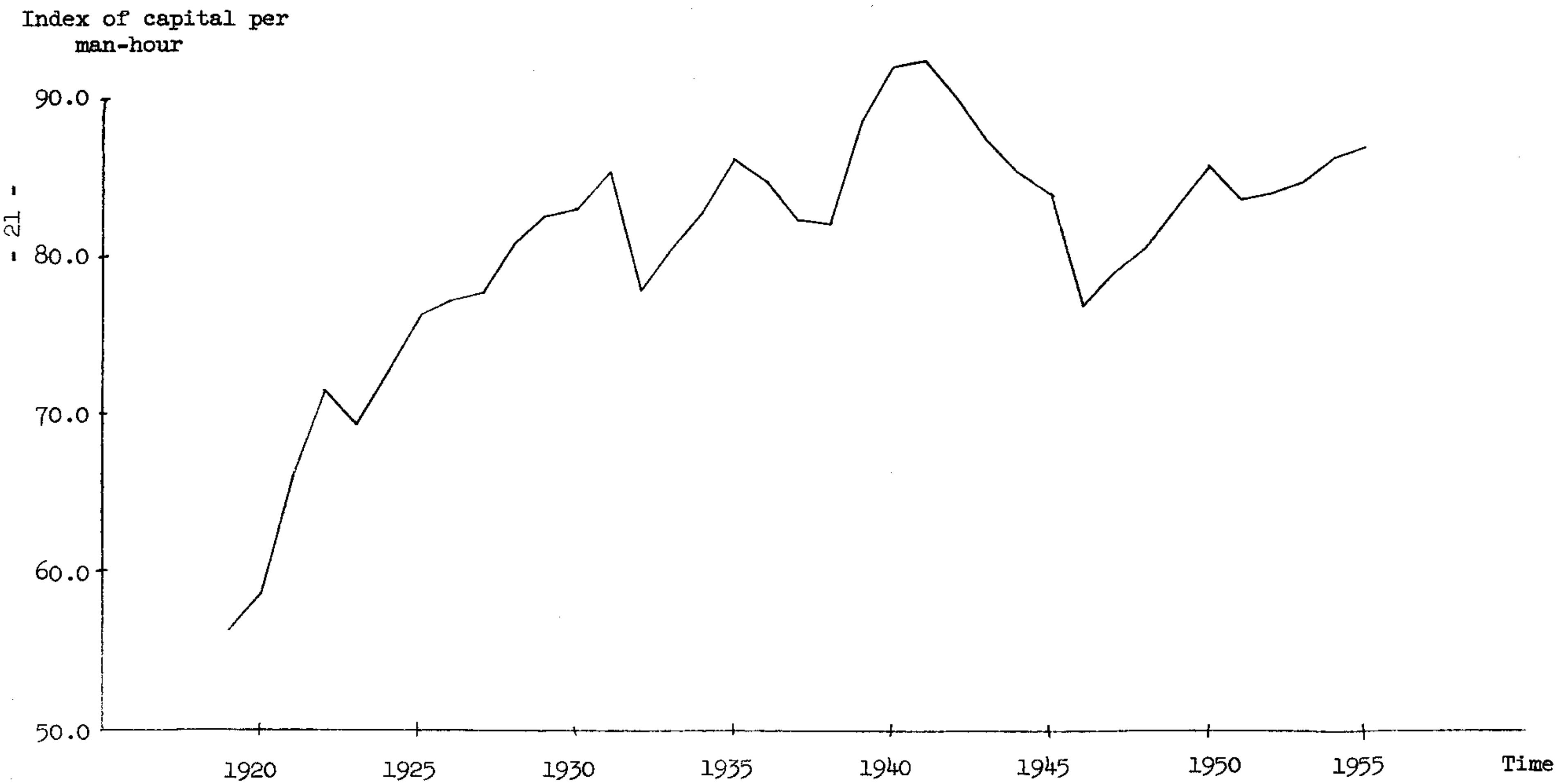


Chart 4. Ratio of Employed Capital to Man-hours of Labor in U.S. Manufacturing, 1919-55



for capital gains and losses.) It is wholly conceivable that, while accounting records indicated a loss to propertied interests, in fact such groups made a positive net gain based on a more "realistic" calculation of depreciation. If our hypothesis is correct, the series of capital's share can be altered to convey more meaningfully the information we want, if depreciation is adjusted along the lines previously discussed, i.e., that it be made to reflect changes in income above and below the expected, normal level. The result will be to increase net income in the depression years, hence increasing capital's share.

