# The Cobb Douglas Marriage Matching Function

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# Motivation

- Since the seventies, there were significant changes in the US marriage market.
  - The marriage rate has decreased.
  - The cohabitation rate has increased.
  - The unmatched rate has increased.
- Economists have investigated different mechanisms including
  - Changes in reproductive, family and labor laws.
  - Changes in earnings inequality and welfare regimes.
  - Social change including peer effects.
  - Technological changes in the household.
- using different methodologies
  - single equation natural experiments/reduced form estimation.
  - structural econometric models.
  - non-transferable (frictional) vs transferable (frictionless) utility models of the marriage market.
  - static vs dynamic models.

# Estimation and identification concerns

- Non-structural single equation reduced form models using natural experiments are popular and relatively easy to estimate. Since the marriage market is a general equilibrium system, it is unclear how to interpret the “causal” estimates for policy or counterfactual experiments.

- With data on only who matches with whom, and who remains unmatched, there is an under appreciation of what structural parameters are identified in structural models.

# The marriage matching function approach

- Consider a marriage market $s$ at time $t$.
- There are $I$, $i = 1, .., I$, types of men and $J$, $j = 1, .. J$, types of women.
- Let $m_{ist}$ and $f_{jst}$ be the population supplies of type $i$ men and type $j$ women respectively.
- Each individual chooses between two types of relationships, marriage or cohabitation, $r = [m; c]$, and a partner (by type) of the opposite sex for relationship $r$.
- Let $\theta$ be a vector of parameters.
- A marriage matching function (MMF) is an $2I \times J \times S \times T$ matrix valued function $\mu(m^{st}, f^{st}; \theta)$ whose typical element is $\mu_{ij}^{rst}$, the number of $(r; i; j)$ relationships.

# Marriage market (accounting) restrictions

- An admissible MMF must satisfy the population constraints:

$$\sum_{j=1}^{J} \mu_{ij}^{\mathcal{M}st} + \sum_{j=1}^{J} \mu_{ij}^{\mathcal{C}st} + \mu_{i0}^{st} = m_i^{st}, \quad 1 \leq i \leq I \tag{1}$$

$$\sum_{i=1}^{I} \mu_{ij}^{\mathcal{M}st} + \sum_{i=1}^{I} \mu_{ij}^{\mathcal{C}st} + \mu_{0j}^{st} = f_j^{st}, \quad 1 \leq j \leq J \tag{2}$$

$$\mu_{0j}^{st}, \mu_{i0}^{st} \geq 0, \quad 1 \leq j \leq J, 1 \leq i \leq I.$$

# A first pass

$$\ln \mu_{ij}^{rst} = \overline{\gamma}_{ij}^{rst} + \alpha^r \ln m_i^{st} + \beta^r \ln f_j^{st} + u_{ij}^{rst}; (r,i,j)$$

$$\ln \frac{\sum_j \mu_{ij}^{rst}}{m_i^{st}} = \sum_j \overline{\gamma}_{ij}^{rst} + u_i^{rst}; (r,i)$$

$$\ln \frac{\sum_i \mu_i^{rst}}{f_j^{st}} = \sum_i \overline{\gamma}_{ij}^{rst} + u_j^{rst}; (r,j)$$

- Advantages:
  - Easy estimation by IV or DD.
  - Flexibly allow for changes in preferences by parameterizing $\gamma_{ij}^{rst}$.
- Disadvantages:
  - May not satisfy accounting identities.
  - Hard to interpret parameters.
  - No spillover effect. I.e. what happens to $\mu_{ij}^{rst}$ if $m_{i'}^{st}$ changes?

# Cobb Douglas MMF

$$\ln \mu_{ij}^{rst} = \overline{\gamma_{ij}}^{rst} + \alpha^r \ln \mu_{i0}^{st} + \beta^r \ln \mu_{0j}^{st} + u_{ij}^{rst}; (r, i, j)$$

- Advantages:
- Easy to estimate by IV DD using $m_i^{st}$ and $f_j^{st}$ as instruments.
  - Flexibly allow for changes in preferences by parameterizing $\gamma_{ij}^{rst}$.
  - Satisfies accounting identities.
  - Admits spillover effects.
  - Given parameters and population supplies, equilibrium exists and is unique.
- Comparative statics are as expected.
- Behavioral mechanims (structural models) include
  - Transferable (frictionless) and non-transferable (frictional) models.
  - Own peer effects
  - Mourifié extends our model to include sibling peer effects and dynamics.

