Bounds on Price Setting

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Abstract

This paper has two parts. In the first part, I demonstrate that, in the absence of bounds on firms’ pricing decisions, monetary models do not have current equilibria - and so lack any predictive content - for a wide range of possible policy rules and/or beliefs about future equilibrium outcomes. This non-existence problem disappears once price-setting firms face (arbitrarily loose) finite upper bounds and positive lower bounds on their choices. In the second part, I study the properties of dynamic monetary models in which prices are fully flexible, except for the presence of these kinds of bounds. Among other results, I show that these models imply that the Phillips curve is L-shaped and are consistent with the existence of permanently inefficiently low output (secular stagnation). I show too that economies with lower price floors have even worse equilibrium outcomes in welfare terms. It follows that commonly used flexible-price or non-monetary models provide poor approximations to the implications of models with the (arbitrarily low) positive price floors needed to ensure existence.

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1 Introduction

This paper has two parts. In the first part of the paper, I analyze a canonical class of monetary models with monopolistically competitive firms. I demonstrate that these models have an important defect: they don’t have an equilibrium in a given period unless there are tight restrictions on the set of monetary policy rules and/or (rational beliefs about) future equilibrium outcomes. It follows that the models are wholly uninformative about what happens if governments and agents don’t obey those restrictions. I demonstrate that this non-existence result disappears when otherwise flexible firms face a finite upper bound and a positive lower bound on their price choices (regardless of how loose these bounds are).

I view the first part of the paper as showing that, because of intrinsic non-existence issues, models without ceilings and floors on price-setting firms are theoretically incomplete.\footnote{This argument is reminiscent of Jackson (1992)’s critique of the use of unbounded mechanisms in implementation theory. (I thank Asen Kochov for pointing out this connection to me.)} In the second part of the paper, I study the implications of a class of models in which prices are otherwise fully flexible except for these bounds. The basic message is that the bounds - needed for existence - mean that these apparently neoclassical models now have a rich mixture of neoclassical/Keynesian implications. More specifically, I prove the following results:\footnote{The models are finite horizon and have a set of equilibria that are indexed by final-period (possibly random) inflation. All of these results are conditional on a particular specification of final period inflation (which might be interpreted as “anchored” inflation expectations). In addition, I limit attention to interest rate rules in which the nominal interest rate converges to infinity as inflation converges to its (presumably high) upper bound. This restriction eliminates equilibria with a positive output gap.}

- Whenever there is a negative output gap (where the gap is defined relative to what happens in a non-monetary equilibrium), the inflation rate is equal to its lowest possible level. When the output gap is zero, the inflation rate varies. In this sense, the models predict an L-shaped Phillips curve that is horizontal when the output gap is negative and vertical when the output gap is zero.

- The models are consistent with a form of secular stagnation in the sense that, under weak conditions, there is a set of equilibria in which the output gap is permanently
negative.

- When the output gap is zero, the output multiplier on government purchases is less than one. When the output gap is negative, the output multiplier on government purchases is one.

- The neo-Fisherian logic doesn’t apply: higher nominal interest rules result in lower inflation.

- There is no “forward guidance puzzle”: if interest rate rules obey the Taylor Principle, the current impact of forward guidance about future interest rates declines exponentially with the horizon of the guidance.  

- Lowering the price floor makes inefficient equilibrium outcomes even worse in a welfare sense.

The theoretical argument in the first part of the paper justifies the imposition of some bounds on prices. Of course, the quantitative implications of a model with such bounds necessarily depend on their magnitudes. But, as described above, the nature of this dependence is somewhat counter-intuitive. Reducing the price floor lowers inflation expectations, raises real interest rates, and (for a given interest rate lower bound) drives down output. This logic means that a world with very low price floors has highly inefficient equilibrium outcomes (even relative to the inefficiency generated by the monopolistic distortion). Such a world is not well-modeled by frameworks in which prices are completely flexible or all trade is non-monetary.

It is important to emphasize that the basic distortion in this class of models is not created by price bounds but by monetary policy. Suppose that the real interest rate on money is kept persistently higher than the real interest rate in a non-monetary equilibrium. The resulting

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4The result resembles the “paradox of toil” described by Eggertsson (2010) and the “paradox of flexibility” described by Eggertsson and Krugman (2012).
low demand for consumption and labor implies that wages are so low that a firm can always gain by cutting its price (to expand production and hire more cheap workers). Without a price floor, this “firms can always gain” situation can’t be an equilibrium. Models without bounds (and their users) can only deal with this kind of policy specification by ruling it out as “impossible”. But, with a price floor (regardless of how low), such low real wages simply mean that firms cut their prices until they are constrained by the floor. In this fashion, the incentive for price-cutting created by poor monetary policy can be a persistent drag on economic activity.

The role of the bounds can be illustrated via this simple game. Consider a game with two players. First, player A chooses an an element $a$ from the set of integers. Next, player B chooses $b$ from the set $\{-1, 0, 1\}$. Both players’ payoffs are given by the product $ab$. If $B$ chooses $-1$ or $1$, player $A$’s decision problem has no solution. Hence, the only pure strategy equilibrium in this game is for both players to choose 0. But this equilibrium has to be viewed as strange: both players are using a weakly dominated strategy.

The non-existence problem here is generated by the non-compactness of player A’s action set. Suppose instead that player A can choose from the integers that are less than or equal to $M$ in absolute value. Then, there are three pure strategy equilibria in this game: $\{(-M, -1), (0, 0), (M, 1)\}$. As noted before, 0 is a weakly dominated strategy, and so the extremal equilibria, based on the bounds, are the natural ones. Note that these extremal equilibria diverge from the original $(0, 0)$ equilibrium as the bounds are made looser: the game without bounds is not a useful way to capture the implications of the game with bounds.

Macroeconomists typically use models in which monetary policy can exert at most a temporary drag on real economic outcomes. But, as argued above, these models are theoretically incomplete in the absence of ceilings and floors on the price-setting firms. A key lesson of this paper is that, once we add the bounds needed to ensure existence, monetary policy can have a material effect on the models’ implications even over the longer run and even when the bounds are loose. Throughout the paper, I’m agnostic about the source of the price
bounds. But it seems clear that, at any point in time, businesses face non-statutory bounds on their price-setting decisions. The important empirical issue with bounds on price-setting is not whether they exist, but rather if and when they bind.

I defer a fuller discussion of the related literature until Section 5. But I close the introduction with a final methodological comment. Throughout the paper, I use finite horizon models rather than infinite horizon model. I do so because of the following discontinuity associated with infinite horizon models. Suppose we know that consumption is bounded from above in any period by $Y_{\text{max}}$. In any finite horizon model (regardless of how long the horizon is), there is a (huge!) set of positive stochastic consumption processes that are bounded from above by $Y_{\text{max}}$ and yet grow at an average rate $g > 0$. In any infinite horizon model, the set of such consumption processes is empty. This observation matters in my analysis because equilibria in which the real interest rate is constant but too high are also equilibria in which growth is constant but too high. These are equilibria which feature “secular stagnation” in the sense that consumption/output is always too low. Such equilibria are potentially feasible in economies with arbitrarily long finite horizons, whereas they are physically impossible in infinite horizon economies.

2 Why Monetary Models Need Pricing Bounds

In this section, I illustrate, through a two-period example, why monetary models with monopolistic competition need to include bounds on prices. I first consider a (standard) flexible price model without bounds. In this model, the government imposes a lump-sum tax in period 2 equal to the average amount of money outstanding. As a result, essentially any price level is an equilibrium in period 2. However, the anticipation of many (possibly almost all) of these period 2 equilibria leads to non-existence of equilibrium in period 1. Put another way, we have to impose an otherwise artificial restriction on the set of period 2 equilibria to ensure that we get existence of equilibrium in period 1.
I should note that there is a key difference between my approach to the definition of equilibrium and the conventional one. I proceed recursively: I define an equilibrium in period 2 and then, conditional on any period 2 equilibrium, define an equilibrium in period 1. The more typical approach is to treat period 1 equilibrium and period 2 equilibrium as (somehow) being determined simultaneously. This approach automatically restricts the set of equilibrium outcomes in period 2 to be those that are consistent with the existence of equilibrium in period 1. But this kind of restriction seems hugely problematic - how exactly is it supposed to be implemented once we get to period 2?

My baseline analysis is for a model with flexible prices. However, I show that the non-existence problem carries over to settings in which almost all prices are fixed. It also generalizes to settings in which markets for risk sharing are incomplete, fiscal policy is being used for price level determination, some money is held only for its liquidity services, and agents are able to store resources.

I then add an upper and lower bound to constrain the firms’ choices of prices. Given these restrictions, I demonstrate there is an equilibrium in period 1 for any period 2 equilibrium.

