There is increasing debate on the role of firms in determining labor market outcomes.
Motivation

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  - e.g. consider AKM (1999) model of log earnings for worker $m$ at firm $i$, time $t$
    \[ w_{mit} = \delta_m + \mu_i + \epsilon_{mit} \]
    - worker effect
    - firm effect
    - residual
  - variance decomposition using employer-employee data (Chile, 2005-2010):
    \[ \text{var}(w_{mit}) = \text{var}(\delta_m) + \text{var}(\mu_i) + 2\text{cov}(\delta_m, \mu_i) + (\epsilon_{mit}) \]
    \[ \begin{array}{c}
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      15\% \\
      15\%
    \end{array} \]

- Why do firms matter for inequality in worker earnings?
  - current literature: firms are different in some innate characteristics, e.g. “productivity”
  - growing evidence of substantial heterogeneity in buyer-seller matching between firms
  - relevance of this for worker outcomes is not well understood
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1. **Firms and earnings inequality**: Davis and Haltiwanger (1991); Abowd, Kramarz, and Margolis (1999); Card, Kline and Heining (2013); Card et al (2018); Borovickova and Shimer (2018); Song et al (2019); Bonhomme et al (2020); Haanwinckle (2020); Lamadon, Mogstad and Setzler (2021); Bonhomme, Lamadon and Manresa (2019)

   - **contribution**: structural representation of earnings variance decomposition allowing for networks

2. **Production networks**: Oberfield (2018); Huneeus (2019); Lim (2019); Dhyne, Kikkawa, Mogstad, and Tintelnot (2020); Kikkawa et al (2020); Acemoglu and Azar (2020); Eaton et al (2018); Demir et al (2020); Alfaro-Urena et al (2019); Adao et al (2020); Bernard et al (2020)

   - **contribution**: add heterogeneous workers and imperfectly competitive labor markets

3. **Labor market power**: Van Reenen (1996); Kline et al (2019); Berger, Herkenhoff and Mongey (2019); Azar, Berry and Marinescu (2019); Chan, Kroft and Mourifie (2019); Dube et al (2020); Jarosch, Nimczik and Sorkin (2021); Kroft, Luo, Mogstad, and Setzler (2020); Lamadon, Mogstad and Setzler (2021)

   - **contribution**: a richer theory of firm production in heterogeneous buyer-seller networks


   - **contribution**: new method for measuring factor prices with heterogeneous workers and inputs
1. **Firm-to-Firm VAT Transactions Data**
   - frequency: annual, 2005-2010
   - coverage: all suppliers of reporting firms, all sectors (≈80\% aggregate value-added)
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   - frequency: monthly, 2005-2018
   - coverage: universe of formal private firms
   - key variables: sales, materials, investment, capital, main industry, HQ location
General Environment

Workers

- heterogeneous in ability $a$, exogenous measure $L(a)$
- derive utility from three sources:
  - consumption goods produced by firms
  - amenities offered by employer
  - idiosyncratic preferences over employers (source of market power)
- observe ability-specific wage offers made by each firm and choose employer
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Firms
- heterogeneous in factor productivities, amenity values, network connections
- produce output by combining workers of different abilities with materials
- set ability-specific wages to hire workers
- source materials from suppliers in production network (exogenous)
- sell output to final consumers and customers in network
Worker preferences

- Utility of a worker with ability \( a \) employed at firm \( i \):

\[
    u_{it}(a) = \log w_{it}(a) + \log g_{i}(a) + \log \tau_{t} + \frac{1}{\gamma} \epsilon_{it}
\]

- workers spend income on consumption goods (CES utility) with CPI as numeraire
- \( \epsilon_{it} \): idiosyncratic Gumbel (GEV-I) preference shock
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Labor supply shifter:

$$\kappa_{it}(a) \equiv \frac{L(a)}{\left[ \sum_j g_j(a) w_{jt}(a) \right]^{\gamma-1}} \times g_i(a)^\gamma$$
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$$\kappa_{it}(a) \equiv \left( \frac{L(a)}{\gamma} \times \left[ \sum_j \left( g_j(a) w_{jt}(a) \right)^{\gamma} \right]^{-1} \times g_{i}(a) \right)^{\gamma}$$

