Social Networks and the Market for News

Rachel Kranton and David McAdams*

May 30, 2019

Abstract

This paper builds a simple model of the market for decision-relevant information (“news”) and examines how expanding social networks impact producers’ incentive to publish true news and consumers’ ability to discern what news is true. Having a highly-connected social network reduces producers’ incentive to invest in story quality if producers are paid when consumers encounter their stories (our baseline model), but increases their incentive to invest if producers are paid to influence consumer behavior. In either case, consumers’ ability to discern the truth is limited in equilibrium, even in the limit as each consumer follows infinitely many others: there are limits to the wisdom of crowds. Contemporary news-media markets provide our main motivation, but the model applies more generally to a variety of settings—such as product marketing, charity fundraising, and political campaigns—in which decision-relevant information is shared socially.

1 Introduction

The 2016 Presidential election in the United States and the subsequent media environment have raised both public and academic interest in “fake news” and overall news quality. As we use the term, “fake news” refers to published information that a news...
producer knows to be false. Many governments have used false propaganda to influence public opinion, at home and abroad.\textsuperscript{1} Tabloid newspapers have long published questionable stories about the lives of celebrities. The current media environment is different, however, in that producers can more easily enter the market using online news-distribution channels and consumers can spread news more easily through social media. This paper builds a simple model of this contemporary market for news and examines how expanding social networks impact the likelihood that published news is true, what we refer to as “news veracity,” and consumers’ ability to discern what is true.

The model specifies the supply and demand for news and applies to a variety of settings in which decision-relevant information is shared socially. The model indicates dual roles of social networks. First, networks spread news. As more people become linked, more people can see a news story even if they did not encounter the story directly from the producer, what we refer to as the “broadcast”. Second, networks filter the news. Consumers receive informative private signals about each story they encounter and only share stories over the social network that they believe are sufficiently likely to be true. Consumers’ sharing behavior therefore can be informative about the likelihood of news truth. As more people become linked, more people can benefit from this evaluative process.

For any fixed news veracity, expanding the social network increases consumer welfare as consumers benefit from increased spread and filtering of the news. As the social network becomes more connected, however, news veracity is not fixed because producers’ incentive to invest in true stories depends on the social network. And as news veracity changes, consumers’ incentive to share stories also changes, which in turn impacts consumers’ ability to learn from others’ sharing behavior. Our key innovation is to endogenize the quality of information that diffuses through a social network, and consumers’ ability to discern what is true, as a function of network structure.

We examine a non-cooperative model of news production in which each news producer

\textsuperscript{1}In 1940, Britain deployed three thousand operatives to the United States to spread (sometimes false) propaganda under the guise of bona fide news reports (Boyd (2006), Cull (1995)), as a way to drum up popular support for entering the war effort against Nazi Germany.
decides whether to invest in news quality, with the interpretation that “high-quality” stories are always true while “low-quality” stories are false with some probability. We characterize all symmetric Nash equilibria of the resulting game, first in the baseline case when producers are paid per consumer who views their story (Sections 3-4), then in an extension allowing for producers also to be paid per consumer who chooses to “adopt” based on their story (Section 5).

We consider a spectrum of news-producer revenue models, reflecting the variety of motivations among news providers operating in the contemporary media market. First, traditional brick and mortar newspapers, such as *The New York Times*, earn revenues from advertising that accompanies their articles both on paper and online. While consumers might base decisions on the news they see, these outlets do not typically earn revenue from those decisions. Second, fictitious-news websites, such as denverguardian.com, which famously published a story about a made-up FBI agent’s death being linked to Hillary Clinton,² earn revenues from advertising that accompanies the articles published online. Third, news outlets such as Fox News, MSNBC, and Breitbart support particular political positions and earn revenues from advertising but can also be supported by owners who care about advancing their own political views (see, e.g., Kroll (2017)³ on Sinclair Media’s owner David Smith). Finally, government-sponsored media, such as the British propagandists of 1940 or the Russian troll factories of today, publish news with the virtually sole aim of inducing consumers to take some desired action.⁴

Our results show how consumers’ social network shapes news veracity, depending on the revenue motives of the producer. Consider a network where each consumer follows many other consumers, what we refer to as a “crowd”. When producers are mostly paid based on views, they have little incentive to invest in story quality since even false stories

²“This is a real news story about fake news stories” by Callum Borchers, *Washington Post*, November 7, 2016.
⁴A well-known example of this category is the Heart of Texas which was revealed to be a fictitious advocacy group created in St. Petersburg that promoted Texas succession from the United States and other provocative positions. When the fake Facebook group called for a rally in against “the adoption of the Sharia law” in Texas, real people showed up to rally and counter rally.
will spread widely enough to be seen by most consumers. Equilibrium news veracity in this case can be just high enough for consumers to sometimes be willing to share, but not higher. By contrast, when producers are mostly paid based on adoption, they have a strong incentive to invest in story quality because consumers can make inferences based on their neighbors’ sharing behavior, to distinguish true stories from false ones and avoid adopting based on false stories. Even in this case, however, consumers’ ability to discern the truth is limited and some false stories are still produced in equilibrium. Indeed, even in the limit as consumers follow infinitely many others, consumers are unable in equilibrium to discern for sure which stories are true, i.e., there is no “wisdom of the crowd”.

The paper contributes to two distinct literatures: the literature on media markets and the literature on social learning, information transmission, and networks. Much of the previous work on media markets studies media bias. In Gentzkow & Shapiro (2006), media bias arises because consumers hold initial beliefs that make them less likely to find contrary news to be credible. News producers who earn revenues based on their reputations for accuracy then have an incentive to slant their news reports. In Besley & Pratt (2006) and Gentzkow, Glaeser & Goldin (2006), news producers face a tradeoff between earning advertising revenue or producing news biased in a sponsor’s favor. In that context, advertising revenue is shown to reduce political bias. In Ellman and Germano (2009), however, newspapers bias their news towards their advertisers.

The present paper considers news outlets that earn an exogenous (reduced form) mix of revenue from advertisements and from content sponsors. We focus on quality per se, with strategic consumers who desire true news, who receive private signals that are informative about news truth, and can share the news over a social network. A key insight that emerges from our analysis is that switching from an advertising-supported revenue model to a sponsor-supported model can increase the equilibrium truthfulness of the news. Several other recent papers also study features of contemporary media markets, such as proliferation of producers and competition for consumers’ limited attention (Chen and Suen (2018)), media bias when consumers have heterogeneous preferences and pass on news to like-minded individuals (Redilicki (2018)), and competition to break a story
that leads to lower quality news (Andreottola and DeMoragas (2018)).

The demand side of our media market involves a simple model of consumer information transmission and social learning that is both similar to and different from other models in the literature. Consumers in our model are Bayesian, receive private signals, and update their beliefs about each news item based on others’ observed choices; so, consumers may in some cases ignore their own private information when deciding whether to “adopt”. However, unlike in the cascades literature (e.g., Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992)), there is only one round of social learning and consumers can observe multiple neighbors’ independent sharing decisions. As in Bloch, Demange and Kranton (2018) and Chatterjee and Dutta (2016), but unlike much of the literature on information diffusion in networks (e.g., Acemoglu, Ozdaglar, and ParandehGheibi (2010) and Banerjee, Chandrasekhar, Duflo, and Jackson (2013)), consumers in our model choose whether or not to pass on information to their neighbors. These decisions account for the network role in filtering the news. Finally, as in Galeotti and Goyal (2009) and Chatterjee and Dutta (2016), there are third parties outside the network, the producing firms, whose optimal decisions are affected by the network structure. These papers study how firms will target advertising, while the present paper endogenizes product quality. To the best of our knowledge, this paper is the first study of product diffusion on a network that considers how firms’ production incentives—and hence the product itself—are shaped by the network.

2 Model: The Market for News

The “market for news” consists of a single\(^5\) news producer and \(M\) news consumers and unfolds in three stages: producer investment at time \(t = 0\), consumer sharing over a social network at time \(t = 1\), and consumer adoption at time \(t = 2\).

Consumers encounter decision-relevant information, what we refer to as “news”. Con-

\(^5\)Our single-producer analysis applies equally to a news market with finitely-many or a continuum of producers, as long as the stories that producers publish are unrelated to one another. See Section ?? for an extension in which there are multiple dissimilar producers.
sumers’ ex ante belief that news items are true, which we refer to as “news veracity,” informs their choices whether to share news over a social network and whether to take an action based on each news item, e.g., whether to buy a product promoted in a marketing message or vote for a candidate featured in a political message. We refer to this action as “adoption”.

The producer decides whether to make a costly investment that increases the likelihood of news truth. In our baseline analysis, we assume that the producer is only paid for views, getting one unit of revenue per consumer who views a published story. In Section 5, we allow the producer to get revenue per consumer who adopts or a combination of revenue from both views and adoptions. The producer lacks commitment power.

At time $t = 0$, the producer has the opportunity to publish a news item, or “story,” and decides whether to invest in story quality. Low-quality stories are costless to produce and false with probability $f \in (0, 1]$, while high-quality stories cost $MC_R > 0$ to produce and are always true. The per-consumer cost of reporting $c_R$ is an atomless random variable, i.i.d. across stories with c.d.f. $H(\cdot)$.

At time $t = 1$, the story is broadcast to consumers, who then decide whether to share it over a social network. Each consumer sees the broadcast with independent probability $b \in (0, 1)$, what we refer to as “broadcast reach”. By assumption, consumers cannot directly observe story quality. However, when first encountering a story, each consumer receives a conditionally i.i.d. private signal $s_i \in \{T, F\}$, where $\Pr(s_i = T|true) = \rho_T$ and $\Pr(s_i = F|false) = \rho_F$. For simplicity, we assume that $\rho_T = \rho_F = \rho > \frac{1}{2}$.

Those who have seen the broadcast decide whether to share the news. Sharing occurs over a directed social network $G$, where $g_{ji} \in \{0, 1\}$ and $g_{ji} = 1$ indicates a communication.

---

6 Consumers may encounter the same news item multiple times but, by assumption, the producer is only paid once per consumer who sees the news. For instance, social-media users may see many friends sharing the same New York Times story but only read it once—and hence only encounter advertisements once on the New York Times website.

7 In the special case when $f = 1$, a producer who chooses not to invest knowingly publishes a false story—“fake news”. Under the population interpretation of the model (see model discussion point (c)), “fake-news providers” are those whose per-consumer reporting cost exceeds the maximal revenue that stories can possibly enjoy, namely, $c_R > 1$. 

6, 7
link from consumer $j$ to $i$, i.e., $i$ “follows” $j$. $N_i$ denotes the set of other consumers from whom $i$ can receive information, with $d_i = \#(N_i)$. For simplicity, we focus on networks in which $d_i = d$ for all $i$. We refer to $d \geq 0$ as “social connectedness” and refer to networks with higher $d$ as being “more connected”. By assumption, consumers would like to share true stories but not false stories. In particular, consumer $i$ earns positive “sharing payoff” $\pi_i^{S} > 0$ when sharing a true story, negative sharing payoff $-\pi_i^{F} < 0$ when sharing a false story, or zero sharing payoff when not sharing. For simplicity, we assume that consumers have symmetric and equal sharing payoffs, i.e., $\pi_i^{S} = \pi_i^{F} = \pi > 0$ for all $i$. Consumers therefore prefer to share whenever they believe that a story’s likelihood of being true exceeds “sharing threshold” $p^S = \frac{1}{2}$.

At time $t = 2$, each consumer $i$ who has seen the news decides whether to take an action referred to as “adopting”. Consumer $i$ earns positive “adoption payoff” $\pi_i^{A} > 0$ when adopting based on a true story, negative adoption payoff $-\pi_i^{F} < 0$ when adopting based on a false story, and zero adoption payoff when not adopting. (After period 2, the story is revealed to be true or false. Consumers’ sharing and adoption payoffs are then realized.) For simplicity, we assume that consumers have symmetric adoption payoffs, i.e., $\pi_i^{A} = \pi_T^{A} > 0$ and $\pi_i^{F} = \pi_F^{A} > 0$ for all $i$. Consumers therefore prefer to adopt whenever they believe that a story’s likelihood of being true exceeds “adoption threshold” $p^A$:

$$p^A = \frac{\pi_F^{A}}{\pi_T^{A} + \pi_F^{A}} \in (0, 1)$$

(1)

Discussion. (a) We focus on a context in which the thing being produced is a factual claim that may or may not be true, but our analysis applies more broadly to settings where consumers care about any unobservable product characteristic, e.g., the entertainment value of a new movie, the effectiveness of a new scientific practice (with “consumers” being scientists), or the viability of a potential political candidate (with “consumers” being political donors).

(b) The analysis can be easily adapted to a setting in which consumers are already aware of an opportunity but not adopting is the optimal action absent any new decision-relevant information. For example, suppose that “consumers” are leading scholars in a field who may be asked to write letters for a tenure case. Suppose further that, absent
new information, the letter-writers will all advise against tenure. The tenure candidate ("producer") has just finished a new working paper. Scholars in the field who read the paper form an impression of its quality and then decide whether to mention it to others at conferences. Those asked to write letters may not have seen or heard about the working paper but, if not, will account for that fact when evaluating its quality—exactly as consumers in our model do when seeing the initial broadcast but then not seeing anyone sharing the news.

(c) The producer in our model can be interpreted as representing a continuum of undifferentiated producers, each of whom makes their own independent choices when the opportunity to publish a story presents itself. Under this population interpretation of the model, our assumption that the producer lacks commitment power is without loss of generality, since no individual producer can significantly change consumers’ beliefs about the likelihood that a published story is true.

(d) The analysis can be extended in a straightforward way to allow for non-linear producer revenue. For example, suppose that a political candidate who is trailing in the polls has an option to spread a nasty rumor about her opponent just before the election. Our analysis can be applied to this sort of context, with "consumers" being voters who prefer the other candidate, "adoption" being to stay home from the polls (or switch their vote), and "revenue" being non-linear: zero if the candidate loses or one if she wins. Whether the candidate chooses to spread a rumor, and how much effort she invests in identifying the most damning possible rumor to spread, will therefore depend on how likely different-quality rumors will be to sway a sufficient number of voters and on the costs associated with conducting opposition research.

3 Equilibrium Outcomes in the Market for News

This section analyzes the market for news in the baseline case when producer revenue depends only on consumer views, namely, that the producer earns one unit of revenue per consumer who views a story. In an extension in Section 5, we allow producer revenue to depend (also or instead) on consumers’ adoption decisions.
Figure 1: Schematic of the market for news when producers are paid for views.