We have not undertaken this adjustment, but have preferred to think of capital's share, not in terms of annually changing values, but in terms of a mean value expressing capital's contribution to output for the period as a whole. Our interest is not in capital's share per se, but in this figure only insofar as it serves as an approximation to capital's contribution to output. Because, as stated earlier, the marginal productivity doctrine expresses only a tendency rather than an instantaneous adjustment, then even were profits to be negative, a negative marginal product of capital is not implied. The marginal product can be zero, if capital is underemployed, but because we are working with a continuous production function, this result is not to be expected. To avoid this difficulty, we have chosen to represent capital's contribution to output for the entire period, i.e., its marginal product divided by its average product, by the unweighted arithmetic mean of the annual ratios of profits to net income. Equation (2) can now be rewritten,

$$(8) \quad \frac{\dot{T}}{T} = \frac{\dot{P}}{P} - \bar{b}\,\frac{\dot{K}}{K}$$

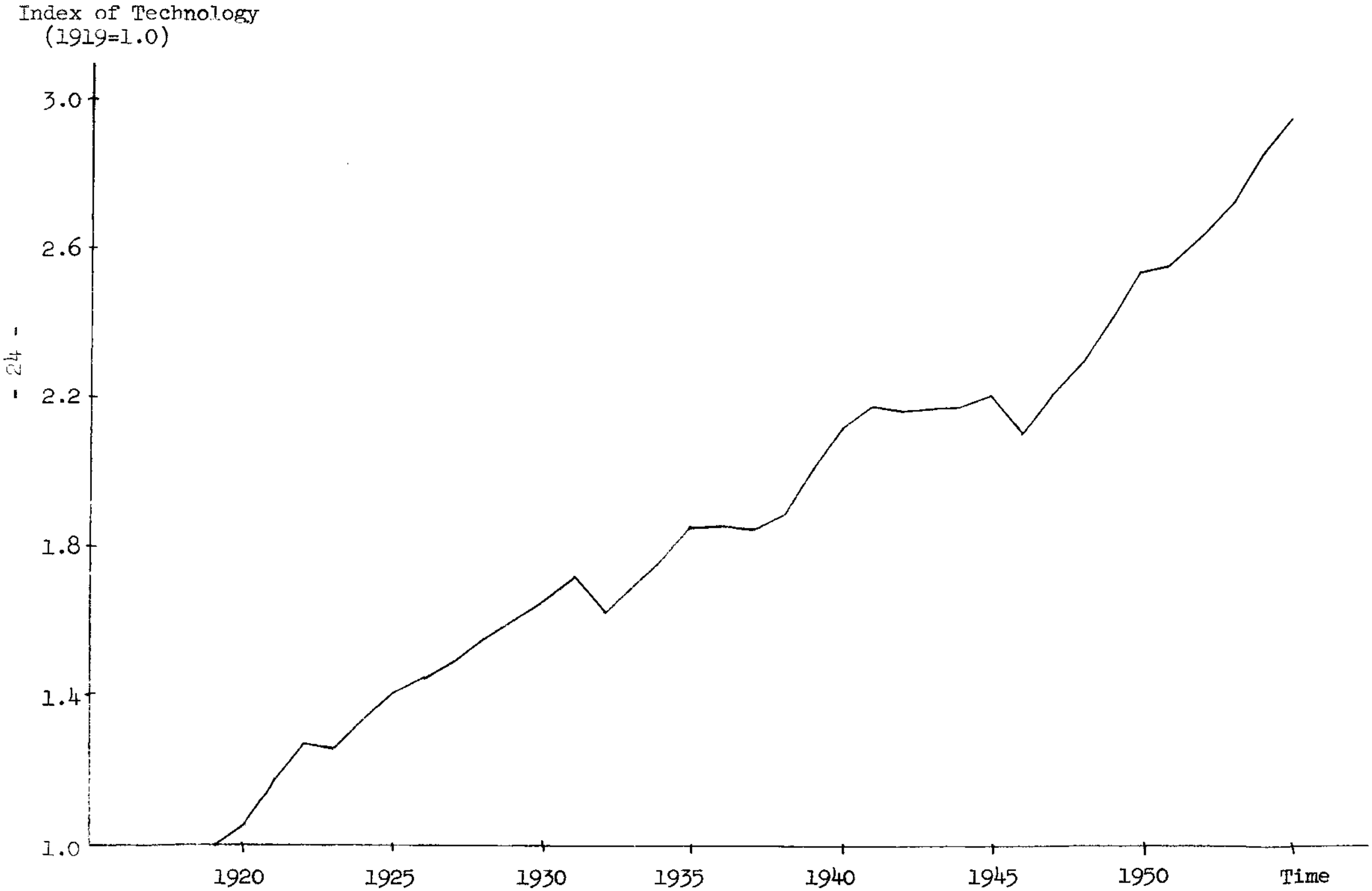
## 3. Results

From our series of output per man-hour, capital input per man-hour, and the mean value of capital's share, we calculated $\frac{\Delta T}{T}$ letting T range from 19 to 55, and setting $T(19) = 1$, we derived the entire $T(t)$ series, shown in chart 5. From the chart the technology index in 1955 is seen to be 2.9. This can be interpreted to mean that the production function has shifted by a factor of 2.9, so that at the prevailing capital-labor ratio, 2.9 times the output per man-hour can be produced in 1955 as in 1919.

Turning back to the output per man-hour figures in chart 1, we note that the 1955 figure is 3.2 times the value for 1919. Letting the ratio of output per man-hour in 1955 to that in 1919 equal $m$, and the ratio of the index of technology in 1955 to that in 1919 equal $s$, then the ratio $s/m$ should indicate what proportion of the increase in output per man-hour is attributable to technical advances. The value of $s/m$ turns out to be .9, so we may say that roughly ninety percent of the increase in output per man-hour is due to technical change.

It was mentioned earlier that the neutrality of shifts in the production function could be tested for. Solow has shown (op. cit., p. 313) that if $\frac{\Delta T}{T}$ is uncorrelated with $K$, then changes in the technology are indeed neutral. We tested for such relationship and found little evidence of correlation, so conclude that shifts in the function were such as not to systematically change the marginal rate of substitution between the inputs. Therefore we are justified in substituting equation (4) for (1), expressing technological change as a multiplicative factor.

Chart 5. Level of Technology in U. S. Manufacturing, 1919-1955