2.1 Two Period Example Setup

There are two periods and a unit measure of agents who all live for two periods. There is a unit measure of goods in period 1 and a single good in period 2. The agents maximize the expectation of a cardinal utility function of the form:

\[ u\left(\int c_1(j)^{1-1/\eta}dj \right)^{\frac{\eta}{\eta-1}} - v(N_1) + u(C_2) \]
Here, $c_1(j)$ is consumption of good $j$ in period 1, $C_2$ is consumption of the single good in period 2, and $N_1$ is labor in period 1. The utility function $u$ satisfies typical restrictions:

\[
\begin{align*}
& u', -u'' > 0 \\
& \lim_{c \to 0} u'(c) = \infty \\
& \lim_{c \to \infty} u'(c) = 0
\end{align*}
\]

As is conventional, I restrict $\eta > 1$ (in order to ensure that monopolistically competitive firms have finite solutions to their maximization problems) and $\chi > 0$.

In period 2, the agents are each endowed with $Y$ units of consumption.

In period 1, each good is produced by a monopolistically competitive firm. Each firm has identical constant returns to scale technologies that transform a measure $n$ units of time in period 1, $n \geq 0$, into $n$ units of consumption goods. The agents have equal ownership of all firms. New firms are not allowed to enter and (perhaps less intuitively) existing firms are not allowed to exit.

### 2.2 Money

I now add money to this model. I treat money as an interest-bearing asset (akin to the interest-bearing reserves that banks hold with the Federal Reserve). Each person is endowed with $M$ dollars of money in period 1. The government commits to an interest rate rule: in period 2, money pays a gross nominal interest rate $R(P_1)$, where $R$ is a continuous function of the period 1 price level $P_1$. In terms of fiscal policy, all agents are required to pay a lump-sum tax of $MR(P_1)$ dollars in period 2.

In period 2, households trade money and goods in a competitive market. Given a period
2 price level $P_2$, the generic household’s problem is:

$$\max_{c_2, M_2} u(c_2)$$

$$s.t. P_2 c_2 + M_2 \leq P_2 Y + M_1' R(P_1^*)$$

$$M_2 \geq MR(P_1^*)$$

where $P_1^*$ is the period 1 price level. (Here, I’m allowing a (measure zero) of households to hold $M_1'$ that differs at least slightly from $M$. However, the average $M_1'$ is equal to the initial per-capita money-holdings $M$.) The last constraint is necessary to ensure that the household has enough money at the end of the period to pay its taxes.

It is straightforward to show that, for any $P_2$, it’s optimal for households to set $M_2 = MR(P_1^*)$ and to set:

$$c_2 = Y + M_1' R(P_1)/P_2 - MR(P_1)/P_2$$

Given these choices, markets clear, because the average of $M_1'$ across households equals the supply of money $M$. It follows that any positive real $P_2$ is an equilibrium.

Now, we move back in time to period 1. It is straightforward to show that all firms set the same price $P_1$, so that they earn a monopoly mark-up over the equilibrium wage:

$$P_1 = W_1(1 - 1/\eta)^{-1}$$

They also hire the same amount of labor $N_1$, which satisfies the households’ marginal labor supply condition:

$$u'(N_1)W = v'(N_1)P_1$$

(Here, we exploit the fact that, in equilibrium, all households consume $N_1$ units of every good.) It follows that in equilibrium $N_1^* = N^{mono}$, where:

$$(1 - 1/\eta) = v'(N^{mono})/u'(N^{mono})$$
This pins down the real equilibrium in period 1.

What about the nominal variables in period 1? Suppose the households rationally expect that the equilibrium period 2 price level $P^*_2$ will equal the equilibrium period 1 price level $P^*_1$ multiplied by (an endogenously determined variable) $\Pi^*$. Given these expectations, they trade in period 1. The households’ money-consumption problem in period 1 is then:

$$
\max_{(c_1, c_2, M_1')} u(c_1) + u(c_2)
\text{s.t.}
P^*_1 c_1 + M_1' = W^* N^{mono} + M + \Phi^*
\Pi^* P^*_1 c_2 = M_1' R(P^*_1) - MR(P_1) + \Pi^* P^*_1 Y
C_1, C_2, , M_1' \geq 0
$$

where $P^*_1$ is the period 1 price level, $W^*$ is the period 1 wage (in terms of dollars), and $\Phi^*$ are the firms’ nominal monopoly profits. The households’ Euler equation is:

$$
u'(C^*_1) = R(P^*_1) u'(C^*_2) / \Pi^*
$$

Putting together the real side and the nominal side, we can conclude that:

**Proposition 1.** Given a monetary policy rule $R$ and an anticipated period 2 gross inflation rate $\Pi^*$, there exists a period 1 monetary equilibrium if and only if there exists some $P^*_1$ such that:

$$
u'(N^{mono}) = \frac{R(P^*_1)}{\Pi^*} u'(Y)
$$

where $N^{mono}$ is defined so that:

$$
(1 - 1/\eta) = u'(N^{mono}) / u'(N^{mono})
$$

Proposition 1 shows that the real interest rate is pinned down in a monetary equilibrium.
by the households’ marginal willingness to hold money in a monetary equilibrium. That real interest rate in turn determines the price level in period 1.

However, Proposition 1 implies that there is an existence problem in period 1. Suppose $R_{min} \leq R(P_1) \leq R_{max}$ for all $P_1$. Consider any $\Pi^*$ such that:

$$u'(N^{mono})\Pi^*/u'(Y) < R_{min}$$

or such that:

$$u'(N^{mono})\Pi^*/u'(Y) > R_{max} \quad (1)$$

Here, as before:

$$(1 - 1/\eta) = u'(N^{mono})/u'(N^{mono})$$

Then, for this combination of interest rate rule and inflation expectations, there is no period 1 equilibrium.

In this way, the criterion of existence in period 1 imposes a constraint on the interest rate rule and/or what agents believe will happen in period 2. But this seems highly problematic. Why should we presume that the government will be informed enough or benevolent enough to choose an interest rate rule that is consistent with existence of equilibrium? Or that people in period 1 will necessarily believe that people in the future will value money in a way that is consistent with the existence of equilibrium today?

2.3 Robustness of the Non-Existence Result

In this subsection, I consider the robustness of the above non-existence result to five perturbations of the basic model: sticky prices, incomplete markets for risk-sharing, other formulations of fiscal policy, the addition of non-interest-bearing currency with liquidity services, and the addition of a physical storage technology.
Sticky Prices

In the above model, all firms can adjust prices freely, and so equilibrium period 1 output is independent of the period 1 price level. We can generalize the non-existence result to allow for arbitrary amounts of price stickiness. Thus, suppose that a fraction $\phi$ of firms have prices fixed at $\bar{P}$ in period 1, while the other $(1 - \phi)$ firms are allowed to adjust their prices freely. The price rigidity creates a relative price distortion between the two kinds of firms, and so equilibrium consumption in period 1 is a function of the period 1 price level. However, there is a limit to the damage that this distortion can do: the worst that can happen is that the fixed price firms produce nothing. This means that we know that equilibrium consumption in period 1 can be no lower than $C_{LB}$, where $C_{LB}$ solves:

$$u'(C_{LB})(1 - \phi)^{\frac{1}{\eta}}(1 - 1/\eta) = v'((1 - \phi)^{\frac{1}{\eta}}C_{LB})$$

Here, $C_{LB}$ is the level of consumption that would be produced if the period 1 price level were close to zero, so that nobody bought goods at the fixed price firms. So, even when almost all firms are unable to change their prices ($\phi$ near 1), we have a non-existence problem: there is no equilibrium for any interest rate rule $R$ and any period 2 inflation $\Pi^*$ such that:

$$u'(C_{LB}) < kR(P_1)/\Pi^*$$

for all $P_1$.

Incomplete Markets

We can generalize the non-existence result to allow for incomplete markets. Thus, suppose that a given agent $i$ has random period 2 endowment $Y_i$, where $Y_i$ is i.i.d. across agents with mean $Y$, and that agents are unable to trade any other assets besides interest-bearing money. It is readily shown that, in this economy, there is no equilibrium for any $\Pi^*$ and interest rate
rule $R$ such that:

$$u'(N_{mono}) \neq kR(P_1) \frac{E(u'(Y_i))}{\Pi^*}$$

for all $P_1$.

**Fiscal Policy**

In the model in section 2.2, the government levies a period 2 purely nominal lump-sum tax equal to the average money-holdings in the economy. This fiscal policy makes the period 2 price level indeterminate. Suppose instead that the government * pegs the price of money in period 2 equal to $\Pi_2 P_1$, where $P_1$ is the endogenous period 1 price level and $\Pi_2$ is an exogenous gross inflation rate. This peg could be accomplished in a number of ways, including a restriction that specifies period 2 taxes to be $M(1 + R(P_1))/\Pi_2 P_1$ units of consumption.\(^5\) Such a peg would eliminate the indeterminacy in period 2 and restrict (rational) period 1 beliefs about period 2 inflation to be concentrated on $\Pi_2$.

