The Cobb Douglas marriage matching function: Marriage matching with peer and scale effects.

Ismael Mourifié and Aloysius Siow*
University of Toronto

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Abstract

Empirical economists often use panel data across states and time to estimate models of marital behavior. In addition to changes in population supplies and policy changes, they are also concerned with the effects of endogenous social influences on such behavior. This paper proposes the Cobb Douglas (CD) marriage matching function which facilitates difference-in-differences estimation, has a behavioral interpretation and also allows for peer and scale effects in marital behavior. The CD marriage matching function encompasses the Choo and Siow (2006a, CS), Dagsvik (2000), Menzel (2015), Chiappori, Salanié and Weiss (1016) marriage matching functions, and CS with peer and scale effects (CSPE). The CD marriage matching function is estimated on marriage and cohabitation of the white population in the US from 1990 to 2010. Scale effects are present in US marriage markets. CSPE is not rejected by the data. However these scale effects are not sufficient to explain the large recent declines in the gains to marriage.

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Introduction

Since the seventies, marital behavior in the United States has changed significantly.[1] First, for most adult groups, marriage rates have fallen. Second, starting from a very low initial rate, cohabitation rates have risen significantly. Because the initial cohabitation rates were so low, the rise in cohabitations did not compensate for the fall in marriages. Thus, the fraction of adults who are unmatched, i.e. not married or cohabiting, have risen significantly. Evidence for these trends for women and men between ages 26-30 and 28-32 respectively are shown in Figure 1 in Appendix A.

Researchers have investigated different causes for these changes including changes in reproductive technologies as well as access to them, changes in family laws, changes in household technologies, changes in earnings inequality and changes in welfare regimes.[2] To date, researchers have not found observable mechanisms which would largely explain the recent changes in marital behavior, leaving room to investigate endogenous mechanisms such as peer effects.

The objective of this paper is to provide an elementary behavioral framework which can be used to analyze both exogenous and endogenous (peer) mechanisms which affect marital behavior. We then use this framework to provide a transparent accounting of changes in recent US marital behavior. We will show that there is quantitatively significant peer and scale effects in US marriage markets. But this endogenous behavior is not sufficient to explain the recent declines in US marriage rates and increases in cohabitation rates.

Consider a static marriage market. There are I, $i = 1, .., I$, types of men and J, $j = 1, .., J$, types of women. Let M be the population vector of men where a typical element is m_i, the supply of type i men. F is the population vector of women where a typical element is f_j, the supply of type j women. Each individual can choose to enter a relationship in the form of either marriage or cohabitation, $r = [\mathcal{M}, \mathcal{C}]$, and a partner (by type) of the opposite sex for the relationship. An unmatched individual chooses a partner of type 0.

Let θ be a vector of parameters. A marriage matching function, is a R_+^{2IJ}

[1] Lundberg and Pollak (2013) has a longer and broader description of marital changes in the US.

[2] E.g. Burtless (1999); Choo and Siow (2006a); Fernandez, Guner and Knowles (2005); Fernandez-Villaverde, et. al. (2014); Goldin and Katz (2002); Greenwood, et. al. (2012, 2014); Lundberg and Pollak (2013); Moffitt, et. al. (1998); Stevenson and Wolderers (2007); Waite and Bachrach (2004).

vector valued function $\mu(M, F, \theta)$ whose typical element is μ_{ij}^r, the number of (r, i, j) relationships. μ_{0j} and μ_{i0} are the numbers of unmatched women and men respectively. μ_{ij}^r have to satisfy the following $I + J$ accounting identities:

$$\sum_{j=1}^{J} \mu_{ij}^{\mathcal{M}} + \sum_{j=1}^{J} \mu_{ij}^{\mathcal{C}} + \mu_{i0} = m_i, \quad 1 \leq i \leq I \tag{1}$$

$$\sum_{i=1}^{I} \mu_{ij}^{\mathcal{M}} + \sum_{i=1}^{I} \mu_{ij}^{\mathcal{C}} + \mu_{0j} = f_j, \quad 1 \leq j \leq J \tag{2}$$

$$\mu_{0j}, \mu_{i0} \geq 0, \quad 1 \leq j \leq J, 1 \leq i \leq I.$$

Recently, Choo and Siow (2006a, 2006b; hereafter CS) used McFadden's (1973) random utility model to model spousal demand in a transferable utility model of the marriage market, in order to obtain an empirically tractable marriage matching function. General equilibrium and population supplies effects on μ_{ij}^r are fully absorbed by the numbers of unmatched men and women of each type, μ_{0j} and μ_{i0}. The CS marriage matching function is:

$$\ln \frac{\mu_{ij}^r}{\sqrt{\mu_{i0}\mu_{0j}}} = \pi_{ij}^r \ \forall\ (r, i, j)$$

CS interprets π_{ij}^r as the expected gain in utility to a randomly chosen (i, j) pair in relationship r relative to the alternative of them remaining unmatched. The CS marriage matching function satisfies constant returns to scale in population supplies (constant returns to scale), meaning that, holding the type distributions of men and women fixed, increasing market size has no effect on the probability of forming a match (i, j) in a relationship r. Also, the marginal effects of μ_{i0} and μ_{0j} on μ_{ij}^r are the same in the CS marriage matching function, i.e. symmetric effect.

Ignoring cohabitation, retaining constant returns to scale, Chiappori, Salanié and Weiss (2016; hereafter CSW) relaxed the symmetric effect of the unmatched in CS to obtain:

$$\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{i0}^{\alpha}\mu_{0j}^{1-\alpha}} = \pi_{ij}^{\mathcal{M}} \ \forall\ (r, i, j).$$

Also ignoring cohabitation, Dagsvik (2000), Dagsvik et al. (2001), and Menzel (2015) study non-transferable utility models of the marriage market to obtain what we denote the DM marriage matching function:

$$\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{i0}\mu_{0j}} = \pi_{ij}^{\mathcal{M}} \; \forall \, (r, i, j).$$

DM has increasing returns to scale in population supplies. The symmetric effect of the unmatched in the marriage matching function is retained.

When we extend the CS, CSW, and DM marriage matching function to additional types of relationships, the log odds of the numbers of different types of relationships, $\ln(\mu_{ij}^{\mathcal{M}}/\mu_{ij}^{\mathcal{C}})$, is independent of the population supplies m_i, f_j. Arciadiacono, et. al. (2010) shows that independence does not hold for sexual versus non-sexual boy girl relationships in high schools. It does not always hold in this paper for marriage versus cohabitation.

Building on the above, this paper proposes the Cobb Douglas (hereafter CD) marriage matching function:

$$\ln \mu_{ij}^{r} = \pi_{ij}^{r} + \alpha_{ij}^{r} \ln \mu_{i0} + \beta_{ij}^{r} \ln \mu_{0j}; \; \alpha_{ij}^{r}, \beta_{ij}^{r} > 0 \; \forall \, (r, i, j) \tag{3}$$

The CD marriage matching function nests a large class of behavioral marriage matching functions and has some useful properties as we will discuss in details in section 2.

While the equations (3) are in the CD form, they are not standard production functions.[3] Rather, they form a set of equilibrium relationships which defines the CD marriage matching function. Compared with the other behavioral marriage matching functions above, the CD marriage matching function relaxes constant returns to scale and symmetry of the unmatched on the marriage matching function and the independence restriction. Our first contribution is to show that there is a class of behavioral marriage matching function which has these properties. The second main contribution of the paper is to show that a marriage matching function with peer effects is in this class. There are two related reasons to incorporate peer effects in marital behavior. First as discussed above, there are conceptual reasons to consider endogenous mechanisms in marital behavior. Second, there is already some empirical evidence to support peer effects in marital behavior (E.g. Adamopoulou (2012); Drewianka (2003); Waite, et. al.

[3]The standard Cobb Douglas model, $\ln \mu_{ij}^{r} = \alpha_{ij}^{r} \ln m_i + \beta_{ij}^{r} \ln f_j + \pi_{ij}^{r}$, is not a well behaved MMF. In general, it will not satisfy the accounting relationships (1) and (2). Nor does it have spillover effects.

(2000); Fernandez-Villaverde, et. al. (2014)).[4]

Our main behavioral specification generates a marriage matching function which is a special testable case of the CD marriage matching function, namely the CS with peer effects marriage matching function (CSPE hereafter). The CD marriage matching function also nests other marriage matching functions as special cases. Since the special cases include frictionless transferable utility models and non-transferable utility models, and although we are partial to CSPE, we propose the CD marriage matching function precisely because we do not want to insist on a particular behavioral model of the marriage market.

CSPE considers the peer specification where individual utilities are affected by the total number of individuals of the same type who choose the same action. We also consider an alternative peer effects specification which is more in the spirit of Brock and Durlauf (2001, hereafter BD). There, the dependence of individual utilities on their peers is captured by the fraction/share (rather than number) of individuals again of the same type who choose the same action.

The BD marriage matching function does not strictly belong to the CD marriage matching function class. It is nested in Mourifié (2016) where its analytic properties are discussed. His model also nests Choo's dynamic marriage matching function.

Our CD marriage matching function is fully consistent with the dynamic Choo marriage matching function and Mourifié (2016). Adding a time superscript t and using a dynamic interpretation, π_{ij}^{rt} in equation (3) is the expected discounted value of the gains to entering relationship r by an (i, j) couple relative to remaining unmatched at time t. If the couple remained unmatched at time t, they can re-enter the marriage market in the future. Thus there is no conceptual difficulty with estimating equation (3) period by period in a dynamic setting.

Another important related paper is Galichon et al. (2014, 2016); who studied a model with an imperfect transfer technology, but does not allow for external-

[4] Marriage and cohabitation are costly individual investments and commitments. Individuals who never married or cohabitated are not likely to be very confident of their payoffs from these relationships. Thus it is reasonable to expect that individuals will be affected by the relationship choices of their peers. Moreover, cohabitation is a relatively new form of socially accepted relationship in the US. The US census first asked about cohabitating relationships in 1990. So peer effects may be more salient for cohabitation compared with marriage (E.g. Thornton and Young-DeMarco (2001)).

ities like peer effects. They proposed a marriage matching function which is qualitatively motivated by their behavioral model and related to our CD marriage matching function but with some clear important distinctions, that we clarify in Appendix B.1.

Furthermore, we provide an elementary difference-in-differences strategy which can be used to analyze and estimate our proposed CD marriage matching function.

Our last contribution is empirical. We estimate the CD and BD marriage matching function with marriage and cohabitation data across states for white women and men between ages 22-52 and 20-50 respectively from the US Censuses in 1990 and 2000, and the American Community Surveys around 2010. Men and women are differentiated by their age range and educational attainment. Our empirical results show that:

1. Cross section variations in state populations are uncorrelated with marriage rates across states which lead to constant returns to scale parameter estimates.[5] Thus it is important to control for state and year effects when estimating scale and peer effects.

2. A simplified CD marriage matching function with relationship match (r, i, j), state and year fixed effects, provides a reasonably complete and parsimonious description of the US marriage market by state from 1990 to 2010.

3. There are estimated scale effects in US marriage markets, close to the DM model. Constant returns to scale is rejected.

4. CSPE is not rejected. The behavioral BD marriage matching function is rejected.

5. Consistent with CSW and other observers, there were large falls in the gains to marriage and large increases in the gains to cohabitation between 1990 and 2010.

Since we estimated large falls in the gains to marriage and large rises in the gains to cohabitation, the estimated quantitatively significant peer and scale

[5] Using only cross section variation in populations in Medieval Italy, China and the US, Botticini and Siow (2008) could not reject CRS in marital behavior. We explain their finding here.

effects were not large enough to explain the decline in marriage and rise of cohabitation.

The empirical work also shows that the CD marriage matching function is a useful framework for testing behavioral models of the marriage market. We do other robustness checks in the paper: (1) For this version of the paper, we use lagged population sizes two decades before to instrument for current populations and sizes of the unmatched. (2) We added population supplies of adjacent types as instruments. (3) We allowed for heterogenous responses of the unmatched by match types. In all cases, we obtain similar estimates as before.

Outline.

The remainder of the paper is organized as follows. Section 1 introduces and discusses the properties of behavioral matching models with peer effects which have the CD marriage matching function form. Section 2 presents the CD marriage matching function and discusses existence and uniqueness of the equilibrium. Section 3 presents our difference-in-difference identification and estimation strategy. Section 5 discusses the empirical application. The last section concludes. Proofs of the main results are collected in the appendix.

1 Marriage matching with peer effects.

The section provides a behavioral model of the marriage market which can rationalize the CD marriage matching function, equation (3). In this model every individual can decide to cohabit, marry or remain unmatched. For a type i man to match with a type j woman in relationship r, he must transfer to her a part of his utility that he values as τ_{ij}^r. The woman values the transfer as τ_{ij}^r. τ_{ij}^r may be positive or negative.

There are $2 \times I \times J$ matching sub-markets for every combination of relationship, and types of men and women. A matching market clears when, given equilibrium transfers τ_{ij}^r, the demand by men of type i for type j women in the relationship r is equal to the supply of type j women for type i men in the relationship r for all (r, i, j). To implement the above framework empirically, we adopt the extreme value random utility model of McFadden (1973) to generate market demands for matching partners. Each individual considers matching with a member of the opposite gender.

Our two-sided random utilities model has one new feature: we will model how marital decisions of peers may affect individual utilities of being matched. However, how to correctly specify the peer effects has always been a challenging question in the social interaction literature. In the spirit of Bramoullé and Kranton (2007), Galeotti et al (2009), and Calvo-Armengol et al (2009), we mainly consider the specification where individual utilities are affected by the total number of individuals of the same type who choose the same action. Subsequently, following Brock and Durlauf (2001), we will also investigate a specification where the dependence of individual utilities on their peers is captured by the fraction/share (rather than number) of individuals again of the same type who choose the same action.[6]

Following Choo and Siow (2006b), the utility of a match (i, j) may differ depending on whether they choose a relationship of type r and r'.

Let the utility of male g of type i who matches a female of type j in a relationship r be:

$$U^r_{ijg} = \tilde{u}^r_{ij} + \phi^r_i \ln \mu^r_{ij} - \tau^r_{ij} + \varsigma^r_{ijg}, \tag{4}$$

where $\tilde{u}^r_{ij} + \phi^r_i \ln \mu^r_{ij}$: Systematic gross return to a male of type i matching to a female of type j in relationship r.

ϕ^r_i: Coefficient of peer effects for relationship r, $0 \leq \phi^r_i \leq 1$.

μ^r_{ij}: Equilibrium number of (r, i, j) relationships.

τ^r_{ij}: Equilibrium transfer made by a male of type i to a female of type j in relationship r.

ς^r_{ijg}: denotes the errors terms (idiosyncratic payoffs) which are assumed to be i.i.d. random variables distributed according to the extreme value Type-I (Gumbel) distribution. It is worth noting that the errors are assumed to be also independent across genders. Due to the peer effects, the net systematic return is increased when more type i men are in the same relationships. It is reduced when the equilibrium transfer τ^r_{ij} is increased.

And $\widetilde{u}_{i0} + \phi^0_i \ln \mu^0_{i0}$ is the systematic payoff that type i men get from remaining unmatched. We allow the peer effect to differ by relationship. For example, unmarried individuals spend more time with their unmarried friends

[6] Galeotti et al. (2009) refer to those two types of specifications as utilities depends on "sums of peer actions" vs "average of peers's actions" (see their Examples 1 and 3). Liua et al. (2014) refer to them as "aggregate" vs "average" peer effect models. See also Ghiglino and Goyal (2010) for a discussion on those two type of specifications.

than married individuals with their married friends. On the other hand, due to their higher shadow cost of time, married individuals may not value interacting with their peers as much.