Inspection of the adjusted data shows the capital-labor ratio in 1955 to have increased by a factor of 1.6 since 1919. (The increase is a little greater than this if observed values are used for the two years being compared, a little less if trend values of the capital-labor ratio are taken.) It is not possible directly to specify how much of an increase in output per man-hour should have been obtained from this increase in capital per man-hour without further restricting the form of the production function. Capital's contribution can be computed, however, if we are willing to employ a Cobb-Douglas function, as in equation (5). Dividing this expression by $L$, we get

$$P = T(t)K^b \quad (9)$$

Taking logs, (9) can be written

$$\ln[T(t)] = \ln[P(t)] - b\ln[K(t)] \quad (10)$$

$$\ln[T(t)/T(0)] = \ln[P(t)] - \ln[P(0)] + b\ln[K(0)] - b\ln[K(t)]. \quad (11)$$

Assume the expression on the left side of (11) = 0; then $\ln[P(t)/P(0)] = b\ln[K(t)/K(0)]$. But this is equal to 1.09, so we conclude that if a Cobb-Douglas function provides a good fit to the data, then the 1.6-fold increase in capital per man-hour of labor accounts for only a 1.1-fold increase in output per man-hour.

It should be observed that the preceding result can be obtained in an independent way without specifying the form of the function. For, because of our classification of causal forces as exhaustive, we must attribute any increase in average labor productivity which is not explained by technological change as being attributable to an increase in capital per man-hour. Thus, dividing 3.2 by 2.9, we get 1.09, which agrees with the previous result.

Inspection of the technology index expressed as a time series reveals that the path of technical change, though containing some irregularities, nevertheless is strongly dominated by a steady upward trend. The trend, which suggests either a linear relationship or an exponential, is interrupted five times, during two of which there is a distinguishable downturn. The first occurs between 1931 and 1932, the other between 1941 and 1946, the two having in common the fact that they represent exceedingly "abnormal" periods. The minor irregularities, i.e., temporary changes in the second derivative of the function, occur in 1922-23, 1935-37, and 1950-51, each denoting a recovery period. It is entirely possible that during a recovery, our index of capital tends to overstate the actual capital input, so that the index of technology is understated, hence leading to the apparent downturn. The large dip in the curve in the '30's may represent the fact that gross investment in this period has been tremendously curtailed, so that there is no way to put into effect whatever inventions were developed (see section 4 regarding this point). Also, during the depression, there may have been little incentive to innovate; while the leveling off during the war probably reflects the fact that during this emergency period there was little scope for replacing obsolescent equipment. Both results are what we should expect.