Under these kinds of policies, there is still a non-existence problem but its source is tied to period 2 policy choices, rather than period 2 equilibrium selection. In particular, suppose that the government specifies $\Pi_2$ and $R(P_1)$ so that:

$$u'(N_{mono}) < kR(P_1)/\Pi_2$$

for all $P_1$. Then, there is no period 1 equilibrium and the model is uninformative about the implications of this (generic) set of policy choices.

**Currency**

The discussion in section 2.2 treats all money as interest-bearing. Of course, in reality, some money (currency) is non-interest-bearing and is held because it provides liquidity services over interest-bearing reserves. We can generalize the above non-existence problem to a model

\(^5\)This kind of fiscal policy, in which the government’s intertemporal budget constraint is not satisfied for all price level processes, is typically termed non-Ricardian (Woodford (1995)).
in which agents can swap (utility-generating) currency in exchange for (interest-bearing) reserves with the government in the first period.

In particular, suppose that agents have preferences of the form:

\[ u(C_1) + kC_2 - v(N_1) + h(X'_1/P_2) \]

where \( X'_1 \) represents the agent’s (non-interest-bearing) currency-holdings at the end of period 1. Here, \( h \) is an increasing and concave function. Suppose too that agents can trade currency and reserves in period 1 with the government, so that their budget constraint in period 1 looks like:

\[ P_1c_1 + M'_1 + X'_1 \leq W_1n_1 + M \]

and money market-clearing in period 1 is given by:

\[ M'_1 + X'_1 = M \]

As before, the agents have to pay a lump-sum tax equal to average money-holdings at the end of period 2.

Suppose that the interest rate rule satisfies:

\[ R_{min} \leq R(P_1) \leq R_{max} \]

for all \( P_1 \). Then, we can generalize the above non-existence result to show that there is no period 1 equilibrium for any period 2 inflation rate \( \Pi^* \) such that:

\[ \Pi^*u'(N^{mono}) > max(kR_{max}, k + h'(0)) \]  \[ \Pi^*u'(N^{mono}) < kR_{min} \]  \[ (2) \]

The restriction (2) is a generalization of (1) that takes into account the new possibility
that the agents can give their reserves to the government in exchange for an equivalent dollar amount of currency. In such an equilibrium, agents set their currency-holdings \( X'_1 = M \) and their reserve-holdings \( M'_1 \) equal to 0. The nominal interest rate is \( (1 + h'(\frac{M}{P_1\Pi^*})/k) > R(P_1) \). But, in this case, the nominal interest rate can be no larger than:

\[
(1 + h'(0)/k)
\]

Note that the restriction (2) only has bite if \( h'(0) < \infty \), so that there is a limit to the willingness of agents to substitute between currency and consumption.

**Storage**

We can generalize the non-existence result to allow for the presence of a technology that transforms current consumption into future consumption. In particular, suppose agents can store \( x \) units of consumption in period 1 to generate \( (1 + \phi)x \) units of consumption in period 2, for any \( x \geq 0 \). I suppose that there exists \( S^{\text{mono}} > 0 \) such that:

\[
\phi'(N^{\text{mono}} - S^{\text{mono}}) = (1 + \phi)k
\]

so that storage is positive in a non-monetary equilibrium allocation.

Let \( R \) be an interest rate rule that satisfies:

\[
R_{\text{min}} \leq R(P_1) \leq R_{\text{max}}
\]

for all \( P_1 \). We can generalize the non-existence result in the presence of storage by noting that there is no equilibrium for any period 2 inflation rate \( \Pi^* \) such that:

\[
(1 + \phi)\Pi^* > R_{\text{max}}
\]
or:

\[(1 + \phi)\Pi^* < R_{\text{min}}\]

The presence of storage guarantees that the real interest rate must equal \((1 + \phi)\) in an equilibrium allocation. But this real interest rate could well be inconsistent with agents’ beliefs about the real return to money.

### 2.4 Price Bounds

In this subsection, I discuss why we obtained the earlier non-existence result. As I did in the introduction, I argue that it is attributable to the non-compactness of the firms’ action sets. I describe and characterize bounded equilibria.

I return to the baseline model (from Sections 2.1-2.2). As before, define \(N^{\text{mono}}\) to be the level of output/labor/consumption in a purely real monopolistically competitive economy:

\[u'(N^{\text{mono}})(1 - 1/\eta) = v'(N^{\text{mono}})\]

First, suppose that the period 2 gross inflation rate \(\Pi^*\) is such that:

\[u'(N^{\text{mono}}) < \frac{R(P_1)}{\Pi^*} u'(Y)\]  \(\text{(3)}\)

for all values of \(P_1\). Consider a putative equilibrium in the price-setting game in which all firms set their prices equal to \(P_1\). That price results in their producing \(C_1^d(P_1)\), where:

\[u'(C_1^d(P_1)) = \frac{R(P_1) u'(Y)}{\Pi^*}\]

In order to hire that many workers, the nominal wage must equal:

\[W(P_1) = P_1 u'(C_1^d(P_1))/u'(C_1^d(P_1))\]
We know from (3) that:
\[ C^d(P_1) < N^{\text{mono}} \]

and that:
\[ W(P_1)/P_1 < (1 - 1/\eta) \]

Now let’s calculate the typical firm’s marginal profit (with respect to its price choice) at that putative equilibrium price \( P_1 \). It’s proportional to:

\[
P_1^{-\eta}(1 - \eta) + \eta W(P_1) P_1^{-\eta - 1}
\]

\[
= P_1^{-\eta}(1 - \eta) + \eta(W(P_1)/P_1) P_1^{-\eta} \]

\[
< P_1^{-\eta}[(1 - \eta) + \eta(1 - 1/\eta)]
\]

\[
= 0
\]

When (3) is satisfied, at any putative equilibrium price \( P_1 \), the typical firm would always gain by cutting its price still further.

Similarly, suppose that \( \Pi^* \) is such that:
\[ u'(N^{\text{mono}}) > \frac{R(P_1)}{\Pi^*} u'(Y) \]

for all \( P_1 \). Then, for any putative equilibrium price level, a typical firm would be able to increase its profits by raising its price further.

These calculations imply that the non-existence of period 1 equilibrium, conditional on beliefs about what will happen in period 2, is attributable to the non-compact nature of the firms’ action sets. When interest rates are “too high” and demand is “too low”, a firm can always gain by cutting its price further. When interest rates are “too low” and demand is “too high”, a firm can always gain by raising its price further.

The non-existence issue is similar to what occurs in a game between two players who are asked to simultaneously name two natural numbers, with a prize being awarded to the player
who names the higher number. There is, of course, no equilibrium to this game because a
player can always increase her chance of winning the prize by choosing a (possibly mixed)
strategy that stochastically dominates her initial one. In contrast, suppose we compactify
the players’ action sets from above by requiring their choices to be less than or equal to $B$.
Then, the unique equilibrium is one in which both players choose $B$ and split the prize.

This analogy suggests that we can resolve the non-existence problem by adding constraints
to the firms’ action sets. With that in mind, I now impose an upper bound $P^{UB}$ and a lower
bound $P^{LB}$ on the firms’ price choices. I term such equilibria bounded equilibria.

**Proposition 2.** Given a monetary policy rule $R$, and a period 2 gross inflation rate $\Pi^*$, an
outcome $(C_1^*, N_1^*, W_1^*, P_1^*)$ is part of a bounded equilibrium outcome if and only if:

$$ u'(C_1^*) = \frac{R(P_1^*)u'(Y)}{\Pi^*} $$

$$ C_1^* = N_1^* $$

$$ W_1^* = P_1^*u'(N_1^*)/u'(N_1^*) $$

and one of the three following sets of conditions are satisfied:

1. $P_1^* = P^{LB}$ and $N_1^* \leq N^{mono}$
2. $P_1^* = P^{UB}$ and $N_1^* \geq N^{mono}$
3. $P^{LB} \leq P_1^* \leq P^{UB}$ and $N_1^* = N^{mono}$

**Proof.** I first show that these cases are, in fact, equilibria. Note first that the households’
Euler equations and labor supply conditions are satisfied. So, we need only check firm
optimization.

