

Misdiagnosing Bank Capital Problems

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Abstract

Banks' reluctance to repair their balance sheets, combined with deposit insurance and regulatory forbearance in recognizing greater risks and losses, can lead to solvency problems that look like liquidity (bank-run) crises. Regulatory forbearance incentivizes banks to both retain risky loans and reject new good opportunities. With sufficient regulatory forbearance, partially-insured banks act exactly as if they are fully insured. Stress tests certify that uninsured creditors will be paid, not that banks are solvent, and have ambiguous effects on the efficiency of investment.

Keywords: Bank Capital, Regulatory Capital, Capital Requirements, Regulatory Forbearance, Bank Runs, Solvency Runs, Liquidity Runs, Stress Tests, Financial Crisis

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1 Introduction

Bankers, regulators, policy makers, and scholars all debate the relative importance of solvency and liquidity in banking crises.¹ The distinction matters. The solution to solvency problems is more capital, while the usual solution to liquidity problems is to transfer risk to taxpayers, through increased insurance or lender of last resort financing. Diagnosing a solvency issue as one of liquidity implies less capital and more insurance, distorting banks' incentives. We will show that insured banks retain risky loans that uninsured banks would sell, and turn down new loans that uninsured banks would make.

But the regulatory system's emphasis has been on more insurance rather than more capital. For example, the ratio of U.S. banks' regulatory capital to their risk-weighted assets rose from 14.28% in 2009 to just 14.65% in 2019.² On the other hand, "volatile liabilities" less cash, a proxy for uninsured short-term debt, fell from 32% of assets in 2007 to 6% in 2019, while domestic deposits, a proxy for insured debt, now far surpass loans outstanding.³

Maybe this reflects a view that banks are now well enough capitalized to deal with solvency problems, and that the primary focus should be on the market failures that cause liquidity shortages and bank runs. But solvency problems can develop well out of proportion to capital losses. The reason is that the amount of uninsured riskless debt that banks can safely issue is very sensitive to value of the collateral backing their loans.

Consider, for example, a multi-period non-recourse real-estate loan. In each period the value of the real estate serving as collateral is multiplied by

¹Beautiful models of liquidity crises (e.g., Diamond and Dybvig 1983, Morris and Shin 1998, 2016, and related papers such as Postlewaite and Vives 1987, Rochet and Vives 2004, and Allen and Gale, 2007) have been used to explain not just short-term squeezes, but also extended periods such as the Financial Crisis of 2008-9. The literature on liquidity crises goes back to Bryant (1980), and that on solvency at least to Mitchell (1941).

²See Table 3, line 8. (2020 data may reflect pandemic-related distortions.) There is controversy about the degree to which capital levels have been strengthened. For example, Atkeson et al. (2019) and Berndt et al. (2020) contend that the value of big banks' "too big to fail" subsidy has declined. Sarin and Summers (2016, 2021) are much more skeptical. Capital requirements have also risen for the largest banks relative to others.

³See Table 3, lines 7, 2, and 3. The U.K.'s approach has been more balanced. The U.K. banking sector's regulatory capital increased from 16.2% of risk-weighted assets in 2014-1 to 21.3% at 2019-4, see Bank of England (2021). However, U.K. share prices remain below book values. Insurance has also increased in the U.K.: short-term funding in the U.K. (similar to U.S. volatile liabilities but excluding repo), fell from 15% of total funding in 2007 to 4% in 2019. Loans exceeded customer deposits by £914 billion at end-2008 but deposits exceeded loans by £238 billion by end-2018. See Bank of England (2019a).

an i.i.d. shock. If, as time passes, the value of the collateral slips only moderately, the loan's value may nevertheless increase, both because of the time value of money and because, with less time left to maturity, the probability of default may be lower. But the amount of short-term riskless⁴ debt that the loan can support is determined by the value of the bank's claim in the "worst case" for the *next* period. When in that worst case the bank receives the value of the collateral, then borrowing capacity will be proportional to the collateral's value.