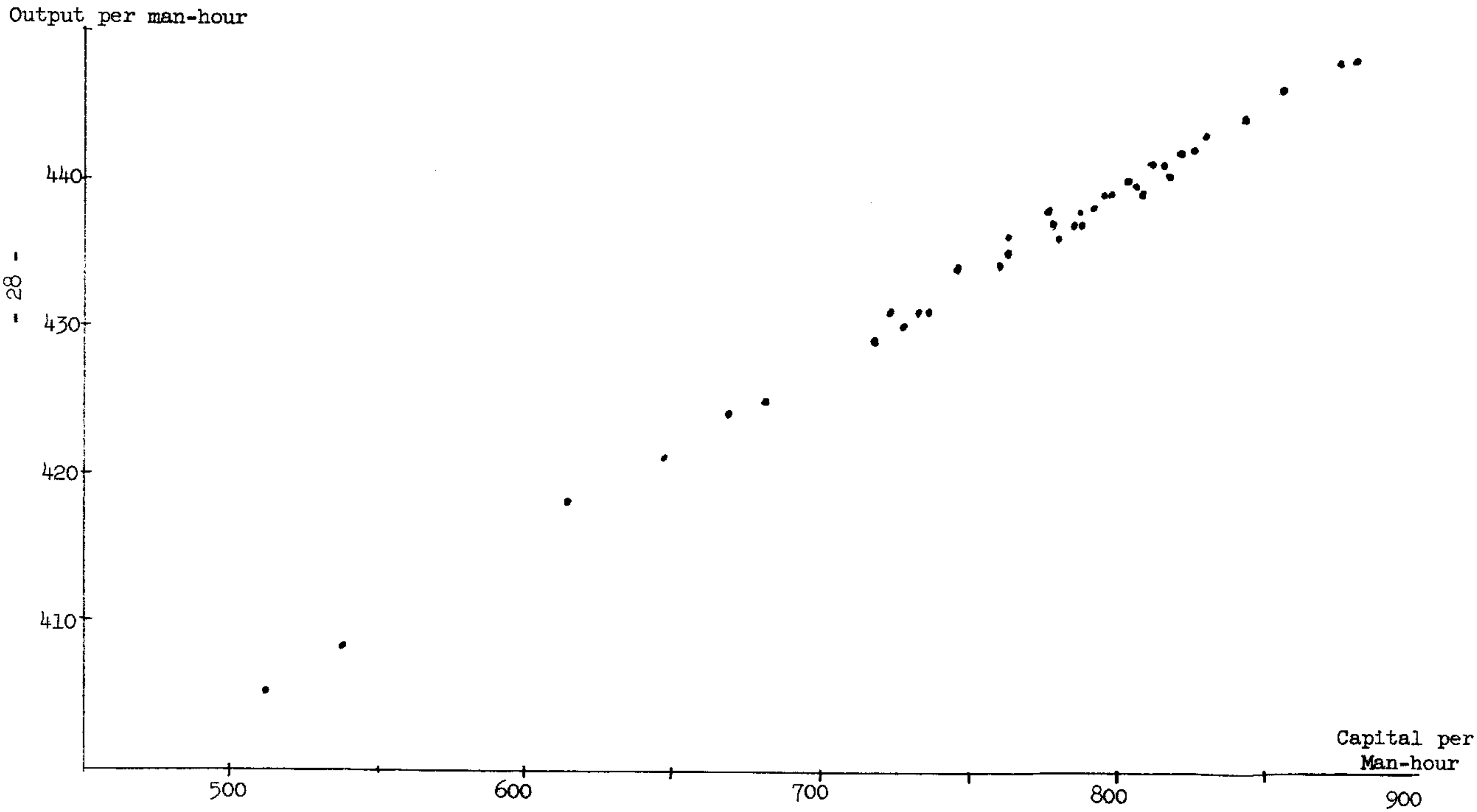
It was found that both linear and exponential functions gave excellent fits to the data, the former yielding an $r^2$ of .97, the latter a value of .96. There is thus little empirical basis on which to choose between the two, though expressing technological change as an exponential function of time may have more intuitive appeal.

As a final step, following Solow, we plotted output per man-hour for each year, divided by the appropriate index of technology, against capital per man-hour, thus getting an aggregate production function corrected for shifts. The scatter, shown in chart 6, was "well-behaved," i.e., it was monotonically increasing and suggested a relationship between the variables which is either linear, or linear in the logs.

A comparison of the results of this paper with those obtained by Solow brings out several interesting points. While his $T(t)$ series also indicated neutral change, the average annual rate of change was considerably less than in our series. Nevertheless, in his paper, technical change accounted for seven-eighths of the increase in output per man-hour, only two-and-one-half percent less than in the present study. (Of course the results are not strictly comparable as the two studies referred to different sectors, and Solow's paper was concerned with the years 1909-49.)