In case 1: Since $N_1^* \leq N^{mono}$, $P_1^* \geq W_1^*(1 - 1/\eta)^{-1}$. The typical firm’s optimization
problem is:

$$ \max_P(P/P_1^*)^{-\eta}P - W_1^*(P/P_1^*)^{-\eta} $$
When $P > P_1^*$, the derivative of the profit function with respect to $P$ is:

$$(1 - \eta)P^{-\eta} + \eta W_1^* P^{-\eta-1}$$

$$= P^{-\eta}[(1 - \eta) + \eta(W_1^*/P_1^*)(P_1^*/P)]$$

$$< P^{-\eta}[(1 - \eta) + \eta(W_1^*/P_1^*)]$$

$$\leq 0$$

where the last step follows from $(W_1^*/P_1^*)$ being less than $(\eta - 1)/\eta$. The optimal choice of $P$ is $P_1^* = P^{LB}$.

In case 2: We can use the same logic as above to prove that the optimal choice of $P$ is $P^{UB}$.

In case 3: All firms are making monopoly profits, and so this is an equilibrium.

Are there other equilibria? It is straightforward to show that the first three restrictions (Euler equation, market-clearing, and household labor supply) have to be satisfied in any equilibrium. If $N_1^* < N_{mono}$, then the real wage is lower than $(1 - 1/\eta)$, and the above argument implies that it is optimal for firms to set their price as low as possible. If $N_1^* > N_{mono}$, the above argument implies that it is optimal for firms to set their price as high as possible.

Proposition 2 describes three kinds of bounded equilibria. In all of them, while they can substitute between consumption and interest-bearing money, the households end up spending their wage income and firm profits in period 1 to buy period 1 goods. Money plays no substantive role in the economy: households simply hold their initial money-holdings $M$ into period 2 and then use that money to pay their taxes.

In the first kind of equilibria, output is even lower than in a purely real monopolistically competitive equilibrium. Because households consume $C_1^* < N_{mono}$ and work $N_1^* < N_{mono}$. Given how low real wages are, the firms would like to cut their prices still further, but can’t. They end up making super-monopoly profits. In the second kind of equilibria (case 2),
households produce and consume more than the monopolistic level $N_{mono}$. Given how high real wages are, firms would gain by raising their prices still further but cannot. They make sub-monopoly profits. Indeed, it is possible that they make negative profits (because exit is barred). The final kind of equilibria (case 3) correspond to the purely real monopolistically competitive equilibria.

### 2.5 Existence of Bounded Equilibria

In this subsection, I prove that for any (continuous) interest rate rule $R$, any period 2 gross inflation rate $\Pi^*$, for any price bounds $(P^{UB}, P^{LB})$, there exists a bounded equilibrium.

**Proposition 3.** For any continuous interest rate rule $R$, any period 2 gross inflation rate $\Pi^*$, price upper bound $P^{UB}$, and price lower bound $P^{LB}$, there exists a bounded equilibrium.

**Proof.** If there isn’t an equilibrium of the case 1 form, then:

$$u'(N_{mono}) \geq \frac{R(P^{LB})}{\Pi^*} u'(Y)$$

If there isn’t an equilibrium of the case 2 form, then:

$$u'(N_{mono}) \leq \frac{R(P^{UB})}{\Pi^*} u'(Y)$$

Since $R$ is continuous, these two inequalities imply via the intermediate value theorem that there is some $P^*_1$ in $[P^{LB}, P^{UB}]$ such that:

$$u'(N_{mono}) = \frac{R(P^*_1)}{\Pi^*} u'(Y)$$

so that there exists some equilibrium of the case 3 form. \[\square\]

There is a bounded equilibrium for all (continuous) interest rate rules and all $\Pi^*$. Note that the proof of existence is valid regardless of how large $P^{UB}$ is or how small $P^{LB}$ is. It
is straightforward to show that the existence proof generalizes to the various model environments considered in Section 2.3.

2.6 Summary

In this section, I illustrated a problem with the standard concept of monopolistically competitive equilibrium with flexible prices: to obtain existence in a given period, we need to restrict the set of future equilibrium outcomes. I showed how to fix this problem by imposing bounds on firm price-setting.

Bounded equilibrium outcomes may be even less efficient than is implied by the monopolistic distortion. The extra inefficiency is created by monetary policy that makes the real return on money overly high, and so leads households to consume too little. Within this equilibrium, households would like to work and consume more. However, a given household can only trade its labor for consumption via firms that own the means of production. Those firms can’t profitably expand their scale of operation because they can’t cut prices.

3 Dynamic Equilibrium

In this section, I extend the above definition of equilibrium to a finite horizon economy. (Henceforth, I use the short-hand term “equilibrium” to refer to a bounded equilibrium.)

3.1 Description of the Economy

Consider a \((T + 1)\) period stochastic version of the above economy, where \(T\) is finite but arbitrarily large. There is a unit measure of goods, indexed by \(j\). Agents have expected utility, with a separable utility function over consumption and labor processes:

\[
E_0\left[\sum_{s=1}^{T} \beta^s [u(C_s; \lambda_s) - v(N_s)] + \beta^T u(C_{T+1}; \lambda_{T+1})\right]
\]
where \( \{\lambda_s\}_{s=1}^{T+1} \) is a stochastic process. Here, consumption and labor are defined as:

\[
C_t = \left( \int c_{jt}^{1-1/\eta} d_j \right)^{\eta}, \eta > 1
\]

\[
N_t = \int n_{jt} d_j
\]

where \( c_{jt} \) is consumption of good \( j \) in period \( t \) and \( n_{jt} \) is labor used in the production of good \( j \) in period \( t \).

In terms of monetary policy, money is again an interest-bearing asset. The one-period nominal interest rate from period \( (t-1) \) to period \( t \) is given by \( R(\pi_t; \varepsilon_t) \), where \( \pi_t \) is the gross inflation rate from period \( (t-1) \) to period \( t \) and \( (\varepsilon_s)_{s=1}^{T} \) is an exogenous stochastic process of monetary policy shocks. (Without loss of generality, I fix \( P_0 = 1 \)). I assume that the interest rate rule \( R \) is strictly increasing and continuous in its first argument. In the final period, the government levies a lump-sum tax equal to the per-capita money supply on each agent.\(^6\)

Each good \( j \) is produced by a firm in each period \( t = 1, ..., T \). Firm \( j \) produces \( \psi_t n \) units of good \( j \) in period \( t \) using \( n \) units of labor, where \( \{\psi_t\}_{t=1}^{T} \) is a stochastic process. They engage in monopolistic competition at each date by choosing prices. Their price choice in period \( t \) is constrained lie in the set \( [\pi^{LB}_t P_{t-1}, \pi^{UB}_t P_{t-1}] \), where \( P_{t-1} \) is last period’s price level.

### 3.2 Definition of Equilibrium

As in the 2-period economy, the gross inflation rate \( \pi_{T+1} \) in the final period \( (T+1) \) is indeterminate. So, the set of equilibria should be seen as being indexed by the (random) inflation rate \( \pi_{T+1} \). At each date \( t = 1, ..., T \), we can define the unconstrained monopolistically competitive equilibrium level of labor \( N^{mono}(\psi_t; \lambda_t) \) to satisfy:

\[
u'(\psi_t N^{mono}; \lambda_t)\psi_t (1 - 1/\eta) = u'(N^{mono})
\]

\(^6\)More generally, I could allow for taxes in all periods. The results rely on the assumption that the government’s final period lump-sum tax is equal to the per-capita residual money supply.
This level of labor is determined only by the marginal utility shock and the productivity shock. It’s (of course) inefficiently low.

Given the exogenous processes \( \{ \varepsilon_s, \lambda_s, \psi_s \}_{s=1}^T \) and the exogenous period \((T + 1)\) shocks \((\pi_{T+1}, Y_{T+1})\), an equilibrium is a joint consumption-labor-inflation process \((C_t, N_t, \pi_t)_{t=1}^T\) that satisfies three sets of restrictions. (All firms hire the same amount of labor and produce the same amount of consumption.) The first set of restrictions link consumption and labor:

\[
C_t = \psi_t N_t, t = 1, ..., T
\]

The second set is the Euler equation for money:

\[
u'(C_t; \lambda_t) = \beta R(\pi_t) E_t \left( \frac{u'(C_{t+1}; \lambda_{t+1})}{\pi_{t+1}} \right), t = 1, ..., T \quad (4)
\]

The final set of restrictions describes how inflation is determined:

\[
P_t = \min(\max(\pi^{LB} P_{t-1}, (1 - 1/\eta)^{-1} W_t \psi_t), \pi^{UB} P_{t-1}), t = 1, ..., T \quad (5)
\]

\[
W_t = P_t v'(N_t)/u'(C_t; \lambda_t) \quad (6)
\]

\[
\pi_t = P_t / P_{t-1} \quad (7)
\]

Without loss of generality, we set \( P_0 = 1 \).