- Assume firms behave atomistically and perceive constant labor supply elasticity $\gamma$
Production combines labor $L_{it}(a)$ and materials $M_{it}(a)$:

$$X_{it} = T_{it} \sum_a f [\phi_i(a) \omega_{it} L_{it}(a), M_{it}(a)]$$

- $f$: CES production function with elasticity of substitution $\epsilon$
- $T_{it}$: TFP; $\omega_{it}$: labor productivity; $\phi_i(a)$: allows for worker-firm complementarities
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Materials produced by combining inputs from suppliers $\Omega^S_{it}$:

$$\sum_a M_{it}(a) \equiv M_{it} = \left[ \Sigma_{j \in \Omega^S_{it}} \psi_{ijt} \frac{1}{\sigma} \left( x_{ijt} \right)^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}$$

- $\psi_{ijt}$: relationship-specific productivity
- market structure: monopolistic competition (CES markups)
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Main departure from standard production network models: **increasing marginal costs**
Wage Determination

Wages are a constant markdown of MRPLs:

\[ w_{it}(a) = \frac{\gamma}{1+\gamma} \times \phi_i(a) W_{it} \]

markdown  
MRPL
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- **Network demand** is the sum of demand shifters across downstream network connections:

\[ D_{it} = E_t + \sum_{j \in \Omega} \Delta_{jt}(\bar{T}_{jt}, g_j, D_{jt}, Z_{jt}) \psi_{jit} \]

**Network cost** is the CES input price index arising from upstream network connections:

\[ Z_{it} = \frac{1-\sigma}{\sigma} - \sum_{j \in \Omega} S_{it} \Phi_{jt}(\bar{T}_{jt}, g_j, D_{jt}, Z_{jt}) \psi_{ijt} \]

In sum: network determines \{\text{D}_it, Z_{it}\}, which then determine \( W_{it} \) and hence wages \( w_{it}(a) \).
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$$\log W_{it} = \text{const.} + \frac{1}{1 + \gamma} \log R_{it} - \frac{1}{1 + \gamma} \log H(g_i, \phi_i) + \frac{1}{1 + \gamma} \log F\left(\frac{Z_{it}}{W_{it}}\right)$$

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Therefore two firms with identical size can have different firm effects through:

- differences in primitives that affect heterogeneous sorting of workers to firms \(\{g_i, \phi_i\}\)
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Decomposing firm size (c.f. Bernard et al (2020)) is not equivalent to decomposing \( W_{it} \)
<table>
<thead>
<tr>
<th></th>
<th>Structural Estimation (Outline)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>worker ability, $a$</td>
</tr>
<tr>
<td></td>
<td>production complementarity, $\phi_i(a)$</td>
</tr>
<tr>
<td>2.</td>
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<td>product substitution elasticity, $\sigma$</td>
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- Other results: network matching, worker-firm sorting, model fit, earnings variance decomposition
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For first-order changes, sufficient to know:

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- observable network sales shares, network cost shares, material shares of cost
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Own passthrough (for the average firm):
\[
\frac{\partial \log W_{it}}{\partial \log T_{it}} = 34\%, \quad \frac{\partial \log W_{it}}{\partial \log \omega_{it}} = 13\%,
\]
\[
\frac{\partial \log W_{it}}{\partial \log D_{it}} = 13\%, \quad \frac{\partial \log W_{it}}{\partial \log Z_{it}} = -21%.
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\end{align*}
\]

- \( \frac{\partial \log W_{it}}{\partial \log T_{it}} > 0 \) due to productivity effect on MPL (requires \( \gamma < \infty, \epsilon < \infty \))
- \( \frac{\partial \log W_{it}}{\partial \log D_{it}} > 0 \) due to scale effect (requires \( \gamma < \infty, \sigma < \infty, \epsilon < \infty \))
- \( \{\omega_{it}, Z_{it}\} \) shocks generate productivity and substitution effects, e.g. \( \text{sgn} \left( \frac{\partial \log W_{it}}{\partial \log Z_{it}} \right) = \text{sgn}(\epsilon - \sigma) \)
Passthrough

- Passthrough from first-degree customers:

\[
\frac{\partial \log W_{it}}{\partial \log T_{ct}} = 33\%,
\quad \frac{\partial \log W_{it}}{\partial \log D_{ct}} = 13\%,
\quad \frac{\partial \log W_{it}}{\partial \log \omega_{ct}} = 6\%,
\quad \frac{\partial \log W_{it}}{\partial \log Z_{ct}} = 8\%.
\]