Solow's $T(t)$ series was considerably less regular than ours, containing more jagged edges. This is very likely due to his method of converting capital in existence to capital input. Also, the acceleration of technical change in the '30's, according to his paper, may well be attributable to the same factor. Both Solow's paper and the present one exhibit a rate of technical advance which, though not in any sense uniform, is nevertheless persistent; and the papers concur in granting to changes in the level of technology a decisively major role in bringing about increases in productivity.

Chart 6. Output per man-hour (corrected for technological change) as a function of capital per man-hour in U.S. manufacturing, 1919-1955.



4. Qualification of Results

The fact that such a large proportion of the increase in output per man-hour is attributable to technological progress should not be taken as proof that the observed increase would not have been significantly smaller had capital formation not occurred. The results need qualification in two important respects. First, although much of what we refer to as technical change consists of organizational changes which require no new inputs, a much greater proportion of changes in the technology are probably embodied in new capital goods. Consequently there is strong reason to believe that the rate of technological advance will be influenced by the rate of capital formation. Probably a more direct relationship exists between changes in the technology and gross investment, for innovations are often embodied in capital goods which represent not additions to the capital stock, but replacements of existing equipment. If net investment is nonpositive there can still be a rapidly changing technology as long as there is a positive rate of gross investment. It would appear however that technical change will flourish most in an economy which is expanding its capital stock. Conversely, we should expect the rate of net capital formation to be greatest when there is a high level of innovational change, for the rate of profit is then likely also to be high.

Another consideration, neglected in the literature, is that the rate of net investment is definitionally determined by the rate of technological progress. Net capital formation is determined by gross investment, physical deterioration, and obsolescence. Given the rate of gross investment (a rate which has been observed to remain relatively constant through time, in many economies), the rate of net investment depends on the rates of deterioration and of obsolescence. The former is largely a technological consideration, while the latter is economically determined. Obsolescence is greater the larger is the rate of technological change; consequently a larger proportion of gross investment will be expended in replacement of existing equipment when there is a rapid rate of innovation. There is thus a link between innovation and capital formation such that the latter is definitionally determined (in part) by the former; so that while we have found technological change of great importance in increasing productivity, further research would be needed to predict, for example, the effect on per-capita output of alternative rates of capital accumulation and of technical change.

## 5. Conclusions and Remarks

The foregoing qualifications do not alter the fact that, conceptually at least, a distinction can be made between innovation and capital formation, and that this study has reinforced the findings of Robert Solow, and Solomon Fabricant,* in finding technical change far more important

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* In "Resources and Output Trends in the U. S. Since 1870," American Economic Review, XLVI, No. 2 (May, 1956), Fabricant has estimated the role of technological change as approximately 90 percent, for the period 1871 to 1951.

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than capital formation. In view of these findings, policy-makers may wish to concern themselves more with the variables which govern the rate at which innovations are injected into the economic system, than with the variables which determine the rate at which additions are made to the capital stock. Such issues as expenditure by business on research, and the policies of firms regarding the replacement of obsolescent equipment will be deemed more important, perhaps, than the rate of net investment. (We believe that one justification for having undertaken this study lies in this very guidance.)

Aside from the policy question, the present inquiry is, we hope, of some theoretical interest. The fact that technical improvements are of such relatively great importance should be no small cause of concern, and possibly embarrassment, to economists, who have traditionally treated technology as exogenous in the theory of production. Current procedure is to include capital, labor, and land, or sub-classifications thereof, as inputs, subsuming in the functional form the technological relationship between these inputs and the product outputs. Changes in the technology are represented by shifts of the curve, i.e., certain parameters are changed in magnitude. Some writers* have protested against the traditional theory of production because (1) this theory appears better suited to a world where labor and land are the only inputs, but becomes unclear upon the introduction of the third factor of production, capital; and (2) strictly speaking, the classical model of production is a static model based on the assumption of a "given state of technological knowledge," which while perhaps useful in the Ricardian era, is of limited applicability today due to the exaggerated influence in our own economy

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* See, for example, Joan Robinson, "The Production Function and the Theory of Capital," Review of Economic Studies, XXI (1), No. 54 (1953-54).

of technical progress on aggregate output. It appears, indeed, as a source of concern, when the factors which are explicitly considered in the model account for only ten percent of the increase in output per man-hour, the remainder being attributed to an exogenous force, one which is little understood, and about which we are able to offer little explanation.