★★Generally, particular behavioral models (structural parameters) are not identified.★★

# Coob Douglas and structural models

- The CD MMF encompasses many different structural matching models:
  - CS, CSW transferable utility MMF
  - Dagsvik Manziel non-transferable utility MMF

| Models and restrictions on $\alpha^r$ and $\beta^r$ | | | | |
|---|---|---|---|---|
| Model | $\alpha^r$ | $\beta^r$ | $\gamma_{ij}^r$ | Restrictions |
| Cobb Douglas MMF | $\alpha^r$ | $\beta^r$ | $\gamma_{ij}^r$ | $\alpha^r \geq 0, \beta^r \geq 0$ |
| CS | $\frac{1}{2}$ | $\frac{1}{2}$ | $\pi_{ij}^r$ | $\alpha^r = \beta^r = \frac{1}{2}$ |
| DM | 1 | 1 | $\pi_{ij}^r$ | $\alpha^r = \beta^r = 1$ |
| CSW | $\frac{\sigma}{\sigma+\Sigma}$ | $\frac{\Sigma}{\sigma+\Sigma}$ | $\frac{\pi_{ij}}{\sigma+\Sigma}$ | $\alpha, \beta > 0; \alpha + \beta = 1$ |

- All those existing behavorial MMF imposes at least one of those restrictions:
  - CRS, i.e., $\alpha + \beta = 1$
  - The sex ratio $\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{ij}^{C}}$ is independent of the population supplies $f_j, m_i$.
- Although those two restrictions are rejected in the data.
- Is there a structural model belong to the CD class that can relax those restrictions?

# A Structural CD MMF: Choo-Siow with peer effect(CSPE)

- Model builds on Choo Siow (2006) we propose the following structural matching model with peer effects.
- Let the utility of male $g$ of type $i$ who matches a female of type $j$ in a relationship $r$ be:

$$U^r_{ijg} = \tilde{u}^r_{ij} + \phi^r_i \ln \mu^r_{ij} - \tau^r_{ij} + \epsilon^r_{ijg}, \text{ where} \tag{4}$$

  - $\tilde{u}^r_{ij} + \phi^r_i \ln \mu^r_{ij}$: Systematic gross return to a male of type $i$ matching to a female of type $j$ in relationship $r$.
  - $\phi^r_i$: Coefficient of peer effect for relationship $r$, $1 \geq \phi^r_i \geq 0$.
  - $\mu^r_{ij}$: Equilibrium number of $(r, i, j)$ relationships.
  - $\tilde{u}_{i0} + \phi^0_i \ln \mu^0_{i0}$ is the systematic payoff that type $i$ men get from remaining unmatched, $1 \geq \phi^0_i \geq 0$.
  - The above empirical model for multinomial choice with peer effects is standard. See Brock and Durlauf (2001)

# CSPE (2)

- Man maximizes utility by choosing:

$$U_{ig} = MAX(U_{i0g}, U^{\mathcal{M}}_{i1g}, ..., U^{\mathcal{M}}_{iJg}, U^{\mathcal{C}}_{i1g}, ..., U^{\mathcal{C}}_{iJg})$$

- Following McFadden's famous result and from above, we obtain a quasi-demand equation by type $i$ men for $(r, i, j)$ relationships.

$$ln \frac{(\mu^r_{ij})^d}{(\mu_{i0})^d} = \tilde{u}^r_{ij} + \phi^r \ln \mu^r_{ij} - \tau^r_{ij} - \tilde{u}_{i0} - \phi \mu_{i0}, \quad (5)$$

- The above equation is a quasi-demand equation by type $i$ men for $(r; i; j)$ relationships