So if the value of collateral had declined by 5%, the loan would support 5% less debt, even if the loan's value was unchanged. If the bank had been able to finance the loan with 90% debt, then the decline would now limit it to 85.5% debt. So, absent deposit insurance, the bank would either have to sell 45% more shares or, if it sold no shares, over one-third of depositors would "run", forcing the bank to sell enough of its loans to reduce its debt ratio to 85.5%.^{5,6} This increase in market capital requirements or "haircuts" as a percentage of market value could be misinterpreted as the liquidity of the borrowing market seizing up.⁷ If the collateral value's decline from the last shock did reduce the loan's value, the necessary dilution or contraction would be even greater.

Moreover, say that the book value of some loans substantially exceeded their market value. For example, Gorton (2009) found the market value of a mortgage index was 20 but the book value of a similar but illiquid pool was

⁴For simplicity, we assume the debt is completely riskless. Assuming (more realistically) that the "riskless" debt that a bank creates has a very tiny chance of default would make no difference to any of our results.

⁵The bank can either sell 45% more shares (since equity has to rise from 10% to 14.5%) or sell $45/145 \approx 31\%$ of its loans at the unchanged value (or some combination). Regardless of whether the bank chose to sell shares or assets, shareholders whose original holdings formerly represented \$100 of equity and \$90 of debt would now have $100-31=69$ in equity and $90-31=59$ in debt, creating the necessary $59/69=85.5\%$ debt ratio. Deposits fall from 90 to 59.

An elaboration of this example is fully worked out in Appendix A. We develop a more detailed example in section 3.5.

⁶In bankers' terms, as the collateral provides less coverage for the loan, the loan requires a higher risk weight. In financial economists' terms, the borrower holds a put option entitling it to return the collateral in lieu of repaying the loan. As the collateral falls in value, the expected cost to the bank of the option rises. Additionally, the option's "delta" increases, so the bank's claim becomes riskier and so able to back less riskless debt. Geanakoplos (2009) shows how collateral has a procyclical effect on haircuts. Kiyotaki and Moore (1997) were early in identifying the importance of collateral.

⁷For junior securities, such as the BBB tranche of mortgage pools, a decline in collateral value could mean that in the worst state the securities will be worthless. In this case, no riskless borrowing would be possible and the haircut would be 100%.

60. Then using the book value would reinforce the misleading impression of a collapse in the liquidity of the borrowing market.⁸

By stark contrast, a bank with deposit insurance could maintain its liabilities while adding little equity, if the regulator's forbearance lets the bank use a book value that reflects little of the decline in market value to calculate *regulatory capital*, and allows it to treat the risk weight of the loan as unchanged.

Regulatory forbearance creates especially low capital requirements for the toxic assets that are the riskiest, encouraging insured banks to retain loans that would otherwise be sold and then, for "debt overhang" reasons, reject good new loans.

Furthermore, if regulators commit to a policy of regulatory forbearance they may create "virtual insurance" that protects some *uninsured* debt against loss—if a bank's assets are sufficiently generously marked, then these uninsured investors can be always confident of being repaid before the bank is closed down. So even if a bank's insured deposits are constrained, it may be able to borrow significant amounts of uninsured debt on the same terms as insured debt, and be able to behave exactly as if it were fully insured. Although uninsured debt is not legally senior to the deposit insurer, all bank credit risk is borne by the insurer.

With a more limited commitment to forbearance, a partially-insured bank faces a "pseudo-liquidity" constraint (PLC) on the minimum amount of capital it must have. Analogous to a liquidity constraint that requires a bank to hold enough capital to protect it against insolvency in case it has to sell assets at fire-sale prices below market value, the PLC requires it to protect against regulatory foreclosure caused by uninsured depositors' withdrawals forcing it to sell assets at market prices that are below book value.