- strength of \{T, D\} shock passthrough similar to own passthrough
Passthrough from second-degree customers:

\[
\frac{\partial \log W_{it}}{\partial \log T_{kt}} = 31\%, \quad \frac{\partial \log W_{it}}{\partial \log \omega_{kt}} = 5\%,
\frac{\partial \log W_{it}}{\partial \log D_{kt}} = 12\%, \quad \frac{\partial \log W_{it}}{\partial \log Z_{kt}} = 7\%
\]

- strength of passthrough decays slowly with downstream relationship degree
Passthrough from first-degree suppliers:

- $\frac{\partial \log W_{it}}{\partial \log T_{st}} = 20\%$
- $\frac{\partial \log W_{it}}{\partial \log \omega_{st}} = 4\%$
- $\frac{\partial \log W_{it}}{\partial \log D_{st}} = -1\%$
- $\frac{\partial \log W_{it}}{\partial \log Z_{st}} = -16\%$

- $\{T, Z\}$ shock passthrough is important, but weaker than own passthrough
- $D$ shock passthrough is negligible because price responds weakly to demand ($\frac{\partial \log p_{st}}{\partial \log D_{st}} \approx 3\%$)
Passthrough from second-degree suppliers:

\[
\frac{\partial \log W_{it}}{\partial \log T_{kt}} = 15\%, \quad \frac{\partial \log W_{it}}{\partial \log \omega_{kt}} = 3\%,
\]

\[
\frac{\partial \log W_{it}}{\partial \log D_{kt}} = -0\%, \quad \frac{\partial \log W_{it}}{\partial \log Z_{kt}} = -12\%
\]

- strength of \( \{ T, Z \} \) shock passthrough decays by about 25% with each upstream relationship degree.
Passthrough from output market competitors:

\[
\frac{\partial \log W_{it}}{\partial \log T_{kt}} = -6\%, \\
\frac{\partial \log W_{it}}{\partial \log \omega_{kt}} = -1\%, \\
\frac{\partial \log W_{it}}{\partial \log D_{kt}} = 0\%, \\
\frac{\partial \log W_{it}}{\partial \log Z_{kt}} = 5\%.
\]

- \{T, Z\} shocks have small but non-zero effects on wages
Passthrough from input market competitors:

\[
\frac{\partial \log W_{it}}{\partial \log T_{kt}} = -1\%, \\
\frac{\partial \log W_{it}}{\partial \log \omega_{kt}} = -0\%, \\
\frac{\partial \log W_{it}}{\partial \log D_{kt}} = -0\%, \\
\frac{\partial \log W_{it}}{\partial \log Z_{kt}} = -0\%
\]

- all effects negligible because mechanisms rely on scale-dependence of output prices
How important is production network heterogeneity for earnings inequality? and how does this compare with importance of other sources of firm heterogeneity?
Decomposing earnings inequality

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Worker earnings depend on the following primitives:

1. worker abilities ($\bar{a}, \hat{a}$)
2. firm productivities ($T, \omega$)
3. production complementarities ($\phi$)
4. amenities ($g$)
5. network linkages ($\Omega^S, \Omega^C, \psi$)
Decomposing earnings inequality

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- Now simulate counterfactual earnings with various dimensions of heterogeneity shut down
  - Shapley value approach used to account for interdependencies in sources of heterogeneity
Decomposing earnings inequality

<table>
<thead>
<tr>
<th>(1) share of earnings variance</th>
<th>(2) worker effect variance</th>
<th>(3) firm effect variance</th>
<th>(4) sorting covariance</th>
<th>(5) interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>earnings variance</td>
<td>100</td>
<td>57.0</td>
<td>10.8</td>
<td>19.8</td>
</tr>
<tr>
<td>worker permanent ability, $\bar{a}_m$</td>
<td>53.8</td>
<td>48.6</td>
<td>-1.5</td>
<td>4.1</td>
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<tr>
<td>worker transient ability, $\tilde{a}_{mt}$</td>
<td>13.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>supplier network, ${m_{ijt}, \psi_{ijt}}_{j \in \Omega^S_t}$</td>
<td>11.9</td>
<td>0.9</td>
<td>7.9</td>
<td>2.7</td>
</tr>
<tr>
<td>customer network, ${m_{jit}, \psi_{jit}}_{j \in \Omega^C_t}$</td>
<td>8.6</td>
<td>-0.1</td>
<td>6.7</td>
<td>1.5</td>
</tr>
<tr>
<td>firm productivities, ${T_{it}, \omega_{it}}$</td>
<td>6.1</td>
<td>7.5</td>
<td>-4.3</td>
<td>3.3</td>
</tr>
<tr>
<td>production complementarities, $\phi_i$</td>
<td>4.6</td>
<td>-4.0</td>
<td>-2.7</td>
<td>8.6</td>
</tr>
<tr>
<td>amenities, $g_i(\cdot)$</td>
<td>1.2</td>
<td>-0.4</td>
<td>3.6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Network heterogeneity accounts for 21% of total earnings variance
- supplier heterogeneity slightly more important than customer heterogeneity
- in contrast, heterogeneity in own-firm characteristics explain around 12%