## CSPE(3)

- Similarly, the utility of woman $k$ of type $j$ who matches a type $i$ man in a relationship $r$ is:
$$V^r_{ijk} = \tilde{v}^r_{ij} + \Phi^r_i \ln \mu^r_{ij} + \tau^r_{ij} + \epsilon^r_{ijk},$$
- Using again McFadden result, the quasi-supply equation of type $j$ women for $(r, i, j)$ relationships is given by:
$$\ln \frac{(\mu^r_{ij})^s}{(\mu_{0j})^s} = \tilde{v}^r_{ij} + \Phi^r \ln \mu^r_{ij} + \tau^r_{ij} - \tilde{v}_{0j} - \Phi^0 \ln \mu_{0j} \qquad (6)$$

# CSPE(4)

- The matching market clears when, given equilibrium transfers $\tau_{ij}^r$, we have for all $(r, i, j)$:
$$(\mu_{ij}^r)^d = (\mu_{ij}^r)^s = \mu_{ij}^r. \tag{7}$$
- Substituting (7) into equations (5) and (6) we get the following MMF: We get the following MMF

$$\ln \mu_{ij}^r = \frac{1-\phi_i^0}{2-\phi_i^r-\Phi_i^r} \ln \mu_{i0} + \frac{1-\phi_i^0}{2-\phi_i^r-\Phi_i^r} \ln \mu_{0j} + \frac{\pi_{ij}^r}{2-\phi_i^r-\Phi_i^r} \tag{8}$$

where $\pi_{ij}^r = \tilde{u}_{ij}^r - \tilde{u}_{i0} + \tilde{v}_{ij}^r - \tilde{v}_{0j}$.

# Empirical Application

- We study the marriage matching behavior of 26-30 years old women and 28-32 years old men with each other in the US for 1990, 2000 and 2010.
- The 1990 and 2000 data is from the 5% US census.
- The 2010 data is from aggregating three years of the 1% American Community Survey from 2008-2010.
- A state year is considered as an isolated marriage market.
- There were 51 states which includes DC.
- Individuals are distinguished by their schooling level: less than high school (L), high school graduate (M) and university graduate (H).
- A cohabitating couple is one where a respondent answered that they are the "unmarried partner" of the head of the household.

# Empirical Application

Table : Summary Statics*.

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| N cohabitations | 1113 | 710.7491 | 1367.211 | 3 | 15362 |
| N marriages | 1283 | 5023.924 | 9981.822 | 6 | 118867 |
| N males | 1377 | 49097.82 | 67662.33 | 174 | 568449 |
| N females | 1377 | 48319.55 | 67411 | 143 | 580493 |
| N unmatched males | 1377 | 20361.38 | 30701.33 | 165 | 262267 |
| N umatched females | 1377 | 18757.32 | 28271.38 | 76 | 236391 |
| Year | 1377 | 2000 | 8.167932 | 1990 | 2010 |

*An observation is a state/year. There are 51 states which includes DC. Observations with 0 cohabitation or marriages are excluded.

# Empirical Application

Table : Ordinary Least Square (OLS).