Put another way, meeting the PLC assures creditors that even if a bank is insolvent it will have enough regulatory capital that uninsured creditors will be able to front-run the deposit insurer and get all their money out before the regulator forecloses.

The PLC, when binding, sets capital requirements for partially-insured banks that are between the regulatory requirements for fully insured banks

⁸Gorton (2009) compared the ABX BBB index for junior mortgage securities, then trading at 20, with a constructed index of Residential Mortgage Backed Securities that he regarded as comparable except for being less liquid and which had an average book value of 60. We will give some of the many other examples of the wide gaps between market and regulatory values during the Financial Crisis, and the projected gaps envisioned in the Stress Tests.

and the market requirements for partly insured banks. However, the distortions in banks' incentives to make and retain loans may be even worse for partially-insured banks than for fully-insured banks. Because it builds on regulatory capital requirements that may discourage new lending, regulatory relaxation of a bank's PLC is sometimes a second-best strategy for encouraging new loans. But because relaxed requirements may also encourage the retention of more "toxic assets" with less equity, relaxing the PLC may transfer risks to taxpayers and discourage new lending.

We outline our model of a bank in section 2.

Section 3 develops preliminary results. We show that, with forbearance and deposit insurance, pooling assets reduces shareholder value—the opposite result to that of a standard model without deposit insurance (such as, e.g., Diamond, 2020). Importantly, we also show that, absent deposit insurance, the minimum equity required to finance a bank loan is intrinsically more volatile than the loan's value, so loans that fall in value (and even some that rise) become riskier. We give an example.

Section 4 analyzes how banks' investment behaviour in bad times depends on whether their short-term debt financing is uninsured, fully insured, or a mixture. We show that although uninsured banks that are subject to the market's discipline always prefer making new loans to retaining underperforming old loans, insured banks prefer the old loans for two reasons. First, overstating the value of these loans, and understating their risks, allows insured banks to retain them using less equity than needed to make lower-risk new loans. Second, the lower equity requirement means that the insurance system subsidizes the retention of old loans and reduces the profitability of new ones.

Partially insured banks, unlike fully-insured banks, have to meet their PLCs to attract uninsured creditors. For those banks benefiting from generous forbearance, virtual insurance is a perfect substitute for formal insurance; virtual insurance unambiguously helps share prices, and encourages banks to retain more risky old loans. But virtual insurance may either improve or worsen incentives for making new loans.

With more limited forbearance, partially insured banks do not benefit from virtual insurance. These banks must either limit their debt to the amount formally insured or, if they wish to issue any uninsured debt, meet their PLCs by operating as if their formal insurance was reduced (though not fully eliminated). This creates a discontinuity in a bank's capital costs at the point where it begins uninsured borrowing, and so can deter a bank for making new loans.

Section 5 describes the twin roles of stress tests. The tests effectively

substitute a *weaker* requirement for the PLC but apply that requirement to *all* banks, including the fully insured and those otherwise not constrained by the PLC. So the tests raise the capital requirements of those banks, benefiting taxpayers and encouraging banks to make more efficient portfolio choices. However, capital requirements are reduced for banks that were previously constrained by the PLC. These reduced requirements may have been the main effect of the original stress tests in the Financial Crisis. Rather than showing that the banks were solvent, they promised that the government would provide enough support that uninsured lenders would be repaid in full regardless of banks' solvency. But while the tests made it easier for these banks to raise money to make new loans, they also increased their incentive to inefficiently retain high-risk old loans.

Section 6 gives examples of phenomena that are commonly attributed to liquidity issues, but can also be explained, and might sometimes be better explained, by solvency problems. For example, our analysis explains why banks have sometimes refused to sell assets at *above* fair value, although in a liquidity-based model, banks would jump at the chance to do this. We show that bankers' arguments for lower capital requirements often depend on questionable assumptions such as mean reversion in asset prices, as in the Last Taxi Fable.