Bernard et al (2020) also find network heterogeneity explains more of var (sales) than own-firm primitives
Conclusion

- Matched employer-employee and firm-to-firm datasets:
  - allow simultaneous study of disaggregated worker and firm outcomes
  - becoming more widely available to researchers (e.g. Turkey, Costa Rica, Ecuador)

- We provide a quantitative framework + estimation methodology for studying these data
  - with heterogeneous firms/workers/network and labor market power

- Network linkages matter for the passthrough into earnings from:
  - productivity and demand shocks to customers
  - productivity and material cost shocks to suppliers
  - productivity and material cost shocks to output market competitors

- Network heterogeneity explains around one-fifth of earnings variance

- Extensions using the model + data:
  - outsourcing (with David Price) – firms hire labor or source labor indirectly from suppliers
  - automation (with Bradley Setzler) – firms have access to imported labor-replacing “robots”
We can write the wage equation as:

$$\log \tilde{w}_{imt} = \theta_i \log a_m + \log W_i + \log \hat{a}_{mt}$$

where \( \log \tilde{w}_{imt} = \log w_{imt} - \frac{1}{1+\gamma} \left( \log E_{it}^L - \mathbb{E}_t [\log E_{it}^L] \right) \) and \( \log W_i \equiv \mathbb{E}_t [\log \eta W_{it}] \)

**Identification: worker and firm effects**
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- BLM show that \( \{\theta_i, W_i\} \) are identified from the following moment condition:

\[
E \left[ \frac{1}{\theta_j} (\log \tilde{w}_{jm,t+1} - \log W_j) - \frac{1}{\theta_i} (\log \tilde{w}_{im,t} - \log W_i) \mid m \in M_{t,t+1}^{i\rightarrow j} \right] = 0
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where \( M_{t,t+1}^{i\rightarrow j} \) is the set of workers that move from \( i \) to \( j \) between \( t \) and \( t + 1 \)
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Given identification of \( \{ \theta_{k(i)}, W_{k(i)} \} \), permanent worker ability is identified from:

\[
\log \bar{a}_m = E \left[ \frac{\log \tilde{w}_{imt} - \log W_{k(i)}}{\theta_{k(i)}} \right]
\]

Time-varying firm effect \( W_{it} \) recovered using \( \log W_{it} = \log W_{k(i)} + \frac{1}{1+\gamma} \left( \log E_{it}^L - \mathbb{E}_t \left[ \log E_{it}^L \right] \right) \)
We can write sales from firm $j$ to firm $i$ as:

$$\log R_{ijt} = \log \tilde{\Delta}_{it} + \log \tilde{\Phi}_{jt} + \log \tilde{\psi}_{ijt}$$

where $\tilde{\Delta}_{it} \equiv \Delta_{it}\psi_{it}$ and $\tilde{\Phi}_{jt} \equiv \Phi_{jt}\psi_{jt}$ are (transformed) buyer and seller effects.
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Following Bernard et al (2019):

- buyer-seller matching is independent of $\tilde{\psi}_{ijt}$, hence $E \left[ \log \tilde{\Delta}_{it} \log \tilde{\psi}_{ijt} \right] = E \left[ \log \tilde{\Phi}_{jt} \log \tilde{\psi}_{ijt} \right] = 0$
- $\tilde{\Delta}_{it}$ identified from purchases by $i$ controlling for total sales of $i$’s suppliers
- $\tilde{\Phi}_{jt}$ identified from sales by $j$ controlling for total material cost of $j$’s customers
Identification: relationship capability