### 3.3 Constructing Equilibrium

It is straightforward to apply backward induction to the two sets of restrictions (4) and (5) to construct the set of equilibria. Fix an arbitrary random period \((T + 1)\) gross inflation rate \(\pi_{T+1}\) and set \(c_{T+1} = Y_{T+1}\). Then, we can use the period \(T\) restrictions (4) to solve for period
Let:

\[
m^H_T = \beta R(\pi^{UB}; \varepsilon_T) E_T \left( \frac{u'(Y_{T+1}; \lambda_{T+1})}{\pi_{T+1}} \right)
\]

\[
m^L_T = \beta R(\pi^{LB}; \varepsilon_T) E_T \left( \frac{u'(Y_{T+1}; \lambda_{T+1})}{\pi_{T+1}} \right)
\]

and then solve for \( C_T \) as:

\[
u'(C_T; \lambda_T) = \min(m^H_T, \max(u'(\psi_T N_{\text{mono}}(\psi_T, \lambda_T); \lambda_T), m^L_T))
\]

We can use the period \( T \) restriction (5) to solve for inflation \( \pi_T \):

\[
u'(C_T; \lambda_T) = \beta R(\pi_T; \varepsilon_T) E_T \frac{u'(Y_{T+1}; \lambda_{T+1})}{\pi_{T+1}}
\]

There is a unique solution for \( \pi_T \) because the interest rate rule \( R \) is strictly increasing and continuous. Finally, labor is simply given by:

\[
N_T = C_T / \psi_T
\]

We can then continue using backward induction to construct the full equilibrium \((C_t, N_t, \pi_t)_{t=1}^T\). In this way, given any (random) terminal inflation \( \pi_{T+1} \), there is a unique equilibrium \((C, N, \pi)\).

Note that in any equilibrium of this form, the (identical) households choose never to trade money and goods. As in the two-period model, money is simply a store of value used to pay their taxes in the final period. Nonetheless, the opportunity to hold interest-bearing money can in fact permanently distort the allocation of resources.
3.4 The Role of the Price Bounds

What if there were no bounds on prices? In that case, the equilibrium would necessarily be equal to the (real) monopolistically competitive outcome and inflation would satisfy the restrictions:

$$\pi_t = R^{-1}(\frac{\beta^{-1} u'(\psi_t N^{\text{mono}}(\psi_t, \lambda_t); \lambda_t)}{E_t(\frac{u'(\psi_{t+1} N^{\text{mono}}(\psi_{t+1}, \lambda_{t+1}); \lambda_{t+1})}{\pi_{t+1}}); \varepsilon_t}), t = 1, \ldots, (T - 1)$$

But these restrictions imply that too that the terminal inflation rate \(\pi_{T+1}\) must be such that for all \(t = 1, \ldots, T\):

$$\frac{\beta^{-1} u'(\psi_t N^{\text{mono}}(\psi_t, \lambda_t); \lambda_t)}{E_t(\frac{u'(\psi_{t+1} N^{\text{mono}}(\psi_{t+1}, \lambda_{t+1}); \lambda_{t+1})}{\pi_{t+1}})}$$

lies in the range of the interest rate rule \(R\). This (potentially complex) restriction on future equilibria to ensure period \(t\) existence is exactly what I criticized in the two-period example.

4 Results

Much work in macroeconomics uses models in which prices are fully flexible. But I view the analysis in Section 2 as showing that, because of their intrinsic non-existence problems, models with fully flexible prices are not theoretically coherent. With that conclusion in mind, I next turn to addressing the question of, “What are the predictions of models in which prices are fully flexible except for the extremal bounds that are necessary to ensure existence?” I answer this question by describing some of the key properties of the bounded equilibria of the dynamic models in Section 3.

In developing these results, I’m primarily interested in low-inflation equilibrium outcomes. With this in mind, I restrict attention to monetary policy rules such that:

$$\lim_{\pi \rightarrow \pi_B} R(\pi_U^B; \varepsilon) = \infty$$  \hspace{1cm} (8)

for all \(\varepsilon\). Under these rules, the central bank is raising the nominal interest rate aggressively.
in response to high levels of inflation. The upper bound on inflation never binds and there are no equilibria with positive output gaps (that is, in which output exceeds the monopolistically competitive level).

4.1 Secular Stagnation

In this section, I show that under weak uniform boundedness conditions, there are equilibria in which output is permanently lower than would be efficient. In my view, this kind of outcome corresponds to what Summers (2013) terms “secular stagnation”. The key to these equilibria is that money is viewed as a highly valuable asset because long-run inflation \( \pi_{T+1} \) is expected to be low.

Proposition 4. Suppose that the interest rate rule \( R \) satisfies the condition:

\[
R(\pi^{LB}; \varepsilon) \geq R_{LB} > 0
\]

for all \( \varepsilon \). Suppose too that for some constants \( L_Y \) and \( L_N \)

\[
u'(Y_{T+1}; \lambda_{T+1}) \geq L_Y > 0 \text{ w.p. 1}
\]

For \( t = 1, \ldots, T \),

\[
u'(\psi_t N^{mono}(\psi_t, \lambda_t); \lambda_t) < L_N \text{ w.p. 1}
\]

Then there is a set of equilibria (with sufficiently low inflation \( \pi_{T+1} \)) in which consumption \( c_t < N^{mono}(\psi_t, \lambda_t) \) with probability one for all \( t \leq T \).

Proof. In Appendix.

The following example is a simple illustration of Proposition 4. The example is based on the premise that \( N^{mono} \) is constant. It then shows that if \( \beta/\pi^{LB} = 1 \), and the nominal

---

7This restriction is somewhat ad hoc in the current class of models. Because of the monopolistic distortion, there are equilibria in which \( \pi_t = \pi^{UB} \) that are welfare-improving over the equilibria in which \( \pi^{LB} \leq \pi_t < \pi^{UB} \). But (based on the premise that \( \pi^{UB} \) is very large), it seems unlikely to me that central banks would ever be willing to use monetary policy to achieve these gains.

8See Eggertsson and Mehrotra (2014) for an overlapping generations model of secular stagnation.
interest rate lower bound equals one, there is a set of constant equilibria.\textsuperscript{9}

**Example 1.** Suppose that there are no marginal utility or technology shocks and that $Y_{T+1} = N^{\text{mono}}$, where:

$$u'(N^{\text{mono}})(1 - 1/\eta) = v'(N^{\text{mono}})$$

Suppose

$$R(\pi^{\text{LB}}, \varepsilon) = 1$$

for all $\varepsilon$ and $\pi^{\text{LB}} = \beta$. Set $\pi_{T+1}$ to be any constant so that:

$$\beta > \pi_{T+1}$$

Then, using backward induction, we can construct an equilibrium in which period $(T + 1)$ inflation equals $\pi_{T+1}$, period $t$ inflation $\pi_t = \pi_{LB}$, and period $t$ consumption satisfies:

$$c_t < N^{\text{mono}}, t = 1, ..., T$$

Begin with period $T$. We know that:

$$u'(c_T) = \max \left( \frac{\beta u'(N^{\text{mono}})}{\pi_{T+1}}, u'(N^{\text{mono}}) \right)$$

Since $\beta/\pi_{T+1} > 1$, it follows that:

$$u'(c_T) = \beta u'(N^{\text{mono}})/\pi_{T+1} > u'(N^{\text{mono}})$$

and $\pi_T = \pi_{LB}$. Now suppose inductively that:

$$u'(c_{t+1}) = \frac{\beta u'(N^{\text{mono}})}{\pi_{T+1}}$$

\textsuperscript{9}Suppose that there (also) exists $\pi^{UB} > \bar{\pi} > \pi^{LB}$ such that $\beta R(\bar{\pi})/\bar{\pi} = 1$. Then, there is also a set of constant equilibria.
and $\pi_{t+1} = \pi_{LB}$. Then:

$$u'(c_t) = \max (\beta u'(c_{t+1})/\pi_{LB}, u'(N_{mono}))$$

$$= u'(c_{t+1})$$

$$= \beta u'(N_{mono})/\pi_{T+1}$$

Since

$$\beta/(\pi_{T+1}) > 1,$$

it follows that $u'(c_t) > u'(N_{mono})$, and that inflation $\pi_t$ must be at its lower bound $\pi_{LB}$. The equilibrium construction follows by induction.

### 4.2 Reducing the Lower Bound on Prices

The main result in this subsection is that reducing the lower bound on prices makes equilibrium outcomes even worse in welfare terms. I first use example 1 to illustrate this claim, and then provide a more general proof.