There is little justification for considering the rate of invention as exogenous, for expenditure on research will certainly affect this variable, and such expenditure is influenced in turn by other economic magnitudes. There is still less justification for considering the rate of innovation as being exogenous, for given the rate of investion, the rate at which new ideas are adopted is a function of such variables as the level of aggregate economic activity, the profit rate, the age composition of the capital stock, etc. Economists would do well to consider these variables as determining, at least in part, the rate at which technical changes occur.

It is believed that the present paper offers evidence to support the view that technological change is of overriding importance in bringing about increased labor productivity over time; and that there is a need for economists to shift emphasis from the theory of capital to the theory of technical progress, as an explanation of the growth in aggregate output.

## 6. Suggestions for Further Research

While the results presented here are believed to indicate the order of magnitude of the respective parts played by technical change and capital formation in increasing labor productivity, they are, however, only a first approximation. If it is believed that further research in this area would prove fruitful a number of refinements suggest themselves as being worthy of further consideration.

One might attempt to consider the inter-industry shift in the composition of the labor force within the manufacturing sector, noting that labor productivity is higher in some areas than in others. An adjustment may be made so as to eliminate that part of the increase in output per man-hour which is attributable to this factor rather than to increases in productivity within the various sectors. Actually two somewhat distinct phenomena are present here; an apparent increase in labor productivity might occur because of a shift in the composition of output such that goods which can be produced with relatively greater labor efficiency are produced in a comparatively greater amount. On the other hand, without any change in the composition of output, there can be a change in the factor proportions such that industries in which labor is more productive become relatively more labor intensive. Either of these changes will give a rise in the output labor ratio which will, in the procedure employed in the present paper, appear as technological change. While this is technical change as defined here, economists generally like to distinguish between such factors as changes in taste, changes in the quality of inputs, and other changes which arise from the necessity to have recourse to index numbers in measurement of the variables, on the one hand; and other "real" changes, which are often referred to as "innovational" in nature. It is the latter which we should like to measure, but it is a mixture of the two which shows up in our results. Therefore, we believe that it would be of interest to attempt the separation of these components of our index of technology, so as to eliminate the "impurities", leaving the "true" index.

Another impurity present in the catch-all measure of technical change is the increase in the value of the labor force, due to increased education, training, and skill. Further investigation in this area might be devoted to obtaining a measure of the increased value of a worker in 1955 as compared to his 1919 counterpart. Perhaps an easier task, but nevertheless a useful one, might be to correct the figures regarding labor input to eliminate for the shift in the composition from low-paying to higher-paying jobs. One might weight each worker by his rate of remuneration, so that labor input will be value-weighted rather than individual-weighted.

The capital per man-hour series may also be adjusted for the changes in the composition of manufacturing output. We might try to determine how much of the increase in capital per man-hour is due to a heavier weighting in more recent years of industries in which the capital-labor ratio is relatively greater. Also, some attempt may be made to correct for the change in the length of the work week, as regards capital input. One could try to determine to what extent the shortening of the work week indicates a decrease in capital input, and to what extent this is offset by the increasing use of multiple shift days, or is irrelevant because capital is in use 24 hours per day.

## Appendix A

The derivation of equation (2) follows: Taking logarithmic derivatives of equation (1), we write

$$(12) \quad \frac{\dot{Q}}{Q} = \frac{1}{T} \cdot \frac{\partial T}{\partial t} + \frac{C}{T} \cdot \frac{\partial T}{\partial C} \cdot \frac{\dot{C}}{C} + \frac{L}{T} \cdot \frac{\partial T}{\partial L} \cdot \frac{\dot{L}}{L}$$

Define $b \equiv \frac{C}{T} \cdot \frac{\partial T}{\partial C}$ ; then, because we have a homogeneous first degree equation,* we may write

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* See footnote page 2

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$$(13) \quad \frac{\dot{Q}}{Q} = \frac{1}{T} \cdot \frac{\partial T}{\partial t} + b \frac{\dot{C}}{C} + (1-b) \frac{\dot{L}}{L}$$

$$(14) \quad \frac{\dot{T}}{T} = \frac{\dot{Q}}{Q} - b \frac{\dot{C}}{C} - (1-b) \frac{\dot{L}}{L}$$

Let $P \equiv \frac{Q}{L}$, and $K \equiv \frac{C}{L}$ . Then

$$\frac{\dot{P}}{P} = \frac{\dot{Q}}{Q} - \frac{\dot{L}}{L} \quad \text{and} \quad \frac{\dot{K}}{K} = \frac{\dot{C}}{C} - \frac{\dot{L}}{L}$$ , so that

$$(2) \quad \frac{\dot{T}}{T} = \frac{\dot{P}}{P} - b \frac{\dot{K}}{K} .$$

If we substitute discrete annual changes for time derivatives, it follows that

$$(3) \quad T(t+1) = T(t) \left(1 + \frac{\Delta T(t)}{T(t)}\right) .$$

Appendix B. Discussion of Data and Their Sources

A. Expenditures for structures and equipment by U. S. manufacturing firms, in millions of current dollars, Lowell J. Chawner, "Capital Expenditures for Manufacturing Plant and Equipment - 1915 to 1940," Survey of Current Business, Vol. 21, No. 3 (March, 1941), p. 10.