Figure 1 illustrates the interaction between consumer and producer decisions in the market for news. Consumers’ sharing decisions impact the producer’s incentive to invest in story quality, by changing the visibility of true and false stories. Producer investment in turn impacts each consumer’s incentive to share, by changing the likelihood that published stories are true. Finally, each consumer’s incentive to adopt is impacted by both producer investment and others’ sharing decisions. However, since the reporter is only paid for views, consumer adoption has no impact on producer investment.

3.1 Consumer sharing

First, we consider the consumer’s incentive to share news stories, depending on their own private signals and the ex ante likelihood of news truth, and the resulting extent to which consumers view true and false stories.

Time-1 belief updating. Let $p_0$ be the ex ante likelihood that published stories are true, what we refer to as “news veracity”. Suppose that consumer $i$ has seen a story’s time-0 broadcast and gets private signal $s_i \in \{T, F\}$. Consumer $i$ updates her belief about the likelihood of news truth to $p_1(s_i; p_0)$. By Bayes’ Rule, $p_1(T; p_0) = \frac{p_0 p}{p_0 p + (1-p_0)(1-\rho)}$ and
\[ p_1(F; p_0) = \frac{p_0(1-\rho)}{p_0(1-\rho)+(1-p_0)\rho} \] or, equivalently,

\[ \frac{p_1(T; p_0)}{1 - p_1(T; p_0)} = \frac{\rho}{1 - \rho} \times \frac{p_0}{1 - p_0} \quad \text{and} \quad \frac{p_1(F; p_0)}{1 - p_1(F; p_0)} = \frac{1 - \rho}{\rho} \times \frac{p_0}{1 - p_0} \] (2)

Figure 2: News-veracity regions with different patterns of consumer sharing.

**Consumer best response.** Since consumer \(i\) gets \(\pi^S\) (or \(-\pi^S\)) when sharing a true (or false) story, her expected sharing payoff is \(p_1(s_i; p_0)\pi^S - (1 - p_1(s_i; p_0))\pi^S\) when sharing or zero when not sharing. Thus, consumer \(i\) strictly prefers to share (or not share) if \(p_1(s_i; p_0) > \frac{1}{2}\) (or \(p_1(s_i; p_0) < \frac{1}{2}\)). Let \(z_iT\) and \(z_iF\) denote consumer \(i\)’s likelihood of sharing after seeing the broadcast and getting, respectively, private signal \(s_i = T\) or \(s_i = F\). We refer to the vector \(z_i = (z_iT, z_iF)\) as consumer \(i\)’s “sharing rule”. To keep the analysis as simple as possible, we restrict attention to equilibria in which all consumers use the same sharing rule, what we call “symmetric sharing”. We say that “sharing is informative” if consumers use a sharing rule \(z = (z_T, z_F)\) such that \(z_T > z_F\) since, in that case, consumers are more likely to share true stories. On the other hand, “sharing is uninformative” if consumers use a sharing rule \(z = (z_T, z_F)\) such that \(z_T = z_F\).

Consumers’ optimal sharing rule depends on news veracity.

- **Always-share region** \((\rho, 1]$$: If news veracity \(p_0 > \rho\), sharing is uninformative as consumers always find it optimal to share.
- **Never-share region** \([0, 1 - \rho)$$: If news veracity \(p_0 < 1 - \rho\), sharing is uninformative as consumers always find it optimal not to share.
- **Filtering region** \((1 - \rho, \rho)$$: If news veracity \(p_0 \in (1 - \rho, \rho)\), sharing is informative as consumers find it optimal to “filter the news,” sharing after a good signal but not after a bad signal. Let \(\bar{z} \equiv (1, 0)\) denote the “filtering sharing rule”.

10
At the boundaries of the filtering region, consumers are sometimes indifferent whether to share. If news veracity $p_0 = \rho$, what we refer to as the “always-share threshold,” consumers are indifferent whether to share after getting a bad signal and hence find it optimal to use any sharing rule of the form $z = (1, z_F)$. Similarly, if news veracity $p_0 = 1 - \rho$, the “never-share threshold,” consumers are indifferent after getting a good signal and have optimal sharing rules of the form $z = (z_T, 0)$. Let $Z(p_0)$ denote consumers’ best-response correspondence, the set of optimal sharing rules for any given news veracity $p_0$.

Proposition 1 summarizes our findings on best-response consumer sharing.

**Proposition 1.** (i) Never-share region: if $p_0 < 1 - \rho$, then $Z(p_0) = (0, 0)$. (ii) Always-share region: if $p_0 > \rho$, then $Z(p_0) = (1, 1)$. (iii) Filtering region: if $p_0 \in (1 - \rho, \rho)$, then $Z(p_0) = \tilde{z} \equiv (1, 0)$. (iv) Never-share threshold: $Z(1 - \rho) = \{(z_T, 0) : z_T \in [0, 1]\}$. (v) Always-share threshold: $Z(\rho) = \{(1, z_F) : z_F \in [0, 1]\}$.

**News visibility.** Each consumer’s likelihood of viewing a story, what we refer to as the story’s “visibility” to that consumer, depends on broadcast reach ($b$), the number of others she follows in the social network ($d$), and the sharing rule $z$ used by her neighbors in the social network. (We use the word “neighbors” to refer to consumers who are linked, with the context indicating the link’s direction. Here, a “neighbor” is anyone that consumer $i$ follows.) Each neighbor $j$ shares iff she sees the broadcast (probability $b$) and gets a private signal that induces sharing (probability $\rho z_T + (1 - \rho) z_F$ if the news is true or $(1 - \rho) z_T + \rho z_F$ if it is false). Each consumer’s ex ante likelihood of viewing a true story, by either seeing the broadcast directly and/or having at least one sharing neighbor, is therefore

$$V_T(z) = 1 - \left(1 - b\right)\left(1 - b\left(\rho z_T + (1 - \rho) z_F\right)\right)^d$$

Similarly, consumer $i$’s likelihood of viewing a false story given sharing rule $z$ is

$$V_F(z) = 1 - \left(1 - b\right)\left(1 - b\left((1 - \rho) z_T + \rho z_F\right)\right)^d$$
Numerical example: news visibility. Suppose that broadcast reach \( b = \frac{1}{12} \) and private-signal precision \( \rho = \frac{2}{3} \). Given any news veracity in the never-share region \( (p_0 < \frac{1}{3}) \), consumers never share; so, consumers only become aware of a story if they see its initial broadcast and visibility \( V_T(0,0) = V_F(0,0) = b = \frac{1}{12} \) for true and false stories. On the other hand, given any news veracity in the always-share region \( (p_0 > \frac{2}{3}) \), consumers who see the broadcast always share; so, each consumer becomes aware of a story whenever she or any of her \( d \) neighbors sees the broadcast and visibility is \( V_T(1,1) = V_F(1,1) = \nabla = 1 - (1 - b)^{d+1} = 1 - (\frac{11}{12})^{d+1} \). Finally, given any news veracity in the filtering region \( (p_0 \in (\frac{1}{3}; \frac{2}{3})) \), consumers who see the broadcast share whenever they get a positive signal; so, each consumer shares true and false stories with ex ante probability \( b\rho = \frac{1}{18} \) and \( b(1 - \rho) = \frac{1}{36} \), respectively, yielding the following visibilities for true and false stories:

\[
V_T(z) = 1 - \frac{11}{12} \left( \frac{17}{18} \right)^d \quad \text{and} \quad V_F(z) = 1 - \frac{11}{12} \left( \frac{35}{36} \right)^d
\]

3.2 Producer investment

Next, we turn to the producer’s incentive to invest in story quality. Let \( R_T(z) \) and \( R_F(z) \) be the per-consumer expected revenue of true and false stories, respectively, when consumers use sharing rule \( z = (z_T, z_F) \). Since the producer is paid for views, \( R_T(z) = V_T(z) \) and \( R_F(z) = V_F(z) \). True stories earn a per-consumer “revenue premium” \( \Delta R(z) = \Delta V(z) \), where \( \Delta V(z) = V_T(z) - V_F(z) \) is the incremental visibility of true stories over false ones to each consumer. Since low-quality stories are false with probability \( f > 0 \) and high-quality stories are always true, investing in story quality increases per-consumer expected revenue by \( f \Delta V(z) \) at per-consumer cost \( c_R \). The producer therefore maximizes expected profit by investing iff \( c_R < f \Delta V(z) \), which occurs with ex ante probability \( H(f \Delta V(z)) \), inducing news veracity

\[
p_0(z) = (1 - f) + f H(f \Delta V(z)) \quad (5)
\]

Note that the producer’s profit-maximizing strategy is uniquely determined at all reporting cost levels except \( c_R = f \Delta V(z) \), which occurs with probability zero since \( c_R \) is drawn from an atomless distribution. Thus, the producer has an essentially-unique
best response to any sharing rule. We refer to the resulting news veracity $p_0(z)$ as the “best-response news veracity”.

Figure 3: Illustration of the best-response news veracity function $p_0(z)$, the maximal best-response news veracity $\tilde{p}_0 \equiv \max_z p_0(z)$, and the filtering news veracity $\hat{p}_0 \equiv p_0(1,0)$.

Figure 3 illustrates key qualitative features of the best-response news veracity.\(^8\) First, the producer never invests if consumers always share or never share, i.e., $p_0(1,1) = p_0(0,0) = 1 - f$. Second, the producer sometimes invests so long as consumer sharing is informative, i.e., $p_0(z) > 1 - f$ whenever $z_T > z_F$. Third, $p_0(z_T,0)$ is continuous and single-peaked in $z_T$. Finally, $p_0(1,z_F)$ is continuous and strictly decreasing in $z_F$.

Figure 3 also illustrates two news-veracity levels that will play an important role in the analysis: the “maximal best-response news veracity” $\tilde{p}_0 \equiv \max_z p_0(z)$ and the “filtering news veracity” $\hat{p}_0 \equiv p_0(1,0)$.

Lemma 1 gathers together useful facts about best-response news veracity.

**Lemma 1.** (i) $p_0(z_T,z_F) > 1 - f$ if $z_T > z_F$ and $p_0(z_T,z_F) = 1 - f$ if $z_T = z_F$. (ii) $p_0(1,z_F)$ is strictly decreasing in $z_F$. (iii) $p_0(z_T,0)$ is strictly increasing in $z_T$ over the

\(^8\)To avoid confusion, note that the x-axis of this figure consists of all sharing rules that could potentially be a best response for consumers, i.e., those of the form $(z_T,0)$ or $(1,z_F)$. 

13
interval $[0, z_T]$ and strictly decreasing in $z_T$ over the interval $[z_T, 1]$ for some $z_T \in (0, 1)$.

(iv) $\arg \max_z p_0(z) = (z_T, 0)$. (v) $p_0(z)$ is continuous in $z$.

Equilibrium concept. In the main text, we focus on Nash equilibria (NE) of the game played by the producer (deciding whether to invest) and consumers (deciding whether to share) in which all consumers use the same sharing rule; such NE are referred to as “symmetric Nash equilibria” (SNE). Moreover, in most of the analysis, we focus on the maximal news veracity that can be supported in any SNE, using notation $p_0^*$ to denote the “maximal equilibrium news veracity”. In an effort to avoid acronyms, we use the term “equilibrium” in the main text when referring to SNE. See the Appendix for a full characterization of all NE.

3.3 Maximal equilibrium news veracity

This section characterizes the maximal news veracity $p_0^*$ that can be supported in any equilibrium, which turns out to depend only on (i) the likelihood $1 - f$ that low-quality stories are true, (ii) private-signal precision $\rho$, (iii) the maximum best-response news veracity $\overline{p}_0$, and (iv) the filtering news veracity $\tilde{p}_0$.

**Theorem 1.** If either $1 - f \geq \rho$ or $\overline{p}_0 < 1 - \rho$, then there is a unique equilibrium with zero investment and $p_0^* = 1 - f$. Otherwise, an equilibrium exists with positive investment and $p_0^* = \max\{1 - \rho, \min\{\overline{p}_0, \rho\}\}$.

We prove Theorem 1 through a series of five main steps.

**Step 1:** In any equilibrium with investment, news veracity cannot be in the always-share region. Suppose for a moment that an equilibrium exists in which the producer sometimes invests, inducing news veracity $\hat{p}_0 > 1 - f$. We begin by observing that $\hat{p}_0$ cannot exceed the always-share threshold $\rho$. To see why, suppose for the sake of contradiction that $\hat{p}_0 > \rho$. Consumers’ unique best response would then be to always share, given which the producer’s best response would be to never invest, contradicting the assumption that the producer sometimes invests.
An immediate implication is that, if low-quality stories are sufficiently likely to be true that \(1 - f \geq \rho\), the producer never invests in any equilibrium. On the other hand, a “zero-investment equilibrium” always exists in this case in which (i) the producer never invests, anticipating that consumers will always share, and (ii) consumers always share, anticipating that all stories are low quality. Thus, whenever \(1 - f \geq \rho\), the maximal equilibrium news veracity \(p_0^* = 1 - f\).

**Step 2:** In any equilibrium with investment, news veracity cannot be in the never-share region. Suppose for the sake of contradiction that an equilibrium exists in which the producer sometimes invests, but invests sufficiently rarely that news veracity \(\hat{p}_0 < 1 - \rho\). Consumers’ unique best response would then be to never share, given which the producer’s best response would be to never invest, contradicting the assumption that the producer sometimes invests.

An immediate implication is that, if the maximal best-response news veracity \(p_0 < 1 - \rho\), then all equilibria must have zero investment. To see why, suppose for the sake of contradiction that \(\bar{p}_0 < 1 - \rho\) and there exists an equilibrium with investment. Let \(\tilde{z}\) denote consumers’ sharing rule in this equilibrium. The resulting news veracity \(\hat{p}_0 = p_0(\tilde{z}) \leq \bar{p}_0\) by definition of \(\bar{p}_0\); so, \(\hat{p}_0 < 1 - \rho\), contradicting Step 2. On the other hand, a zero-investment equilibrium always exists in this case in which (i) the producer never invests, anticipating that consumers will never share, and (ii) consumers never share, anticipating that all stories are low quality.\(^9\) Thus, whenever \(\bar{p}_0 < 1 - \rho\), the maximal equilibrium news veracity \(p_0^* = 1 - f\).