Our peer effects specification is chosen for analytic and empirical convenience. We want CSPE to be nested in the CD marriage matching function. We also want CSPE to be testable.

Individual g will choose according to:

$$U_{ig} = \max_{j,r}\{U_{i0g}, U_{i1g}^{\mathcal{M}}, ..., U_{ijg}^{\mathcal{M}}, ..., U_{iJg}^{\mathcal{M}}, U_{i1g}^{\mathcal{C}}, ..., U_{ijg}^{\mathcal{C}}, ..., U_{iJg}^{\mathcal{C}}\}.$$

Let $(\mu_{ij}^r)^d$ be the number of (r, i, j) matches demanded by i type men and $(\mu_{i0})^d$ be the number of unmatched i type men. Following the well known McFadden result, we have:

$$\begin{aligned} \frac{(\mu_{ij}^r)^d}{m_i} &= \mathbb{P}(U_{ijg}^r - U_{ikg}^{r'} \geq 0, k = 1, ..., J; r' = (\mathcal{M}, \mathcal{C})) \\ &= \frac{e^{\tilde{u}_{ij}^r + \phi_i^r \ln \mu_{ij}^r - \tau_{ij}^r}}{e^{\tilde{u}_{i0} + \phi_i^0 \ln \mu_{i0}} + \sum_{r' \in \{\mathcal{M}, \mathcal{C}\}} \sum_{k=1}^J e^{\tilde{u}_{ik}^{r'} + \phi_i^{r'} \ln \mu_{ik}^{r'} - \tau_{ik}^{r'}}}, \end{aligned} \tag{5}$$

where m_i denotes the number of men of type i. Using (5) we can easily derive the following relationship:

$$ln \frac{(\mu_{ij}^r)^d}{(\mu_{i0})^d} = \tilde{u}_{ij}^r - \tilde{u}_{i0} + \phi_i^r \ln \mu_{ij}^r - \phi_i^0 \mu_{i0} - \tau_{ij}^r. \tag{6}$$

The above equation is a quasi-demand equation by type i men for (r, i, j) relationships.

The random utility function for women is similar to that for men except that in matching with a type i man in an (r, i, j) relationship, a type j woman receives the transfer, τ_{ij}^r. Let $\tilde{v}_{ij}^r + \Phi_j^r \ln \mu_{ij}^r$ denote the systematic gross gain that type j women get from matching with type i men in the relationship r. Φ_j^r, $0 \leq \Phi_j^r \leq 1$, is the woman peer effect coefficient in relationship (r, i, j). And $\tilde{v}_{0j} + \Phi_j^0 \ln \mu_{0j}^0$ is the systematic payoff that type j women get from remaining unmatched. Let $(\mu_{ij}^r)^s$ be the number of (i, j) matches offered by j type women for the relationship r and $(\mu_{0j})^s$ the number of type j women who want to remain unmatched. The quasi-supply equation of type j women for (r, i, j) relationships is given by:

$$\ln \frac{(\mu_{ij}^r)^s}{(\mu_{0j})^s} = \tilde{v}_{ij}^r - \tilde{v}_{0j} + \Phi_j^r \ln \mu_{ij}^r - \Phi_j^0 \ln \mu_{0j} + \tau_{ij}^r. \tag{7}$$

The matching market clears when, given equilibrium transfers τ_{ij}^r, the demand of type i men for (r,i,j) relationships is equal to the supply of type j women for (r,i,j) relationships for all (r,i,j):

$$(\mu_{ij}^r)^d = (\mu_{ij}^r)^s = \mu_{ij}^r. \tag{8}$$

Substituting (8) into equations (6) and (7) we get:

$$\ln \mu_{ij}^r = \frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r} \ln \mu_{i0} + \frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r} \ln \mu_{0j} + \pi_{ij}^r \tag{9}$$

where $\pi_{ij}^r = \frac{\tilde{u}_{ij}^r - \tilde{u}_{i0} + \tilde{v}_{ij}^r - \tilde{v}_{0j}}{2-\phi_i^r-\Phi_j^r}$.

The above is the CS model with peer effects, the CSPE marriage matching function. Now, let's present different properties of the CSPE marriage matching function.

1.1 Properties of the CSPE.

First, using equation (9) we have the following result.

Proposition 1 *The CSPE marriage matching function imposes the following testable restriction on the CD marriage matching function parameters:*

$$\frac{\alpha_{ij}^r}{\alpha_{ij}^{r'}} = \frac{\beta_{ij}^r}{\beta_{ij}^{r'}}. \tag{10}$$

This restriction also implies that if the coefficient on unmatched men, $\ln \mu_{i0}$, is larger (smaller) than the coefficient on unmatched women, $\ln \mu_{0j}$, in the CD marriage equation, (i.e. $\ln \mu_{ij}^{\mathcal{M}} = \alpha_{ij}^{\mathcal{M}} \ln \mu_{i0} + \beta_{ij}^{\mathcal{M}} \ln \mu_{0j} + \pi_{ij}^{\mathcal{M}}$) then the coefficient on unmatched men, $\ln \mu_{i0}$, is larger (smaller) than the coefficient on unmatched women, $\ln \mu_{0j}$, in the CD cohabitation equation, (i.e. $\ln \mu_{ij}^{\mathcal{C}} = \alpha_{ij}^{\mathcal{C}} \ln \mu_{i0} + \beta_{ij}^{\mathcal{C}} \ln \mu_{0j} + \pi_{ij}^{\mathcal{C}}$). Without a behavioral model, there is no reason to expect Proposition 1 to be true.

Now, we will show how several previous marriage matching functions can be interpreted as special case of the CSPE model.

1.1.1 CS or Homogenous peer effects model.

The CS marriage matching function is observationally equivalent to having no peer effect coefficients or having all peer effect coefficients are the same:

Indeed, when

$$\phi_i^0 = \Phi_j^0 = \phi_i^r = \Phi_j^r$$

we recover the CS marriage matching function. That is, we have the following result:

Proposition 2 *No peer effect, or homogenous peer effects, generates observationally equivalent marriage matching functions.*

Unlike the standard reflection problem introduced by Manski, non-homogenous peer effects under CSPE are generically detectable:

Corollary 1 *When* $\frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r} \neq \frac{1}{2}$ *and/or* $\frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r} \neq \frac{1}{2}$*, non-homogenous peer effects are present.*

This corollary is related to identification of linear models with non-homogenous peer effects.[7]

1.1.2 CSW or relationship type-independent peer effects.

The CSW marriage matching function is observationally equivalent to peer effects coefficients that are relationship independent, i.e.

$$\phi_i^0 = \phi_i^r, \;\; \Phi_j^0 = \Phi_j^r.$$

Indeed, if you consider that peer effects are gender-specific, we can write $\sigma_i \equiv 1-\phi_i^0 = 1-\phi_i^r$, and $\Sigma_j \equiv 1-\Phi_j^0 = 1-\Phi_i^r$ where σ_i, Σ_j can be interpreted as the standard deviations of idiosyncratic payoffs of type i men and type j women, respectively. In this case, we recover the CSW marriage matching function.

The CSW can also be recovered under the following weaker condition:

$$\phi_i^0 + \Phi_j^0 = \phi_i^r + \Phi_j^r = \phi_i^{r'} + \Phi_j^{r'}.$$

The CSPE marriage matching function with the above corresponding restrictions is observationally equivalent to the CS or CSW marriage matching function.

[7] Blume, et. al. (2015) has a state of the art survey. Also see Djebbari, et. al. (2009).

1.1.3 DM or weaker peer effect for unmatched.

Here we show that the CSPE can also nest some marriage matching functions with non-transferable utilities. Indeed, whenever

$$1 + \Phi_j^0 = 1 + \phi_i^0 = \phi_i^r + \Phi_j^r$$

we have

$$\frac{1 - \phi_i^0}{2 - \phi_i^r - \Phi_j^r} = \frac{1 - \Phi_j^0}{2 - \phi_i^r - \Phi_j^r} = 1$$

and we recover then DM marriage matching function. The above condition imposes $\Phi_j^0 = \phi_j^0 < \min\{\phi_i^r; \Phi_i^r\}$ meaning that the peer effects on relationships are more powerful than peer effects for remaining unmatched. E.g. $\phi_i^0 = \Phi_j^0 = 0$ and $\phi_i^r = \Phi_j^r = \frac{1}{2}$. The DM obey increasing return to scale in population supplies.[8]

Now, it will be convenient to summarize different marriage matching functions existing in the literature and to clarify their relations to the CD marriage matching function.[9]

Models and restrictions on α^r and β^r of CD marriage matching function				
Model	α^r	β^r	π_{ij}^r	Restrictions
CD marriage matching function	α^r	β^r	π_{ij}^r	$\alpha^r \geq 0, \beta^r \geq 0$
CS	$\frac{1}{2}$	$\frac{1}{2}$	π_{ij}^r	$\alpha^r = \beta^r = \frac{1}{2}$
DM	1	1	π_{ij}^r	$\alpha^r = \beta^r = 1$
CSW	$\frac{\sigma}{\sigma+\Sigma}$	$\frac{\Sigma}{\sigma+\Sigma}$	$\frac{\pi_{ij}^r}{\sigma+\Sigma}$	$\alpha, \beta > 0; \alpha + \beta = 1$
CSPE	$\frac{1-\phi^0}{2-\phi^r-\Phi^r}$	$\frac{1-\Phi^0}{2-\phi^r-\Phi^r}$	$\frac{\pi_{ij}^r}{2-\phi^r-\Phi^r}$	$\alpha^r, \beta^r \geq 0, \frac{\alpha^{\mathcal{M}}}{\alpha^{\mathcal{C}}} = \frac{\beta^{\mathcal{M}}}{\beta^{\mathcal{C}}}$

1.1.4 Behavioral interpretation of the CSPE marriage matching function.

As discussed earlier, Proposition 1 is the main restriction of CSPE on the CD marriage matching function. Although individual peer effect coefficients, i.e.,

[8] A reader pointed out that this property follows from results in Dagvik (p. 40; 2000).

[9] Other behavioral MMFs can also be nested in the Cobb Douglas MMF. Dagsvik (2000, Page 43) provides another example of MMF which allows correlation between idiosyncratic payoffs. However, this extension still does not relax the independence assumption, and imposes $1 < \alpha + \beta \leq 2$.

Φ^0, ϕ^0, ϕ^r, and Φ^r are not point identified, economically meaningful information can be learned through the reduced form parameters α^r, β^r. We consider that the coefficient are type-independent to ease the notation.

Corollary 2 *Under CSPE, i.e.,* $\frac{\alpha^r}{\beta^r} = \frac{\alpha^{r'}}{\beta^{r'}}$, $\frac{\alpha^r}{\beta^r} \begin{cases} = 1 \Leftrightarrow \Phi^0 = \phi^0 \\ > 1 \Leftrightarrow \Phi^0 > \phi^0 \\ < 1 \Leftrightarrow \Phi^0 < \phi^0. \end{cases}$

With this result, we can know which gender's value of being unmatched is more sensitive to peer effects. For instance, if the coefficient on unmatched males (α^r) is smaller than that for unmatched females (β^r) for both relationships, then the value that women derive from being unmatched will be more sensitive to peer and scale effects than for men.

Corollary 3 *Under CSPE, i.e.,* $\frac{\alpha^r}{\alpha^{r'}} = \frac{\beta^r}{\beta^{r'}}$, $\frac{\alpha^r}{\alpha^{r'}} \begin{cases} = 1 \Leftrightarrow \phi^{r'} + \Phi^{r'} = \phi^r + \Phi^r \\ > 1 \Leftrightarrow \phi^r + \Phi^r > \phi^{r'} + \Phi^{r'} \\ < 1 \Leftrightarrow \phi^r + \Phi^r < \phi^{r'} + \Phi^{r'}. \end{cases}$

This latter lemma says which type of relationship is more affected by the peer and scale effects. For instance, if the ratio of the coefficient of unmatched men (women) in marriage is larger than the coefficient of unmatched men (women) in cohabitation, then the value that a couple derives from marriage will be more affected by peer and scale effects than for cohabitation.

1.1.5 CSPE relaxes the log odds independence restriction.

As we discussed in the introduction, when we extend the CS, CSW, and DM marriage matching functions to additional types of relationships, the log odds of the numbers of different types of relationships is independent of the population supplies m_i, f_j. Indeed, we have:

$$\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{ij}^{\mathcal{C}}} = \frac{\pi_{ij}^{\mathcal{M}}}{\pi_{ij}^{\mathcal{C}}}.$$

This implies that the ratio between the number of (i, j) marriage matches over the number of (i, j) cohabitation matches remains totally unchanged with the new arrival of any type of men or women. In addition, this is supposed to hold for every pair of relationships, making this implication unlikely to hold in many applications as in Arciadiacono, et. al. (2010).

In fact, we would expect that an important change in the population supplies would affect differently the transfers (prices) mechanism, i.e. $\tau_{ij}^{\mathcal{C}}$, $\tau_{ij}^{\mathcal{M}}$ and that this would lead to a change of this ratio. The CSPE relaxes this restriction. Indeed, using Eq (9) we have:

$$\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{ij}^{\mathcal{C}}} = \frac{(\phi_i^{\mathcal{M}} + \Phi_j^{\mathcal{M}} - \phi_i^{\mathcal{C}} - \Phi_j^{\mathcal{C}})}{(2 - \phi_i^{\mathcal{M}} - \Phi_j^{\mathcal{M}})(2 - \phi_i^{\mathcal{C}} - \Phi_j^{\mathcal{C}})} \left[(1 - \phi_i^0) \ln \mu_{i0} + (1 - \Phi_j^0) \ln \mu_{0j}\right] + \pi_{ij}^{\mathcal{M}} - \pi_{ij}^{\mathcal{C}}. \tag{11}$$

Since μ_{i0} and μ_{0j} appear on the right hand side of (11), the log odds of the number of r to r' relationships will no longer be independent of the populations supplies. Notice that a change of adjacent population supplies $m_{i'}$ or $f_{j'}$ would also affect the ratio through their impact on μ_{i0} and μ_{0j} as showed in the comparative statistics (Theorem 2 in Appendix). However, because the coefficients on unmatched men and women have the same sign, this independence is restricted. We will now study the positive assortative matching (positive assortative matching) patterns.

1.1.6 CSPE and positive assortative matching.

Let the heterogeneity across males (females) be one dimensional and ordered. Without loss of generality, let male (female) ability be increasing in i (j).

As we discussed earlier, we still consider type-independent peer effects:

$$\phi_i^0 = \phi^0; \Phi_j^0 = \Phi^0; \phi_i^r = \phi^r; \Phi_j^r = \Phi^r \tag{12}$$

Then using (9), the local log odds for (r, i, j) is:

$$\begin{aligned} l(r,i,j) = \ln \frac{\mu_{ij}^r \mu_{i+1,j+1}^r}{\mu_{i+1,j}^r \mu_{i,j+1}^r} &= \pi_{ij}^r + \pi_{i+1,j+1}^r - \pi_{i+1,j}^r - \pi_{i,j+1}^r \\ &= \frac{\tilde{u}_{ij}^r + \tilde{v}_{ij}^r + \tilde{u}_{i+1,j+1}^r + \tilde{v}_{i+1,j+1}^r - (\tilde{u}_{i+1,j}^r + \tilde{v}_{i+1,j}^r) - (\tilde{u}_{i,j+1}^r + \tilde{v}_{i,j+1}^r)}{2 - \phi^r - \Phi^r} \end{aligned} \tag{13}$$

According to (13), if the marital output function, $\tilde{u}_{ij}^r + \tilde{v}_{ij}^r$, is supermodular in i and j, then the local log odds, $l(r, i, j)$, are positive for all (i, j), or totally positive of order 2. Statisticians use the latter as a measure of stochastic positive assortative matching. Thus even when peer effects are present, we can test for supermodularity of the marital output function, a cornerstone of Becker's theory of positive assortative matching in marriage. This result generalizes Siow (2015), CSW and Graham (2011).