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- To recover $\psi_{it}$, use share of $i$’s total sales $s_{it}^{\text{net}}$ from network (excluding final sales):

\[
\psi_{it} = E_t \left( \frac{s_{it}^{\text{net}}}{1 - s_{it}^{\text{net}}} \right) \frac{1}{\sum_{j \in \Omega} \Delta_{jt} \psi_{jit}}
\]

- Given $\psi_{it}$, can recover network demand $\Delta_{it}$ and efficiency $\Phi_{it}$, and hence cost $Z_{it}$
Standard CES production function with labor-augmenting productivity $\omega_{it}$ implies:

$$\log \left( \frac{E_{it}^M}{E_{it}^L} \right) = \text{const.} + (1 - \epsilon) \log \left( \frac{P_{it}^M}{P_{it}^L} \right) + (1 - \epsilon) \log \omega_{it}$$

relative M-L expenditure  
relative M-L unit price
Identification: labor-materials substitution elasticity $\epsilon$

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- Given input prices $\{P^M_{it}, P^L_{it}\}$, Doraszelski and Jaumandreu (2018) develop approach to identify $\{\epsilon, \omega_{it}\}$
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- Current literature approach to measurement of input prices:
  - $P_{it}^L =$ avg. local market wage (e.g. Oberfield-Raval (2020)), avg. firm wage (e.g. DJ (2018))
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What are the correct price measures when both workers and inputs are heterogeneous?
"Price of labor" can be estimated from decomposition of worker earnings into worker and firm effects:

\[
\log w_{mit} = \log \eta + \theta_i \log \bar{a}_m + \log W_{it} + \log \hat{a}_{mt}
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- theoretically correct price of labor is \( P^L_{it} = W_{it} \), i.e. the firm effect
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\]

- theoretically correct price of materials is \( P_{it}^M = Z_{it} = \left[ \sum_{j \in \Omega_i} \Phi_{jt} \psi_{ijt} \right]^{1-\sigma} \), i.e. aggregation of seller effects across suppliers adjusted by relationship productivity
Pass-through of changes in wage bill for firm $i$ to wages for employee $m$:

$$
\Delta \log w_{mit} = \frac{1}{1 + \gamma} \Delta \log E_{it}^L + \frac{1}{1 + \gamma} \Delta \log e_{it}^L + \Delta \log \hat{a}_{mt}
$$

- $\Delta \log w_{mit}$: change in wage bill
- $\Delta \log E_{it}^L$: wage bill measurement error
- $\Delta \log e_{it}^L$: worker shock

Since worker and firm shocks are orthogonal, so are $\Delta \log E_{it}^L$ and $\Delta \log \hat{a}_{mt}$.

If measurement error is MA($k$) and greater lags of $\Delta \log E_{it}^L$ are valid instruments, results are robust to omitting worker earnings from firm wage bill.
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- Results are robust to omitting worker $m$ earnings from firm wage bill
Identification: elasticity of substitution $\sigma$

The first-order conditions from the firm’s profit maximization problem imply:

$$p_{it}x_{it} = \frac{\sigma}{\sigma - 1} \times \left[ \frac{1}{\eta} - E_{it}^L + E_{it}^M \right]$$

where $\eta$ corrects for wage markdown and increasing marginal cost.
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Hence we identify \( \sigma \) from the following moment condition:

\[
\sigma = \mathbb{E} \left[ \frac{R_{it}}{R_{it} - \frac{1}{\eta} E_{it}^L + E_{it}^M} \right]
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interpreting empirical deviations from the FOC as measurement error.
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Intuition: if firms make high profit fixing output, then demand must be inelastic

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Intuition: if firms make high profit fixing output, then demand must be inelastic.

- when \( \gamma \to \infty, \eta \to 1 \) and \( \sigma \) is identified from the population average sales-profit ratio.

Note that leveraging firm FOCs allows identification of \( \sigma \) without needing identification of:

- firm-specific demand shifters \( D_{it} \)
- firm-specific prices and marginal costs

Approach is robust to additional firm-specific final demand heterogeneity (another component of \( D_{it} \)).
Follow LMS (2021) in restricting amenities as follows:

\[ g_i(a) = \bar{g}_i \bar{g}_{k(i)}(\bar{a}) \]
Follow LMS (2021) in restricting amenities as follows:

\[ g_i(a) = \tilde{g}_i \tilde{g}_{k(i)}(\bar{a}) \]