So far, we have seen that all equilibria must exhibit zero investment whenever either low-quality stories are sufficiently likely to be true that \(1 - f \geq \rho\) (Step 1) or the maximal best-response news veracity \(\bar{p}_0 < 1 - \rho\), which can only happen if low-quality stories are sufficiently unlikely to be true that \(1 - f < 1 - \rho\) (Step 2). This completes the proof of the first part of Theorem 1. It remains to show that, when neither of these conditions holds, an equilibrium with positive investment exists and the maximal news veracity that can be supported in equilibrium equals \(p_0^* = \max\{1 - \rho, \min\{\bar{p}_0, \rho\}\}\). Fortunately, this

---

\(^9\)Since \(\bar{p}_0 \geq 1 - f\), \(\bar{p}_0 < 1 - \rho\) is only possible if \(1 - f < 1 - \rho\). Thus, whenever \(\bar{p}_0 < 1 - \rho\), consumers never share a story that they believe to be definitely low quality.
complicated-looking expression can be written more simply in terms of three separate cases, depending on the filtering news veracity \( \tilde{p}_0 \): (i) if \( \tilde{p}_0 \in [1 - \rho, \rho] \), then \( p_0^* = \tilde{p}_0 \) (Step 3); (ii) if \( \tilde{p}_0 > \rho \), then \( p_0^* = \rho \) (Step 4); and (iii) if \( \tilde{p}_0 < 1 - \rho \), then \( p_0^* = 1 - \rho \) (Step 5).

**Step 3:** If the filtering news veracity \( \tilde{p}_0 \in [1 - \rho, \rho] \), then the maximal equilibrium news veracity \( p_0^* = \tilde{p}_0 \). If consumers filter the news, using sharing rule \( \tilde{z} = (1, 0) \), the producer’s best response induces news veracity \( p_0(1, 0) = \tilde{p}_0 \). If \( \tilde{p}_0 \in [1 - \rho, \rho] \),\(^\text{10}\) then consumers find it to be a best response to filter the news and an equilibrium exists with news veracity \( \tilde{p}_0 \). Moreover, no equilibrium can exist with higher news veracity. Why? First, by Step 1, no equilibrium can have news veracity greater than \( \rho \). Second, no equilibrium can have news veracity in the interval \( (\tilde{p}_0, \rho) \) since, if there were such an equilibrium, consumers would all filter the news and the producer would only invest enough to support news veracity \( \tilde{p}_0 \). Finally and least obviously, no equilibrium can have news veracity equal to \( \rho \) when \( \tilde{p}_0 < \rho \). In any such equilibrium, consumers must use a sharing rule of the form \( z = (1, z_F) \) for some \( z_F \in [0, 1] \). The producer’s best response then induces news veracity \( p_0(1, z_F) \). However, by Lemma 1(ii), \( p_0(1, z_F) \) is strictly decreasing in \( z_F \), implying \( p_0(1, z_F) \leq p_0(1, 0) = \tilde{p}_0 < \rho \), a contradiction. Overall, we conclude that the maximal equilibrium news veracity \( p_0^* = \tilde{p}_0 \).

Figure 4 illustrates the set of equilibria in the most interesting subcase when low-quality stories are sufficiently unlikely to be true that \( 1 - f < 1 - \rho \): the x-axis depicts the set of potentially-optimal sharing rules; the y-axis is news veracity; the thick line depicts consumers’ best-response correspondence; and the thin line depicts best-response news veracity (as in Figure 3). Each equilibrium corresponds to a crossing point of these curves, with the maximal crossing-point in this case being at the filtering rule \( \tilde{z} = (1, 0) \) and filtering news veracity \( \tilde{p}_0 \). (In the Appendix, we show that two other equilibria also exist with news veracities equal to \( 1 - f \) and \( 1 - \rho \), and that the equilibrium with news veracity \( 1 - \rho \) is dynamically unstable with respect to simple adaption dynamics.)

**Step 4:** If the filtering news veracity \( \tilde{p}_0 > \rho \) and \( 1 - f < \rho \), then the maximal equilibrium news veracity \( p_0^* = \rho \). Suppose next that \( \tilde{p}_0 > \rho \). This is always true if \( 1 - f \geq \rho \), in

\(^{10}\)Since \( \tilde{p}_0 > 1 - f \) and \( \tilde{p}_0 \leq \tilde{p}_0 \), \( \tilde{p}_0 \in [1 - \rho, \rho] \) is only possible if \( 1 - f < \rho \) and \( \tilde{p}_0 \geq 1 - \rho \); so, Steps 1-2 do not apply.
which case Step 1 implies that all equilibria have zero investment. We therefore focus on the more interesting case in which $\tilde{p}_0 > \rho$ and $1 - f < \rho$. No equilibrium exists with news veracity in the filtering region since, if it did, consumers’ unique best response would be to filter the news, given which the producer’s best response would induce news veracity $\tilde{p}_0 > \rho$, a contradiction. However, an equilibrium does exist with news veracity equal to $\rho$. Since $p_0(1, 0) = \tilde{p}_0 > \rho$ and $p_0(1, z_F)$ is continuous and strictly decreasing in $z_F$ (Lemma 1(ii,v)), there exists a unique $z^*_F \in (0, 1)$ such that $p_0(1, z^*_F) = \rho$. An equilibrium therefore exists in which consumers use sharing rule $z^* = (1, z^*_F)$ and the producer invests just often enough to induce news veracity equal to $\rho$. Since no equilibrium can have news veracity greater than $\rho$ by Step 1, we conclude that the maximal equilibrium news veracity $p^*_0 = \rho$.

**Step 5:** If the filtering news veracity $\tilde{p}_0 < 1 - \rho$ and $p_0 \geq 1 - \rho$, then the maximal equilibrium news veracity $p^*_0 = 1 - \rho$. Suppose finally that $\tilde{p}_0 < 1 - \rho$. This is always
true if \( p_0 < 1 - \rho \), in which case Step 2 implies that all equilibria have zero investment. We therefore focus on the more interesting case in which \( \tilde{p}_0 < 1 - \rho \) and \( p_0 \geq 1 - \rho \). No equilibrium exists with news veracity in the filtering region since, if it did, consumers’ unique best response would be to filter the news, given which the producer’s best response would induce news veracity \( \tilde{p}_0 < 1 - \rho \), a contradiction. In the same way, because \( p_0(1, z_F) \leq \tilde{p}_0 \) for all \( z_F \in [0, 1] \) (Lemma 1(ii)), no equilibrium exists with news veracity equal to \( \rho \). However, an equilibrium does exist with news veracity equal to \( 1 - \rho \). Since \( \bar{p}_0 = p_0(\tilde{z}_T, 0) \geq 1 - \rho \) for some \( \tilde{z}_T \in (0, 1) \) (Lemma 1(iv)), \( p_0(1, 0) = \bar{p}_0 < 1 - \rho \), and \( p_0(z_T, 0) \) is continuous in \( z_T \) (Lemma 1(v)), there exists \( z_T^* \in (\tilde{z}_T, 1) \) such that \( p_0(z_T^*, 0) = 1 - \rho \). An equilibrium therefore exists in which consumers use sharing rule \( z^* = (z_T^*, 0) \) and the producer invests just often enough to induce news veracity equal to \( 1 - \rho \). We conclude that the maximal equilibrium news veracity \( p_0^* = 1 - \rho \).

Figure 5 illustrates the set of equilibria in the case when \( \tilde{p}_0 < 1 - \rho < p_0 \). (In the Appendix, we show that two other equilibria also exist with news veracities equal to
1 − f and 1 − ρ, and that the other equilibrium with news veracity 1 − ρ is dynamically unstable.)

3.4 Equilibrium Comparative Statics

In this section, we consider how the amount of investment that can be supported in equilibrium, as captured by the maximal equilibrium news veracity, varies with model parameters, focusing especially on social connectedness (d), private-signal precision (ρ), and broadcast reach (b).\footnote{Other equilibrium comparative statics include that, holding all else fixed, (i) \( p^*_0 \) is non-decreasing in market size M (due to economies of scale in reporting) and (ii) \( p^*_0 \) is non-increasing in the reporting-cost distribution \( H(\cdot) \).} For clarity, we add notation in each subsection to indicate how the parameter of interest impacts endogenous variables, e.g., the maximal equilibrium news veracity is denoted as \( p^*_0(d) \) in Section 3.4.1, \( p^*_0(b) \) in Section 3.4.2, and \( p^*_0(\rho) \) in Section 3.4.3.

3.4.1 Social connectedness

The effect of increasing social connectedness \( d \) on equilibrium news veracity \( p^*_0(d) \) depends on how well consumers are already connected.\footnote{For ease of exposition, we focus here and in the next subsection on the case in which \( 1 − f \in (1 − \rho, \rho) \), so that consumers would prefer to filter the news if they believed that all stories were low quality. This case is especially convenient since, by Theorem 1, there is a unique symmetric Nash equilibrium and news veracity in this equilibrium takes the relatively simple form \( p^*_0(d) = \min\{\tilde{p}_0(d), \rho\} \).}

**Proposition 2.** In the case when \( 1 − f \in (1 − \rho, \rho) \), \( p^*_0(d) \) is single-peaked in \( d \), i.e. there exists \( \overline{d} \) such that \( p^*_0(d) \leq p^*_0(d') \) for all \( d < d' \leq \overline{d} \) and all \( d > d' \geq \overline{d} \).

**Proof.** Since \( 1 − f \in (1 − \rho, \rho) \), Theorem 1 implies that \( p^*_0(d) = \min\{\tilde{p}_0(d), \rho\} \), where \( \tilde{p}_0(d) = p_0(\tilde{z}; d) \) is the filtering news veracity. Note that, for any \( d', d \), \( p^*_0(d') > p^*_0(d) \) implies \( \tilde{p}_0(d') > \tilde{p}_0(d) \) and \( \tilde{p}_0(d') > \tilde{p}_0(d) \) implies \( p^*_0(d') \geq p^*_0(d) \). To show that \( p^*_0(d) \) is single-peaked in \( d \), it therefore suffices to show that \( \tilde{p}_0(d) \) is single-peaked in \( d \).

By definition, \( \tilde{p}_0(d) \) is the news veracity that results if all consumers use the filtering sharing rule \( \tilde{z} = (1, 0) \) and the producer invests optimally given such sharing behavior. By
equation (5), \( \tilde{p}_0(d) = 1 - f + fH(f \Delta V(\tilde{z}; d)) \), where \( H(\cdot) \) is the c.d.f. of the producer’s per-consumer reporting cost and \( \Delta V(\tilde{z}; d) \) is the extra visibility of true stories when consumers filter the news. Note that \( \tilde{p}_0(d) \) depends on \( d \) only through \( \Delta V(\tilde{z}; d) \) and that \( \tilde{p}_0(d) \) is strictly increasing in \( \Delta V(\tilde{z}; d) \). Thus, to show that \( \tilde{p}_0(d) \) is single-peaked in \( d \), it suffices to show that \( \Delta V(\tilde{z}; d) \) is single-peaked in \( d \).

By equations (3,4), \( \Delta V(\tilde{z}; d) = (1-b) ((1-b(1-\rho))^d - (1-b\rho)^d) \) implying \( \frac{d\Delta V(\tilde{z}; d)}{dd} = (1-b) (\ln(1-b(1-\rho))(1-b(1-\rho))^d - \ln(1-b\rho)(1-b\rho)^d) \). Re-arranging terms, we conclude that \( \frac{d\Delta V(\tilde{z}; d)}{dd} \geq 0 \) iff

\[
\left( \frac{1-b\rho}{1-b(1-\rho)} \right)^d \geq \frac{\ln(1-b(1-\rho))}{\ln(1-b\rho)} \in (0,1)
\]  

(6)

Since \( \rho > \frac{1}{2} \), \( \frac{1-b\rho}{1-b(1-\rho)} \) and the left-hand-side of (6) is exponentially decreasing in \( d \), while the right-hand-side of (6) does not depend on \( d \). We conclude that \( \Delta V(\tilde{z}; d) \) is strictly increasing in \( d \) up to some critical level \( \bar{d} \) and strictly decreasing after \( \bar{d} \), i.e., \( \Delta V(\tilde{z}; d) \) is single-peaked in \( d \).

Discussion: When consumers are relatively poorly connected, adding more links increases the producer’s incentive to invest. Intuitively, this is due to the “viral” amplifying effect that social sharing has for true stories, since true stories are more likely to be shared. For instance, suppose that broadcast reach \( b \approx 0 \), private-signal precision \( \rho = \frac{2}{3} \), consumers filter the news, and each consumer is followed by \( d \) others. Each consumer who sees the broadcast will expose approximately \( \frac{2d}{3} \) others to the story when it is true but only about \( \frac{d}{3} \) when it is false. On the other hand, when consumers are already well-connected, increasing social connectedness decreases the producer’s incentive to invest. For instance, suppose that \( b = \frac{1}{2}, \rho = \frac{9}{10} \), and consumers filter the news. Increasing social connectedness from \( d = 10 \) to \( d = 20 \) increases the likelihood that consumers see true stories from 99.8% to 99.9997% and false stories from 70% to 82%. The extra likelihood that consumers see true stories, which determines the producer’s incentive to invest, therefore falls from about 30% to about 18%.
3.4.2 Broadcast reach

The effect of increasing broadcast reach on equilibrium news veracity depends on how widely the broadcast is already seen. As in the last subsection, we focus for convenience on the case in which $1 - f \in (1 - \rho, \rho)$.

**Proposition 3.** In the case when $1 - f \in (1 - \rho, \rho)$, $p_0^*(b)$ is non-monotone with $\frac{dp_0^*(0)}{db} > 0$ and $\frac{dp_0^*(1)}{db} < 0$.