| | 1a | 1b | 2a | 2b | 3a | 3b |
|---|---|---|---|---|---|---|
| Dep. Var. | LCOH | LMAR | LCOH | LMAR | LCOH | LMAR |
| LU_M ($\alpha$) | 0.562 | 0.536 | 0.320 | 0.244 | 0.557 | 0.626 |
| | (0.041)** | (0.048)** | (0.075)** | (0.061)** | (0.084)** | (0.056)** |
| LU_F ($\beta$) | 0.531 | 0.665 | 0.630 | 0.609 | 0.827 | 0.939 |
| | (0.040)** | (0.047)** | (0.076)** | (0.058)** | (0.078)** | (0.051)** |
| $L\left[\frac{HH*MM}{HM*MH}\right]$ | | | 2.31 | 2.44 | 2.290 | 2.412 |
| | | | (0.078)** | (0.069)** | (0.071)** | (0.048)** |
| $L\left[\frac{MM*LL}{LM*ML}\right]$ | | | 1.78 | 2.52 | 1.781 | 2.464 |
| | | | (0.087)** | (0.082)** | (0.080)** | (0.063)** |
| $L\left[\frac{HM*ML}{MM*HL}\right]$ | | | 0.784 | 1.43 | 0.796 | 1.426 |
| | | | (0.147)** | (0.092)** | (0.145)** | (0.084)** |
| $L\left[\frac{MH*LM}{MM*LH}\right]$ | | | 1.33 | 1.37 | 1.344 | 1.379 |
| | | | (0.141)** | (0.101)** | (0.135)** | (0.083)** |
| Y2000 | | | 0.289 | -0.287 | 0.313 | -0.277 |
| | | | (0.042)** | (0.032)** | (0.037)** | (0.022)** |
| Y2010 | | | 0.627 | -0.604 | 0.615 | -0.667 |
| | | | (0.041)** | (0.037)** | (0.040)** | (0.030)** |
| STATE | | | | | Y | Y |
| _cons | -4.788 | -3.981 | -2.842 | 1.172 | -7.359 | -5.015 |
| | (0.383)** | (0.441)** | (0.196)** | (0.158)** | (0.772)** | (0.543)** |
| $R^2$ | 0.51 | 0.45 | 0.90 | 0.95 | 0.92 | 0.97 |
| $N$ | 1,113 | 1,283 | 1,113 | 1,283 | 1,113 | 1,283 |
| $\frac{\alpha}{\beta}$ | 1.058 | 0.806 | 0.508 | 0.400 | 0.673 | 0.667 |
| | (0.137) | (0.115) | (0.178) | (0.138) | (0.146) | (0.082) |
| $\alpha+\beta$ | 1.093 | 1.202 | 0.950 | 0.853 | 1.384 | 1.57 |
| | (0.039) | (0.045) | (0.018) | (0.016) | (0.079) | (0.057) |
| $\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$ | | 0.762 | | 0.788 | | 0.991 |
| | | (0.147) | | (0.387) | | (0.246) |
| $prob\left[\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}=\frac{\alpha^{\mathcal{C}}}{\beta^{\mathcal{C}}}\right]$ | | 0.091 | | 0.001 | | 0.117 |

* $p<0.05$; ** $p<0.01$

# Empirical Application

Table : IV with time varying match effects.