B. Depreciation charges for manufacturing firms, in millions of book value dollars, Solomon Fabricant, Capital Consumption and Adjustment, pp. 32-33.

C. Value of inventory in manufacturing firms, in millions of 1929 dollars, end of year values, Simon Kuznets, National Income and Its Composition, 1919-38, Volume II, p. 904.

D. Ratio of current prices to average prices underlying historical cost depreciation for all American business, for structures and equipment, in percentage form, M.A.P.I., Statistical Notes to Capital Goods Review, No. 29.

E. Expenditures for structures and equipment for manufacturing establishments, in billions of current dollars, D. G. Wooden and R. C. Wasson, "Manufacturing Investment Since 1929 in Relation to Employment, Output, and Income," Survey of Current Business, Vol. 36, No. 11 (November 1956), p. 9.

F. Depreciation on privately owned structures and equipment in manufacturing establishments, in billions of dollars, original cost, op. cit., p. 11.

G. Ratio of current year cost to original cost (computed from unrounded figures), op. cit., p. 11.

H. Real net value of privately owned inventories in manufacturing establishments, end of years, billions of 1947 dollars, op. cit., p. 14.

I. Real net value of privately owned structures and equipment in manufacturing establishments, end of years, billions of 1947 dollars, op. cit., p. 14.

J = H + I. Total capital, billions of 1947 dollars (see Wooden and Wasson, op. cit.).

K. Expenditures by manufacturing firms for structures and equipment, in millions of 1939 dollars, Chawner, op. cit., p. 11.

L. Implicit price indexes for structures and equipment, computed from unrounded figures, 1947-100, Wooden and Wasson, op. cit., p. 9.

M. Net income originating in manufacturing, adjusted, in millions of current dollars (excludes gains and losses from changes in inventory), Simon Kuznets, op. cit., volume I, pp. 310-311.

N. Compensation of employees in manufacturing, in millions of current dollars, op. cit. pp. 314-315.

O. Indexes of man-hours worked in manufacturing, 1947-49-100, Joint Economic Committee, Productivity, Prices, and Incomes, p. 148.

P = A - .1 billion, for comparability with E, and rounded to nearest .1 billion. Expenditures for plant and equipment, adjusted, in billions of dollars. E excludes, while P includes expenditures by the government. This may be more significant in 1919 than other years, but Chawner states that for years 1916-19, the total government expenditure for manufacturing plant and equipment was only approximately .5 billion dollars.

Q = B/1.2, to make series comparable with F. Depreciation charges, billions of dollars. Series is rounded to the nearest .1 billion dollars.

R = 1.29C, to make series comparable with H, which in effect converts to 1947 dollars, and rounded to nearest .1 billion. Value of inventory, billions of 1947 dollars. From 1928 to 1955, figures are taken directly from H.

S = (Q) (D), to convert depreciation charges from historic (book) cost to billions of current dollars, thus achieving comparability with P .

T = P - S, net expenditure for plant and equipment, billions of dollars.

U = A/K. Implicit price index, 1939 = 100.

V = .56U, making series comparable with L. Implicit price index, 1947 = 100.

W = T/V, net expenditure for plant and equipment, billions of 1947 dollars.

X. End of year value of structures and equipment, billions of 1947 dollars, computed from I, 1928 figure, and W for other years.

Y. Indexes of real output per man-hour, 1919-55, in the manufacturing sector, with 1947-49=100, ibid, p. 148.

$$Z(t) = \frac{Y(t+1) - Y(t)}{Y(t)}$$

AA = R + X, $t < 1929$. AA = J, $1929 < t < 1956$. Real net value of structures, equipment, and inventories in manufacturing at beginning of years, 1919-56, billions of 1947 dollars. See Wooden and Wasson, op. cit., for explanation of comparability with labor and output series, especially p. 14.