*Proof.* The maximal equilibrium news veracity $p_0^*(b) = \min\{\tilde{p}_0(b), \rho\}$, where $\tilde{p}_0(b) = 1 - f + fH(f\Delta V(\tilde{z}; b))$ and

$$
\Delta V(\tilde{z}; b) = (1 - b) \left( (1 - b(1 - \rho))^d - (1 - b\rho)^d \right)
$$

(7)

is the extra visibility of true news when consumers filter the news. Note that $\Delta V(\tilde{z}; 0) = \Delta V(\tilde{z}; 1) = 0$; so, $\tilde{p}_0(0) = \tilde{p}_0(1) = 1 - f$ which in turn implies $p_0^*(1) = p_0^*(0) = 1 - f$ since $1 - f < \rho$. In order to show that $\frac{dp_0^*(0)}{db} > 0$ and $\frac{dp_0^*(1)}{db} < 0$, it therefore suffices to show that $\frac{dp_0(0)}{db} > 0$ and $\frac{dp_0(1)}{db} < 0$. By equation (7) and by our assumption that the c.d.f. $H(\cdot)$ is smooth, it suffices further to show that $\frac{d\Delta V(\tilde{z}; 0)}{db} > 0$ and $\frac{d\Delta V(\tilde{z}; 1)}{db} < 0$, where

$$
\frac{d\Delta V(\tilde{z}; b)}{db} = - \left( (1 - b(1 - \rho))^d - (1 - b\rho)^d \right) + (1 - b)d \left( \rho (1 - b\rho)^{d-1} - (1 - \rho) (1 - b(1 - \rho))^{d-1} \right)
$$

(8)

By (8), $\frac{d\Delta V(\tilde{z}; 0)}{db} = d(2\rho - 1) > 0$ and $\frac{d\Delta V(\tilde{z}; 1)}{db} = - \left( \rho^d - (1 - \rho)^d \right) < 0$. □

**Discussion:** Proposition 3 considers the relatively simple case in which low-quality stories have an intermediate likelihood of being true ($1 - f \in (1 - \rho, \rho)$), but qualitatively similar results hold in other more complex cases. For example, consider the case in which low-quality stories are always false ($f = 1$) but with the simplifying assumptions that each consumer follows one other person ($d = 1$) and the producer’s per-consumer reporting cost is uniformly distributed on $[0, 1]$ ($H(x) = x$). For any given sharing rule $z$, the extra visibility of true news takes the form $\Delta V(z; b) = (1 - b)(2\rho - 1)(z_T - z_F)$. This is maximized when consumers filter the news, using sharing rule $\tilde{z} = (1, 0)$; so, the
filtering news veracity \( \tilde{p}_0(b) \) equals the maximal best-response news veracity \( p_0(b) \) and, after simplifying equation (5), takes the form

\[
\tilde{p}_0(b) = \bar{p}_0(b) = (1 - b)(2\rho - 1)
\]

By Theorem 1: if \( \tilde{p}_0(b) < 1 - \rho \), then no investment is possible in any equilibrium, i.e., \( p_0^*(b) = 0 \); if \( \tilde{p}_0(b) \in (1 - \rho, \rho) \), then \( p_0^*(b) = \tilde{p}_0(b) \); and if \( \tilde{p}_0(b) > \rho \), then \( p_0^*(b) = \rho \). For instance, suppose further that \( \rho = \frac{7}{8} \). If \( b < \frac{1 - \sqrt{1/3}}{2} \approx 0.21 \) or \( b > \frac{1 + \sqrt{1/3}}{2} \approx 0.79 \), then \( \tilde{p}_0(b) = \frac{3(1 - b)b}{4} < 1 - \rho = \frac{1}{8} \) and the unique equilibrium has zero investment and hence news veracity \( p_0^*(b) = 0 \). On other hand, if \( b \in (0.21, 0.79) \), then some investment can be supported in equilibrium and \( p_0^*(b) = \tilde{p}_0(b) \in (1/8, 3/16) \).

### 3.4.3 Private-signal precision

The effect of increasing private-signal precision \( \rho \) on equilibrium news veracity \( p_0^*(\rho) \) depends on how precise consumers’ private signals already are and on whether low-quality stories are more likely to be true or false.

**Proposition 4.** Signal-precision thresholds \( \bar{\rho} \geq \rho \geq \frac{1}{2} \) exist such that (i) \( p_0^*(\rho) = 1 - f \) for all \( \rho \in \left( \frac{1}{2}, \bar{\rho} \right) \), (ii) \( p_0^*(\rho) = 1 - \rho \) and \( p_0^*(\rho) \) is strictly decreasing in \( \rho \) for all \( \rho \in \left( \rho, \bar{\rho} \right) \), and (iii) \( p_0^*(\rho) > 1 - \rho \) and \( p_0^*(\rho) \) is strictly increasing in \( \rho \) for all \( \rho \in (\bar{\rho}, 1) \).

**Proof.** We break the proof down into two basic cases, depending on whether low-quality stories are more likely to be true \( (1 - f \geq \frac{1}{2}) \) or false \( (1 - f < \frac{1}{2}) \). First, we establish a preliminary result that the filtering news veracity \( \tilde{p}_0(\rho) \) and the maximal best-response news veracity \( \bar{p}_0(\rho) \) are each strictly increasing in \( \rho \). Consider any fixed sharing rule \( z = (z_T, z_F) \) that favors true-story sharing, i.e., such that \( z_T > z_F \). Increasing \( \rho \) increases the visibility of true stories and decreases the visibility of false stories; see equations (3-4). This increases the producer’s incentive to invest and the resulting best-response news veracity \( p_0(z; \rho) \); see equation (5). Consequently, both \( \tilde{p}_0(\rho) = p_0(z; \rho) \) and \( \bar{p}_0(\rho) = \max_z p_0(z; \rho) \) are increasing in \( \rho \).

**Case 1:** \( 1 - f \geq \frac{1}{2} \). Define \( \bar{\rho} = \rho = \frac{1}{2} \). Prop 4(iii) hold vacuously with respect to these thresholds and, because \( 1 - f \geq \frac{1}{2} > 1 - \rho \), \( p_0^*(\rho) > 1 - \rho \) holds automatically; so, it
suffices to show that \( p_0^*(\rho) \) is strictly increasing in \( \rho \). But this follows immediately from the fact that 
\[ p_0^*(\rho) = \min\{\tilde{p}_0(\rho), \rho\} \] 
(by Theorem 1) and that \( \tilde{p}_0(\rho) \) and \( \rho \) are each strictly increasing in \( \rho \).

**Case 2:** \( 1 - f < \frac{1}{2} \). Define \( \bar{p} \) and \( \underline{\rho} \) implicitly by the conditions
\[ \bar{p}_0(\rho) = 1 - \rho \] 
\[ \tilde{p}_0(\bar{p}) = 1 - \bar{p} \] 
\( \bar{p} \) and \( \underline{\rho} \) are uniquely defined by (9-10), with \( \bar{p} \geq \rho > \frac{1}{2} \). To see why, observe first that, if private signals were completely uninformative \( (\rho = \frac{1}{2}) \), the producer would have zero incentive to invest no matter what sharing rule consumers used; so, \( \bar{p}_0(\frac{1}{2}) = \tilde{p}_0(\frac{1}{2}) = 1 - f < \frac{1}{2} \). On the other hand, since \( \bar{p}_0(\rho) \geq \tilde{p}_0(\rho) > 1 - f \) for all \( \rho > \frac{1}{2} \), \( \bar{p}_0(\rho) > 1 - \rho \) and \( \tilde{p}_0(\rho) > 1 - \rho \) for all \( \rho > f \). Since \( \bar{p}_0(\rho) \) and \( \tilde{p}_0(\rho) \) are each strictly increasing and continuous in \( \rho \) (by Step 1; continuity is obvious), we conclude that equations (9-10) each have a unique solution, greater than \( \frac{1}{2} \) and less than 1. Finally, \( \bar{p} \geq \underline{\rho} \) follows from the fact that \( \bar{p}_0(\rho) \geq \tilde{p}_0(\rho) \) for all \( \rho \).

By Theorem 1, either \( \bar{p}_0(\rho) < 1 - \rho \) and the maximal equilibrium news veracity 
\( p_0^*(\rho) = 1 - f \) or \( \bar{p}_0(\rho) \geq 1 - \rho \) and 
\( p_0^*(\rho) = \max\{1 - \rho, \min\{\tilde{p}_0(\rho), \rho\}\} \). Suppose first that \( \rho < \underline{\rho} \). By definition of \( \underline{\rho} \), \( \bar{p}_0(\rho) < 1 - \rho \); so, \( p_0^*(\rho) = 1 - f \). This completes the proof of Prop 4(i). Suppose next that \( \rho \in (\underline{\rho}, \bar{p}) \). By definition of \( \underline{\rho} \) and \( \bar{p} \), \( \tilde{p}_0(\rho) < 1 - \rho < \bar{p}_0(\rho) \); so, \( p_0^*(\rho) = 1 - \rho \), which is obviously decreasing in \( \rho \). This completes the proof of Prop 4(ii). Finally, suppose that \( \rho > \bar{p} \). In this case, \( \tilde{p}_0(\rho) > 1 - \rho \) and hence 
\( p_0^*(\rho) = \min\{\tilde{p}_0(\rho), \rho\} > 1 - \rho \), which is increasing in \( \rho \) (shown in Case 1). This completes the proof of Prop 4(iii).

**Discussion:** Increasing private-signal precision \( \rho \) has two main direct effects. First, the filtering region expands as the always-share threshold increases and the never-share threshold decreases (see Figure 6). This reflects the fact that consumers make use of more-informative private signals given a wider range of prior beliefs. Second, consumers are more likely to share true stories and less likely to share false stories when signal precision is higher. Although higher signal precision induces more investment for any fixed sharing rule, increasing \( \rho \) also changes consumers’ equilibrium sharing behavior in ways that can
Figure 6: Increasing private-signal precision $\rho$ increases maximal equilibrium news veracity $p^*_0(\rho)$ if filtering news veracity $\bar{p}_0(\rho) > 1 - \rho$ (Scenario 1, upper panel) but decreases $p^*_0(\rho)$ if $\bar{p}_0(\rho) < 1 - \rho < \overline{p}_0(\rho)$ (Scenario 2, lower panel).

reduce the producer’s incentive to invest. For this reason, the overall effect of increasing $\rho$ on equilibrium news veracity $p^*_0(\rho)$ is ambiguous. If the filtering news veracity is initially high enough that $\bar{p}_0(\rho) > 1 - \rho$ (see the upper panel of Figure 6), increasing private-signal precision to $\hat{\rho} > \rho$ increases equilibrium news veracity from $\min\{\bar{p}_0(\rho), \rho\}$ to $\min\{\bar{p}_0(\hat{\rho}), \hat{\rho}\}$. On the other hand, if $\bar{p}_0(\rho) < 1 - \rho < \overline{p}_0(\rho)$ (see the lower panel of Figure 6) and $\hat{\rho}$ is sufficiently close to $\rho$ that $\bar{p}_0(\hat{\rho}) < 1 - \rho$, then increasing private-signal precision to $\hat{\rho} = \rho$ decreases equilibrium news veracity from $1 - \rho$ to $\max\{\bar{p}_0(\rho), 1 - \hat{\rho}\}$.

\[\text{In the last main possibility, when } \overline{p}_0(\rho) < 1 - \rho, \text{ slightly increasing } \rho \text{ has no effect on the unique}\]
4 The Curse (and Limited Wisdom) of the Crowd

In “Vox Populi” (Galton (1907)), Francis Galton examined hundreds of entries in a contest at the “West of England Fat Stock and Poultry Fair,” in which people guessed the dressed weight of an ox. Although individual guesses varied widely, the average guess (1997 lbs) was very close to the ox’s true weight (1998 lbs). In the same way, in our model, consumers in a large news market could almost-certainly discern which stories are true if their private signals could somehow be aggregated.

Consumer sharing over a highly-connected social network, what we refer to as a “crowd,” provides one means by which consumers’ information could potentially be aggregated in a decentralized way. If consumers all follow many others and all filter the news, a “wisdom of the crowd” will emerge and consumers will almost always act on true stories and almost never act on false stories. However, increasing the number of consumers’ social sources of information can also have indirect equilibrium effects, as consumers may change how often they share stories and the producer may invest more or less frequently in story quality.

In this section, we examine equilibrium outcomes in the market for news when consumers are in a crowd, focusing on the limit case of a complete network as network size goes to infinity and the cost of reporting scales with network size, what we refer to as “the crowd limit.” More precisely, we consider a sequence of news markets in which (i) market size $M$ goes to infinity, (ii) social connectedness $d = M - 1$, meaning that equilibrium, which exhibits zero investment, zero sharing, and news veracity $p^*_0 = 1 - f < 1 - \rho$.

$^{14}$Golub and Jackson (2010) and Mueller-Frank (2014) show how dispersed information can be successfully aggregated through repeated communication over a social network.

$^{15}$Our analysis extends easily to a richer setting in which there are both fixed and marginal (per-consumer) costs of investment, in which per-consumer reporting cost takes the form $c_R = X + Y/M$ where $X \geq 0, Y \geq 0$, and $(X,Y)$ is i.i.d. across stories. In the special case in which all reporting costs are fixed ($X = 0$), our crowd-limit analysis simplifies substantially as even very small differences in true- and false-news visibility are enough to induce maximal investment. Consequently, consumer sharing becomes uninformative in the crowd limit but, nonetheless, some investment can be supported; in particular, $p^*_0 = \rho$ in the case when the producer is paid for views (our focus here) and $p^*_0 = \bar{p}^4 \geq \rho$ in the case when the producer is paid for adoptions (Section 5).
everyone follows everyone, and (iii) the distribution of per-consumer reporting cost $c_R$ is held fixed, i.e., $c_R$ is drawn from c.d.f. $H(\cdot)$ for all $M$. We will refer to this sequence of markets as “the crowd-limit sequence”.

This section’s analysis addresses two main questions. First, how does equilibrium news veracity change as consumers grow increasingly connected? We find that, in the crowd limit, equilibrium investment either vanishes entirely or remains positive but falls to the lowest level consistent with consumer sharing. Either way, being in a crowd exposes consumers to more false news than if they had been in a less-well-connected social network—a finding that we refer to as “the curse of the crowd”.

Second, to what extent are consumers in a crowd able to learn from others’ sharing behavior? We find that, for some model parameters, consumers in a crowd filter the news, allowing consumers to discern which stories are true and avoid acting on false information. However, in such cases when a wisdom of the crowd emerges, equilibrium investment always vanishes in the crowd limit. Interesting, a wisdom of the crowd never emerges in the case in which low-quality stories are always false ($f = 1$). In that case, either equilibrium investment is zero and consumers never share in all sufficiently large markets or equilibrium investment does not vanish but individuals’ likelihood of sharing any given story goes to zero at such a rate that consumers typically have a small number of sharing neighbors and cannot determine with high confidence which stories are true and which are false—a finding that we refer to as “the limited wisdom of the crowd”.

Some terminology is helpful for stating our main findings: “investment vanishes” if the producer’s likelihood of investing in the maximal-veracity equilibrium converges to zero in the crowd limit; “individual sharing vanishes” if each consumer’s equilibrium likelihood of sharing any given story converges to zero; “the crowd is wise” if, in the crowd limit, consumers are able to perfectly discern which stories are true from others’ equilibrium sharing behavior; and “the crowd is ignorant” if consumers are able to infer nothing from others’ equilibrium sharing behavior.

---

16 When individual sharing vanishes, consumers’ likelihood of having at least one sharing neighbor may not go to zero, since the number of neighbors is going to infinity. So, there may still be meaningful sharing in the crowd limit even when individual sharing vanishes.
Finally, let $\rho_\infty = \lim_{M \to \infty} \rho^M$, where $\rho^M$ denotes the threshold signal precision (implicitly defined in equation (9) for any fixed $M$) below which no equilibrium investment can be supported in the market of size $M$.

**Theorem 2.** In the maximal-veracity equilibrium in the crowd limit: (i) If $1 - f \geq \rho$, then investment is zero, consumers always share, and the crowd is ignorant. (ii) If $1 - f \in [1 - \rho, \rho)$, then investment vanishes, consumers filter the news, and the crowd is wise. (iii) If $1 - f < 1 - \rho$ and $\rho < \rho_\infty$, then investment is zero, consumers never share, and the crowd is ignorant. (iv) If $1 - f < 1 - \rho$ and $\rho > \rho_\infty$, then investment does not vanish, individual sharing vanishes, and the crowd is neither wise nor ignorant.