2 The CD marriage matching function.

The CSPE marriage matching function described in (9) is a special case of a more general class of marriage matching functions defined by equation (3), that we name the Cobb Douglas (CD) marriage matching function.

We now discuss some of its general properties below which are independent of the behavioural models in the previous section:

1. Given population supplies and parameters, the equilibrium marriage matching distribution $\mu(M, F, \theta)$ exists and is unique. It is easy to simulate for policy evaluations.

2. Without restrictions on π_{ij}^r, the marriage matching function fits any observed marital behavior in a single marriage market. In fact, the model must be restricted to obtain identification even with multi-market data. Luckily, identification is transparent. Due to the log-linear estimating equations (3), we do not need to add any identifying restrictions over and above what is common in the empirical literature, which uses state and time variation to estimate different aspects of US marriage market behavior.[10]

3. Estimation is easy. The parameters of the marriage matching function can be estimated using multi-market data by difference in differences and using population supplies as instruments for the unmatched. This shall be clearer in section 3.

The matching equilibrium in this model is characterized by the CD marriage matching function (3) and the population constraint equations (1,2). Appendix B proves the existence and uniqueness of this model.

Following CS, an important simplification in the proof is to first reduce the $2r \times I \times J$ system of non-linear equations to an $I + J$ system of the numbers

[10] E.g. Bitler, et. al. (2004); Chiappori, Fortin and Lacroix (2002); Dahl (2010); Mechoulan (2011), Stevenson and Wolfers (2006); Wolfers (2006).

of unmatched individuals by substituting the CD marriage matching function in equation (3) into the population constraints, (1) and (2), to get:

$$m_i = \mu_{i0} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^{\mathcal{M}}} \mu_{0j}^{\beta_{ij}^{\mathcal{M}}} e^{\pi_{ij}^{\mathcal{M}}} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^{\mathcal{C}}} \mu_{0j}^{\beta_{ij}^{\mathcal{C}}} e^{\pi_{ij}^{\mathcal{C}}}, \text{ for } 1 \leq i \leq I, \tag{14}$$

$$f_j = \mu_{0j} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^{\mathcal{M}}} \mu_{0j}^{\beta_{ij}^{\mathcal{M}}} e^{\pi_{ij}^{\mathcal{M}}} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^{\mathcal{C}}} \mu_{0j}^{\beta_{ij}^{\mathcal{C}}} e^{\pi_{ij}^{\mathcal{C}}}, \text{ for } 1 \leq j \leq J. \tag{15}$$

Although there are $2 \times I \times J$ elements in μ, the analyst only has to first solve a sub-system of $I + J$ non-linear equations whose solution is unique (see Theorem 1 below). Using this two steps approach, the marriage matching function is easy to simulate for policy evaluations.[11]

The following theorem summarizes our existence and uniqueness results:

Theorem 1 *[Existence and Uniqueness of the Equilibrium matching of the CD marriage matching function] For every fixed matrix of relationship gains and coefficients $\beta_{ij}^r; \alpha_{ij}^r \geq 0$, the equilibrium matching of the CD marriage matching function model exists and is unique.*

Notice that using (3),

$$\ln \frac{\mu_{ij}^r}{\mu_{ij}^{r'}} = (\alpha_{ij}^r - \alpha_{ij}^{r'}) \ln(\mu_{i0}) + (\beta_{ij}^r - \beta_{ij}^{r'}) \ln(\mu_{0j}) + \pi_{ij}^r - \pi_{ij}^{r'} \; \forall \, (r, i, j).$$

So, we have the following result:

Lemma 1 *When $(\alpha_{ij}^r - \alpha_{ij}^{r'}) = (\beta_{ij}^r - \beta_{ij}^{r'}) = 0$ as in CS, CSW and DM, the log odd of μ_{ij}^r to $\mu_{ij}^{r'}$ is independent of the sex ratio m_i/f_j. Otherwise the log odd is not independent of the sex ratio.*

Arciadiacono et. al. (2010) show that independence does not hold for sexual versus non-sexual boy girl relationships in high schools. As can be seen in the next section our proposed CS with peer effects model provides a behavioral model which relaxes independence. We can also relax independence under CSW by letting α and $(1 - \alpha)$ be dependent on r.

[11] Feedback from users of the CS MMF (a special case) suggest that a one step numerical solution is difficult to achieve.

Notice that with multi-market data, β^r_{ij} and α^r_{ij} are identified under usual restrictions which will be stated later. However, β^r_{ij} and α^r_{ij} cannot be estimated precisely in datasets where we do not observe enough different markets. Often, researchers will assume that the exponents on the CD marriage matching function are gender and relationship specific but independent of the types of couples, (i, j): $\beta^r_{ij} = \beta^r$ and $\alpha^r_{ij} = \alpha^r$. With multi-market data, type independence, $\beta^r_{ij} = \beta^r$ and $\alpha^r_{ij} = \alpha^r$, is in principle a testable restriction.

Whenever the log odds is not independent of the sex ratio it is possible in some cases to analytically predict how the log odds vary with the sex ratio: Building on Graham (2013), we derive in Theorem 2 (relegated in Appendix C.2) various comparative statics results for the CD marriage matching function model. We may state the following results:

Proposition 3 *Variation of the log ratio* $\ln \frac{\mu^{\mathcal{M}}_{ij}}{\mu^{\mathcal{C}}_{ij}}$*:*
If $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ *and* $\beta^{\mathcal{C}} > \beta^{\mathcal{M}}$ *we have*

1. $\frac{\partial}{\partial m_i}[\ln \frac{\mu^{\mathcal{M}}_{kj}}{\mu^{\mathcal{C}}_{kj}}] \geq \begin{cases} > 0 & \text{if } k \neq i \\ > \alpha^{\mathcal{M}} - \alpha^{\mathcal{C}} & \text{if } k = i, \end{cases} \quad 1 \leq k \leq I$

2. $\frac{\partial}{\partial f_j}[\ln \frac{\mu^{\mathcal{M}}_{ik}}{\mu^{\mathcal{C}}_{ik}}] \leq \begin{cases} < 0 & \text{if } k \neq j \\ < -(\alpha^{\mathcal{M}} - \alpha^{\mathcal{C}}) & \text{if } k = j, \end{cases} \quad 1 \leq k \leq J.$

Observers have often conjectured that men prefer cohabitation to marriage and women prefer the reverse (e.g. Guzzo (2006)). An increase in the sex ratio reduces the bargaining power of men in the marriage market and we would expect the ratio of marriages to cohabitations to increase, and vice versa. Proposition 3 provides sufficient conditions for this conjecture. If we reject independence of the log odds of marriage versus cohabitation with respect to the sex ratio, we can check if the estimated parameters satisfy Proposition 3.

Since different cases of the CD marriage matching function imply the presence of scale effects, we provide a restriction for scale effects. Then:

Proposition 4 (Constant return to scale) *The equilibrium matching distribution of the CD marriage matching function model satisfies the constant returns to scale property if* $\beta^r + \alpha^r = 1$ *i.e.,*

$$\beta^r + \alpha^r = 1 \text{ for } r \in \{\mathcal{M}, \mathcal{C}\} \Rightarrow \sum_{i=1}^{I} \frac{\partial \mu}{\partial m_i} m_i + \sum_{j=1}^{J} \frac{\partial \mu}{\partial f_j} f_j = \mu.$$

The result claims that the CD marriage matching function model exhibits constant return to scale if $\beta^r + \alpha^r = 1$. The proposition generalizes to the result that $\beta_{ij}^r + \alpha_{ij}^r = 1$ for all (r, i, j) implies constant returns to scale.

Notice that further comparative statics results for the CD marriage matching function model provide the following results:

1. For any admissible k and l, the unmatched rate for type l individuals is increasing in the supply of type k individuals of the same gender.

2. For any admissible k and l, the unmatched rate type l individuals is decreasing in the supply of type k individuals of the opposite gender.

These two results show how the number of unmatched at the equilibrium varies with respect to population supply changes. These results were anticipated by Decker et. al. (2012) for the CS marriage matching function and by Graham (2013) for the CSW marriage matching function.

2.1 Identification and Estimation.

Consider the general CD marriage matching function in the presence of independent multi-market data where an isolated marriage market is defined by the state s and time t:

$$\ln \mu_{ij}^{rst} = \alpha_{ij}^r \ln \mu_{i0}^{st} + \beta_{ij}^r \ln \mu_{0j}^{st} + \pi_{ij}^{rst}. \tag{16}$$

This section provides flexible specifications which are identified and can be estimated using a DID instrumental variables methodology. As discussed above, many studies use variations across state and time in marriage markets to estimate models of marital behavior. A maintained assumption in these studies is that the variation in (lagged) population supplies is orthogonal to variation in the payoffs to marital behavior. Otherwise most of the estimates of marital behavior using state and time variation will be inconsistent. In agreement with the empirical research that relies on this assumption, we recognize that there is migration across states. The large number of studies, on different marital outcomes, which have obtained behaviorally plausible estimates, suggest that the orthogonality assumption is empirically reasonable.

Even with multi-market data, the most general CD marriage matching function is not identified. There are $2 \times I \times J \times S \times T$ elements in the observed matching distribution (i.e. μ_{ij}^{rst}) and there are $2\times I\times J\times S\times T+4\times I\times J$ parameters i.e. (π_{ij}^{rst}, α_{ij}^{r}, and β_{ij}^{r}). Therefore, to obtain identification of the general CD marriage matching function we will impose additional standard restrictions on the structure of the gains, i.e. π_{ij}^{rst}.

Assumption 1 1. *(Additive separability of the gain).* $\pi_{ij}^{rst}=\pi_{ij}^{r}+\eta_{ij}^{rs}+\zeta_{ij}^{rt}+\epsilon_{ij}^{rst}$ *where* π_{ij}^{r} *represents the type fixed effect,* η_{ij}^{rs} *the state fixed effect,* ζ_{ij}^{rt} *the time fixed effect, and* ϵ_{ij}^{rst} *the residual terms.*

2. *(Instrumental Variable (IV)).* $\mathbb{E}[\epsilon_{ij}^{rst}|z_{ij}^{11},...,z_{ij}^{ST}]=0$*, where* $z_{ij}^{st}=(m_i^{st},f_j^{st})'$.

Assumption 1 (1) decomposes π_{ij}^{rst} into a match type fixed effect, state fixed effect and time fixed effect, and the error term of the regression, ϵ_{ij}^{rst}. π_{ij}^{r}, η_{ij}^{rs} and ζ_{ij}^{rt} are identified. ϵ_{ij}^{rst} is not identified. Assumption 1 (1) allows us to reduce the number of parameters. When ϵ_{ij}^{rst} increases, the gain to the match increases which will increase μ_{ij}^{rst} and therefore likely reduces μ_{i0}^{st} and μ_{0j}^{st}. Thus μ_{i0}^{st} and μ_{0j}^{st} and the error term ϵ_{ij}^{rst} are likely negatively correlated. So in general, using ordinary least square (OLS) to estimate equation (16) is inconsistent. Assumption 1 (2) allows us to use the population supplies, m_i^{st} and f_j^{st} as instruments for μ_{i0}^{st} and μ_{0j}^{st}. The assumption says that the population supplies must be orthogonal to ϵ_{ij}^{rst}. As discussed in the introduction to this section, Assumption 1 does not impose any additional restrictions over and above what is standard in the empirical literature on US marriage markets which uses state and time variation for estimation. And just like that literature, we cannot identify parameters which vary by s and t without additional restrictions.[12] Under Assumption 1 (1), equation (16) becomes:

$$\ln\mu_{ij}^{rst}=\alpha_{ij}^{r}\ln\mu_{i0}^{st}+\beta_{ij}^{r}\ln\mu_{0j}^{st}+\pi_{ij}^{r}+\eta_{ij}^{rs}+\zeta_{ij}^{rt}+\epsilon_{ij}^{rst}. \tag{17}$$

Notice that for a fixed (i,j,r) type we have now $3+S+T$ parameters and ST observations. Therefore, the parameter of interests will be identified whenever $2+S+T<ST$.

Since μ_{i0}^{st} and μ_{0j}^{st} are potentially correlated with the residual terms ϵ_{ij}^{rst}, the ordinary least squares (OLS) will not be able to identify $\widetilde{\lambda_{ij}^{r}}$. Therefore, we will instrument μ_{i0}^{st} and μ_{0j}^{st} respectively with m_i^{st} and f_j^{st}. Notice that to be a valid

[12]Cornelson and Siow (2015) provides an example in which the effect of covariates which vary by (i,j,s,t) on π_{ij}^{rst} can be estimated.

instrument μ_{i0}^{st} and μ_{0j}^{st} should be respectively correlated with μ_{i0}^{st} and μ_{0j}^{st} and respect the exogeneity condition summarized in Assumption 1 (2).

As can be seen in Theorem 2, the comparative statics show the correlation between m_i^{st} and f_j^{st} and the unmatched. Therefore, $\widetilde{\lambda_{ij}^r}$ can be identified using the IV estimand if $\mathbb{E}[z_{ij}^{st}\widetilde{x_{ij}^{st}}']$ is of full column rank. The identification result is summarized in the following proposition.

Proposition 5 *Under Assumption 1, the general CD marriage matching function is identified if $\mathbb{E}[z_{ij}^{st}\widetilde{x_{ij}^{st}}']$ is of full column rank. The identification equation is given by $\widetilde{\lambda_{ij}^r} = \{\mathbb{E}[z_{ij}^{st}\widetilde{x_{ij}^{st}}']\}^{-1}\mathbb{E}[z_{ij}^{st}\widetilde{y_{ij}^{rst}}]$.*

We have a few comments. First, whenever $\widetilde{\lambda_{ij}^r}$ is identified, we can identify the gain matrix π_{ij}^{rst} using equation (16). Second, this model can also be estimated using the generalized method of moments (GMM). Third, whenever the number of states S and periods T are not high, we do not need to do the double differentiation. We can use a sequence of state and time fixed-effects.

3 Analyzing changes in marital patterns using difference-in-difference (DID) strategy.

Much of the empirical research that studies changes in marital patterns uses panel data with variation across state and time in marital behavior. For instance, Angrist and Evans (2000), Choo and Siow (2006a) used DID estimators to study the effect of the legalization of abortion on marital behavior. Brandt, Siow and Vogel (2016) also used the Choo and Siow (2006a) DID estimator to study the marital behavior of famine born cohorts in rural Sichuan. Bronson and Mazzocco (2013) studied the effect of changes in the sizes of birth cohorts on their subsequent marital behavior using variations across US states and time. Kearney and Wilson (2017) used also a DID estimator to study the marital behavior of low-skilled men who experienced an increase in earnings due to fracking. They showed that the increase in earnings did not increase their marriage rate, unlike an earlier episode of increased earnings due to the Appalachian mining boom in the seventies and eighties. They attribute the difference in marital behavior due to changes in the social context. Again using the Choo and Siow (2006a) estimator, Cornelson and Siow (2016) showed that changes in labor earnings cannot explain the recent decline in marriage rates in the United States.[13]

The diverse set of above studies show the usefulness of the DID estimator using across states and time variation to study changes in marital behavior. To date, there is no DID estimator which accommodate changes in sex ratios, policy changes as well as endogenous mechanism on marital behavior such as peer effects.

Here we provide an elementary DID framework which can be used to analyze both exogenous and endogenous (peer) mechanisms which affect marital behavior.