**Example 2.** Consider the parameter setting in example 1, except that the lower bound $\pi_{LB}$ is set to be less than $\beta$. (Note that $R(\pi_{LB}; \varepsilon)$ still equals one for this lower value of $\pi_{LB}$.) Then, we can use reverse induction as before to show that, given long-run inflation $\pi_{T+1}$, the equilibrium marginal utility in period $t$ is given by:

$$u'(c_t) = \frac{\beta^{T-t+1} u'(N_{mono})}{(\pi_{LB})^{T-t} \pi_{T+1}}.$$

This equilibrium marginal utility is an exponentially increasing function of $\pi_{LB}$, meaning that the equilibrium becomes arbitrarily less efficient as the lower bound $\pi_{LB}$ is made smaller.

The intuition behind Example 2 is simple. Reducing the lower bound in period $(t + 1)$ also reduces expected inflation in period $t$. Because the lower bound on the interest rate rule has been left unchanged, the fall in expected inflation results in a higher real interest rate,
lower demand, and lower output.

The following proposition generalizes Example 2.

**Proposition 5.** Consider two economies that are identical except that they have distinct inflation lower bounds \( \pi_H^{LB} > \pi_L^{LB} \) and interest rate rules \( R, R' \) such that:

\[
R'(\pi_L^{LB}; \varepsilon) = R(\pi_H^{LB}; \varepsilon)
\]

for all \( \varepsilon \). Suppose \( (c_t, \pi_t)_{t=1}^T \) is an equilibrium in the former economy given random period \((T + 1)\) inflation \( \pi_{T+1} \) such that \( C_t < \gamma \text{N}^{\text{mono}}(\psi_t, \lambda_t) \) with probability one for all \( t \leq T \). Then, given the same random period \((T + 1)\) inflation \( \pi_{T+1} \), there is an equilibrium \((C'_t, \pi'_t)\) in the latter economy such that:

\[
u'(C'_t; \lambda_t)(\frac{\pi_H^{LB}}{\pi_L^{LB}})^{T-t-1} \leq u'(C_t; \lambda_t)
\]

with probability one for all \( t \leq T \).

**Proof.** In Appendix.

Proposition 4 established that, for a wide class of economies, there is a “secular stagnation” equilibrium in which output is lower than the monopolistically competitive outcome in all periods. Proposition 5 shows that reducing the price floor, without changing the minimum nominal interest rate, makes this secular stagnation equilibrium even worse. The intuition is the same as in Example 2: The smaller value of the inflation floor translates into lower inflation expectations. With a fixed lower bound on the nominal interest rate, the lower inflation expectations translate into higher real interest rates, lower demand and lower output.

### 4.3 L-Shaped Phillips Curve

In this subsection, we show that the Phillips curve relating the output gap to inflation is L-shaped.
Proposition 6. Consider an equilibrium \((C, N, \pi)\). In this equilibrium, \(\pi_t = \pi^{LB}(\pi^{UB})\) in any date and state in which \(C_t < (>)\psi_t N^{\text{mono}}(\psi_t, \lambda_t)\). As well, \(C_t = \psi_t N^{\text{mono}}(\psi_t, \lambda_t)\) in any date and state in which \(\pi^{LB} < \pi_t\).

Proof. Straightforward implication of the definition of equilibrium.

In any equilibrium, the Phillips curve is horizontal with inflation equal to \(\pi^{LB}\) when there is a negative output gap\(^{10}\)(in the sense that output is lower than the monopolistically competitive outcome.). In these dates and states, firms find it optimal to cut prices to the common lower bound. Their markups are higher than in the monopolistically competitive equilibrium. In contrast, when the output gap is zero, current inflation is determined by expectations about future inflation and future consumption:

\[
\pi_t = R^{-1}(\beta^{-1} \frac{1}{E_t\{u'(C_{t+1}; \lambda_{t+1})\}}; \varepsilon_t)
\]

In this vertical portion of the curve, higher future expected inflation and higher future expected consumption is associated with higher current inflation. This L-shaped Phillips curve is quite different from the log-linear Phillips curve that emerges from the New Keynesian paradigm.

Friedman (1968) famously argued that, in the long-run, the Phillips curve is necessarily vertical. But in the models that I study in this paper, there is no force that ensures that the economy converges to the vertical portion of the Phillips curve. Instead, as Proposition 4 and Example 1 illustrate, the economy may remain stuck permanently on the horizontal portion of the Phillips curve with a negative output gap.

Proposition 6 relies on the central bank’s rule satisfying (8). In the absence of this restriction, there is another flat portion to the Phillips curve in which inflation is stuck at \(\pi^{UB}\) and the output gap is positive. In these dates and states, firms find it optimal to keep

\(^{10}\)I have done some preliminary work to extend the results to an economy with capital. The relevant notion of an output gap in this dynamic economy is what is sometimes called the static labor market wedge (taking into account the monopolistic distortion). Thus, the Phillips curve is flat if \(u'(C_t; \lambda_t)MPL_t(1-1/\eta) > v'(N_t)\), where \(MPL_t\) is the marginal product of labor in period \(t\).
raising prices until they are constrained by the upper bound $\pi^{UB}$. Their markups are lower than in a monopolistically competitive equilibrium.

### 4.4 Fiscal Multipliers

In this subsection, I discuss how fiscal multipliers work in this class of models. The main finding is that the multiplier is larger when the Phillips curve is flat than when it is vertical.

Suppose that the government buys an exogenously specified process $g = \{g_t\}_{t=1}^T$ of private consumption goods; we can view them as either discarded or entering utility separably from private consumption and leisure. The purchases could be financed in a number of ways. To be explicit, suppose that the government issues debt that pays off only in period $(T + 1)$ and pays off that debt using lump-taxes.) This purchase process affects the non-monetary monopolistically competitive equilibrium in the usual way:

$$u'(\psi_t N_{\text{mono}}(\psi_t, \lambda_t, g_t) - g_t; \lambda_t)(1 - 1/\eta)\psi_t = u'(N_{\text{mono}}(\psi_t, \lambda_t, g_t))$$

It is readily shown that, because of income effects on labor supply, an increase in $g_t$ pushes up $N_{\text{mono}}(\psi_t, \lambda_t, g_t)$, but by less than the increase in $g_t$.  

The equilibrium conditions for private consumption, in the monetary economy with bounded prices, become:

$$u'(c_t; \lambda_t) = \max(\beta R(\pi^{LB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1}}), u'(\psi_t N_{\text{mono}}(\psi_t, \lambda_t, g_t) - g_t; \lambda_t)), t = 1, \ldots, T$$  \hspace{1cm} (9)$$

Now suppose that we perturb the government purchases process by increasing period $t$ purchases $g_t$ by a small positive $\Delta$, while keeping final period inflation $\pi_{T+1}$ remaining unchanged. This increase in government purchases has no effect on private consumption if $c_t < (N_{\text{mono}}(\psi_t, \lambda_t) - g_t)$. In this case, the output multiplier is one. The impact on welfare depends on how government purchases enter into agents’ utility functions. There is no effect
on period $t$ inflation (if $\Delta$ is small).

In contrast, if $c_t + g_t = \psi_t N^{\text{mono}}(\psi_t, \lambda_t, g_t)$, then raising $g_t$ by $\Delta$ leads to a fall in private consumption $c_t$. There is at least some crowding out, and the output multiplier is less than one. The period $t$ inflation rate is given by:

$$\pi_t = R^{-1}(\beta^{-1} \frac{1}{E_t\{\psi_t N^{\text{mono}}(\psi_t, \lambda_t, g_t) - g_t - \Delta, \lambda_t, \pi_{t+1}\}}; \varepsilon_t$$

and so it is an increasing function of $\Delta$.

### 4.5 Failure of Neo-Fisherian Logic

In recent papers, Cochrane (2016) and Schmitt-Grohe and Uribe (2014) have argued that increasing the nominal interest rate rule will result in higher inflation. In this subsection, I consider this claim in the context of the models with price bounds studied in this paper. I consider two policy rules $(R, R')$ such that $R' > R$ for all $\pi, \varepsilon$. The following proposition shows that, given a random variable $\pi_{T+1}$, the implied equilibrium $(C', N', \pi')$ under $R'$ is no larger than the implied equilibrium under $R$.

**Proposition 7.** Consider two interest rate rules $R, R'$ such that $R'(\pi; \varepsilon) > R(\pi; \varepsilon)$ for all $(\pi, \varepsilon)$. If $(c^*, \pi^*)$ is an equilibrium given $R$ with (random) period $(T+1)$ inflation $\pi_{T+1}$, and $(c', \pi')$ is an equilibrium given $R'$ with the same (random) period $(T+1)$ inflation, then $c'_t \leq c^*_t$ and $\pi'_t \leq \pi^*_t$ for all $t$ with probability one.