The rest of this section characterizes crowd-limit investment and consumer sharing in the maximal-veracity equilibrium; in doing so, we will prove Theorem 2.

**Shorthand notation.** For each market size $M$ in the crowd-limit sequence, let $p^*_M$ denote the equilibrium news veracity and let $z^*_M$ denote the equilibrium sharing rule in the maximal equilibrium news veracity. Also: let $\tilde{p}^*_M$ denote the filtering news veracity, induced when consumers use sharing rule $\tilde{z} = (1, 0)$; let $\bar{p}^*_M$ denote the maximal best-response news veracity, induced when consumers use sharing rule $z^*_M$; and let $p^*_\infty = \lim_{M \to \infty} p^*_M$, $z^*_\infty = \lim_{M \to \infty} z^*_M$, $\tilde{p}^*_\infty = \lim_{M \to \infty} \tilde{p}^*_M$, $\bar{p}^*_\infty = \lim_{M \to \infty} \bar{p}^*_M$, and $\tilde{z}^*_\infty = \lim_{M \to \infty} \tilde{z}^*_M$ denote the crowd limits of these variables.

We organize the rest of this section’s analysis into four parts.

**Part One:** Investment vanishes ($p^*_\infty = 1 - f$) and/or individual sharing vanishes ($z^*_\infty = (0, 0)$). For each market size $M$, each consumer views each true story with probability $V^*_T(z^*_M)$ and each false story with probability $V^*_F(z^*_M)$ where, by equations (3-4),

\[
V^*_T(z^*_M) = 1 - (1 - b) \left( 1 - b \left( \rho z^*_T z^*_M + (1 - \rho) z^*_F \right) \right)^{M-1} \tag{11}
\]

\[
V^*_F(z^*_M) = 1 - (1 - b) \left( 1 - b \left( (1 - \rho) z^*_T z^*_M + \rho z^*_F \right) \right)^{M-1} \tag{12}
\]

Note that both of $V_T(z^*_M)$ and $V_F(z^*_M)$ converge to one so long as either of $z^*_T$ or $z^*_F$ does not converge to zero. Therefore, the extra visibility of true stories $\Delta V(z^*_M) = V_T(z^*_M) - V_F(z^*_M)$ must vanish in the crowd limit—so that equilibrium producer investment also vanishes—unless individual sharing vanishes.
This leaves three possibilities: (i) $p_0^\infty = 1 - f$ and $z^{\infty} > (0, 0)$ (investment vanishes but individual sharing does not vanish); (ii) $p_0^{\infty} > 1 - f$ and $z^{\infty} = (0, 0)$ (investment does not vanish but individual sharing vanishes or (iii) $p_0^{\infty} = 1 - f$ and $z^{\infty} = (0, 0)$ (investment and individual sharing both vanish). As we will see, each of these possibilities can occur depending on whether low-quality stories are sufficiently likely to be true that $1 - f \geq 1 - \rho$ and whether the precision of consumers’ private signals is sufficiently high that $\rho > \rho^\infty$ (which, by definition, implies that $p_0^\infty > 1 - \rho$).

**Part Two:** When $1 - f \geq 1 - \rho$, individual sharing does not vanish and investment vanishes. Suppose first that low-quality stories are sufficiently likely to be true that $1 - f \geq \rho$. By Theorem 1, consumers always share and the producer never invests no matter what the social network, i.e., $p_0^M = 1 - f$ and $z^M = (1, 1)$ for all $M$. Since others always share, consumers cannot infer anything about news truth from others’ sharing behavior, i.e., the crowd is ignorant. This completes the proof of Theorem 2(i).

Next, suppose that low-quality stories have an intermediate likelihood of being true, so that $1 - f \in [1 - \rho, \rho)$. By equations (5, 7), $p_0^M = 1 - f + f H(f \Delta V^M(\tilde{z}))$ where $\Delta V^M(\tilde{z}) = (1 - b) \left( (1 - b(1 - \rho))^M - (1 - b \rho)^M \right)$ is the extra visibility of true stories when consumers in the market of size $M$ filter the news. Note that $\Delta V^M(\tilde{z}) > 0$ for all $M$ but $\lim_{M \to \infty} \Delta V^M(\tilde{z}) = 0$; thus, $p_0^M > 1 - f$ for all $M$ but $p_0^\infty = 1 - f$. Theorem 1 therefore implies that $p_0^M \in (1 - \rho, \rho)$ for all sufficiently large $M$ and that $p_0^\infty = 1 - f$. We conclude that investment vanishes but that individual sharing does not vanish. In particular, because consumers use the filtering sharing rule in all arbitrarily large markets, $z^{\infty} = \tilde{z} = (1, 0)$. Moreover, because consumers in large markets filter the news, they can in the limit perfectly discern whether any given news story is true or false (by the Law of Large Numbers); so, the crowd is wise. This completes the proof of Theorem 2(ii).

**Part Three:** When $1 - f < 1 - \rho$ and $\rho < \rho^\infty$, individual sharing and investment both vanish. For the remainder of the analysis, we focus on the case in which low-quality stories are sufficiently unlikely to be true that $1 - f < 1 - \rho$. For each market size $M$, Proposition 4(i) implies that there is zero equilibrium investment if $\rho < \rho^M$ but that a positive-investment equilibrium exists if $\rho > \rho^M$. In the case when $\rho < \rho^\infty$, $\rho < \rho^M$
and hence $p^*_M = 1 - f$ for all sufficiently large market sizes $M$. Since $1 - f < 1 - \rho$, consumers must also never share in all sufficiently large markets. It follows immediately that $p^*_0 = 1 - f$ and $z^*_0 = (0, 0)$. Moreover, since there is no sharing in all sufficiently large markets, the crowd is obviously ignorant. This completes the proof of Theorem 2(iii).

Part Four: When $1 - f < 1 - \rho$ and $\rho > \overline{\rho}^\infty$, individual sharing vanishes and investment does not vanish. Finally, consider the case in which $1 - f < 1 - \rho$ and private-signal precision is high enough that $\rho > \overline{\rho}^\infty$. For any sufficiently large market size $M$, $\rho > \overline{\rho}^M$ and hence, by definition of $\overline{\rho}^M$, $p^*_0 > 1 - \rho$. By Theorem 1, we conclude that $p^*_M = 1 - \rho$ and consumer sharing in the maximal-veracity equilibrium takes the form $z^*_M = (z^*_T, 0)$ for all sufficiently large $M$. An immediate implication is that $p^*_0 = 1 - \rho > 1 - f$; so, investment does not vanish in the crowd limit.

Next, we show that individual sharing must vanish in the crowd limit. Suppose for the sake of contradiction that $\lim_{M \to \infty} z^*_M = X > 0$. By equations (11,12), true stories’ extra visibility in the crowd limit would then be

$$
\lim_{M \to \infty} \Delta V^M(z^*_T, 0) = (1 - b) \lim_{M \to \infty} \left( (1 - b(1 - \rho)X)^{M-1} - (1 - b\rho X)^{M-1} \right),
$$

which equals zero since $(1 - b(1 - \rho)X)^{M-1}$ and $(1 - b\rho X)^{M-1}$ each converge to zero. But then investment must vanish in the crowd limit, a contradiction.

Finally, we establish a Goldilocks-type result that consumers in a crowd cannot learn too little (i.e., the crowd cannot be ignorant) but also cannot learn too much (i.e., the crowd cannot be wise) in the maximal-veracity equilibrium.\textsuperscript{17} We have shown that $p^*_M = 1 - \rho$ for all sufficiently large $M$; so, the producer must invest with ex ante probability $\frac{f - \rho}{f}$ in any sufficiently large market. Since investing generates extra expected per-consumer revenue $f\Delta V^M(z^*_T, 0)$, it must be that $\hat{H}(f\Delta V^M(z^*_T, 0)) = \frac{f - \rho}{f}$, which in turn requires that the extra visibility of true stories

$$
\Delta V^M(z^*_T, 0) = H^{-1} \left( \frac{f - \rho}{f} \right) > 0
$$

\textsuperscript{17}\text{When } 1 - f < 1 - \rho, another equilibrium always exists in which the producer never invests, consumers never share, and the crowd is ignorant.
for all sufficiently large $M$. In particular, true stories must be viewed more often than false stories in the crowd limit.

Using the basic mathematical fact that $\lim_{M \to \infty} (1 - Y/M)^M = e^{-Y}$, equation (13) can be re-written as

$$\lim_{M \to \infty} \Delta V^M(z_T^*, 0) = (1 - b) \left( e^{-b(1-\rho) \lim_{M \to \infty} Mz_T^M} - e^{-b\rho \lim_{M \to \infty} Mz_T^M} \right)$$

Moreover, since equation (14) must hold for all sufficiently large $M$, it must be that

$$\lim_{M \to \infty} \Delta V^M(z_T^*, 0) = H^{-1} \left( \frac{f}{f - \rho} \right) / (1 - b).$$

Equation (15) therefore implies that

$$\lim_{M \to \infty} Mz_T^* = C^*, \text{ where } C^* > 0 \text{ solves } e^{-b(1-\rho)C^*} - e^{-b\rho C^*} = H^{-1} \left( \frac{f}{f - \rho} \right) / (1 - b).$$

Having characterized the rate at which consumer sharing vanishes in the crowd limit, we can now characterize the crowd-limit visibility of true and false stories:

$$V_T^{\infty} \equiv \lim_{M \to \infty} V_T^M(z_T^M, 0) = 1 - (1 - b) e^{-b\rho C^*}$$

$$V_F^{\infty} \equiv \lim_{M \to \infty} V_F^M(z_T^M, 0) = 1 - (1 - b) e^{-b(1-\rho)C^*}$$

The fact that the crowd cannot be ignorant and also cannot be wise follows immediately from (16-17). In the crowd limit, each consumer is more likely to have zero sharing neighbors when a story is false (since $e^{-b(1-\rho)C^*} > e^{-b\rho C^*}$). Seeing that no one has shared is therefore informative about the likelihood of news truth; hence, the crowd is not ignorant. On the other hand, since true stories also sometimes fail to be shared, a consumer seeing no one share cannot discern for certain whether the story is true or false; hence, the crowd also is not wise. This completes the proof of Theorem 2(iv).

Discussion: Equilibrium investment in the crowd limit. Theorem 2 establishes conditions under which at least some equilibrium investment can be supported in the crowd limit. In particular, $p_0^{\infty} > 1 - f$ whenever (i) low-quality stories are sufficiently likely to be false that $1 - f < 1 - \rho$ and (ii) consumers’ private signals are sufficiently precise that $\rho > \rho^\infty$. Recall that $\rho^\infty = \lim_{M \to \infty} \rho^M$, where the signal-precision thresholds $\{\rho^M\}$ were defined implicitly in Proposition 4. Proposition 5 (proven in the Appendix) provides a more explicit, equivalent condition in terms of model primitives.

18Such a solution always exists when $p_0^{\infty} > 1 - \rho$. In fact, there are two solutions: the larger solution corresponds to the limit of dynamically-stable equilibria and the smaller solution corresponds to the limit of dynamically-unstable equilibria. See Figure 5 for a visualization.
Proposition 5. \( \rho > \overline{p}^\infty \) if and only if \( H\left(f\Delta V^\infty\right) > 1 - \frac{\rho}{\rho} \), where

\[
\Delta V^\infty = \lim_{M \to \infty} \Delta V(z^M_T, 0) = H\left(f(1-b)\left(\frac{1-\rho}{\rho}\overline{z}_{\overline{z}}^{-\frac{1}{\overline{z}}} - \frac{1-\rho}{\rho}\overline{z}_{\overline{z}}^{-\frac{1}{\overline{z}}}\right)\right)
\]

is the maximal extra visibility of true stories in the crowd limit.

5 Extension: Sponsor-Supported (Partisan) Content

We have thus far focused on a news market in which the producer is paid each time a consumer encounters ("views") a news item, as in the ad-supported revenue models of many online news outlets. However, those producing decision-relevant information often do so because they want to influence others’ decisions. For instance, a company launching a social-media campaign promoting a new product ultimately wants consumers to buy their product (i.e., "adopt"), not just to see and share the message. Similarly, a political candidate with a new campaign video wants people to vote for her, not just watch or talk about the video.

In this section, we extend our previous analysis to a setting in which the producer is paid each time a consumer chooses to adopt based on a news item. Stories produced under this pay-for-adoptions revenue model can be interpreted as "sponsor-supported content" (in the commercial context) or "partisan content" (in the political context). Our key finding is that more equilibrium investment can be supported in equilibrium in the crowd limit—resulting in more true stories being provided—when the producer is paid for adoptions than when paid for views (corollary to Theorem 3).

Figure 7 illustrates the market for news when the producer is paid for adoptions. Consumers’ sharing decisions still impact the producer’s incentive to invest, but now indirectly through two channels: by impacting how widely true and false stories are seen and by affecting how much consumers can infer about a story’s truthfulness from others’ sharing behavior.
Figure 7: Schematic of the market for news when producers are paid for adoptions.

5.1 Consumer adoption

This section characterizes optimal consumer adoption. For ease of exposition, we focus here on news-veracity levels \( p_0 \not\in \{1 - \rho, \rho\} \),\(^{19}\) given which there is a unique optimal sharing rule, denoted \( z(p_0) \). By Proposition 1: if \( p_0 > \rho \), then \( z(p_0) = (1, 1) \) (“always sharing”); if \( p_0 \in (1 - \rho, \rho) \), then \( z(p_0) = \tilde{z} = (1, 0) \) (“filtering”); and if \( p_0 < 1 - \rho \), then \( z(p_0) = (0, 0) \) (“never sharing”).

Likelihood of adoption based on true and false stories. For any “observation” \( o_i = (s_i, \sigma_i) \), let \( l_T(o_i; z) \) and \( l_F(o_i; z) \) denote the ex ante likelihood that a consumer becomes aware of a story and observes \( o_i \) when the story is true or false, respectively, and all consumers use sharing rule \( z \). Let \( \Delta l(o_i; z) = l_T(o_i; z) - l_F(o_i; z) \) be the extra likelihood of observation \( o_i \) when the story is true.