Cluster-ability component of amenities can be identified from:

\[ \tilde{g}_{k(i)}(\bar{a}) = (\bar{a})^{-\theta_k} [\Lambda_{kt}(\bar{a})]^{\frac{1}{\gamma}} \]

where \( \Lambda_{kt}(\bar{a}) \) is share of workers of permanent ability \( \bar{a} \) employed by firms in cluster \( k \).
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Firm-specific component of amenities can be identified from:

\[ \tilde{g}_i = \frac{1}{W_{it}} \left( \frac{\tilde{\Lambda}_{it}}{\tilde{\Lambda}_{k(i)t}} \right)^{\frac{1}{\gamma}} \]

where \( \tilde{\Lambda}_{it} \) and \( \tilde{\Lambda}_{k(i)t} \) are shares of employment (of all worker types) accounted for by firm \( i \) and cluster \( k(i) \)
We can express the time-varying firm effects that we recover from BLM as:

\[ W_{it} = F_i \left[ \{ T_{jt} \}_{j \in \Omega^F} | \Theta - T \right] \]

- \( \Theta - T \): set of model primitives other than TFPs
- \( \{ F_i \}_{i \in \Omega^F} \): set of known functions that depend on structural relationships of model
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Given identification of \( \Theta - T \), this provides a set of \(|\Omega^F|\) moments for exact identification of TFP

Note that without intermediates (\( \lambda \to 1 \)), \( \log W_{it} \) is linear in \( \log T_{it} \) and identification is straightforward

With intermediates, \( F_i \) is generally not log-linear and depends on \( T_{jt} \) for \( j \neq i \)

- hence need numerical approach in practice for estimation
Worker-firm sorting:

- Worker-firm sorting:

- note: “BLM cluster” indicates $k$-means cluster of firm based on percentiles of within-firm earnings distribution.

- worker ability quantile: 1 2 3 4 5

- model

- data

- firm BLM cluster

- employment share (%)
### Model Fit

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>model</th>
</tr>
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<tbody>
<tr>
<td><strong>aggregate:</strong></td>
<td></td>
<td></td>
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<tr>
<td>labor share of value-added</td>
<td>0.24</td>
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<td>value-added share of sales</td>
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<td>earnings 75/25 ratio</td>
<td>2.91</td>
<td>2.76</td>
</tr>
<tr>
<td>earnings 75/50 ratio</td>
<td>1.81</td>
<td>1.79</td>
</tr>
<tr>
<td>earnings 50/25 ratio</td>
<td>1.61</td>
<td>1.54</td>
</tr>
</tbody>
</table>
## Model Fit

### firm-level:

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>model</th>
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</thead>
<tbody>
<tr>
<td>log sales s.d.</td>
<td>1.60</td>
<td>1.27</td>
</tr>
<tr>
<td>avg. log wage s.d.</td>
<td>0.57</td>
<td>0.50</td>
</tr>
<tr>
<td>corr(log sales, avg. log wage)</td>
<td>0.53</td>
<td>0.73</td>
</tr>
<tr>
<td>corr(log sales, s.d. log wage)</td>
<td>0.46</td>
<td>0.74</td>
</tr>
<tr>
<td>corr(log sales, log out-degree)</td>
<td>0.54</td>
<td>0.70</td>
</tr>
<tr>
<td>corr(log sales, log in-degree)</td>
<td>0.78</td>
<td>0.75</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>corr(log sales, cust. log sales)</td>
<td>-0.28</td>
<td>-0.57</td>
</tr>
<tr>
<td>corr(log in-degree, cust. log in-degree)</td>
<td>-0.37</td>
<td>-0.54</td>
</tr>
<tr>
<td>corr(avg. log wage, cust. avg. log wage)</td>
<td>0.12</td>
<td>0.15</td>
</tr>
</tbody>
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<tr>
<td><strong>firm-to-firm-level:</strong></td>
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Earnings Decomposition

Earnings: \( \log w_{\text{int}} = \tilde{\theta} (\log \bar{a}_m - \log \bar{a}) + \log W_{it} \omega_{it} + \theta_i \log \bar{a} + (\theta_i - \bar{\theta}) (\log \bar{a}_m - \log \bar{a}) + \log \hat{a}_{mt} \)

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1. var(worker effect)</td>
<td>57</td>
<td>53</td>
</tr>
<tr>
<td>2. var(firm effect)</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>3. cov(worker effect,firm effect)</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>4. interactions</td>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>5. residual</td>
<td>14</td>
<td>22</td>
</tr>
</tbody>
</table>