Sections 3-6 show the need for more market-based capital requirements. But market-based capital requirements are hard to implement if banks cannot or will not raise additional equity in bad times. So in section 7 we explain how to create self-repairing balance sheets. Doing so privatizes both liquidity and solvency risk, and improves incentives for banks at less cost to taxpayers, while still preventing bank failures.

2 The Model

2.1 Modelling Banking

The Role of Banking

Banks would have no role in an economy with a complete Arrow-Debreu securities market. Potential borrowers and lenders would simply trade for the contingent claims that maximized their welfare, with no need for a middleman (bank). But with incomplete markets bankers give borrowers and lenders opportunities that they cannot access on their own.

We model banks as adding value by their ability to identify and service both short-term lenders/depositors and long-term borrowers interested in "liquidity". For depositors, liquidity means providing riskless access to their

money at any time. For borrowers, the bank identifies good risks desiring long-term financing that protects them from having to put up more money if their collateral falls in value.⁹ The bank is able to earn a (risk-adjusted) spread from these customers, and if loans are safe enough this spread exceeds the bank's costs.

Even without government insurance a bank is able to borrow short and safe while lending long and risky if (1) it raises an initial cushion of equity to protect against loss and (2) when losses occur it repairs its balance sheet to assure depositors' future safety, through some combination of stock sales and asset sales. Essentially the bank is able to make a profit by sharing its superior access to the capital markets with its customers.

For simplicity we model the ability of the bank to earn a margin by assuming that all risky assets earn the same expected return while, as in Stein (2012), investors who want a safe demand-deposit are willing to accept a lower rate (normalized to zero). One may think of investors as risk-neutral but for a willingness to sacrifice expected return for a completely riskless claim.¹⁰

Banks' Comparative Advantage

We assume as, for example, William Diamond (2020), banks have a comparative advantage in holding low-risk loans that can be used to create safe short-term debt using relatively little equity. Conversely, they are disadvantaged at holding high-risk loans, which are less good as collateral. To model this as simply as possible, we assume banks are uniquely able to issue riskless short-term debt, but that their costs of managing assets are higher than non-banks'.

We further assume that the government has no advantage or disadvantage over the market in providing insurance, so a bank's contribution to welfare is the value of its portfolio to an uninsured bank, less its riskless debt and tangible equity.

Importantly, banks are therefore efficient holders of low-risk loans. However, they are also inefficient holders of old bank loans after a sufficiently bad shock lowers the value of the collateral backing them, because this makes the loans riskier and able to support less safe debt.

⁹Home buyers would have little use for a mortgage that required them to supplement their downpayment if housing prices dropped.

¹⁰Stein (2012) cites Sidrauski (1967) and Krishnamurthy and Vissing-Jorgenson (2012) as papers that similarly put money directly into consumers' utility function. We follow this approach to creating non-Arrow Debreu securities because of its simplicity.

Capital Requirements

We model the need for capital requirements by assuming the bank (and regulators) can only react to economic shocks at the beginning of each discrete period, but allow it to react fully at those times. So while our banks' assets are not perfectly liquid, a solvent bank cannot be bankrupted by a liquidity crisis.¹¹

2.2 Model Details

We consider a bank which maximises shareholder wealth, and in which all actions take place at the discrete times $t = 0$, $t = 1$, and $t = 2$.

At times $t = 0, 1$, the bank finances a loan portfolio with insured debt (deposits), D_i , riskless uninsured debt, U_i , and tangible equity, Q_i .¹² We will consider different cases: $D_i \equiv 0$; D_i is unconstrained; and the bank is constrained to $D_i \leq \bar{D}$, in which $0 < \bar{D} < \infty$. All loans come due at $t = 2$.