By Bayes’ Rule, a consumer who observes \( o_i \) and believes that other are using sharing rule \( z \) will update her belief about the likelihood of news truth from \( p_0 \) to \( p_2(o_i; p_0, z) \),

\(^{19}\)Matters are more complex when news veracity equals the always-share threshold (\( p_0 = \rho \)) or the never-share threshold (\( p_0 = 1 - \rho \)) since, in those cases, consumers have a continuum of optimal sharing rules, each of which induces a different pattern of news awareness and optimal adoption. However, these complexities have no impact on equilibrium outcomes in the crowd limit.
implicitly defined by
\[
\frac{p_2(o_i; p_0, z)}{1 - p_2(o_i; p_0, z)} = \frac{p_0}{1 - p_0} \times \frac{l_T(o_i; z)}{l_F(o_i; z)}
\] (19)

Let \( O(p_0) \) be the set of observations given which consumers strictly prefer to adopt when others use a best-response sharing rule \( z = z(p_0) \), i.e., such that \( p_2(o_i; p_0, z(p_0)) > p^A \), where \( p^A = \frac{\pi^A}{\pi^A + \pi^F} \) is the adoption threshold defined in equation (1).

Let \( A_T(p_0) \) and \( A_F(p_0) \) denote the ex ante likelihood that consumers adopt based on true and false stories, respectively, when consumers use the individually-optimal sharing rule \( z(p_0) \). Since consumer \( i \) adopts whenever seeing an observation \( o_i \in O(p_0) \), these adoption likelihoods can be expressed simply as
\[
A_T(p_0) = \sum_{o_i \in O(p_0)} l_T(o_i; z(p_0)) \quad \text{and} \quad A_F(p_0) = \sum_{o_i \in O(p_0)} l_F(o_i; z(p_0)).
\]
The extra likelihood that true stories induce adoption is therefore
\[
\Delta A(p_0) = \sum_{o_i \in O(p_0)} \Delta l(o_i; z(p_0)) \quad \text{for} \quad p_0 \in (1 - \rho, \rho)
\] (20)

Always-share region \((p_0 > \rho)\): Suppose first that stories are sufficiently likely to be true that news veracity is in the always-share region. Given that consumers always share, all stories enjoy the maximum possible visibility \( \bar{V} = 1 - (1 - b)^{d+1} \). Because true and false stories are equally likely to be shared, consumers infer nothing from others’ sharing behavior; so, \( p_2(s_i; \sigma_i; p_0) = p_1(s_i; p_0) \), where \( p_1(s_i; p_0) \) is defined in equation (2).

Let \( \overline{p}^A \) and \( \overline{p}^A \) be the ex ante beliefs given which consumers are indifferent whether to adopt after getting, respectively, a negative or positive private signal. These thresholds are implicitly characterized by the conditions \( p_1(F; \overline{p}^A) = p^A \) and \( p_1(T; \overline{p}^A) = p^A \), or
\[
\overline{p}^A = \frac{\rho p^A}{\rho p^A + (1 - \rho)(1 - p^A)} \quad \text{and} \quad \overline{p}^A = \frac{(1 - \rho)p^A}{(1 - \rho)p^A + \rho(1 - p^A)}.
\]
Recall that, by assumption, 20 consumers require at least as much evidence of news truth to adopt than to share, i.e., \( p^A \geq 1/2 \); thus, \( \overline{p}^A \geq \rho \) and \( \overline{p}^A \geq 1 - \rho \).

Consumer \( i \) strictly prefers to adopt whenever either \( p_0 > \overline{p}^A \) (regardless of private signal) or \( p_0 \in (\overline{p}^A, \overline{p}^A) \) and she has gotten a positive private signal. There are three main possibilities. (i) If \( p_0 > \overline{p}^A \), then consumers always adopt based on any story

\[20\]The alternative case in which \( p^A < 1/2 \) differs in that consumers may sometimes adopt when news veracity is in the never-share region, but does not provide any significant new insight.
they encounter, resulting in equal adoption of all stories: 

\[ A_T(p_0) = A_F(p_0) = V \] and 

\[ \Delta A(p_0) = 0, \] where 

\[ V = 1 - (1 - b)^{d+1} \] is the maximal news visibility. (ii) If 

\[ p_0 \in (\rho, \bar{p}_A) \], then consumers adopt after a positive private signal only, resulting in greater adoption of true stories: 

\[ A_T(p_0) = \rho V, \quad A_F(p_0) = (1 - \rho) V, \quad \text{and} \quad \Delta A(p_0) = (2\rho - 1)V > 0. \] (iii) If 

\[ p_0 < \rho \], then consumers never adopt: 

\[ A_T(p_0) = A_F(p_0) = 0. \] We conclude:

\[ \Delta A(p_0) = (2\rho - 1)(1 - (1 - b)^{d+1}) \text{ for all } p_0 \in (\rho, 1] \cap (\rho, \bar{p}_A) \] (21)

\[ = 0 \text{ for all } p_0 \in (\bar{p}_A, 1] \text{ and all } p_0 \in (\rho, \bar{p}_A) \]

Never-share region \((p_0 < 1 - \rho)\): Suppose next that stories are sufficiently unlikely to be true that news veracity is in the never-share region. Given that consumers never share, all stories enjoy the minimum possible visibility 

\[ V = b. \] Moreover, because consumers infer nothing from others’ failure to share and 

\[ p_0 < 1 - \rho \leq \bar{p}_A, \] consumers who see the broadcast prefer not to adopt even after getting a positive private signal; so, 

\[ A_T(p_0) = A_F(p_0) = 0. \] We conclude:

\[ \Delta A(p_0) = 0 \text{ for all } p_0 \in [0, 1 - \rho) \]

Filtering region \((p_0 \in (1 - \rho, \rho))\): Suppose finally that stories have an intermediate likelihood of being true, so that news veracity lies in the filtering region and 

\[ z(p_0) = \bar{z} = (1, 0). \] Each consumer shares true stories with probability \(b\rho\), shares false stories with probability \(b(1 - \rho)\), and encounters a story so long as she has at least one sharing neighbor and/or sees the broadcast. Thus, for each signal \(s_i \in \{T,F\}, \)

\[ l_T(s_i, 0; \bar{z}) = b \Pr(s_i|\text{true}) (1 - b\rho)^d \]

\[ l_F(s_i, 0; \bar{z}) = b \Pr(s_i|\text{false}) (1 - b(1 - \rho))^d \]

\[ l_T(s_i, \sigma; \bar{z}) = \Pr(s_i|\text{true}) (b\rho)^\sigma (1 - b\rho)^{d-\sigma} \text{ for all } \sigma = 1, 2, ..., d \]

\[ l_F(s_i, \sigma; \bar{z}) = \Pr(s_i|\text{false}) (b(1 - \rho))^\sigma (1 - b(1 - \rho))^{d-\sigma} \text{ for all } \sigma = 1, 2, ..., d \]

In particular, the likelihood ratio for each possible observation \(o_i = (s_i, \sigma_i)\) takes the form

\[ \frac{l_T(s_i, \sigma_i; \bar{z})}{l_F(s_i, \sigma_i; \bar{z})} = \frac{\Pr(s_i|\text{true})}{\Pr(s_i|\text{false})} \times \left( \frac{\rho}{1 - \rho} \right)^{\sigma_i} \times \left( \frac{1 - b\rho}{1 - b(1 - \rho)} \right)^{d-\sigma_i}, \]

34
which in turn determines consumer $i$’s “ex post belief” $p_2(s_i, \sigma_i; p_0)$ by equation (19).

Let $\sigma(s_i; p_0)$ be the threshold number of sharing neighbors given which a consumer would be indifferent whether to adopt when getting private signal $s_i$, implicitly defined by the condition $p_2(s_i, \sigma_i = \sigma(s_i; p_0); p_0, \tilde{z}) = p^A$. Consumer $i$ strictly prefers to adopt whenever either she gets a good private signal and has more than $\sigma(T; p_0)$ sharing neighbors or she gets a bad private signal and has more than $\sigma(F; p_0)$ sharing neighbors. We conclude that,

$$
\Delta A(p_0) = \sum_{\sigma = \lceil \sigma(T; p_0) \rceil}^d \Delta l(T, \sigma; \tilde{z}) + \sum_{\sigma = \lceil \sigma(F; p_0) \rceil}^d \Delta l(F, \sigma; \tilde{z}) \text{ for } p_0 \in (1 - \rho, \rho)
$$

for all but finitely many $p_0 \in (1 - \rho, \rho)$.

Figure 8: Extra likelihood that true stories induce adoption in the crowd limit, depending on news veracity.

---

21 By construction, $\sigma(T; p_0)$ and $\sigma(F; p_0)$ are potentially-negative real numbers, generally satisfying the conditions $\sigma(F; p_0) > \sigma(T; p_0)$ and $\sigma(F; p_0) < \sigma(T; p_0) + 2$.

22 Let $O^= (p_0)$ be the set of observations given which consumers are indifferent whether to adopt when others share optimally. Since $p_2(o_i; p_0, z)$ is strictly increasing in $p_0$ for all $o_i$, and since $z(p_0)$ equals $(0, 0), (1, 0), \text{ or } (1, 1)$ for all $p_0 \notin \{1 - \rho, \rho\}$, $O^= (p_0) = \emptyset$ for all but finitely many news-veracity levels. If $O^= (p_0) \ni o_i = (s_i, \sigma)$, then consumer $i$ may randomize whether to adopt after getting signal $s_i$ and seeing $\sigma$ neighbors sharing, causing $\Delta A(p_0)$ to take on an interval of values, from $\lim_{\epsilon \to 0} \Delta A(p_0 - \epsilon)$ to $\lim_{\epsilon \to 0} \Delta A(p_0 + \epsilon)$. 
Consumer adoption in the crowd limit. Consider the “crowd-limit sequence” of news markets introduced in Section 4 where, in the market of size $M$, each consumer follows $M - 1$ others. The previous analysis characterizes $\Delta A^M(p_0)$ for each market size $M$. The crowd limit $\Delta A^\infty(p_0) \equiv \lim_{M \to \infty} \Delta A^M(p_0)$ takes an especially simple form:

$$\Delta A^\infty(p_0) = 1 \text{ for all } p_0 \in (1 - \rho, \rho)$$

$$= 2\rho - 1 \text{ for all } p_0 \in (\underline{p}^A, \overline{p}^A)$$

$$= 0 \text{ for all } p_0 \in [0, 1 - \rho) \cup (\rho, \underline{p}^A) \cup (\overline{p}^A, 1]$$

See Figure 8, which shows the case in which consumers’ adoption threshold $\underline{p}^A$ is sufficiently high that $\underline{p}^A > \rho$.

5.2 Equilibrium news veracity in the crowd limit

In this section, we characterize the maximal crowd-limit equilibrium news veracity ($p_0^{*\infty}$) in the case when the producer is paid for adoptions. To simplify exposition, we focus on the case in which low-quality stories are always false, i.e., $f = 1$.

Our main finding is that, so long as any equilibrium investment can be supported in the crowd limit when the producer is paid for views, strictly more crowd-limit equilibrium investment can be supported when the producer is paid for adoptions. Recall that, by Theorem 2 and Proposition 5, $p_0^{*\infty} > 0$ when the producer is paid for views only if $H(\overline{\Delta V}^\infty) \geq 1 - \rho$ and, in that case, $p_0^{*\infty} = 1 - \rho$. Since $\overline{\Delta V}^\infty < 1$, this implies that $H(1) > 1 - \rho$; Theorem 3(ii-iv) then implies that $p_0^{*\infty} > 1 - \rho$ when the producer is paid for adoptions.

**Theorem 3.** Suppose that $f = 1$ and the producer is paid for adoptions. (i) $p_0^{*\infty} = 0$ if $H(1) < 1 - \rho$; (ii) $p_0^{*\infty} = H(1)$ if $H(1) \in (1 - \rho, \rho)$; (iii) $p_0^{*\infty} = \rho$ if $H(1) > \rho$ and $H(2\rho - 1) < \max\{\rho, \underline{p}^A\}$; and (iv) $p_0^{*\infty} > \rho$ if $H(2\rho - 1) > \max\{\rho, \underline{p}^A\}$.

**Proof of Theorem 3(i):** Consider the crowd-limit-sequence market of size $M$. Recall that $A^M_T(p_0)$ and $A^M_F(p_0)$ denote the respective adoption likelihoods of true and false stories, $\Delta A^M(p_0) = A^M_T(p_0) - A^M_F(p_0)$, and $p_0^{*M}$ denotes the maximal equilibrium news veracity. Since every observation that induces a consumer to adopt sometimes occurs even when
the story is false, \( A_M^T(p_0) > 0 \) implies \( A_F^M(p_0) > 0 \); so, it must be that \( \Delta A^M(p_0) < 1 \). The producer must therefore invest with probability strictly less than \( H(1) \); in particular, news veracity must be strictly less than \( H(1) \) in any equilibrium, no matter what the market size \( M \). We conclude that, if \( H(1) < 1 - \rho \), then every equilibrium must have news veracity strictly less than \( 1 - \rho \). Moreover, any such equilibrium must have zero investment since consumers never share—and hence never adopt; so, \( p_0^{*M} = 0 \) for all \( M \) and hence therefore \( p_0^{*\infty} \equiv \lim_{M \to \infty} p_0^{*M} = 0 \).

**Proof of Theorem 3(ii):** Suppose next that \( H(1) \in (1 - \rho, \rho) \). We will show that, in all sufficiently large markets, \( p_0^{*M} \in (1 - \rho, \rho) \) and hence that consumers filter the news. Since \( H(1) > 1 - \rho \), the producer will invest often enough to induce news veracity strictly greater than \( 1 - \rho \) so long as \( \lim_{\epsilon \to 0} \Delta A^M(1 - \rho + \epsilon) \) is sufficiently close to one. Moreover, since consumers who follow many others are able to almost perfectly discern which stories are true when others filter the news (by the Law of Large Numbers), \( \lim_{M \to \infty} \Delta A^M(1 - \rho + \epsilon) = 1 \) for all \( \epsilon \approx 0 \). We conclude that, if consumers believe that news veracity is in the filtering region and market size \( M \) is sufficiently large, the producer will optimally respond by investing sufficiently to induce some news-veracity level \( \hat{p}_0^{M} \) that converges to \( H(1) \) as \( M \to \infty \). Because \( H(1) \in (1 - \rho, \rho) \), this implies that \( \hat{p}_0^{M} \in (1 - \rho, \rho) \) for all sufficiently large \( M \). We conclude that \( \hat{p}_0^{M} \) is indeed the maximal equilibrium news veracity given market size \( M \), and hence that \( p_0^{*\infty} = \lim_{M \to \infty} \hat{p}_0^{M} = H(1) \in (1 - \rho, \rho) \).