$$BB(t) = \frac{AA(t) + AA(t+1)}{2}$$

CC = 1 - N/M, $t < 1929$. Figures from 1929 to 1955 are from Wooden and Wasson, op. cit., p. 20. Property income as a percentage of national income originating in manufacturing, based on current value depreciation, both numerator and denominator expressed in current dollars. No adjustment is made for inclusion of income of self-employed proprietors, nor for unpaid family workers, but it is felt that their inclusion would not significantly change the results.

$$DD = BB/O$$

$$EE = \frac{DD}{Y}$$

FF. Cyclically adjusted values for the ratio of capital input to man-hours worked, obtained by multiplying Y by trend value of capital-output ratio.

$$GG(t) = \frac{FF(t+1) - FF(t)}{FF(t)}$$

$$HH = Z - (\bar{C}\bar{C})(GG)$$

$$II(t+1) = II(t)\,[1 + HH(t)]$$

$$JJ(t) = \frac{Y(t)}{II(t)}$$

Table of Data Used in Paper

| Year | Y | CC | DD | EE | FF | II |
|---|---|---|---|---|---|---|
| 1919 | 40.5 | .23 | 56.4 | 139 | 55.9 | 1.000 |
| 1920 | 43.0 | .26 | 59.0 | 137 | 58.5 | 1.053 |
| 1921 | 49.4 | .22 | 87.7 | 178 | 66.2 | 1.183 |
| 1922 | 54.2 | .20 | 76.5 | 141 | 71.5 | 1.279 |
| 1923 | 53.3 | .22 | 68.2 | 128 | 69.3 | 1.265 |
| 1924 | 56.8 | .21 | 78.4 | 138 | 72.7 | 1.336 |
| 1925 | 60.6 | .23 | 75.4 | 124 | 76.4 | 1.412 |
| 1926 | 62.4 | .25 | 75.2 | 121 | 77.4 | 1.450 |
| 1927 | 63.8 | .21 | 78.0 | 122 | 77.8 | 1.480 |
| 1928 | 67.4 | .23 | 79.6 | 118 | 80.9 | 1.551 |
| 1929 | 70.0 | .23 | 77.8 | 111 | 82.6 | 1.605 |
| 1930 | 71.6 | .22 | 97.3 | 136 | 83.1 | 1.640 |
| 1931 | 74.9 | .11 | 119.0 | 159 | 85.4 | 1.706 |
| 1932 | 69.7 | -.08 | 140.6 | 202 | 78.1 | 1.617 |
| 1933 | 73.4 | -.06 | 120.5 | 164 | 80.7 | 1.691 |
| 1934 | 77.0 | .09 | 110.7 | 144 | 83.2 | 1.764 |
| 1935 | 81.4 | .16 | 97.4 | 120 | 86.3 | 1.852 |
| 1936 | 81.6 | .20 | 84.6 | 104 | 84.9 | 1.854 |
| 1937 | 80.7 | .20 | 81.6 | 101 | 82.3 | 1.845 |
| 1938 | 82.1 | .15 | 107.2 | 131 | 82.1 | 1.876 |
| 1939 | 89.7 | .18 | 91.1 | 102 | 88.8 | 2.020 |
| 1940 | 95.1 | .25 | 86.2 | 91 | 92.2 | 2.125 |
| 1941 | 97.5 | .29 | 70.0 | 72 | 92.6 | 2.176 |
| 1942 | 96.9 | .28 | 59.3 | 61 | 90.1 | 2.174 |
| 1943 | 96.2 | .26 | 49.3 | 51 | 87.5 | 2.172 |
| 1944 | 96.2 | .25 | 50.0 | 52 | 85.6 | 2.181 |
| 1945 | 96.7 | .22 | 58.9 | 61 | 84.1 | 2.201 |
| 1946 | 90.5 | .17 | 71.1 | 79 | 76.9 | 2.098 |
| 1947 | 95.4 | .21 | 72.8 | 76 | 79.2 | 2.199 |
| 1948 | 99.8 | .24 | 77.4 | 78 | 80.8 | 2.291 |
| 1949 | 105.4 | .23 | 88.6 | 84 | 83.3 | 2.406 |
| 1950 | 111.8 | .26 | 82.1 | 73 | 86.1 | 2.536 |
| 1951 | 111.6 | .26 | 81.5 | 73 | 83.7 | 2.546 |
| 1952 | 115.3 | .22 | 86.7 | 75 | 84.2 | 2.627 |
| 1953 | 119.7 | .21 | 85.9 | 72 | 85.0 | 2.722 |
| 1954 | 125.6 | .19 | 97.9 | 78 | 86.7 | 2.844 |
| 1955 | 130.0 | .22 | 92.9 | 71 | 87.1 | 2.941 |

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