**Proof.** In Appendix.

The key neo-Fisherian premise is that, for any interest rate rule, the long run real interest rate is necessarily efficient. Given this premise, the Fisher equation then implies that the long-run inflation rate has to move one-for-one with the long-run nominal interest rate. But, as Proposition 4 demonstrates, this presumption of a policy-invariant long-run real interest rate is not valid in models with price bounds. In these models, there is a set of equilibria
indexed by the long-run inflation rate, and the long-run real interest rate can vary both across and within these equilibria.

4.6 The Forward Guidance Puzzle

Del Negro, et. al. (2015) and MacKay, et. al. (2016) demonstrate that forward guidance about future monetary policy is puzzlingly powerful in the New Keynesian modeling paradigm. In this subsection, I analyze the effect of forward guidance on current output within the class of models studied in this paper. I make two main points:

- The effect of forward guidance is completely summarized through its impact on the inflation rate during the period in which output returns to an efficient level.

- If the (logged) interest rate rule obeys the Taylor Principle, forward guidance becomes exponentially less effective with respect to the horizon.

Transition Inflation as a Summary Statistic

Suppose that the interest rate rule satisfies \( R(\pi^{LB}; \varepsilon) = R_{LB} \) for all \( \varepsilon \). Consider an equilibrium such that, in some event \( \Xi_t \), the output gap is known to be negative in periods \((t + s), s = 0, \ldots, \tau \), and known to be zero in all periods after \((t + s)\). This is a description of a deterministic liquidity trap, in which the nominal interest rate is known to be pinned at its lowest level for the next \( \tau \) periods.

In any equilibrium of this kind, we know that:

\[
u'(c_t; \lambda_t) = \beta R_{LB} E_t \left\{ \frac{u'(c_{t+1}; \lambda_{t+1})}{\pi^{LB}_{t+1}} \right\} \\
= (\beta R_{LB}/\pi^{LB}) \tau R_{LB} E_t \left\{ \frac{u'(\psi_{t+\tau+1}; \lambda_{t+\tau+1})}{\pi_{t+\tau+1}} \right\}
\]

This restriction implies that the impact of any form of post-trap forward guidance is completely summarized through its impact on the inflation rate \( \pi_{t+\tau+1} \), during the single period
in which the economy exits the trap. Note too that the effect of changes in this transition inflation rate \( \pi_{t+\tau+1} \) on prior consumption is independent of the anticipated duration \( \tau \) of the liquidity trap.

**Decaying Effect of Forward Guidance**

We’ve seen that post-liquidity trap forward guidance affects outcomes in the trap only through the inflation rate during the period in which the economy transits from the trap. How is this transition inflation rate affected by the level of future (that is, post-trap) interest rates? The answer to this question depends on the interest rate rule that maps realized inflation into interest rates.

By way of example, return to the liquidity trap described in the prior subsection. Suppose that, after the liquidity trap ends, the marginal utility of consumption is equal to a constant \( MU^{\text{mono}} \) (for all \( s > \tau \)). Suppose too that, after the liquidity trap ends, the interest rate rule takes the form:

\[
R(\pi; \varepsilon) = B\pi^\gamma, \gamma > 1
\]

The logged version of this interest rate rule obeys the Taylor Principle (so that the nominal interest rate adjusts more than one-for-one with the inflation rate).

Now consider a form of forward guidance in which the central bank lowers \( B \) to \( B' = \xi B \), \( 0 < \xi < 1 \), in a single period \( (t + k) \), where \( k > (\tau + 1) \). This change in policy in a future period increases the inflation rate in that period:

\[
\pi'_{t+k} = \left( \frac{B' - 1}{E_{t+k}(1/\pi_{t+k+1})} \right)^{1/\gamma} = \pi_{t+k} \xi^{-1/\gamma}
\]

This increase in period \( (t + k) \) inflation feeds back into prior inflation rates, so that:

\[
\pi'_{t+k-r} = \xi^{-1/\gamma} \pi_{t+k-r}, r > 1
\]
But, since $\gamma > 1$, $\xi^{-1/\gamma}$ converges to 1 as $r$ converges to infinity. Unlike in the New Keynesian model, the impact of forward guidance declines exponentially with the relevant horizon.

5 Literature

In this section, I discuss related literature.

5.1 Flat Phillips Curve

The data from at least the past nine years suggest that there is little connection between resource underutilization and inflation.\(^{11}\) Thus, most measures of labor market slack rose sharply from 2008 to 2009 and inflation fell relatively little over that same time period. Similarly, inflation has remained essentially unchanged while most measures of labor market slack have fallen considerably over the past four years (2013-17).

These observations about inflation don’t seem all that surprising when viewed through the lens of the bounded equilibrium models analyzed in this paper. As long as there is a negative output gap, the Phillips curve is flat: there is no connection between the magnitude of the gap and inflation. The Phillips curve becomes vertical only when the output gap rises back to zero. And, when the Phillips curve is vertical, inflation is determined by the interaction of the nominal interest rate rule, the efficient real interest rate, and expected inflation.

The above emphasizes the underutilization of labor when the output gap is negative. But the underutilization of labor is in fact due to excessive product market power. Hence, if the models included other inputs to production, these inputs would also be underutilized when the output gap is negative. In this sense, the notion of a negative output gap in this paper is consistent with the evidence in Bils, Klenow, and Malin (forthcoming).

\(^{11}\)This lack of connection may well go back much further in time - see, for example, Stock and Watson (2009).
5.2 Sticky Prices and Wages

In the models that I study in Section 4, prices are completely flexible except for the bounds. There is considerable evidence that prices and wages are not completely flexible, although the degree of inflexibility remains a subject of much empirical study (see Nakamura and Steinsson (2013) for a recent survey of the relevant evidence). Section 2.3 shows that, even if some firms are unable to adjust their prices, there is a non-existence problem when the other firms can choose their prices from the entire real line. Hence, it would be of interest to extend Section 4 to consider the properties of models with conventional pricing frictions (like Calvo stickiness or menu costs) and bounds on the flexible firms’ pricing decisions.

5.3 Indeterminacy

In the class of finite horizon models studied in this paper, the final period inflation rate is not pinned down. By construction, the price bounds guarantee that there is a dynamic equilibrium associated with each of these possible final period outcomes. Equilibrium indeterminacy is intrinsic to this class of economies.

This indeterminacy should not be entirely surprising. Cochrane (2011) describes how, even under active Taylor Rules, there is a set of equilibrium outcomes in New Keynesian models. It is typical practice to discard all but one of these equilibria because they lead to explosive inflationary paths. But, as Cochrane rightly emphasizes, there is no economics to justify that practice.

However, there is a key difference between the finite horizon indeterminacy highlighted in this paper and the infinite horizon indeterminacy that Cochrane discusses. The set of equilibria in this paper is indexed by the final period random inflation. This is a large set, because it consists of all random variables that are measurable with respect to past realizations of the exogenous processes in the economy. In contrast, the infinite horizon indeterminacy is indexed by a one dimensional variable: initial inflation.

How can it be that the indeterminacy in these finite horizon models is so much larger than
the indeterminacy in the infinite horizon models? One main reason is that users of the infinite horizon models typically impose an auxiliary (non-economic) restriction that equilibria be time homogeneous.

6 Conclusions

This paper makes two points. The first is technical. It is well-understood that there may be non-existence issues in games with non-compact action sets. This paper demonstrates that macroeconomics is not immune to this general criticism.\footnote{See Bassetto and Phelan (2015) for another illustration of how compactifying action sets can affect conclusions in macroeconomic models.} It shows that for a wide set of policy rules and beliefs about future equilibrium outcomes, monetary models without bounds on firms’ pricing decisions have no equilibria. The basic intuition is simple. Suppose that the monetary policy rule and inflation beliefs are such that the real interest rate paid by money must be higher than what would occur in a non-monetary equilibrium. Then, the demand for labor and the real wage are lower than would occur in a non-monetary equilibrium. Firms can always gain by cutting their prices (so as to expand output). The only way that this “gain by cutting prices’ situation can be an equilibrium is if firms face some positive floor on prices.

The second point is that this seemingly minor technical change has large effects on the empirical and policy implications of flexible-price models. Among other results, I show that the models with pricing bounds imply that the Phillips curve is L-shaped, are consistent with the existence of permanent secular stagnation, and do not imply that forward guidance is surprisingly effective. Perhaps most importantly, I prove that lowering the price floor toward zero leads to less efficient outcomes emerging as equilibria. It is not possible to use non-monetary models or flex-price models to understand a world with the (arbitrarily low) price floors needed to ensure equilibrium existence.\footnote{This “discontinuity” result (which of course is no such thing) echoes the findings of Kocherlakota (2016).}

As the above indicates, my goals in this paper have been theoretical. As a consequence,
I’ve deliberately kept the class of models simple in many respects. To better engage with the data, it would be useful to extend the analysis in a number of directions such as:

- exploring the consequences of adding dimensions of heterogeneity, like different price bounds across firms
- including worker-firm matching impediments in the labor market.
- endogenizing the price bounds

As I show in Section 2, and argue above, the non-existence result is highly robust. In particular, it applies to models in which only some firms can change their prices at any date, and to models in which markets for risk-sharing are incomplete. It is important to understand how adding pricing bounds would affect the implications of recent models that incorporate both of these frictions (as in Kaplan, Moll, and Violante (2018)). My conjecture is that such investigations are likely to be more informative in (long) finite horizon settings than in infinite horizon settings.