4 An extension: The Brock Durlauf marriage matching function.

As discussed earlier, instead of CSPE peer effects, we may also consider a Brock Durlauf peer effects (BD) type of specification:

$$U_{ijg}^{r} = \tilde{u}_{ij}^{r} + \phi_{i}^{r} \ln \frac{\mu_{ij}^{r}}{m_{i}} - \tau_{ij}^{r} + \varsigma_{ijg}^{r},\ j = 0, 1.., J$$
$$V_{ijk}^{r} = \tilde{v}_{ij}^{r} + \Phi_{j}^{r} \ln \frac{\mu_{ij}^{r}}{f_{j}} + \tau_{ij}^{r} + \varrho_{ijk}^{r},\ i = 0, 1.., I.$$

With this specification the dependence of individual utilities on their peers is captured by the fraction/share (rather than number as used in the CSPE) of the same type of individuals who choose the same action. An increase of new arrivals of type i man increases U_{ijg}^{r} only if i is in a (r, i, j) match. Otherwise it decreases U_{ijg}^{r}, while an increase of new arrivals of individuals of type i, has always a monotone (non-negative) impact on the U_{ijg}^{r} in the aggregate specification. Also, in the BD specification it is more costly to deviate from the social norms, which make it very costly to try a new type of relationship. Indeed, assume that we observe a very low cohabitation rate in a specific state: with the BD specification the probability of forming such a match in this state approaches zero even if the total gain of this specific match $\tilde{u}_{ij}^{\mathcal{C}} + \tilde{v}_{ij}^{\mathcal{C}} < \infty$ is

[13]Additional studies include Wolfers (2006), Galichon and Salanié (2015), Bitler, et. al. (2004)), Dahl (2010); Kerwin and Luoh (2010). Mechoulan (2011), Chiappori, Fortin and Lacroix (2002)).

very high. However, this is not the case for CSPE where even if the rate of cohabitation is very low in the population, there is a relatively high probability of forming a match $(\mathcal{C}, i, j)$ whenever the total gain of the match $(\mathcal{C}, i, j)$ is high. Similar interpretation of the behavioral implications of those two specifications have also been well discussed in Liu et al. (2014). The BD specification therefore does not encourage as much development of new forms of relationships compared to the aggregate specification presented earlier.

In terms of the number of parameters, the BD parameterization of an individual's utility from an action is the same as that of CSPE, so the two models are not nested in each other.

Following the derivation in CSPE, the above results in the following BD marriage matching function:

$$\begin{aligned}\ln \mu_{ij}^r = & \frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r}\ln \mu_{i0} + \frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r}\ln \mu_{0j} \\ & + \frac{\phi_i^0-\phi_i^r}{2-\phi_i^r-\Phi_j^r}\ln m_i + \frac{\Phi_j^0-\phi_i^r}{2-\phi_i^r-\Phi_j^r}\ln f_j + \pi_{ij}^r, \end{aligned} \tag{18}$$

where $\pi_{ij}^r = \frac{\tilde{u}_{ij}^r - \tilde{u}_{i0} + \tilde{v}_{ij}^r - \tilde{v}_{0j}}{2-\phi_i^r-\Phi_j^r}$.

This specification suggests the following unrestricted BD marriage matching function:

$$\ln \mu_{ij}^r = \pi_{ij}^r + \alpha_{ij}^r \ln \mu_{i0} + \beta_{ij}^r \ln \mu_{0j} + \delta_{ij}^r \ln m_i + \sigma_{ij}^r \ln f_j, \tag{19}$$

with $\delta_{ij}^r, \sigma_{ij}^r > 0 \; \forall \; (r, i, j)$.

Notice that in addition of respecting the population constraints, a marriage matching function should respect some basics properties. For instance, we might expect that an increase of availability should not decrease the number of marriages, i.e. μ_{ij} should be non-decreasing in m_i and f_j.

In the CD marriage matching this is ensured since $\alpha_{ij}^r, \beta_{ij}^r$ are restricted to be positive. In the BD marriage matching function because of the presence of m_i and f_j and that $\delta_{ij}^r, \sigma_{ij}^r > 0$, $\alpha_{ij}^r, \beta_{ij}^r$ must not necessarily need to be positive as we will see in the empirical application when estimating the BD marriage matching function.

Mourifié (2016) shows that the equilibrium matching distribution of the unrestricted BD marriage matching function, defined by equations (19), exists under mild restrictions but it is not always unique.[14]

The behavioral BD marriage matching function implies

$$\frac{\alpha_{ij}^{r}}{\alpha_{ij}^{r'}} = \frac{\beta_{ij}^{r}}{\beta_{ij}^{r'}}; \tag{20}$$

$$1 = \alpha_{ij}^{r} + \beta_{ij}^{r} + \delta_{ij}^{r} + \sigma_{ij}^{r} = \alpha_{ij}^{r'} + \beta_{ij}^{r'} + \delta_{ij}^{r'} + \sigma_{ij}^{r'}. \tag{21}$$

The unrestricted BD marriage matching function can also be estimated by difference in differences instrumental variables. Since the covariates in the unrestricted BD marriage matching function include m_i^{st} and f_j^{st}, we also add the population supplies of substitute partners, $m_{i'}^{st}$ and $f_{j'}^{st}$ as instruments.

5 Empirical results.

We study the marriage matching behavior of 20-50 years old white women and 22-52 years old white men with each other in the US for 1990, 2000 and 2010. We group women into 5 age groups: 20-25, 26-30, 31-35, 36-40, 41-50. We group men into 5 age groups: 22-27, 28-32, 33-37, 38-42, 43-52.

The 1990 and 2000 data is from the 5% US census. The 2010 data is from aggregating five years of the 1% American Community Survey from 2008-2012. A state year is considered as an isolated marriage market. There were 51 states which includes Washington DC. Individuals are distinguished by their schooling level: less than high school (L or 1), high school graduate (M or 2) and university graduate (H or 3).

Each individual can be unmatched, married or cohabitating. A cohabitating couple is one where a respondent answered that they are the "unmarried partner" of the head of the household.[15] Individuals who are married or cohabitating with non-white or partners outside the age ranges are treated as unmatched.

An observation in the dataset is the number of (r, i, j) relationships in a state year. An age range and an education level is a type of individual. With five age ranges and three schooling levels, there are fifteen types of men and women. So

[14] The equilibrium still unique if $\alpha_{ij}^{r}, \beta_{ij}^{r} > 0$.

[15] A very small number of households had more than one member claiming that they were the "unmarried partner" of the head of household. We assigned everyone involved to be unmarried in that case.

there are potentially 225 types of matches for each type of relationship, marriage versus cohabitation.

Table 1a in Appendix A provides some summary statistics. There are 20429 and 27373 types of non-zero cohabitations and marriages respectively (a type of relationship is indexed by the relationship, types of partners, time and state). There are close to an average of 68,000 males and females of each type. There are close to an average of 26,000 unmatched individuals of each type. The aggregate marriage rate is around 56% and the aggregate cohabitation rate is around 5%.

Table 2 presents estimates of equation (17) by weighted instrumental variables.[16,17] To mitigate the concern that own population, due to endogenous migration, would not be a valid instrument, we instrument each current unmatched population with its population two decades earlier.[18] E.g. the number of unmatched high school female graduates in a state is instrumented by the number of high school female graduates in that state twenty years earlier.

In order to reduce the number of parameters, we characterize an (i, j) match effect as additive in its age range interaction effect (5×5) and its education interaction effect (3×3). So in each relationship r, there are 34 parameters which capture the 225 (i, j) match effects. The smallest model which only includes year effects, model 1, is in columns (1a) and (1b) where $\pi_{ij}^{srt} = \pi_{ij}^{rt}$. Compared with later specifications, the goodness of fit of this model is poor, so we will not discuss its estimated properties further.

Model 2, in columns (2a) and (2b) add unrestricted match and year effects. Compared with model 1, the R^2s jump significantly for both relationships which says that different types of individuals prefer to match with different types of partners. The estimated year effects show that compared with 1990, the gains to cohabitation increased in 2000 and again in 2010, whereas the gains to marriage fell in 2000 and again in 2010.

Since the estimates of the match effects are difficult to interpret, we present instead the local log odds for educational matching, equation (13). With three

[16]Each observation is weighted by the average of m_i^{st} and f_j^{st}. The unweighted point estimates were similar with larger standard errors.

[17]The OLS precision and fit are very similar to the IV results. So we dispense with them for convenience.

[18]The first draft of this paper used contemporaneous populations as instruments. The results are qualitatively similar to that reported here.

education groups by gender, there are four local log odds. In columns (2a) and (2b), all the local log odds are significantly positive. Consistent with the literature, there is strong evidence for positive assortative matching by educational attainment in both cohabitation and marriage. positive assortative matching is present in both cohabitation and marriage in all our empirical models. Every local log odds is larger for marriage than for cohabitation. From a behavioral point of view, positive assortative matching by education has higher average payoff in marriage than in cohabitation. Such a finding is consistent with the hypothesis that couples who are dissimilar in educational attainment choose to cohabit rather than marry because separation is easier under cohabitation than marriage.

Constant returns to scale is statistically rejected in model (2) at the 1% significance level. However, the quantitative magnitude of the departure from constant returns to scale is not large. As we will clarify below, we should be skeptical about the test for constant returns to scale without including state fixed effects.

We cannot reject the hypothesis that $\alpha^{\mathcal{M}} + \beta^{\mathcal{M}} = \alpha^{\mathcal{C}} + \beta^{\mathcal{C}}$ at the 5% level which means that peer/scale effects for both types of relationships are quantitatively similar.

The test of CSPE, that $\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}} = 1$, is in the second last row of the table. CSPE is not rejected at any conventional significance level.

In the last row of the Table 2, using Lemma 1, we test for the independence of the log odds of cohabitation versus marriage with respect to the sex ratio. Independence is not rejected at the 5% significance level.

In summary, model 2 includes match and year effects, and without state effects. There is no evidence against CSPE. The quantitative departure from constant returns to scale is modest. CSW would be a relevant model in this case. Finally, there is no evidence against independence of the log odds of cohabitation versus marriage with respect to the sex ratio. A goodness of fit measure, CSPE with constant returns to scale (equivalently CSW) provides a parsimonious summary of recent US marital behavior.

Model 3, in columns (3a) and (3b) we add state effects to the covariates. In model 1, the R^2s are in the 0.2 range. The R^2s increase to 0.84 and 0.92 by adding match and year effects in models 2a and 2b respectively. Although we cannot reject the hypothesis that the state effects are statistically significant as a group at the 1% level, there is only a 1% increase in R^2 in the marriage equation in model 3. So adding state effects contributes marginally to the increase in goodness of fit.

The estimated local log odds of positive assortative matching by education is quantitatively similar to those in model 2. Again positive assortative matching is stronger under marriage than cohabitation.

Unlike model 2, there is strong evidence that constant returns to scale is rejected in model 3. Because we include state fixed effects in model 3, the variation used to identify changes in population sizes are across time within state, so this is a very different variation used to test constant returns to scale in model 2.

Unsurprisingly, we reject the hypothesis that $\alpha^{\mathcal{M}} + \beta^{\mathcal{M}} = \alpha^{\mathcal{C}} + \beta^{\mathcal{C}}$ at the 1% level which means that peer/scale effects for both types of relationships are quantitatively different.

CSPE is not rejected at the 5% significance level. In the last row of the Table 2, independence of the log odds of cohabitation versus marriage with respect to the sex ratio is rejected at the 1% significance level in model 3.

Positive assortative matching in education is similar to that in model 2. We also do not reject CSPE but we reject constant returns to scale and independence. Our point estimates suggest $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ and $\beta^{\mathcal{M}} > \beta^{\mathcal{C}}$ which is compatible with the CSPE restriction.

We think that model 3 is the most accurate for at least two reasons. First, the gains to marriage in a marriage market may depend on both state and year, so ignoring these state effects will result in inconsistent estimates of the parameters. Second, if we do not include state effects, the variation in populations across states are large. Since large states do not have systematically different marriage and cohabitation rates than small states, the across state variation in populations will "impose" constant returns to scale in our parameter estimates. But individuals in a marriage market are not responding to across state variation in population supplies. Rather, individuals in a marriage market respond to within state differences in population supplies of different individuals in that state year. So for consistent estimates of parameters, we believe that analyst must use both state and year effects in their estimation. Moreover, our conceptual framework assumes that each (s, t) marriage market is isolated from other marriage markets. Since each individual lives in one particular marriage market, the individual only considers peer effects in their own marriage market when making their own relationship decision. Thus from both an econometric and a behavioral point of view, model 3, which includes state and time effects, is the relevant empirical model of behavior.

5.1 Heterogenous effects.

Table 3 presents results where we allow for heterogeneous effects, α^r_{ij} and β^r_{ij}. We allow for different coefficients for "unmatched "effects for the youngest and oldest age groups of our sample. In addition to lagged own populations by two decades, we also included lagged populations of adjacent types as instruments.

The first two columns present estimates of α^r_{ij} and β^r_{ij} which only include year effects. As before, the goodness of fit is poor and we will not discuss the estimates further. Columns 2a and 2b add match effects. The last two columns present results which also include state effects.

Although some of the estimated heterogenous parameters are significantly different from 0, from a R^2 point of view, Table 3 results in similar fit of the data relative the corresponding model in Table 2. As before, without state effects in columns 2a and 2b, constant returns to scale is statistically rejected but the quantitative departure from constant returns to scale is small. With state effects in columns 3a and 3b, constant returns to scale is again consistently rejected. In general, the point estimates of α^r and β^r in Table 3 are qualitatively similar the corresponding estimated model in Table 2.

With or without state effects, there is no evidence against CSPE.

Looking at the last row, without state effects, there is no evidence against independence of the log odds of cohabitation versus marriages with respect to the sex ratio. With state effects, independence is rejected at the 1% significance level. The point estimates for model 3 suggest $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ and $\beta^{\mathcal{M}} > \beta^{\mathcal{C}}$ which is again compatible with the CSPE.

From our reading of the results in Table 2 and 3, allowing for heterogeneous effects is marginally useful. With or without state effects, there is no evidence against CSPE. The departure from constant returns to scale and thus CSW is modest without state effects; it is quantitatively significant with state effects. Without state effects, we cannot reject independence of the ratio of marriages to cohabitation with respect to changes in the sex ratio. With state effects, independence is rejected.

In results not reported here, we also estimated the model with all races rather than the white only sample presented here. Our earlier draft mostly estimated models with current populations as instruments rather than lagged instruments. Those results are also similar to that presented here. We also estimated the models by OLS, i.e. without instruments. The results are quantitatively similar to what we obtain here.

We also experimented with unrestricted match effects rather than additive match effects with spousal age interactions and spousal education interactions. We also allowed for time varying match effects. Again, those results are similar to that presented here.

Finally, we also allowed for time varying unmatched coefficients, α^{rt} and β^{rt}. Although the estimated unmatched coefficients are sometimes smaller in the later decades compared with 1990, constant returns to scale is still rejected in every year when we include state effects. There is also no difference in the tests of CSPE or independence of the log odds.

Using also primarily cross sectional data and the CS model, Botticini and Siow (2008) also found constant returns to scale in the marriage market.[19] This is consistent with our estimates without state effects which also uses across states variation to estimate the returns to scale. Our results here show that the test of constant returns to scale is sensitive to where the variation population sizes comes from. In cross section data, the variation in population sizes across states is much larger than the variation in population sizes within states across time. On the other hand, using other methodologies, Fernandez-Villaverde, et al. (2014), Adamopoulou (2012) and Drewianka (2003) argue that peer effects in the marriage market are empirically important.