| | 1a | 1b | 2a | 2b |
|---|---|---|---|---|
| Dependent variable | LCOH | LMAR | LCOH | LMAR |
| LU_M ($\alpha$) | 0.415 | 0.357 | 0.576 | 0.754 |
| | (0.071)** | (0.064)** | (0.077)** | (0.057)** |
| LU_F ($\beta$) | 0.528 | 0.524 | 0.688 | 0.885 |
| | (0.071)** | (0.063)** | (0.073)** | (0.050)** |
| $L\left[\frac{HH*MM}{HM*MH}\right]$ | 2.288 | 2.458 | 2.278 | 2.440 |
| | (0.148)** | (0.087)** | (0.145)** | (0.045)** |
| $L\left[\frac{MM*LL}{LM*ML}\right]$ | 1.504 | 2.255 | 1.514 | 2.198 |
| | (0.116)** | (0. .121)** | (0.103)** | (0.067)** |
| $L\left[\frac{HM*ML}{MM*HL}\right]$ | 1.169 | 1.698 | 0.787 | 1.702 |
| | (0.283)** | (0.136)** | (0.211)** | (0.110)** |
| $L\left[\frac{MH*LM}{MM*LH}\right]$ | 0.693 | 1.305 | 1.177 | 1.313 |
| | (0.260)** | (0.134)** | (0.266)** | (0.119) |
| $L\left[\frac{HH*MM}{HM*MH}\right]*Y2000$ | 0.259 | -0.011 | 0.663 | -0.097 |
| | (0.181) | (0.191) | (0.230)** | (0.119) |
| $L\left[\frac{MM*LL}{LM*ML}\right]*Y2000$ | 0.346 | 0.346 | 0.257 | 0.337 |
| | (0.323) | (0.186) | (0.153) | (0.124)** |
| $L\left[\frac{HM*ML}{MM*HL}\right]*Y2000$ | 0.032 | -0.279 | 0.332 | -0.274 |
| | (0.330) | (0.187) | (0.317) | (0.151) |
| $L\left[\frac{MH*LM}{MM*LH}\right]*Y2000$ | 1.133 | -0.042 | 0.040 | -0.052 |
| | (0.252) | (0.193) | (0.311) | (0.152) |
| $L\left[\frac{HH*MM}{HM*MH}\right]*Y2010$ | 1.133 | -0.208 | 1.062 | -0.396 |
| | (0.252)** | (0.199) | (0.232)** | (0.128) |
| $L\left[\frac{MM*LL}{LM*ML}\right]*Y2010$ | 0.692 | 0.534 | 0.679 | 0.540 |
| | (0.190)** | (0.200)** | (0.181)** | (0.162)** |
| $L\left[\frac{HM*ML}{MM*HL}\right]*Y2010$ | -0.384 | -0.609 | -0.323 | -0.655 |
| | (0.261) | (0.268)** | (0.261) | (0.246)** |
| $L\left[\frac{MH*LM}{MM*LH}\right]*Y2010$ | 0.394 | 0.264 | 0.391 | 0.237 |
| | (0.392) | (0.234) | (0.376) | (0.203) |
| Y2000 | 0.693 | 0.014 | 0.669 | -0.071 |
| | (0.104)** | (0.075) | (0.091)** | (0.054) |
| Y2010 | 1.122 | -0.097 | 1.061 | -0.274 |
| | (0.102)** | (0.079) | (0.092)** | (0.057)** |
| STATE | | | Y | Y |
| _cons | -3.075 | 0.618 | -6.512 | -5.900 |
| | (0.185)** | (0.161)** | (0.735)** | (0.506)** |
| $R^2$ | 0.91 | 0.96 | 0.93 | 0.98 |
| $N$ | 1,113 | 1,283 | 1,113 | 1,283 |
| $\frac{\alpha}{\beta}$ | 0.786 | 0.681 | 0.836 | 0.852 |
| | (0.239) | (0.203) | (0.174) | (0.096) |
| $\alpha+\beta$ | 0.943 | 0.881 | 1.264 | 1.640 |
| | (0.017) | (0.016) | (0.076) | (0.055) |
| $\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$ | | 0.866 | | 1.019 |
| | | (0.369) | | (0.241) |
| $prob\left[\begin{matrix}\alpha^{\mathcal{M}}=\alpha^{\mathcal{C}}\\ \beta^{\mathcal{M}}=\beta^{\mathcal{C}}\end{matrix}\right]$ | | 0.051 | | 0.000 |

# Empirical application: Results

1. From a descriptive (goodness of fit) point of view, a simplified Cobb Douglas MMF with relationship match $(r, i, j)$, state and year fixed effects, provides a reasonably complete and parsimonious description of the US marriage market by state from 1990 to 2010.
2. There are scale effects in US marriage markets.
3. CS, CSW and DM are rejected by the data.
4. CSPE is not rejected by the data. Homogenous peer effects *à* la Manski (2003) is rejected.

# Empirical application: Results (1)

1. The value of cohabitation is less sensitive to peer effects than the value for marriage.
2. The value of remaining unmatched is less sensitive to peer effects for women than for men.
3. Consistent with CSW and Siow (2015), to a first order, there is no general increase in positive assortative matching (PAM) by educational attainment from 1990 to 2010.
4. Consistent with CSW and many other observers, gains to marriage declined from 1990 to 2010. We further show that gains to cohabitation increased. Both findings are consistent with the observation that the average age of first marriage has increased over this period.

# Conclusion

- The CD MMF is estimated on US marriage and cohabitation data by states from 1990 to 2010.
- CS with peer effects is not rejected.
- There are peer and scale effects in the US marriage markets.
- We find evidence against all existing MMFs present so far in the literature.
- Positive assortative matching in marriage and cohabitation by educational attainment are relatively stable from 1990 to 2010.