**Proof of Theorem 3(iii):** Suppose next that \( H(1) > \rho \) but \( H(2\rho - 1) < \max\{\rho, p^A\} \). We begin by showing that equilibrium news veracity cannot exceed \( \rho \). Suppose for the sake of contradiction that \( p_0^{*M} > \rho \). By (21), for any \( p_0 > \rho \), true stories’ extra adoption likelihood \( \Delta A^M(p_0) \) equals either zero or \( (2\rho - 1)(1 - (1 - b)^{M-1}) < 2\rho - 1 \). When the producer invests optimally, the resulting induced news veracity is bounded above by \( H(2\rho - 1) \) which, by presumption, is less than \( \max\{\rho, p^A\} \). This leaves two possibilities: either \( H(2\rho - 1) < \rho \), in which case a contradiction is immediately reached, or \( p^A > \rho \) and \( H(2\rho - 1) \in (\rho, p^A) \). But in that second case, \( \Delta A^M(p_0) = 0 \) since consumers never adopt, also a contradiction since then induced news veracity must equal zero.

Next, we need to show that \( p_0^{*M} = \rho \) for all sufficiently large \( M \). Let \( \Delta A^M(\rho; z_F) \)
denote the extra adoption likelihood of true stories in the market of size $M$, given news veracity $p_0 = \rho$ and consumer sharing according to the rule $z = (1, z_F)$ for some $z_F \in [0, 1]$, and let $\Delta A^\infty(\rho; z_F) = \lim_{M \to \infty} \Delta A^M(\rho; z_F)$. Since $\Delta A^\infty(\rho; z_F)$ is continuous in $z_F$ (straightforward details omitted), to prove that an equilibrium exists with news veracity equal to $\rho$, it suffices to show that $\Delta A^\infty(\rho; 0) > \rho > \Delta A^\infty(\rho; 1)$. The case with $z_F = 0$ corresponds to news filtering; so, $\Delta A^\infty(\rho; 0) = 1$ due to the Law of Large Numbers. The other extreme $z_F = 1$ corresponds to always sharing, generating either $\Delta A^\infty(\rho; 1) = 0$ if $\underline{p}^A > \rho$ (since then consumers never adopt when others always share) or $\Delta A^\infty(\rho; 1) = 2\rho - 1$ if $\underline{p}^A < \rho$ (since then consumers adopt after good private signal only when others always share). Either way, $\Delta A^\infty(\rho; 1) < \rho$. This completes the proof that $p_0^M = \rho$ for all sufficiently large $M$ and hence that $p_0^\infty = \rho$.

**Proof of Theorem 3(iv):** Suppose finally that $H(2\rho - 1) > \max\{\rho, \underline{p}^A\}$. If consumers believe that news veracity $p_0 \in (\max\{\rho, \underline{p}^A\}, \overline{p}^A)$, then they will optimally respond by (i) always sharing and (i) adopting only after getting a good private signal, given which $\Delta A^M(p_0) = (2\rho - 1)\overline{V}^M$, where $\overline{V}^M = 1 - (1 - b)^M$ is the likelihood of becoming aware of a story in a completely-connected market of size $M$ when everyone always shares. The producer’s best response to such consumer behavior is to invest with probability $H((2\rho - 1)\overline{V}^M)$, which is increasing in $M$ and converges to $H(2\rho - 1)$ as $M \to \infty$. Since $\hat{H}(2\rho - 1) > \max\{\rho, \underline{p}^A\}$, this leaves two relevant cases. First, if $H(2\rho - 1) \leq \underline{p}^A$, then an equilibrium exists with news veracity $p_0^*M \in (\max\{\rho, \underline{p}^A\}, \overline{p}^A)$. Second, if $\hat{H}(2\rho - 1) > \overline{p}^A$, then an equilibrium exists with news veracity $p_0^*M = \overline{p}^A$.23 Either way, we conclude that $p_0^*\infty > \rho$. \hfill \square

6 Concluding remarks

This paper is, to the best of our knowledge, the first to consider how the structure of consumers’ social network impacts the quality of the information that spreads through

23We omit full details to save space but, in such equilibria with news veracity $\overline{p}^A$, consumers randomize whether to adopt after getting a bad private signal, increasing the likelihood that any story is adopted but decreasing the extra likelihood that true stories are adopted.
the network. A key insight that emerges from our analysis, in the context of media markets, is that increased social connection may strengthen or weaken the quality of news, depending on the density of the network and on how news producers are paid. In large networks in which each consumer follows many others, the equilibrium quality of news is relatively poor in markets where news producers are paid when consumers see their stories and relatively good in markets where news producers are paid when consumers believe the story enough to take an action. Even in that case, however, there is a limit to the overall news quality that can emerge in equilibrium and a limit to the amount of information aggregation that even an arbitrarily large network of consumers can achieve—the ideal of “the wisdom of crowds” is beyond reach.

References


A  Omitted proofs and supplementary analysis

A.1 Proof of Lemma 1

Parts (i, ii, v). By equation (5), \( p_0(z_T, z_F) = 1 - f + f H(f \Delta V(z_T, z_F)) \), where \( \Delta V(z_T, z_F) = (1 - b) \left( (1 - b((1 - \rho)z_T + \rho z_F))^d - (1 - b(\rho z_T + (1 - \rho)z_F))^d \right) \) by equations (3,4). Recall that, by assumption, the producer’s reporting cost has a continuous c.d.f. and support on the interval \([0, M]\), implying that \( H(0) = 0 \) and that \( H(c_R) \) is continuous and strictly increasing in \( c_R \) over the relevant range.\(^{24}\) Parts (i, ii) of Lemma 1 follow immediately from the fact that \( \Delta V(z_T, z_F) = 0 \) when \( z_T = z_F \) and \( \Delta V(z_T, z_F) > 0 \) when \( z_T > z_F \). Part (v) is also immediate, following from the continuity of \( H(\cdot) \) and the easily-checked continuity of \( \Delta V(\cdot) \).

Part (iii). Define \( x(z_T) = \frac{\Delta V(z_T, 0)}{1 - b} \). To prove part (iii), it suffices to show that \( x(z_T) \) is strictly increasing in \( z_T \) over the interval \([0, \overline{z}_T]\) and strictly decreasing in \( z_T \) over \([\overline{z}_T, 1]\) for some \( \overline{z}_T \in (0, 1) \). Note that \( x'(z_T) = db \left( (\rho(1 - b\rho z_T)^{d-1} - (1 - \rho)(1 - b(1 - \rho)z_T)^{d-1}) \right. \). Suppose first that \( d = 1 \). Since \( x'(z_T) = b(2\rho - 1) > 0 \), \( x(z_T) \) is strictly increasing over the whole interval \( z_T \in [0, 1] \), establishing the desired result with respect to \( \overline{z}_T = 1 \). Suppose next that \( d \geq 2 \). \( x'(z_T) > 0 \) iff \( \frac{\rho}{1 - \rho} > \frac{1 - b(1 - \rho)z_T}{1 - b\rho z_T} \) which, after re-arranging, can be written as \( z_T < \hat{z}_T \equiv \frac{\frac{x_T}{\rho(x_T)}^{\frac{d-1}{d}} - 1}{b\left(\frac{x_T}{\rho(x_T)}\right)^{\frac{1}{d}} - 1} \). So, \( x(z_T) \) is strictly increasing in \( z_T \) over the interval \([0, \min\{\hat{z}_T, 1\}]\) and, if \( \hat{z}_T < 1 \), strictly decreasing over the interval \([\hat{z}_T, 1]\), establishing the desired result with respect to \( \overline{z}_T \equiv \min\{\hat{z}_T, 1\} \).

Part (iv). Define \( y(z_T, z_F) = \frac{\Delta V(z_T, z_F)}{1 - b} \). To prove part (iv), we need to show that \( y(\overline{z}_T, 0) \geq y(z_T, z_F) \) for all \( z_T, z_F \in [0, 1] \). First, note that \( y(z_T, z_F) \leq 0 \) whenever \( z_T \leq z_F \) but \( y(\overline{z}_T, 0) > 0 \). Restricting attention to sharing rules with \( z_T > z_F \), note that

\[
\frac{\partial y(z_T, z_F)}{\partial z_F} = db \left( (1 - \rho)(1 - b(\rho z_T + (1 - \rho)z_F)^{d-1} - (1 - (1 - \rho)z_T + \rho z_F)^{d-1}) < 0 \right.
\]

since \( 1 - \rho < \rho \) and \( \rho z_T + (1 - \rho)z_F > (1 - \rho)z_T + \rho z_F \). Finally, \( y(\overline{z}_T, 0) \geq y(z_T, 0) \) for all \( z_T \in [0, 1] \) by definition of \( \overline{z}_T \). We conclude \( y(\overline{z}_T, 0) \geq y(z_T, z_F) \) for all \( z_T, z_F \).

\(^{24}\)Because \( f \leq 1, b > 0 \), and the producer is paid one unit of revenue per consumer who views the story, \( 0 \leq f \Delta V(z_T, z_F) < M \) no matter how consumers share the news.
A.2 Proof of Proposition 5

Proof. Let $\mathbf{z}^M = (z^M_T, 0)$ be the consumer sharing rule that maximizes the extra visibility of true stories, for any given market size $M$. By equation (7),

$$\Delta V^M(\mathbf{z}^M) = (1 - b) \left( (1 - b(1 - \rho)z^M_T)^{M-1} - (1 - b\rho z^M_T)^{M-1} \right)$$

(23)

For each market size $M$, $p^M_0 = 1 - f + f H\left(f \Delta V^M(\mathbf{z}^M)\right)$, which exceeds $1 - \rho$ if and only if $H\left(f \Delta V^M(\mathbf{z}^M)\right) > 1 - \rho$. We conclude that $p^\infty_0 > 1 - \rho$ if and only if $H\left(f \Delta V^\infty\right) > 1 - \rho$, where $\Delta V^\infty = \lim_{M \to \infty} \Delta V^M(\mathbf{z}^M)$ is the maximal extra visibility of true stories in the crowd limit.

To complete the proof, it remains to derive $\Delta V^\infty$. Using the fact that $\lim_{M \to \infty} (1 - X/M)^M = e^{-X}$,

$$\Delta V^\infty = (1 - b) \left( e^{-b(1 - \rho)\lim_{M \to \infty} M z^M_T} - e^{-b\rho \lim_{M \to \infty} M z^M_T} \right)$$

(24)

so long as $\lim_{M \to \infty} M z^M_T$ exists. The expression in (24) converges to zero if either $\lim_{M \to \infty} M z^M_T = 0$ or $\lim_{M \to \infty} M z^M_T = \infty$, but converges to a positive number if $\lim_{M \to \infty} M z^M_T = C$ for any finite $C$. Because $z^M_T$ maximizes the expression in (23) for all $M$, we conclude that $\lim_{M \to \infty} M z^M_T$ must exist, and that $M z^M_T$ must converge to $\overline{C} = \arg \max_{C > 0} e^{-b(1 - \rho)C} - e^{-b\rho C}$.

To solve for $\overline{C}$, we use the first-order condition $d \left[ e^{-b(1 - \rho)C} - e^{-b\rho C} \right] / dC = 0$, which can be re-written as $b(1 - \rho)e^{-b(1 - \rho)\overline{C}} = b\rho e^{-b\rho \overline{C}}$ or, equivalently,

$$\frac{\rho}{1 - \rho} = e^{b(2\rho - 1)\overline{C}}.$$  

(25)

Finally, note that (25) implies $\left(\frac{\rho}{1 - \rho}\right)^{\frac{1}{2\rho - 1}} = e^{-b(1 - \rho)\overline{C}}$ and $\left(\frac{\rho}{1 - \rho}\right)^{\frac{\rho}{2\rho - 1}} = e^{-b\rho \overline{C}}$. Equation (24) can therefore be re-written as desired:

$$\Delta V^\infty = (1 - b) \left( (1 - \rho) \left(\frac{1 - \rho}{\rho}\right)^{\frac{1 - \rho}{2\rho - 1}} \right) - (1 - \rho) \left(\frac{1 - \rho}{\rho}\right)^{\frac{\rho}{2\rho - 1}}$$

(26)
A.3 Supplementary analysis related to Theorem 1

In Theorem 1 and the surrounding text, we focused on the question of whether an equilibrium exists with positive investment and on characterizing the maximal news veracity that can be supported in any symmetric Nash equilibrium (SNE). Here we augment that analysis by characterizing all Nash equilibria in the baseline case when the producer is paid only for views.\textsuperscript{25} Recall by Steps 1-2 of the proof of Theorem 1 that (i) no Nash equilibrium exists with positive investment and news veracity in the always-share region and (ii) no Nash equilibrium exists with positive investment and news veracity in the never-share region. In what follows, we catalogue all equilibria in the remaining possibilities: Nash equilibria with no investment and hence news veracity $p_0 = 1 - f$; and Nash equilibria with positive investment and news veracity $p_0 \in (1 - \rho, \rho)$, $p_0 = \rho$, or $p_0 = 1 - \rho$.

**Nash equilibria with minimal news veracity.** Whether a Nash equilibrium exists with no investment depends on the likelihood $1 - f$ that low-quality stories are true, what we refer to as the “minimal news veracity.” There are three main cases.

First, suppose that $1 - f \geq \rho$. In this case, a Nash equilibrium exists in which consumers always share and the producer never invests; moreover, this is the unique Nash equilibrium. Uniqueness is obvious in the subcase when $1 - f > \rho$, since then consumers strictly prefer to always share no matter how what the producer does, giving the producer zero incentive to invest. Consider now the subcase when $1 - f = \rho$. The producer cannot invest with positive probability in any Nash equilibrium since, if she did, news veracity would exceed $\rho$, consumers would always invest, and the producer would have zero incentive to invest, a contradiction. Any Nash equilibrium must therefore have news veracity equal to $\rho$, making consumers indifferent whether to share after getting a bad private signal, i.e., each consumer $i$ must use a sharing rule of the form $z_i = (1, z_{iF})$ for some $z_{iF} \in [0, 1]$. Moreover, consumers must always share since, if any consumer(s) mixed after a bad signal with probability $z_{iF} < 1$, true stories would enjoy greater

\textsuperscript{25}Characterizing all Nash equilibria in the more general case when the reporter may also be paid for adoptions is substantially more complex and omitted.
visibility and the producer would sometimes prefer to invest, a contradiction.

Second, suppose that $1 - f \in (1 - \rho, \rho)$. In this case, all Nash equilibria exhibit positive investment. To see why, suppose that a Nash equilibrium exists in which the producer never invests. News veracity in this equilibrium equals $1 - f$, giving consumers an incentive to filter the news. This causes true stories to enjoy greater visibility, giving the producer an incentive to sometimes invest, a contradiction.

Finally, suppose that $1 - f \leq 1 - \rho$. In this case, a Nash equilibrium exists in which consumers never share and the producer never invests.