5.2 Behavioral interpretation of estimated peer effects.

Since CSPE is not rejected in Table 2 or 3, this section proceeds with a behavioral interpretation of our estimates. We will use the estimates with state fixed effects in our interpretation. First, there is evidence for CSPE with increasing returns to scale.

[19]Using variation across cities in the US, pre-reform China and medieval Tuscany, Botticini and Siow's (2008) could not reject constant returns to scale with an aggregate marriage rate.

Second, the condition for Corollary 2 is satisfied, $\beta^r > \alpha^r$ which implies $\Phi^0 < \phi^0$. Having more unmatched women does not increase the utilities of unmatched women as much as the same exercise for men. This result is apparently contrary to Adamopolou (2012). Adamopolou's estimates of peer effects can be viewed as a direct peer effects because she uses small peer groups to estimate her effects. There is little variation in her peer group size because the survey data which she uses only allows a small number of peers.[20]

Finally, we cannot reject the hypothesis that $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ which is the same as $\beta^{\mathcal{M}} > \beta^{\mathcal{C}}$ under CSPE with either homogeneous or heterogeneous unmatched effects. According to Corollary 3, the value a typical couple derives from marriage was more affected by peer and scale effects than for cohabitation.

5.3 BD marriage matching function estimates.

Table 4 provides weighted IV estimates of the unrestricted BD marriage matching function (equation (19)). The instruments are the same as that for Table 3, lagged own and adjacent population types. As with the CD marriage matching function estimates in Table 2 and 3, the goodness of fit of the unrestricted BD model without match effects is much worse than with match effects.

From hereon, we discuss the models which include match effects. In terms of goodness of fit, the unrestricted BD marriage matching function fits as well as the CD marriage matching function.

Except for one case, the estimated coefficients on own unmatched are all positive and less than one for all four columns. This means that, for each relationship, the sum of the matched peer effect coefficients is smaller than the peer effect coefficients for remaining unmatched. The coefficients on own populations do not have to be positive and the estimated coefficients are both positive and negative.

Restriction (20) is not rejected without state effects. It is rejected with state

[20] In our context, our peer effect coefficient can be viewed as an aggregate of a direct effect and an indirect effect. The direct peer effect captures how an individual g's utility is affected when he *observes* how many others like him choose the same action. The indirect effect is a market level effect. As there are more (i, j, r) relationships in a community, local firms will provide services to them (e.g. Compton and Pollak (2007); Costa and Kahn (2000)). This community response will make it cheaper for g to choose (i, j, r) relationships. Marriage market participants do not necessarily recognize the impact of their aggregate actions on the prices of goods and services which they face. Thus the indirect peer effect is a scale effect.

effects. In general, restriction (21) is rejected at the 1% significance level. It is not rejected for marriage without state effects. In general, the quantitative departures from restriction (21) are not large. Restrictions (20) and (21) are jointly rejected with and without state effects at the 1% significance level. Since both restrictions must hold for BD, the behavioral BD marriage matching function is not a good behavioral model for this data. This conclusion differs from CSPE which is a good behavioral model within the CD class.

However, at this stage, it is premature to choose the CD marriage matching function over the unrestricted BD marriage matching function or the reverse. A comprehensive comparison of the two models will have to deferred to future research.

6 Conclusion.

This paper presented two easy ways to estimate and simulate marriage matching functions, the CD and unrestricted BD marriage matching functions. Several behavioral marriage matching functions are special cases including CSPE and BD marriage matching functions. Our empirical results show that the CD marriage matching function provides a reasonably complete and parsimonious characterization of the recent evolution of the US marriage market. Peer and scale effects are quantitatively important. With or without state effects, CSPE is not rejected. With state effects constant returns to scale and independence are rejected. As discussed in the paper, we prefer the estimates with state effects.

We show that the BD marriage matching function is nested within an unrestricted BD marriage matching function. The empirical evidence does not support the BD marriage matching function.

Using the CD marriage matching function to study particular mechanisms for marital change is an important topic for future research. For e.g., Cornelson and Siow (2015) used a special case of the above framework to show that increased earnings inequality cannot explain the decline of the marriage rate of young Americans from 1970 to 2010.

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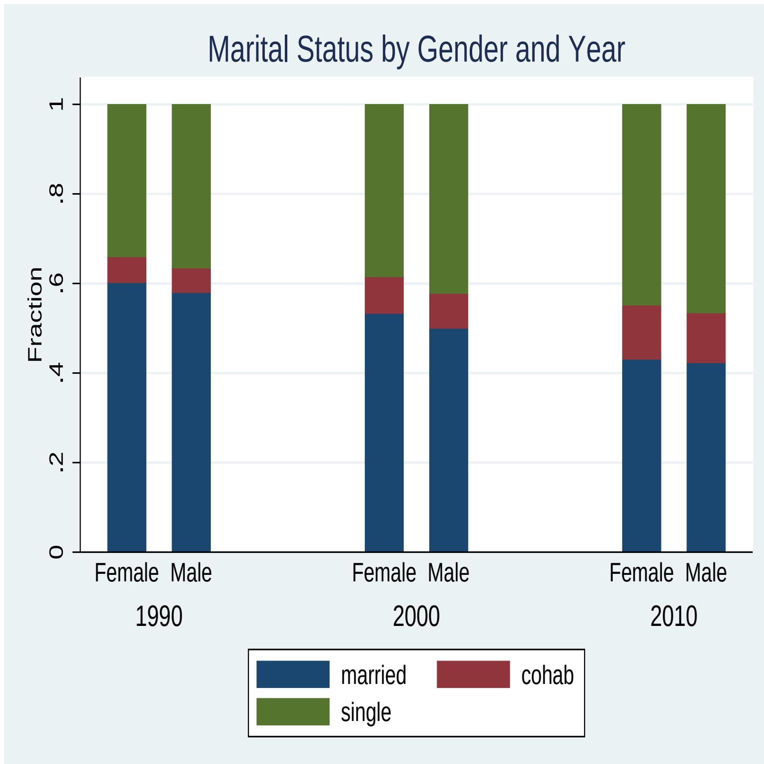
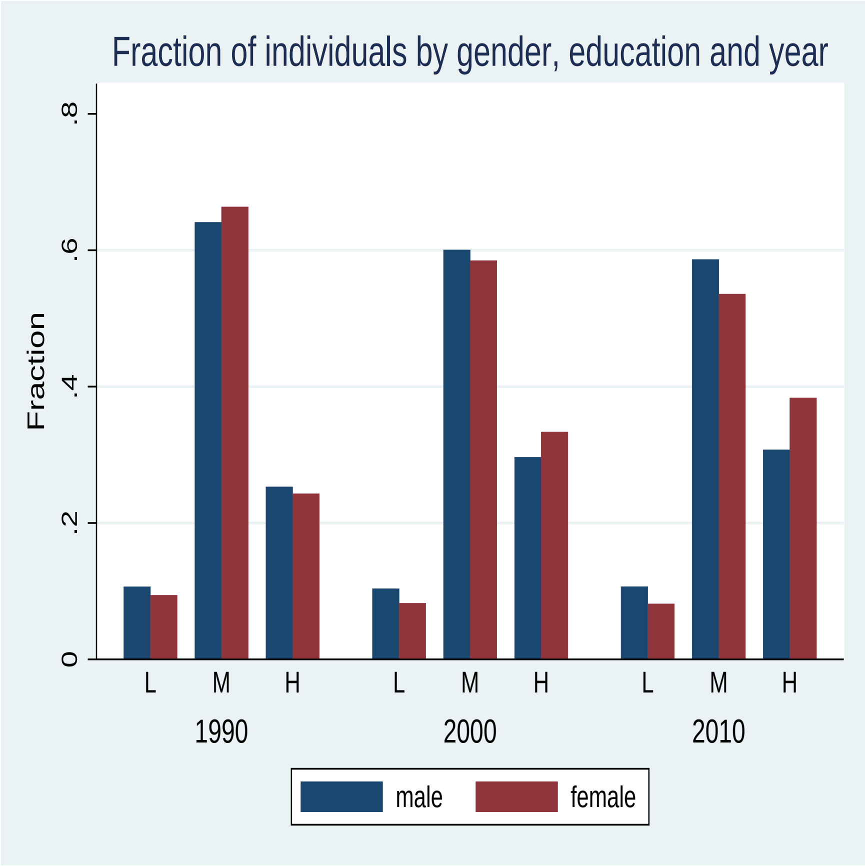
Figure 1: Marital Status by Gender and Year.



A Figures and Tables

A.1 Figures

Figure 2: Fraction of individual by gender, education and year.



A.2 Tables

Table 1: Summary Statistics

Variables	Mean	Std	Max	Min
Total males	68060.15	90320.82	777439	62
Total females	67996	97365.03	962248	16
Total marriages	2754.476	9900.458	285067	1
Total cohabitations	314.9492	861.7351	22570	2
Male age	38.28	8.439013	52	27
Female age	36.47681	9.031399	52	25
Male schooling	2.009119	0.7869205	3	1
Female schooling	2.026016	0.7866299	3	1

Each observation is a year, state. Observations with zero match are dropped.
Total males or females include all individuals who can potentially choose a relationship.

Table 2: Weighted IV estimates with lagged instruments.

Model	1a	1b	2a	2b	3a	3b
Dep Var	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$
$\ln \mu_{i0}(\alpha)$	0.404 (0.015)**	0.339 (0.019)**	0.330 (0.033)**	0.348 (0.026)**	0.701 (0.043)**	0.978 (0.033)**
$\ln \mu_{0j}(\beta)$	0.445 (0.016)**	0.546 (0.021)**	0.588 (0.033)**	0.553 (0.026)**	0.927 (0.039)**	1.091 (0.032)**
$\ln \frac{\mu_{HH}^r \mu_{MM}^r}{\mu_{HM}^r \mu_{MH}^r}$			1.89 (0.030)**	1.99 (0.028)**	1.86 (0.029)**	1.95 (0.029)**
$\ln \frac{\mu_{MM}^r \mu_{LL}^r}{\mu_{LM}^r \mu_{ML}^r}$			1.30 (0.035)**	1.88 (0.032)**	1.21 (0.034)**	1.73 (0.031)**
$\ln \frac{\mu_{HM}^r \mu_{ML}^r}{\mu_{MM}^r \mu_{HL}^r}$			0.774 (0.052)**	1.12 (0.035)**	0.766 (0.051)**	1.15 (0.035)**
$\ln \frac{\mu_{MH}^r \mu_{LM}^r}{\mu_{MM}^r \mu_{LH}^r}$			0.588 (0.072)**	1.06 (0.040)**	0.616 (0.072)**	1.10 (0.041)**
$Y2000$	1.197 (0.036)**	-0.182 (0.048)**	0.101 (0.014)**	-0.357 (0.012)**	0.069 (0.014)**	-0.426 (0.012)**
$Y2010$	0.224 (0.037)**	-0.383 (0.049)**	0.063 (0.015)**	-0.760 (0.013)**	0.003 (0.016)**	-0.877 (0.014)**
State fixed effects					Y	Y
R^2	0.29	0.16	0.84	0.92	0.85	0.92
N	20,429	27,373	20,429	27,373	20,429	27,373
$\alpha^r + \beta^r - 1$	-0.151 (0.021)**	0.115 (0.025)**	-0.082 (0.008)**	-0.098 (0.006)**	0.628 (0.052)**	1.07 (0.041)**
$Pr(\alpha^{\mathcal{M}} + \beta^{\mathcal{M}} = \alpha^{\mathcal{C}} + \beta^{\mathcal{C}} = 1)$		0.000**		0.000**		0.000**
$Pr\left(\frac{\alpha^{\mathcal{M}} \beta^{\mathcal{C}}}{\beta^{\mathcal{M}} \alpha^{\mathcal{C}}} = 1\right)$		0.015*		0.554		0.062
$Pr(\alpha^{\mathcal{M}} - \alpha^{\mathcal{C}} = \beta^{\mathcal{M}} - \beta^{\mathcal{C}} = 0)$		0.000**		0.220		0.000**

*Significantly different from 0 at 5% level. **Significantly different from 0 at 1% level.

Table 3: Heterogeneous Weighted IV estimates with lagged instruments.

Model	1a	1b	2a	2b	3a	3b
Dep Var	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$
$\ln \mu_{i0}(\alpha)$	0.513 (0.015)**	0.572 (0.018)**	0.315 (0.039)**	0.311 (0.033)**	0.839 (0.053)**	1.043 (0.040)**
$\ln \mu_{0j}(\beta)$	0.502 (0.014)**	0.643 (0.017)**	0.608 (0.041)**	0.598 (0.030)**	0.999 (0.044)**	1.141 (0.034)**
$\alpha 1\{22-27\}$	−0.0490 (0.004)**	−0.138 (0.005)**	−0.025 (0.021)	−0.018 (0.020)	−0.072 (0.022)**	−0.078 (0.019)**
$\alpha 1\{43-52\}$	−0.026 (0.004)**	−0.124 (0.005)**	−0.059 (0.018)**	−0.017 (0.014)**	−0.099 (0.018)**	−0.169 (0.014)**
$\beta 1\{20-25\}$	−0.025 (0.004)**	−0.009 (0.005)	0.015 (0.030)	0.015 (0.025)	0.104 (0.032)**	0.124 (0.025)**
$\beta 1\{41-50\}$	−0.053 (0.004)**	−0.063 (0.005)**	0.057 (0.026)*	0.091 (0.023)**	0.077 (0.024)**	0.081 (0.022)**
$\ln \frac{\mu^r_{HH}\mu^r_{MM}}{\mu^r_{HM}\mu^r_{MH}}$			1.88 (0.030)**	1.88 (0.032)**	1.86 (0.029)**	1.95 (0.029)**
$\ln \frac{\mu^r_{MM}\mu^r_{LL}}{\mu^r_{LM}\mu^r_{ML}}$			1.30 (0.035)**	1.99 (0.032)**	1.21 (0.034)**	1.72 (0.039)**
$\ln \frac{\mu^r_{HM}\mu^r_{ML}}{\mu^r_{MM}\mu^r_{HL}}$			0.775 (0.052)**	1.12 (0.035)**	0.766 (0.052)**	1.14 (0.035)**
$\ln \frac{\mu^r_{MH}\mu^r_{LM}}{\mu^r_{MM}\mu^r_{LH}}$			0.588 (0.072)**	1.06 (0.040)**	0.617 (0.72)**	1.10 (0.041)**
Year fixed-effects	Y	Y	Y	Y	Y	Y
State fixed-effects					Y	Y
R^2	0.33	0.28	0.84	0.92	0.84	0.92
N	20,429	27,373	20,429	27,373	20,429	27,373
$\alpha^r + \beta^r - 1$	0.015 (0.017)	0.215 (0.020)**	−0.077 (0.011)**	−0.091 (0.010)**	0.838 (0.059)**	1.18 (0.047)**
$Pr(\alpha^{\mathcal{M}} + \beta^{\mathcal{M}} = \alpha^{\mathcal{C}} + \beta^{\mathcal{C}} = 1)$		0.000**		0.000**		0.000**
$Pr\left(\frac{\alpha^{\mathcal{M}}\beta^{\mathcal{C}}}{\beta^{\mathcal{M}}\alpha^{\mathcal{C}}} = 1\right)$		0.034*		0.985		0.402
$Pr(\alpha^{\mathcal{M}} - \alpha^{\mathcal{C}} = \beta^{\mathcal{M}} - \beta^{\mathcal{C}} = 0)$		0.000**		0.182		0.000**

*Significantly different from 0 at 5% level. **Significantly different from 0 at 1% level.

Table 4: Weighted BD's IV estimates with lagged instruments.