Equity can be raised at any time at the price that yields the market expected return of $r > 0$ per period. Riskless short-term debt earns 0.¹³

Loans

Borrowers at $t = 0$ each post an asset worth $K > 1$ as collateral against a loan of \$1, and promise to repay P at $t = 2$.¹⁴ All collateral is affected multiplicatively, and identically, by two random shocks, θ_1 (at $t = 1$) and θ_2 (at $t = 2$), so absent management costs the loans would be worth $\min(K\theta_1\theta_2, P)$ at $t = 2$.

The shocks θ_1 and θ_2 are realisations of the random variables $\tilde{\theta}_1$ and $\tilde{\theta}_2$, respectively; $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are distributed independently and identically, with expectation $E(\tilde{\theta}_1) = E(\tilde{\theta}_2) = (1 + r)$, minimum value $\underline{\theta} > 0$, and maximum value $\bar{\theta}$. We assume $K\underline{\theta} < 1$, and $K\bar{\theta} < P$.

¹¹Analogous assumptions are made by, e.g., Stein (2012) and Geanakoplos (2009). So market capital requirements (but not necessarily regulatory requirements) will be determined by the worst that can happen before the bank has time to react.

¹²Tangible equity equals the cost of new assets plus the market value of old assets if sold to a non-bank, less total debt outstanding. The market value of the bank's equity may exceed its tangible equity because of good investment opportunities and insurance subsidies.

¹³The assumption that the bank's balance sheet only contains riskless debt and equity is without loss of generality: since *all* risky securities have the same required return, the bank's cost of capital will be minimized by using riskless securities to finance as much of its holdings as possible.

¹⁴The loans might include not only secured loans but also, e.g., limited liability loans that a company might take out and are backed by the value of its business.

Similarly, borrowers at $t = 1$ each post K as collateral against a \$1 loan, and promise to repay P' at $t = 2$, so the $t = 1$ loans would be worth $\min(K\theta_2, P')$ at $t = 2$, if there were no costs of managing them.¹⁵

Timing

At $t = 0$, the bank receives $N_0 > 0$ opportunities to make new two-period collateralized, non-recourse, loans of \$1 each. Still at $t = 0$ it makes L_0 , of these loans and finances them by raising enough tangible equity to satisfy the *regulatory capital requirement* (RCR). (We describe regulatory (or “book”) values, and the RCR, below.)¹⁶

At $t = 1$, shock θ_1 affects the value of the collateral and therefore the market and regulatory value of the loans. Immediately thereafter, the bank receives $N_1 > 0$ opportunities to make new one-period collateralized, non-recourse, loans of \$1 each. It sells $L_0 - L_0^R$ of its old $t = 0$ loans (that is, it retains L_0^R old loans), makes L_1 of the new loans, and finances its new portfolio with enough equity to both satisfy the RCR and also make U_1 riskless. When loans are sold the bank’s regulatory capital is immediately reduced by the difference between their regulatory value and the price received for them. (All these things happen at $t = 1$.)¹⁷

At $t = 2$, a shock θ_2 affects the market value and regulatory value of all loans. The bank then sells enough loans at fair (i.e., market) value to raise U_1 , selling them in the order of their ratios of market value to regulatory value (highest ratio first, but all at $t = 2$).¹⁸

We will see later (section 3.3) that the preceding assumption means that, in the cases of interest, new ($t = 1$) loans will be sold first to repay uninsured debt. Furthermore, as we will also see (sections 4.2-4.3), in these cases old loans are subsidized by deposit insurance while new loans are not. Because of this, we can think of insured debt as being used as much as possible to finance the bank’s old loans, and we will sometimes describe financing accordingly—even though the bank’s balance sheet does not technically attribute any

¹⁵We show in the Appendix (Lemma 0) that $P' < P/(1+r)$ is required by our later assumptions about the competitiveness of markets. (The intuition is that one-period loans are less risky than two-period loans.)

¹⁶At $t = 1$ banks may need to hold more capital than the regulatory requirement to persuade uninsured creditors that their claims are safe. However, our assumptions below mean this will be unnecessary for a bank that holds only new loans, as at $t = 0$.