**Best-response news veracity in a general directed graph.** Let $\vec{z} = (z_i : i = 1, \ldots, M)$ denote consumers’ sharing-rule profile and let $N_i$ be the set of others that consumer $i$ follows. Each consumer $i$’s likelihood of viewing true and false stories is

\[
V_{IT}(\vec{z}) = 1 - (1 - b)\Pi_{j \in N_i}(1 - b(\rho z_{jT} + (1 - \rho)z_{jF}))
\]

\[
V_{IF}(\vec{z}) = 1 - (1 - b)\Pi_{j \in N_i}(1 - b((1 - \rho)z_{jT} + \rho z_{jF}))
\]

so that the extra visibility of true stories to consumer $i$ is

\[
\Delta V_i(\vec{z}) = (1 - b)\left(\Pi_{j \in N_i}(1 - b((1 - \rho)z_{jT} + \rho z_{jF})) - \Pi_{j \in N_i}(1 - b(\rho z_{jT} + (1 - \rho)z_{jF}))\right).
\]

The extra expected revenue associated with investing in story quality is now $f \sum_i \Delta V_i(\vec{z})$, inducing best-response news veracity $p_0(\vec{z}) = 1 - f + fH\left(f \frac{\sum_i \Delta V_i(\vec{z})}{M}\right)$.

**Nash equilibria with news veracity in the filtering region.** Suppose for a moment that a Nash equilibrium exists with positive investment and news veracity $p_0 \in (1 - \rho, \rho)$. Consumers strictly prefer to filter the news, using the sharing rule $\vec{z} = (1, 0)$ and resulting in “filtering news veracity” $\tilde{p}_0 = p_0(\vec{z}, \ldots, \vec{z})$. Thus, such a Nash equilibrium can only exist if $\tilde{p}_0 \in (1 - \rho, \rho)$; moreover, if that is the case, there is a unique Nash equilibrium with news veracity in the filtering region, in which all consumers filter the news and news veracity equals $\tilde{p}_0$.

**Nash equilibria with news veracity at the always-share threshold.** Suppose for a moment that a Nash equilibrium exists with positive investment and news veracity
$p_0 = \rho$. Consumers’ optimal sharing rules take the form $z_i = (1, z_{iF})$ where $z_{iF} \in [0, 1]$ for all $i$. Since $z_{iT} = 1$ for all $i$ and $\rho > 1/2$,

$$\frac{\partial \Delta V_i(\bar{z})}{\partial z_{jF}} = (1 - b)((1 - \rho)\Pi_{k \neq j \in N_i}(1 - b(\rho + (1 - \rho)z_{kF})) - \rho \Pi_{k \neq j \in N_i}(1 - b(1 - \rho + \rho z_{kF}))) < 0$$

for all $i$ and all neighbors $j \in N_i$. Thus, $\frac{\partial p_0(\bar{z})}{\partial z_{jF}} < 0$ for all $j \in N_i$. We conclude that news veracity in any such Nash equilibrium cannot exceed the filtering news veracity $\tilde{p}_0$ and, in particular, that no such Nash equilibrium exists if $\tilde{p}_0 < \rho$. Similarly, if $\tilde{p}_0 = \rho$, there is a unique equilibrium with news veracity $\rho$, in which all consumers use the filtering sharing rule $\tilde{z} = (1, 0)$. Finally, if $\tilde{p}_0 > \rho$, the fact that $\frac{\partial p_0(\bar{z})}{\partial z_{jF}} < 0$ and $p_0((1, 1), ..., (1, 1)) = 1 - f < \rho$ implies that there is a $(M - 1)$-dimensional set of Nash equilibria with news veracity equal to $\rho$, in each of which consumers use a profile of sharing rules that creates the same overall incentive to invest in story quality.

This $(M - 1)$-dimensional set of Nash equilibria includes a unique Nash equilibrium in which all consumers use the same sharing rule. In the main text, we characterized this unique symmetric equilibrium in the special case with all consumers follow $d$ others.

**Nash equilibria with news veracity at the never-share threshold.** Suppose for a moment that a Nash equilibrium exists with positive investment and news veracity $p_0 = 1 - \rho$. Consumers’ optimal sharing rules take the form $z_i = (z_{iT}, 0)$ where $z_{iT} \in [0, 1]$ for all $i$, resulting in extra visibility

$$\Delta V_i(\bar{z}) = (1 - b)((1 - (1 - \rho))\Pi_{j \in N_i}(1 - b(1 - \rho)z_{jT}) - \Pi_{j \in N_i}(1 - b(1 - \rho z_{jT})))$$

for true stories. This results in extra revenue $\Delta R(\bar{z}) = f \sum_i \Delta V_i(\bar{z})$ for true stories and hence best-response news veracity $p_0(\bar{z}) = 1 - f + fH(\Delta R(\bar{z}))$. Building on the notational shorthand used in the main text, let $\bar{p}_0 = \max_{\bar{z}: z_{iF} = 0 \text{ for all } i} p_0(\bar{z})$ denote the maximal equilibrium news veracity that can be supported when consumers use sharing rules of the form $z_i = (z_{iT}, 0)$. As can be easily checked, $p_0(\bar{z})$ is continuous in $z_{iT}$ for all $i$ and $p_0((0, 0), ..., (0, 0)) = 1 - f < 1 - \rho$. Thus, a Nash equilibrium with news veracity $1 - \rho$ exists iff $\bar{p}_0 \geq 1 - \rho$ and such an equilibrium exists with sharing-rule profile $\bar{z}$ iff $p_0(\bar{z}) = 1 - \rho$.

45
The set of solutions to $p_0((z_i T, 0) : i = 1, ..., M) = 1 - \rho$ is not easily characterized in the general case, because $p_0((z_i T, 0) : i = 1, ..., M)$ is non-monotone in $(z_i T : i = 1, ..., M)$. However, consider for a moment the special case examined in the main text: each consumer follows the same number $d$ of others and uses the same sharing rule $(z_T, 0)$. Viewed now as a function of $z_T$, the extra visibility of true stories takes the form $\Delta V(z_T) = (1 - b) \left( (1 - b(1 - \rho)z_T)^d - (1 - b\rho z_T)^d \right)$, with derivative $\Delta V'(z_T) \geq 0$ iff $z_T \geq \tilde{z}_T$, where $\tilde{z}_T$ was defined in the proof of Lemma 1(iv). Consequently, there may be up two symmetric Nash equilibria with news veracity equal to $1 - \rho$, one in which consumers share with likelihood greater than $\tilde{z}_T$ after a good signal and another in which they share less than $\tilde{z}_T$ after a good signal. (As we discuss later in Appendix B, of these two symmetric Nash equilibria, the one with more sharing is dynamically stable while the one with less sharing is dynamically unstable.)

B Dynamic stability

Thm 1 characterizes all SNE, but not all of these equilibria are dynamically stable with respect to small changes in consumers’ sharing rule. Let $p_0^*(z; \hat{z}) = p_0(z(1 - \epsilon) + \hat{z}\epsilon)$ be the best-response news veracity that would result if consumers randomized between using sharing rule $z$ and sharing rule $\hat{z}$, with weight $\epsilon \in (0, 1)$ on $\hat{z}$.

Definition 1 (Dynamic stability). A symmetric sharing rule $z$ is “dynamically stable” (or simply “stable”) if, for all $\hat{z}$ and all $\epsilon \approx 0$, $z$ is a strictly better response for consumers than $\hat{z}$ given news veracity $p_0^*(z; \hat{z})$. Similarly, $z$ is “dynamically unstable” (or simply “unstable”) if there exists $\hat{z}$ such that, for all $\epsilon \approx 0$, $\hat{z}$ is a strictly better response for consumers than $z$ given news veracity $p_0^*(z; \hat{z})$.

Lemma 2 (Dynamic stability). (i) Any SNE with news veracity $p_0^* \notin \{1 - \rho, \rho\}$ is dynamically stable. (ii) Any SNE with news veracity $p_0^* = \rho$ is dynamically stable. (iii)\footnote{Implicit in this definition is a simplifying assumption that producers adapt immediately to any change in consumers’ sharing strategies while consumers adapt gradually over time to changes in producers’ investment strategies. However, this is not essential. Our results hold under any monotone co-adaptation dynamics (Samuelson and Zhang (1982)); straightforward details omitted to save space.}
Any SNE with news veracity $p^*_0 = 1 - \rho$ in which consumers use sharing rule $(z^*_T, 0)$ for some $z^*_T > 0$ is dynamically unstable if $z^*_T \in [0, p_0]$ and dynamically stable if $z^*_T \in (p_0, 1]$.

Proof: Given a SNE with sharing rule $\mathbf{z}^*$, say that sharing rule $\check{z}$ can “successfully invade” if $\check{z}$ is a better reply than $\mathbf{z}^*$ given “perturbed news veracity” $p_0^*(\mathbf{z}; \check{z}) = p_0(\mathbf{z}(1 - \epsilon) + \check{z}\epsilon)$ for all $\epsilon \approx 0$. By definition, a SNE with sharing rule $\mathbf{z}^*$ is dynamically stable iff no other sharing rule $\check{z}$ can successfully invade.

**Part One:** $p^*_0 \not\in \{\rho, \rho\}$. Consumers have a unique best response $\mathbf{z}(p^*_0)$ equals $(1, 1)$ if $p^*_0 > \rho$, $(1, 0)$ if $p^*_0 > (1 - \rho, \rho)$, or $(0, 0)$ if $p^*_0 < 1 - \rho$. For any given $\check{z} \neq \mathbf{z}(p^*_0)$ and $\epsilon \approx 0$, $p_0^*(\mathbf{z}; \check{z}) \approx p_0^*$. (Note by equation (5) that $p_0(\mathbf{z})$ is continuous in $\mathbf{z}$, a fact we used repeatedly throughout the proof.) Thus, for all $\epsilon \approx 0$, $\mathbf{z}(p^*_0)$ continues to be consumers’ unique best response; in particular, $\mathbf{z}(p^*_0)$ is a better reply than $\check{z}$ and hence $\check{z}$ cannot successfully invade. This completes the proof of (i).

**Part Two:** $p^*_0 = \rho$. Consumers strictly prefer to share given signal $s_i = T$ and are indifferent whether to share given signal $s_i = F$; so, the equilibrium sharing rule must take the form $\mathbf{z}^* = (1, z^*_F)$ for some $z^*_F \in [0, 1]$. For any $\check{z} \neq (1, z^*_F)$, perturbed news veracity $p_0^*(\mathbf{z}^*; \check{z}) \approx \rho$, given which consumers still strictly prefer to share when $s_i = T$ and are approximately indifferent whether to share when $s_i = F$. The rest of the proof that $\check{z}$ cannot successfully invade has three steps. First, consider any $\check{z}$ with $\hat{z}_T < 1$. After getting signal $s_i = T$ (probability $\Pr(s_i = T|p_0 = \rho) > 0$), a consumer who shares with probability $\hat{z}_T$ loses approximately $(1 - \hat{z}_T)\pi s(2\rho - 1) > 0$ relative to the best response of always sharing. By contrast, after getting signal $s_i = F$, the benefit (if any) that a consumer gets by sharing with probability $\hat{z}_F$ rather than probability $z^*_F$ goes to zero as $\epsilon$ goes to zero. Overall, then, $\check{z}$ is a worse reply than $\mathbf{z}^*$ for all small enough $\epsilon$ and hence $\check{z}$ cannot successfully invade. Second, consider any $\check{z} = (1, \hat{z}_F)$ with $\hat{z}_F < z^*_F$, inducing perturbed news veracity $p_0^*(\mathbf{z}^*; \check{z}) = p_0(1, z^*_F - \epsilon(z^*_F - \hat{z}_F))$. Because $p_0 = (1, z_F)$ is strictly decreasing in $z_F$ (Lemma 1(ii)), $p_0^*(\mathbf{z}^*; \check{z}) > \rho$ and consumers have a strict incentive to share after signal $s_i = F$. Since $\hat{z}_F < z^*_F$, $\check{z}$ is therefore a worse reply than $\mathbf{z}^*$ and so cannot successfully invade. Third and finally, consider any $\check{z} = (1, \hat{z}_F)$ with $\hat{z}_F > z^*_F$, 47
inducing perturbed news veracity \( p'_0(z^*; \hat{z}) = p_0(1, z^*_F + \epsilon(\hat{z}_F - z^*_F)) \). Because \( p_0(1, z_F) \) is strictly decreasing in \( z_F \), \( p'_0(z^*; \hat{z}) < \rho \), giving consumers a strict incentive not to share after signal \( s_i = F \) and making \( \hat{z} \) a worse reply than \( z^* \) since \( \hat{z}_F > z^*_F \). We again conclude that \( \hat{z} \) cannot successfully invade. This completes the proof of (ii).

**Part Three:** \( p^*_0 = 1 - \rho \). Consumers are indifferent whether to share after getting a positive signal \( s_i = T \) and strictly prefer not to share after a negative signal \( s_i = F \); so, the equilibrium sharing rule must take the form \( z^* = (z^*_T, 0) \) for some \( z^*_T \in [0, 1] \). As in Part Two, one can easily show that any \( \hat{z} \) with \( \hat{z}_F > 0 \) cannot successfully invade; so, we will only consider sharing rules of the form \( \hat{z} = (\hat{z}_T, 0) \). Suppose first that \( z^*_T \leq \bar{z}_T \) and consider the perturbing sharing rule \( \hat{z} = (\hat{z}_T, 0) \). Suppose first that \( z^*_T \leq \bar{z}_T \) and consider the perturbing sharing rule \( \hat{z} = (\hat{z}_T, 0) \). By Lemma 1(iii), \( p_0(z_T, 0) \) is strictly increasing over the range \([0, \bar{z}_T]\); so, \( p_0(z^*_T - \epsilon z^*_T, 0) < p_0(z^*_T, 0) = p^*_0 = 1 - \rho \). Since consumers have a strict incentive not to share given private signal \( s_i = F \) after the perturbation, sharing rule \( \hat{z} = (0, 0) \) can successfully invade; so, the SNE in question is dynamically unstable. Suppose next that \( z^*_T > \bar{z}_T \) and consider any \( \hat{z} = (\hat{z}_T, 0) \). By Lemma 1(iii), \( p_0(z_T, 0) \) is strictly decreasing over the range \((\bar{z}_T, 1]\); so, \( p'_0(z^*; \hat{z}) > 1 - \rho \) whenever \( \hat{z}_T < z^*_T \) (making \( z^* \) a better reply than \( \hat{z} \)) and \( p'_0(z^*; \hat{z}) < 1 - \rho \) whenever \( \hat{z}_T > z^*_T \) (again making \( z^* \) a better reply than \( \hat{z} \)). We conclude in this case that the SNE in question is dynamically stable. This completes the proof of (iii).