Model	1a	1b	2a	2b	3a	3b
Dep Var	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$	$\ln \mu_{ij}^{\mathcal{C}}$	$\ln \mu_{ij}^{\mathcal{M}}$
$\ln \mu_{i0}(\alpha)$	0.350 (0.064)**	0.475 (0.053)**	0.625 (0.076)**	0.365 (0.064)**	0.589 (0.033)**	0.300 (0.041)**
$\ln \mu_{0j}(\beta)$	0.423 (0.078)**	0.259 (0.069)**	0.546 (0.080)**	−0.083 (0.071)**	0.475 (0.026)**	0.464 (0.032)**
$\ln m_i(\sigma)$	0.027 (0.037)**	−0.009 (0.032)**	0.029 (0.031)	0.239 (0.026)**	−0.215 (0.035)**	0.063 (0.044)**
$\ln f_i(\delta)$	0.102 (0.047)**	0.162 (0.044)**	0.158 (0.037)	0.515 (0.034)**	−0.024 (0.029)**	0.092 (0.034)**
Match-type fixed effect (ij)			Y	Y	Y	Y
Year fixed-effects	Y	Y	Y	Y	Y	Y
State fixed-effects					Y	Y
R^2	0.30	0.16	0.84	0.92	0.85	0.92
N	20,429	27,373	20,429	27,373	20,429	27,373
$Pr\left(\frac{\alpha^{\mathcal{M}}\beta^{\mathcal{C}}}{\beta^{\mathcal{M}}\alpha^{\mathcal{C}}} = 1\right)$		0.129		0.48		0.000**
$\alpha^r + \beta^r + \sigma^r + \delta^r - 1$	−0.098 (0.010)	−0.113 (0.008)**	0.358 (0.065)**	0.037 (0.056)	−0.175 (0.017)**	−0.081 (0.017)**
$Pr(\alpha^r + \beta^r + \delta^r + \sigma^r = 1, r = \mathcal{M}, \mathcal{C})$		0.000**		0.000**		0.000**

*Significantly different from 0 at 5% level. **Significantly different from 0 at 1% level.

B Existence and Uniqueness of the Matching Equilibrium.

To ease the notation, denote $\mathcal{M} \equiv a$ and $\mathcal{C} \equiv b$ in the rest of the paper. The matching equilibrium in this model is characterized by the Cobb Douglas MMF (3) and the population constraint equations

$$\sum_{j=1}^{J} \mu_{ij}^{a} + \sum_{j=1}^{J} \mu_{ij}^{b} + \mu_{i0} = m_i, \quad 1 \leq i \leq I \tag{22}$$

$$\sum_{i=1}^{I} \mu_{ij}^{a} + \sum_{i=1}^{I} \mu_{ij}^{b} + \mu_{0j} = f_j, \quad 1 \leq j \leq J \tag{23}$$

$$\mu_{0j}, \mu_{i0} \geq 0, \quad 1 \leq j \leq J, 1 \leq i \leq I.$$

Let $m \equiv (m_1, ..., m_I)'$, $f \equiv (f_1, ..., f_J)'$,
$\mu \equiv (\mu_{10}, ..., \mu_{I0}, \mu_{01}, ..., \mu_{0J})'$, $\pi^r \equiv (\pi_{11}^r, ..., \pi_{1I}^r, ..., \pi_{I1}^r, ..., \pi_{IJ}^r)'$ for $r \in \{a, b\}$, $\beta^r \equiv (\beta_{11}^r, ..., \beta_{1I}^r, ..., \beta_{I1}^r, ..., \beta_{IJ}^r)'$, $\alpha^r \equiv (\alpha_{11}^r, ..., \alpha_{1I}^r, ..., \alpha_{I1}^r, ..., \alpha_{IJ}^r)'$ $\beta \equiv ((\beta^a)', (\beta^b)')'$, $\alpha \equiv ((\alpha^a)', (\alpha^b)')'$ and $\theta \equiv ((\pi^a)', (\pi^b)', \alpha', \beta')'$. Let π be a closed and bounded subset of $\mathbb{R}^{2IJ}$ such that $\theta \in \pi \times (0, \infty)^2$. Equation (3) can be written as follows:

$$\mu_{ij}^{r} = \mu_{i0}^{\alpha_{ij}^{r}} \mu_{0j}^{\beta_{ij}^{r}} e^{\pi_{ij}^{r}} \qquad \text{for } r \in \{a, b\}. \tag{24}$$

And finding the equilibrium matching distribution is equivalent to solve the following system of $I + J$ equations with $I + J$ unknowns.

$$m_i = \mu_{i0} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^{a}} \mu_{0j}^{\beta_{ij}^{a}} e^{\pi_{ij}^{a}} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^{b}} \mu_{0j}^{\beta_{ij}^{b}} e^{\pi_{ij}^{b}}, \text{ for } 1 \leq i \leq I, \tag{25}$$

$$f_j = \mu_{0j} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^{a}} \mu_{0j}^{\beta_{ij}^{a}} e^{\pi_{ij}^{a}} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^{b}} \mu_{0j}^{\beta_{ij}^{b}} e^{\pi_{ij}^{b}}, \text{ for } 1 \leq j \leq J. \tag{26}$$

B.1 Proof of Theorem 1.

In the first version of this paper Mourifié and Siow (2014) available online, we propose a lengthy proof for the existence and uniqueness of the equilibrium matching distribution of the CD MMF.

1. Our first version was using the Brouwer fixed point theorem to show the existence and the Hadamard's theorem (see Krantz and Park (2003, Theorem 6.2.8 p 126)) for the uniqueness of the equilibrium.

2. Simultaneously and independently, Galichon et al (2014, 2016) studied a matching model with imperfect transfer and propose the aggregate matching function (AMF), i.e. $\mu_{ij} = g_{ij}(\mu_{i0}, \mu_{0j})$ such that g is a non-negative isotone function and homogeneous of degree 1 (meaning that $g_{ij}(a\mu_{i0}, a\mu_{0j}) = ag_{ij}(\mu_{i0}, \mu_{0j})$). Galichon et al. (2014, 2016) use instead the Tarski theorem for the existence and invoke Gale and Nikaido (1965) for the uniqueness. While Galichon et al. (2014, 2016) allow for a wider set of functional forms, the homogeneity restriction rules out some important MMFs such that DM and CSPE. However, their proof of existence and uniqueness does not rely on the homogeneity assumption and can also be extended for allowing multiple type of relationships.
3. In the companion paper, Mourifié (2016) studies a very general form of MMF, $g_{ij}(\mu_{10}, ..., \mu_{I0}, \mu_{01}, ..., \mu_{0J}, m, f)$, where $g_{ij}(.)$ is a non-negative differentiable function. Notice that the function $g_{ij}(.)$ considered by Mourifié does not require to be monotone in its arguments. Mourifié (2016) shows using again the Brouwer fixed point theorem that the equilibrium always exists. And using Gale and Nikaido (1965) he derives the conditions under which the equilibrium is unique. Conditions which he shows trivially holds for the AMF and the CD MMF. Please see Mourifié (2016, Section 4).

Because of this general result of Mourifié (2016) which was motivated by this present paper, we will not repeat the proof here. The reader which is interested can refer to our previous version, Galichon et al (2014, 2016) or to Mourifié (2016).

C Comparative Statics.

C.1 Fixed point representation of the equilibrium of the Cobb Douglas MMF

After rearranging equation (24) we have four equalities that holds for all (i, j) pairs:

$$\frac{\mu^r_{ij}}{\mu_{i0}} = exp\Big[\pi^r_{ij} + (\alpha^r - 1)\ln\mu_{i0} + \beta^r \ln\mu_{0j}\Big] \equiv \eta^r_{ij} \qquad \text{for } r \in \{a, b\}, \qquad (27)$$

$$\frac{\mu^r_{ij}}{\mu_{0j}} = exp\Big[\pi^r_{ij} + \alpha^r \ln\mu_{i0} + (\beta^r - 1)\ln\mu_{0j}\Big] \equiv \zeta^r_{ij} \qquad \text{for } r \in \{a, b\}. \qquad (28)$$

Using equations (27) and (28) we have:

$$\sum_{j=1}^{J}\mu^a_{ij} + \sum_{j=1}^{J}\mu^b_{ij} = \mu_{i0}\sum_{j=1}^{J}\Big[\eta^a_{ij} + \eta^b_{ij}\Big], \quad 1 \le i \le I,$$
$$\sum_{i=1}^{I}\mu^a_{ij} + \sum_{i=1}^{I}\mu^b_{ij} = \mu_{0j}\sum_{i=1}^{I}\Big[\zeta^a_{ij} + \zeta^b_{ij}\Big], \quad 1 \le j \le J.$$

Manipulating the population constraints (22), (23) we have the following:

$$\mu_{i0} = \frac{m_i}{1 + \sum_{j=1}^{J} \left[\eta_{ij}^a + \eta_{ij}^b \right]} \equiv B_{i0}, \quad 1 \le i \le I \tag{29}$$

$$\mu_{0j} = \frac{f_j}{1 + \sum_{i=1}^{I} \left[\zeta_{ij}^a + \zeta_{ij}^b \right]} \equiv B_{0j}, \quad 1 \le j \le J. \tag{30}$$

Let $B(\mu; m, f, \theta) \equiv (B_{10}(.), ..., B_{I0}(.), B_{01}(.), ..., B_{0J}(.))'$. For a fixed θ we have shown that the $(I + J)$ vector μ of the number of agents of each type who choose not to match is a solution to $(I + J)$ vector of implicit functions

$$\mu - B(\mu; m, f, \theta) = 0. \tag{31}$$

Let $\mathbb{T}_\epsilon = \{\epsilon \le \mu_{10} \le m_1, ..., \epsilon \le \mu_{I0} \le m_I, \epsilon \le \mu_{01} \le f_1, ..., \epsilon \le \mu_{0J} \le f_J\}$ be a closed and bounded rectangular region in $\mathbb{R}^{I+J}$ with ϵ some arbitrarily small positive constant. We know from Theorem 1 that the fixed point representation has a unique solution $\mu^{eq} > 0$. We can verify that $\mu^{eq} \in \mathbb{T}_\epsilon$. Now, let $J(\mu) = I_{I+J} - \bigtriangledown_\mu B(\mu; m, f, \theta)$ with $\bigtriangledown_\mu B(\mu; m, f, \theta) = \frac{\partial B(\mu; m, f, \theta)}{\partial \mu'}$ be the $(I+J) \times (I+J)$ Jacobian matrix associated with (32). For a fixed θ we have shown that the $(I + J)$ vector μ of the number of agents of each type who choose not to match is a solution to $(I + J)$ vector of implicit functions

$$\mu - B(\mu; m, f, \theta) = 0. \tag{32}$$

C.2 Comparative Statistics

Theorem 2 *Let μ be the equilibrium matching distribution of the Cobb Douglas MMF model. If the coefficients β^r and α^r respect the restrictions*

1. $0 < \beta^r; \alpha^r \le 1$ *for* $r \in \{\mathcal{M}, \mathcal{C}\}$;
2. $\max(\beta^{\mathcal{C}} - \alpha^{\mathcal{C}}, \beta^{\mathcal{M}} - \alpha^{\mathcal{M}}) < \min_{i \in I} \left(\frac{1-\rho_i^m}{\rho_i^m} \right)$;
3. $\min(\beta^{\mathcal{C}} - \alpha^{\mathcal{C}}, \beta^{\mathcal{M}} - \alpha^{\mathcal{M}}) > -\max_{j \in J} \left(\frac{1-\rho_j^f}{\rho_j^f} \right)$;

where ρ_i^m is the rate of matched men of type i and ρ_j^f is the rate of matched women of type j, then the following inequalities hold in the neighbourhood of μ^{eq}:

1. *Type-specific elasticities of unmatched.*

   (a) $$\frac{m_i}{\mu_{k0}} \frac{\partial \mu_{k0}}{\partial m_i} \ge \begin{cases} \frac{1}{m_i^*} \frac{m_k}{m_k^*} \sum_{j=1}^{J} \frac{[\alpha^{\mathcal{M}} \mu_{kj}^{\mathcal{M}} + \alpha^{\mathcal{C}} \mu_{kj}^{\mathcal{C}}][\beta^{\mathcal{M}} \mu_{kj}^{\mathcal{M}} + \beta^{\mathcal{C}} \mu_{kj}^{\mathcal{C}}]}{f_j^*} > 0 & \text{if } k \ne i \\ \frac{m_i}{m_i^*} [1 + \frac{1}{m_i^*} \sum_{j=1}^{J} \frac{[\alpha^{\mathcal{M}} \mu_{ij}^{\mathcal{M}} + \alpha^{\mathcal{C}} \mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}} \mu_{ij}^{\mathcal{M}} + \beta^{\mathcal{C}} \mu_{ij}^{\mathcal{C}}]}{f_j^*}] > 1 & \text{if } k = i, \end{cases}$$

   $1 \le k \le I$.

(b) $\frac{f_j}{\mu_{0k}}\frac{\partial \mu_{0k}}{\partial f_j} \geq \begin{cases} \frac{1}{f_j^*}\frac{f_k}{f_k^*}\sum_{i=1}^{I}\frac{[\alpha^{\mathcal{M}}\mu_{ik}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ik}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ik}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ik}^{\mathcal{C}}]}{m_i^*} > 0 & \text{if } k \neq j \\ \frac{f_j}{f_j^*}[1+\frac{1}{f_j^*}\sum_{i=1}^{I}\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^*}] > 1 & \text{if } k = j, \end{cases}$
$1 \leq k \leq J$,

(c)

$$\frac{m_i}{\mu_{0j}}\frac{\partial \mu_{0j}}{\partial m_i} \leq -\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*}m_i < 0, \text{ for } 1 \leq i \leq I \text{ and } 1 \leq j \leq J,$$

(d)

$$\frac{f_j}{\mu_{i0}}\frac{\partial \mu_{i0}}{\partial f_j} \leq -\frac{[\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*}f_j < 0, \text{ for } 1 \leq i \leq I \text{ and } 1 \leq j \leq J,$$

2. *Variation of the log ratio* $\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{ij}^{\mathcal{C}}}$:
If $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ *and* $\beta^{\mathcal{C}} > \beta^{\mathcal{M}}$ *we have*

(a) $\frac{1}{\partial m_i}[\ln \frac{\mu_{kj}^{\mathcal{M}}}{\mu_{kj}^{\mathcal{C}}}] \geq \begin{cases} \frac{\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}}}{m_i^* m_i}\frac{m_k}{m_k^*}\sum_{j=1}^{J}\frac{[\alpha^{\mathcal{M}}\mu_{kj}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{kj}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{kj}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{kj}^{\mathcal{C}}]}{f_j^*} \\ \qquad +(\beta^{\mathcal{M}}-\beta^{\mathcal{C}})\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} > 0 & \text{if } k \neq i \\ \frac{\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}}}{m_i^*}\left[1+\frac{1}{m_i^*}\sum_{j=1}^{J}\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{f_j^*}\right] \\ \qquad +(\beta^{\mathcal{M}}-\beta^{\mathcal{C}})\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} > \alpha^{\mathcal{M}}-\alpha^{\mathcal{C}} & \text{if } k = i, \end{cases}$
$1 \leq k \leq I$

(b) $\frac{1}{\partial f_j}[\ln \frac{\mu_{ik}^{\mathcal{M}}}{\mu_{ik}^{\mathcal{C}}}] \leq \begin{cases} \frac{\beta^{\mathcal{M}}-\beta^{\mathcal{C}}}{f_j^* f_j}\frac{f_k}{f_k^*}\sum_{i=1}^{I}\frac{[\alpha^{\mathcal{M}}\mu_{ik}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ik}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ik}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ik}^{\mathcal{C}}]}{m_i^*} \\ \qquad -(\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}})\frac{[\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} < 0 & \text{if } k \neq j \\ \frac{\beta^{\mathcal{M}}-\beta^{\mathcal{C}}}{f_j^*}[1+\frac{1}{f_j^*}\sum_{i=1}^{I}\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^*}] \\ \qquad -(\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}})\frac{[\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*}f_j < -(\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}}) & \text{if } k = j, \end{cases}$
$1 \leq k \leq J$

where

$$m_i^* \equiv m_i - \sum_{j=1}^{J}[(1-\alpha^{\mathcal{M}})\mu_{ij}^{\mathcal{M}} + (1-\alpha^{\mathcal{C}})\mu_{ij}^{\mathcal{C}}], \text{ for } 1 \leq i \leq I,$$

$$f_j^* \equiv f_j - \sum_{i=1}^{I}[(1-\beta^{\mathcal{M}})\mu_{ij}^{\mathcal{M}} + (1-\beta^{\mathcal{C}})\mu_{ij}^{\mathcal{C}}], \text{ for } 1 \leq j \leq J.$$

It is worth noting that the restriction imposed on β^r and α^r are only necessary and would be very mild depending on the model. For instance, those restrictions directly holds for the CS and DM model; Graham (2013) shows that those restrictions are not necessary to derive the comparative statistics in the CSW model.