References


Appendix

In this appendix, I gather the remaining proofs.

Proof of Proposition 4

Given the restrictions on the efficient marginal utility process and on the interest rate rule, there exists $L$ such that for all $t = 1, ..., T$:

$$
\frac{\beta^{T-t+1} R_{LB}^{T-t+1}}{\left(\pi^{LB}\right)^{T-t}} E_t u'(Y_{T+1}; \lambda_{T+1}) > L
$$

with probability one. Pick any positive constant $\pi_{T+1}$ that is less than $L$. I proceed by reverse induction to show that, with probability one:

$$
u'(c_t) > \frac{\beta^{T-t+1} R_{LB}^{T-t+1}}{\left(\pi^{LB}\right)^{T-t} \pi_{T+1}} E_t u'(Y_{T+1}; \lambda_{T+1}) > u'(\psi_t N_{mono}^* (\psi_t, \lambda_t); \lambda_t)
$$

and $\pi_t = \pi^{LB}$.

Note first that:

$$
u'(c_T) \geq \beta R(\pi^{LB}; \varepsilon_T) E_T \{u'(Y_{T+1}; \lambda_{T+1})/\pi_{T+1}\}
$$

$$
> Lu'(\psi_T u'(\psi_T N_{mono}^* (\psi_T, \lambda_T); \lambda_T)/\pi_{T+1})
$$

$$
> u'(\psi_T N_{mono}^* (\psi_T, \lambda_T); \lambda_T)
$$
with probability one, which implies that $\pi_T = \pi^{LB}$ with probability one.

Now, inductively assume that:

$$u'(c_{t+1}; \lambda_{t+1}) > \frac{\beta T^{-t} R_{LB}^{T-t}}{(\pi^{LB})^{T-t-1} \pi_{T+1}} E_{t+1} u'(Y_{T+1}; \lambda_{T+1})$$

with probability one and $\pi_{t+1} = \pi^{LB}$ with probability one. Then, if we roll back one period, we can show that:

$$u'(c_t; \lambda_t) \geq \beta R(\pi^{LB}; \varepsilon_t) E_t \frac{u'(c_{t+1}; \lambda_{t+1})}{\pi^{LB}}$$

$$> (\beta R_{LB}) \frac{\beta^{T-t} R_{LB}^{T-t}}{(\pi^{LB})^{T-t} \pi_{T+1}} E_t u'(Y_{T+1}; \lambda_{T+1})$$

$$= \frac{\beta^{T-t+1} R_{LB}^{T-t+1}}{(\pi^{LB})^{T-t} \pi_{T+1}} E_t u'(Y_{T+1}; \lambda_{T+1})$$

with probability one. It follows that:

$$u'(c_t; \lambda_t) > Lu'(\psi_t \lambda^{mono}(\psi_t, \lambda_t); \lambda_t)/\pi_{T+1}$$

$$> u'(\psi_t \lambda^{mono}(\psi_t, \lambda_t); \lambda_t)$$

with probability one, which in turn shows that $\pi_t = \pi^{LB}$ with probability one.

**Proof of Proposition 5**

We proceed by reverse induction. Note first that:

$$u'(c_t; \lambda_t) = R'(\pi^{LB}_t; \varepsilon_T) E_T u'(Y_{T+1}; \lambda_{T+1})/\pi_{T+1}$$

$$= R(\pi^{LB}_T; \varepsilon_T) E_T u'(Y_{T+1}; \lambda_{T+1})/\pi_{T+1}$$

$$= u'(c_T; \lambda_T)$$
Now suppose inductively that:

\[ u'(c_{t+1}; \lambda_{t+1}) \left( \frac{H}{L} \right)^{T-t-1} \leq u'(c_{t+1}'; \lambda_{t+1}) \]

with probability one for some \( t \leq (T-1) \). Then, \( u'(c_{t+1}'; \lambda_{t+1}) > u'(\psi_{t+1}N^{\text{mono}}(\psi_{t+1}, \lambda_{t+1}); \lambda_{t+1}) \)

with probability one and \( \pi_{t+1}' = \pi_{LB}L \) with probability one. Similarly, since \( c_{t+1} < \psi_{t+1}N^{\text{mono}}(\psi_{t+1}, \lambda_{t+1}) \)

with probability one, \( \pi_{t+1} = \pi_{LB}L \) with probability one.

Next move backwards in time to period \( t \). We can show that with probability one:

\[
\begin{align*}
 u'(c_t; \lambda_t) &= \max(\beta R(\pi_{LB}^H; \varepsilon_t)E_t\{u'(c_{t+1}; \lambda_{t+1})/\pi_{t+1}\}, u'(\psi_tN^{\text{mono}}(\psi_t, \lambda_t); \lambda_t)) \\
 &= \beta R(\pi_{LB}^H; \varepsilon_t)E_t\{u'(c_{t+1}; \lambda_{t+1})/\pi_{LB}^H\} \\
 &\leq \beta R'(\pi_{LB}^H; \varepsilon_t)E_t\{u'(c_{t+1}'; \lambda_{t+1})(\pi_{LB}^H/\pi_{LB}^H)^{T-t-1}/\pi_{LB}^H\}(\pi_{LB}^H/\pi_{LB}^H) \\
 &\leq u'(c_{t}'; \lambda_{t})(\pi_{LB}^H/\pi_{LB}^H)^{T-t}
\end{align*}
\]

which implies that:

\[ u'(c_t; \lambda_t)(\pi_{LB}^H/\pi_{LB}^H)^{T-t} \leq u'(c_t'; \lambda_t) \]

with probability one.

We have established that \( u'(c_t'; \lambda_T) = u'(c_T; \lambda_T) \) with probability one, and that if \( c_{t+1}' \leq c_{t+1} \) with probability one for \( t \leq (T-1) \), then \( c_t' < c_t \) with probability one. The proposition is proved.
Proof of Proposition 7

We can prove the proposition via reverse induction. Suppose inductively that \( c'_{t+1} \leq c^*_t \) and \( \pi'_{t+1} \leq \pi^*_t \) with probability one. Then:

\[
u'(c'_t; \lambda_t) = \max(u'(\psi_tN_{t}^{mono}(\psi_t; \lambda_t); \lambda_t), \beta R'(\pi^{LB}; \varepsilon_t)E_t\{u'(c'_{t+1}; \lambda_{t+1})/\pi'_{t+1}\})
\]

\[\geq \max(u'(\psi_tN_{t}^{mono}(\psi_t; \lambda_t); \lambda_t), \beta R(\pi^{LB}; \varepsilon_t)E_t\{u'(c^*_t; \lambda_{t+1})/\pi^*_t\})
\]

\[= u'(c^*_t; \lambda_t)
\]

which implies that \( c'_t \leq c^*_t \). In terms of inflation, consider any event in which \( \pi^*_t = \pi^{LB} \). In that event:

\[\beta R(\pi^{LB}; \varepsilon_t)E_t\{u'(c^*_t; \lambda_{t+1})/\pi^*_t\} \geq 1,
\]

and it follows that:

\[\beta R'(\pi^{LB}; \varepsilon_t)E_t\{u'(c'_{t+1}; \lambda_{t+1})/\pi'_{t+1}\} > 1
\]

which implies that \( \pi'_t = \pi^{LB} \) in that event.

Next, consider any event in which \( \pi^*_t = \pi^{UB} \). In that event, \( \pi'_t \leq \pi^{UB} = \pi^*_t \).

Finally, consider any event in which \( \pi^{LB} < \pi^*_t < \pi^{UB} \) with probability one. Then:

\[\beta R(\pi^*_t; \varepsilon_t)E_t\{u'(c^*_t; \lambda_{t+1})/\pi^*_t\} = 1
\]

In that event:

\[\beta R'(\pi^*_t; \varepsilon_t)E_t\{u'(c'_{t+1}; \lambda_{t+1})/\pi'_{t+1}\} > 1
\]

which implies that \( \pi'_t \leq \pi_t \).

Note that \( \pi_{T+1} \) is the same in the two equilibria, and \( c_{T+1} = Y_{T+1} \) in the two equilibria.
Hence, the reverse induction above implies that:

\[ c_t' \leq c_t^* \]
\[ \pi_t' \leq \pi_t^* \]

for all \( t \) and with probability one.