C.3 Proof of Theorem 2

All derivation in this section will be done at the matching equilibrium μ^{eq}. However, to ease notation we will use the notation μ.

Proof.

Step 0: Derivation of the $J(\mu)$ matrix.

To ease the notation, in the following we will use $B(\mu)$ to denote $B(\mu; m, f, \theta)$ whenever no confusion is possible.

$J(\mu) = I_{I+J} - \bigtriangledown_\mu B(\mu)$. After tedious but simple manipulations we can show that
$\bigtriangledown_\mu B(\mu) = \begin{pmatrix} E_{11}(\mu) & E_{12}(\mu) \\ E_{21}(\mu) & E_{22}(\mu) \end{pmatrix}$
with
$E_{11}(\mu) = diag\Big\{ \sum_{j=1}^J e_{j|1}(\mu), ..., \sum_{j=1}^J e_{j|I}(\mu) \Big\}$,
$E_{22}(\mu) = diag\Big\{ \sum_{i=1}^I g_{i|1}(\mu), ..., \sum_{i=1}^I g_{i|J}(\mu) \Big\}$ where
$e_{j|i} = \frac{m_i}{\mu_{i0}} \Big[\frac{(1-\alpha^a)\eta_{ij}^a + (1-\alpha^b)\eta_{ij}^b}{\Big(1+\sum_{j=1}^J [\eta_{ij}^a + \eta_{ij}^b]\Big)^2} \Big]$, $g_{i|j} = \frac{f_j}{\mu_{0j}} \Big[\frac{(1-\beta^a)\zeta_{ij}^a + (1-\beta^b)\zeta_{ij}^b}{\Big(1+\sum_{i=1}^I [\zeta_{ij}^a + \zeta_{ij}^b]\Big)^2} \Big]$.
$E_{12}(\mu) = - \begin{pmatrix} \frac{\mu_{10}}{\mu_{01}} \hat{e}_{1|1} & \cdots & \frac{\mu_{10}}{\mu_{0J}} \hat{e}_{J|1} \\ \vdots & \ddots & \vdots \\ \frac{\mu_{I0}}{\mu_{01}} \hat{e}_{1|I} & \cdots & \frac{\mu_{I0}}{\mu_{0J}} \hat{e}_{J|I} \end{pmatrix}$, $E_{21}(\mu) = - \begin{pmatrix} \frac{\mu_{01}}{\mu_{10}} \hat{g}_{1|1} & \cdots & \frac{\mu_{01}}{\mu_{I0}} \hat{g}_{I|1} \\ \vdots & \ddots & \vdots \\ \frac{\mu_{0J}}{\mu_{10}} \hat{g}_{1|J} & \cdots & \frac{\mu_{0J}}{\mu_{I0}} \hat{g}_{I|J} \end{pmatrix}$ where
$\hat{e}_{j|i} = \frac{m_i}{\mu_{0j}} \Big[\frac{\beta^a \eta_{ij}^a + \beta^b \eta_{ij}^b}{\Big(1+\sum_{j=1}^J [\eta_{ij}^a + \eta_{ij}^a]\Big)^2} \Big]$, $\hat{g}_{i|j} = \frac{f_j}{\mu_{i0}} \Big[\frac{\alpha^a \zeta_{ij}^a + \alpha^b \zeta_{ij}^b}{\Big(1+\sum_{i=1}^I [\zeta_{ij}^a + \zeta_{ij}^b]\Big)^2} \Big]$.

Now, it is important to remark that at the **equilibrium** when (32) holds, we get simplified versions of $e_{j|i}$, $g_{i|j}$, $\hat{e}_{j|i}$, and $\hat{g}_{i|j}$ which are the following:
$e_{j|i} = \frac{(1-\alpha^a)\eta_{ij}^a + (1-\alpha^b)\eta_{ij}^b}{1+\sum_{j=1}^J [\eta_{ij}^a + \eta_{ij}^b]} = \frac{1}{m_i} [(1-\alpha^a)\mu_{ij}^a + (1-\alpha^b)\mu_{ij}^b]$;
$g_{j|i} = \frac{(1-\beta^a)\zeta_{ij}^a + (1-\beta^b)\zeta_{ij}^b}{1+\sum_{i=1}^I [\zeta_{ij}^a + \zeta_{ij}^b]} = \frac{1}{f_j} [(1-\beta^a)\mu_{ij}^a + (1-\beta^b)\mu_{ij}^b]$;
$\hat{e}_{j|i} = \frac{\beta^a \eta_{ij}^a + \beta^b \eta_{ij}^b}{1+\sum_{j=1}^J [\eta_{ij}^a + \eta_{ij}^b]} = \frac{1}{m_i} [\beta^a \mu_{ij}^a + \beta^b \mu_{ij}^b]$;
$\hat{g}_{j|i} = \frac{\alpha^a \zeta_{ij}^a + \alpha^b \zeta_{ij}^b}{1+\sum_{i=1}^I [\zeta_{ij}^a + \zeta_{ij}^b]} = \frac{1}{f_j} [\alpha^a \mu_{ij}^a + \alpha^b \mu_{ij}^b]$;

An appropriate adaptation of the supplement calculation of Graham (2013) (not published) would help the reader to understand some details of the calculations, that we have done here. Note that $0 < \sum_{j=1}^J e_{j|i}(\mu) < 1$, for all $1 \leq i \leq I$, and $0 < \sum_{i=1}^I g_{i|j}(\mu) < 1$ for all $1 \leq j \leq J$ whenever $0 < \beta^r < 1$ and $0 < \alpha^r < 1$ for $r \in \{a, b\}$. Now, we can write $J(\mu)$ at the equilibrium. We have the following:
$J(\mu) = \begin{pmatrix} J_{11}(\mu) & J_{12}(\mu) \\ J_{21}(\mu) & J_{22}(\mu) \end{pmatrix}$
where $J_{11}(\mu) = I\{I\} - E_{11}(\mu)$, $J_{22}(\mu) = I\{J\} - E_{22}(\mu)$, $J_{12}(\mu) = -E_{12}(\mu)$, $J_{21}(\mu) = -E_{21}(\mu)$

Step 1: Factorization of the $J(\mu)$ matrix

Recall $J(\mu) = \begin{pmatrix} J_{11}(\mu) & J_{12}(\mu) \\ J_{21}(\mu) & J_{22}(\mu) \end{pmatrix}$, where

$$J_{12}(\mu) = diag(m)^{-1}\left\{ \beta^a \begin{pmatrix} \frac{\mu_{10}}{\mu_{01}}\mu_{11}^a & \cdots & \frac{\mu_{10}}{\mu_{0J}}\mu_{1J}^a \\ \vdots & \ddots & \vdots \\ \frac{\mu_{I0}}{\mu_{01}}\mu_{I1}^a & \cdots & \frac{\mu_{I0}}{\mu_{0J}}\mu_{IJ}^a \end{pmatrix} + \beta^b \begin{pmatrix} \frac{\mu_{10}}{\mu_{01}}\mu_{11}^b & \cdots & \frac{\mu_{10}}{\mu_{0J}}\mu_{1J}^b \\ \vdots & \ddots & \vdots \\ \frac{\mu_{I0}}{\mu_{01}}\mu_{I1}^b & \cdots & \frac{\mu_{I0}}{\mu_{0J}}\mu_{IJ}^b \end{pmatrix} \right\}$$

Define $diag(\mu_{\cdot 0}) = diag(\mu_{10}, ..., \mu_{I0})$, $diag(\mu_{0\cdot}) = diag(\mu_{01}, ..., \mu_{0J})$ and $R^r = \begin{pmatrix} \mu_{11}^r & \cdots & \mu_{1J}^r \\ \vdots & \ddots & \vdots \\ \mu_{I1}^r & \cdots & \mu_{IJ}^r. \end{pmatrix}$.

Therefore,

$$J_{12}(\mu) = diag(\mu_{.0}) diag(m)^{-1}[\beta^a R^a + \beta^b R^b] diag(\mu_{0.})^{-1}$$

Similarly, we can show that $J_{21}(\mu)$ can be factored as follows:

$$J_{21}(\mu) = diag(\mu_{0.}) diag(f)^{-1}[\alpha^a (R^a)' + \alpha^b (R^b)'] diag(\mu_{.0})^{-1}$$

We also factor also $J_{11}(\mu)$ and $J_{22}(\mu)$ as follows:

$$J_{11}(\mu) = I_I - diag(m)^{-1}[(1-\alpha^a)R_{\iota J}^a + (1-\alpha^a)R_{\iota J}^b],$$

$$J_{22}(\mu) = I_J - diag(f)^{-1}[(1-\beta^a)(R^a)'_{\iota I} + (1-\beta^b)(R^b)'_{\iota I}].$$

where $R_{\iota J}^r = (\sum_{j=1}^J \mu_{1j}^r, ..., \sum_{j=1}^J \mu_{Ij}^r)'$ and $(R^r)_{\iota I} = (\sum_{i=1}^I \mu_{i1}^r, ..., \sum_{i=1}^I \mu_{iJ}^r)$. After rearranging we can show that:

$$J(\mu) = C(\mu)^{-1}[A(\mu) + U(\mu)B_0(\mu)U(\mu)^{-1}]$$

where

$C(\mu) = \begin{pmatrix} diag(m) & 0 \\ 0 & diag(f) \end{pmatrix}$

$A(\mu) = \begin{pmatrix} diag(m - (1-\alpha^a)R_{\iota J}^a - (1-\alpha^b)R_{\iota J}^b) & 0 \\ 0 & diag(f - (1-\beta^a)(R^a)'_{\iota I} - (1-\beta^b)(R^b)'_{\iota I}) \end{pmatrix}$

$U(\mu) = \begin{pmatrix} diag(\mu_{.0}) & 0 \\ 0 & diag(\mu_{0.}) \end{pmatrix}$

$B_0(\mu) = \begin{pmatrix} 0 & \beta^a R^a + \beta^b R^b \\ \alpha^a (R^a)' + \alpha^b (R^b)' & 0) \end{pmatrix}$.

Therefore, $J(\mu)$ can be equivalently rewritten as:

$$\begin{aligned} J(\mu) &= U(\mu)C(\mu)^{-1}[A(\mu) + B_0(\mu)]U(\mu)^{-1} \\ &= U(\mu)H(\mu)U(\mu)^{-1} \end{aligned}$$

where

$$H(\mu) = C(\mu)^{-1}[A(\mu) + B_0(\mu)].$$

■ Let us write $H(\mu)$ in detail:

$$H(\mu) = \begin{pmatrix} H_{11}(\mu) & H_{12}(\mu) \\ H_{21}(\mu) & H_{22}(\mu) \end{pmatrix}$$

with

$$H_{11}(\mu) = \begin{pmatrix} 1 - \frac{\sum_{j=1}^{J}[(1-\alpha^a)\mu_{1j}^a + (1-\alpha^b)\mu_{1j}^b]}{m_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - \frac{\sum_{j=1}^{J}[(1-\alpha^a)\mu_{Ij}^a + (1-\alpha^b)\mu_{Ij}^b]}{m_I} \end{pmatrix}, H_{12}(\mu) =$$

$$\begin{pmatrix} \frac{\beta^a\mu_{11}^a + \beta^b\mu_{11}^b}{m_1} & \cdots & \frac{\beta^a\mu_{1J}^a + \beta^b\mu_{1J}^b}{m_1} \\ \vdots & \ddots & \vdots \\ \frac{\beta^a\mu_{I1}^a + \beta^b\mu_{I1}^b}{m_I} & \cdots & \frac{\beta^a\mu_{IJ}^a + \beta^b\mu_{IJ}^b}{m_I} \end{pmatrix}, H_{21}(\mu) = \begin{pmatrix} \frac{\alpha^a\mu_{11}^a + \alpha^b\mu_{11}^b}{f_1} & \cdots & \frac{\alpha^a\mu_{I1}^a + \alpha^b\mu_{I1}^b}{f_1} \\ \vdots & \ddots & \vdots \\ \frac{\alpha^a\mu_{1J}^a + \alpha^b\mu_{1J}^b}{f_J} & \cdots & \frac{\alpha^a\mu_{IJ}^a + \alpha^b\mu_{IJ}^b}{f_J} \end{pmatrix},$$

$$H_{22}(\mu) \;=\; \begin{pmatrix} 1 - \frac{\sum_{i=1}^{I}[(1-\beta^a)\mu_{i1}^a + (1-\beta^b)\mu_{i1}^b]}{f_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - \frac{\sum_{i=1}^{I}[(1-\beta^a)\mu_{iJ}^a + (1-\beta^b)\mu\alpha^b b_{iJ}]}{f_J} \end{pmatrix}.$$

Similar to Graham (2013, p 16), we observe that all elements of $H(\mu)$ are non-negative whenever $0 < \beta^r; \alpha^r \leq 1$.

Step 2: Derivation of M-matrix property

The main goal of this step is to show that the Schur complements of $H(\mu)$ the upper $I \times I$ (H_{11}) and lower $J \times J$ (H_{22}) diagonal blocks, (i.e. $SH_{11} = H_{22} - H_{21}H_{11}^{-1}H_{12}$ and $SH_{22} = H_{11} - H_{12}H_{22}^{-1}H_{21}$) are M-matrices which implies $SH_{11}^{-1} \geqq 0$ and $SH_{22}^{-1} \geqq 0$. To show that, we first need to show that $H(\mu)$ is row diagonally dominant. In other terms, if we denote the element of $H(\mu)$, h_{ij} with $1 \leq i, j \leq I + J$ we need to show that there exist $d_i > 0$ such that $d_i|h_{ii}| > \sum_{j \neq i}^{I+J} d_j|h_{ij}|$. This will be difficult to show without further restrictions on β^r and α^r. Graham (2013, p15) showed this result in the particular case where the two following restrictions hold simultaneously: $\beta^r + \alpha^r = 1$ and $\beta^a = \beta^b$. Here, we will impose some conditions on the coefficients β^r and α^r that ensure $H(\mu)$ to be row diagonally dominant. Let first assume that $0 < \beta^r; \alpha^r < 1$, then $h_{ij} \geq 0$ for $1 \leq i, j \leq I + J$.

Case 1: $1 \leq i \leq I$

$$|h_{ii}| > \sum_{j \neq i}^{I+J} |h_{ij}| \Leftrightarrow \sum_{j=1}^{J} \Big((1 - \alpha^a + \beta^a)\mu_{ij}^a + (1 - \alpha^b + \beta^b)\mu_{ij}^b\Big) < m_i. \quad (33)$$

Notice that

$$\max\Big((1 - \alpha^a + \beta^a), (1 - \alpha^b + \beta^b)\Big) \sum_{j=1}^{J} \Big(\mu_{ij}^a + \mu_{ij}^b\Big) < m_i \Rightarrow$$

$$\sum_{j=1}^{J} \Big((1 - \alpha^a + \beta^a)\mu_{ij}^a + (1 - \alpha^b + \beta^b)\mu_{ij}^b\Big) < m_i,$$

and

$$\max\Big((1-\alpha^a+\beta^a),(1-\alpha^b+\beta^b)\Big)\sum_{j=1}^{J}\Big(\mu^a_{ij}+\mu^b_{ij}\Big) < m_i \quad \Leftrightarrow$$
$$\max\Big((1-\alpha^a+\beta^a),(1-\alpha^b+\beta^b)\Big)\rho^m_i < 1,$$

where $\rho^m_i \equiv \frac{m_i-\mu_{i0}}{m_i}$ is the rate of matched men of type i. The latter inequality is equivalent to $\max(\beta^b-\alpha^b,\beta^a-\alpha^a) < \frac{1-\rho^m_i}{\rho^m_i}$. Therefore, if $\max(\beta^b-\alpha^b,\beta^a-\alpha^a) < \frac{1-\rho^m_i}{\rho^m_i}$ for all i then $|h_{ii}| > \sum_{j\neq i}^{I+J}|h_{ij}|$.

Case 2: $I+1 \leq i \leq I+J$.

Similarly, we can show that if $\min(\beta^b-\alpha^b,\beta^a-\alpha^a) > -\frac{1-\rho^f_j}{\rho^f_j}$ for all j where $\rho^f_j \equiv \frac{f_j-\mu_{0j}}{f_j}$ is the rate of matched women of type j, then we have $|h_{ii}| > \sum_{j\neq i}^{I+J}|h_{ij}|$.

Assume that the two latter restrictions on β^r and α^r hold in the rest of the proof. The Schur complements of the $H(\mu)$ upper $I\times I$ and lower $J\times J$ diagonal blocks are $SH_{11} = H_{22} - H_{21}(H_{11})^{-1}H_{12}$ and $SH_{22} = H_{11} - H_{12}(H_{22})^{-1}H_{21}$. Since H has been showed to be diagonally dominant, Theorem 1 of Carlson and Markham (1979 p 249) implies that the two schur complements are also diagonally dominant. Therefore, SH_{11} and SH_{22} are also row diagonally dominant. We can easily see that SH_{11} and SH_{22} are also Z-matrices (i.e., members of the class of real matrices with nonpositive off-diagonal elements). By applying Theorem 4.3 of Fiedler and Ptak (1962) it follows that they are M-matrices and then $SH_{11}^{-1} \geqq 0$ and $SH_{22}^{-1} \geqq 0$. These results are sufficient to establish the sign structure of $H^{-1}(\mu)$. $H^{-1}(\mu) = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22}\end{pmatrix} = \begin{pmatrix} + & \vdots & - \\ \cdots & & \cdots \\ - & \vdots & + \end{pmatrix}$ where W_{ij} are exactly defined as defined in Graham (2013. p 16).

Step 3: Derivation of $H^{-1}(\mu)$

Following Graham we can show the following inequalities:

$$\begin{aligned}
W_{11} &\geq H_{11}^{-1} + H_{11}^{-1}H_{12}H_{22}^{-1}H_{21}H_{11}^{-1} = LW_{11} \\
W_{22} &\geq H_{22}^{-1} + H_{22}^{-1}H_{21}H_{11}^{-1}H_{12}H_{22}^{-1} = LW_{22} \\
W_{12} &\leq -H_{11}^{-1}H_{12}H_{22}^{-1} = UW_{12} \\
W_{21} &\leq -H_{22}^{-1}H_{21}H_{11}^{-1} = UW_{21}.
\end{aligned}$$

Using the expression of the matrix $H(\mu)$ and after some tedious calculations we can show the following: $LW_{11} = H_{11}^{-1} +$

$$\begin{pmatrix}
\frac{1}{m_1^*}\frac{m_1}{m_1^*}\sum_{j=1}^{J}\frac{[\alpha^a\mu^a_{1j}+\alpha^b\mu^b_{1j}][\beta^a\mu^a_{1j}+\beta^b\mu^b_{1j}]}{f_j^*} & \cdots & \frac{1}{m_1^*}\frac{m_I}{m_I^*}\sum_{j=1}^{J}\frac{[\alpha^a\mu^a_{Ij}+\alpha^b\mu^b_{Ij}][\beta^a\mu^a_{1j}+\beta^b\mu^b_{1j}]}{f_j^*} \\
\vdots & \ddots & \vdots \\
\frac{1}{m_I^*}\frac{m_1}{m_1^*}\sum_{j=1}^{J}\frac{[\alpha^a\mu^a_{1j}+\alpha^b\mu^b_{1j}][\beta^a\mu^a_{Ij}+\beta^b\mu^b_{Ij}]}{f_j^*} & \cdots & \frac{1}{m_I^*}\frac{m_I}{m_I^*}\sum_{j=1}^{J}\frac{[\alpha^a\mu^a_{Ij}+\alpha^b\mu^b_{Ij}][\beta^a\mu^a_{Ij}+\beta^b\mu^b_{Ij}]}{f_j^*}
\end{pmatrix}$$

where

$$m_i^* \equiv m_i - \sum_{j=1}^{J} [(1-\alpha^a)\mu_{ij}^a + (1-\alpha^b)\mu_{ij}^b], \text{ for all } 1 \leq i \leq I$$

and

$$f_j^* \equiv f_j - \sum_{i=1}^{I} [(1-\beta^a)\mu_{ij}^a + (1-\beta^b)\mu_{ij}^b], \text{ for all } 1 \leq j \leq J.$$

Moreover, we can show that:

$$\begin{aligned}(LW_{11})_{ii} &= \frac{m_i}{m_i^*}\Big[1 + \frac{1}{m_i^*}\sum_{j=1}^{J}\frac{[\alpha^a\mu_{ij}^a + \alpha^b\mu_{ij}^b][\beta^a\mu_{ij}^a + \beta^b\mu_{ij}^b]}{f_j^*}\Big] \\ &> 1,\end{aligned}$$

for all $1 \leq i \leq I$. Therefore we have $LW_{11} > I_I$. Similarly, we have also the following: $LW_{22} = H_{22}^{-1} +$

$$\begin{pmatrix} \frac{1}{f_1^*}\frac{f_1}{f_1^*}\sum_{i=1}^{I}\frac{[\alpha^a\mu_{i1}^a+\alpha^b\mu_{i1}^b][\beta^a\mu_{i1}^a+\beta^b\mu_{i1}^b]}{m_i^*} & \cdots & \frac{1}{f_1^*}\frac{f_J}{f_J^*}\sum_{i=1}^{I}\frac{[\alpha^a\mu_{i1}^a+\alpha^b\mu_{i1}^b][\beta^a\mu_{iJ}^a+\beta^b\mu_{iJ}^b]}{m_i^*} \\ \vdots & \ddots & \vdots \\ \frac{1}{f_J^*}\frac{f_1}{f_1^*}\sum_{i=1}^{I}\frac{[\alpha^a\mu_{iJ}^a+\alpha^b\mu_{iJ}^b][\beta^a\mu_{i1}^a+\beta^b\mu_{i1}^b]}{m_i^*} & \cdots & \frac{1}{f_J^*}\frac{f_J}{f_J^*}\sum_{i=1}^{I}\frac{[\alpha^a\mu_{iJ}^a+\alpha^b\mu_{iJ}^b][\beta^a\mu_{iJ}^a+\beta^b\mu_{iJ}^b]}{m_i^*} \end{pmatrix}$$

Moreover, we can show that:

$$\begin{aligned}(LW_{22})_{jj} &= \frac{f_j}{f_j^*}\Big[1 + \frac{1}{f_j^*}\sum_{i=1}^{I}\frac{[\alpha^a\mu_{ij}^a + \alpha^b\mu_{ij}^b][\beta^a\mu_{ij}^a + \beta^b\mu_{ij}^b]}{m_i^*}\Big] \\ &> 1,\end{aligned}$$

for all $1 \leq j \leq J$. Therefore, we have $LW_{11} > I_J$. Now, let us look at the off-diagonal blocks of $H(\mu)^{-1}$.

$$UW_{12} = -\begin{pmatrix} \frac{[\beta^a\mu_{11}^a+\beta^b\mu_{11}^b]}{m_1^* f_1^*}f_1 & \cdots & \frac{[\beta^a\mu_{1J}^a+\beta^b\mu_{1J}^b]}{m_1^* f_J^*}f_J \\ \vdots & \ddots & \vdots \\ \frac{[\beta^a\mu_{I1}^a+\beta^b\mu_{I1}^b]}{m_I^* f_1^*}f_1 & \cdots & \frac{[\beta^a\mu_{IJ}^a+\beta^b\mu_{IJ}^b]}{m_I^* f_J^*}f_J \end{pmatrix}$$

and $UW_{21} = -\begin{pmatrix} \frac{[\alpha^a\mu_{11}^a+\alpha^b\mu_{11}^b]}{m_1^* f_1^*}m_1 & \cdots & \frac{[\alpha^a\mu_{I1}^a+\alpha^b\mu_{I1}^b]}{m_1^* f_J^*}m_I \\ \vdots & \ddots & \vdots \\ \frac{[\alpha^a\mu_{1J}^a+\alpha^b\mu_{1J}^b]}{m_I^* f_1^*}m_1 & \cdots & \frac{[\alpha^a\mu_{IJ}^a+\alpha^b\mu_{IJ}^b]}{m_I^* f_J^*}m_I \end{pmatrix}$

Step 4: Main results

Case 1: Type specific elasticities of single hood

By applying the implicit function theorem to the equation (32) we have: $\frac{\partial\mu}{\partial m_i} = J(\mu)^{-1}\frac{\partial B}{\partial m_i}$ for $1 \leq i \leq I$ and $\frac{\partial\mu}{\partial f_j} = J(\mu)^{-1}\frac{\partial B}{\partial f_j}$ for all $1 \leq j \leq J$, where $\frac{\partial B}{\partial m_i} = (0, ..., 0, \frac{\mu_{i0}}{m_i}, 0, ..., 0)'$ and $\frac{\partial B}{\partial f_j} = (0, ..., 0, \frac{\mu_{0j}}{f_j}, 0, ..., 0)'$ are $(I+J)$ vectors such that the non-zero entries are respectively at the i^{th} row and the $(I+j)^{th}$ row. Let $h_k =$

$(0, ..., 0, 1, 0, ..., 0)'$ be a $(I + J)$ vector such that the non-zero entry is at the k^{th} row. We have the following:

$$\begin{aligned} U(\mu)^{-1}\frac{\partial \mu}{\partial m_i} m_i &= U(\mu)^{-1} J(\mu)^{-1} \frac{\partial B}{\partial m_i} m_i \\ &= H(\mu)^{-1} U(\mu)^{-1} h_i \mu_{i0} \\ &= H(\mu)^{-1} h_i \\ &= [H(\mu)^{-1}]_{\cdot i} \end{aligned} \tag{34}$$

for $1 \le i \le I$, where $[H(\mu)^{-1}]_{\cdot i}$ represents the i^{th} column of the matrix $H(\mu)^{-1}$. Similarly, we can show that $U(\mu)^{-1} \frac{\partial \mu}{\partial f_j} f_j = [H(\mu)^{-1}]_{\cdot (I+j)}$ for $1 \le j \le J$. Putting these results together, we get the following inequalities:

$$\frac{m_i}{\mu_{k0}} \frac{\partial \mu_{k0}}{\partial m_i} \ge \begin{cases} \frac{1}{m_i^*} \frac{m_k}{m_k^*} \sum_{j=1}^{J} \frac{[\alpha^a \mu_{kj}^a + \alpha^b \mu_{kj}^b][\beta^a \mu_{kj}^a + \beta^b \mu_{kj}^b]}{f_j^*} > 0 & \text{if } k \neq i \\ \frac{m_i}{m_i^*} [1 + \frac{1}{m_i^*} \sum_{j=1}^{J} \frac{[\alpha^a \mu_{ij}^a + \alpha^b \mu_{ij}^b][\beta^a \mu_{ij}^a + \beta^b \mu_{ij}^b]}{f_j^*}] > 1 & \text{if } k = i, \end{cases} \quad \text{for } 1 \le k \le I.$$

$$\frac{f_j}{\mu_{0k}} \frac{\partial \mu_{0k}}{\partial f_j} \ge \begin{cases} \frac{1}{f_j^*} \frac{f_k}{f_k^*} \sum_{i=1}^{I} \frac{[\alpha^a \mu_{ik}^a + \alpha^b \mu_{ik}^b][\beta^a \mu_{ik}^a + \beta^b \mu_{ik}^b]}{m_i^*} > 0 & \text{if } k \neq j \\ \frac{f_j}{f_j^*} [1 + \frac{1}{f_j^*} \sum_{i=1}^{I} \frac{[\alpha^a \mu_{ij}^a + \alpha^b \mu_{ij}^b][\beta^a \mu_{ij}^a + \beta^b \mu_{ij}^b]}{m_i^*}] > 1 & \text{if } k = j, \end{cases} \quad \text{for } 1 \le k \le J.$$

$$\frac{m_i}{\mu_{0j}} \frac{\partial \mu_{0j}}{\partial m_i} \le -\frac{[\alpha^a \mu_{ij}^a + \alpha^b \mu_{ij}^b]}{m_i^* f_j^*} m_i < 0$$

and

$$\frac{f_j}{\mu_{i0}} \frac{\partial \mu_{i0}}{\partial f_j} \le -\frac{[\beta^a \mu_{ij}^a + \beta^b \mu_{ij}^b]}{m_i^* f_j^*} f_j < 0$$

for $1 \le i \le I$ and $1 \le j \le J$.

C.4 Proof of Proposition 4

Recall, from the result of Theorem 1 we know that the fixed point representation (32) admits a unique solution. Therefore, $\mu - B(\mu; m, f, \theta)$ must be at least locally invertible at the equilibrium. This ensures that it jacobian matrix $J(\mu)$ does not vanish at the equilibrium. Then, $det(J(\mu)) \neq 0$ for all $\beta^r, \alpha^r > 0$. Since we shown within Step 1 of proof of Theorem 2 that $J(\mu) = U(\mu)H(\mu)U(\mu)^{-1}$ for all $\beta^r, \alpha^r > 0$, we have then $det(H(\mu)) \neq 0$. Moreover, we have shown that

$$\sum_{i=1}^{I} U(\mu)^{-1} \frac{\partial \mu}{\partial m_i} m_i + \sum_{j=1}^{J} U(\mu)^{-1} \frac{\partial \mu}{\partial f_j} f_j = \sum_{i=1}^{I} [H(\mu)^{-1}]_{\cdot i} + \sum_{j=1}^{J} [H(\mu)^{-1}]_{\cdot (I+j)}.$$

If $\beta^r + \alpha^r = 1$, we observe that all elements of $H(\mu)$ are non-negative and the rows sum to one. Therefore, $H(\mu)$ is a row stochastic matrix, see Horn and Johnson (2013, p.547), with an inverse whose rows also sum to one. Then,

$$[H(\mu)^{-1}]_{\cdot i} + \sum_{j=1}^{J} [H(\mu)^{-1}]_{\cdot (I+j)} = \iota_{I+J}.$$

where $\iota_{I+J} = (1, ..., 1)'$. The last equality holds since the rows of $[H(\mu)^{-1}]$ sum to one.