¹⁷If the bank has more equity than it needs to both meet its regulatory capital requirement, and assure that U_1 will be paid in full, then it may pay out the excess, at $t = 1$.

¹⁸This order means the total regulatory value of the remaining loans is as high as possible.

specific debts to any specific assets.

If, after the sales to raise U_1 , the regulatory value of the remaining loans is greater than or equal to D_1 (as must be the case, if U_1 is really riskless), then the uninsured creditors are paid in full. In this case, the remaining loans are then resolved, and if their proceeds exceed D_1 , the surplus goes to shareholders; if their proceeds are less than D_1 , then the shareholders get nothing and the deposit-insurer (i.e., the government) pays the deficit.

If (out of equilibrium) the regulatory value of remaining loans becomes negative after the sales to raise U_1 (or the bank cannot raise U_1 even by selling all its loans), then the bank is immediately placed in receivership.¹⁹ In this case, any remaining loans would be sold and the losses allocated pro rata to the uninsured and insured depositors (so the uninsured debt would not be paid in full),²⁰ with the deposit insurer bearing the losses assigned to the insured depositors.²¹

There will be no loss of generality in assuming both N_0 and N_1 are known at $t = 0$.

Loan Management Costs

Banks' costs of managing assets correspond to multiplying the holdings of each asset they manage by $\delta < 1$ in each period but, as discussed in Section 2.1, a non-bank incurs no such costs. So at $t = j$ the bank owns only fraction δ^{j-i} of each loan made at $t = i$.²² For simplicity, when we refer to a bank as retaining or selling L loans, we will mean the retention or sale of the bank's remaining interest in L loans (*not* the bank's interest in L/δ^{j-i} loans). We make an assumption, (A1), on the size of δ in Section 3, below.

So, after both shocks are realised, the value (to the bank) of a $t = 0$

¹⁹In actuality "Prompt Corrective Action" could put the bank into receivership if its regulatory value were positive but sufficiently small and, moreover, this could happen (out of equilibrium) even before completing the sales to raise U_1 —ignoring these details has no important effect on our results.

Allowing the bank to avoid receivership by adding more capital would also not affect the results—in our model the bank would never do this.

²⁰The precise sharing of losses has no effect on our results—all that matters is that uninsured debt that was supposed to be riskless bears strictly positive losses in receivership.

²¹The minimum regulatory capital requirements described below ensure that at $t = 1$ the bank will be solvent.

²²This is analogous to an index fund which will own δ as great a share in each of its investments after paying its expenses. Note that the management cost therefore does not affect the value of the underlying loan, or its associated collateral.

Our "iceberg loans" assumption follows Samuelson's (1954) use of iceberg transport costs (in which the cost of shipping goods was deducted from the volume of goods arriving).

loan that was held by the bank is $M_{0,2}(\theta_1, \theta_2) \equiv \delta^2 \min(K\theta_1\theta_2, P)$, and of a $t = 1$ loan that was held by the bank is $M_{1,2}(\theta_2) \equiv \delta \min(K\theta_2, P')$.

We assume non-banks are perfectly competitive, so require a per-period expected return r . Since non-banks also have no costs, we therefore have $E\{\min(K\tilde{\theta}_1\tilde{\theta}_2, P)\} = (1+r)^2$, and $E\{\min(K\tilde{\theta}_2, P')\} = (1+r)$. It also follows that the bank can sell (its interest in) a $t = 0$ loan to a non-bank at $t = 1$ (after θ_1 is realised), for $M_{0,1}(\theta_1) \equiv E\{\delta \min(K\theta_1\tilde{\theta}_2, P)\}/(1+r)$.

We write $\underline{M}_{0,1}$, $\underline{M}_{1,2}$, and $\underline{M}_{0,2}(\theta_1)$ for $M_{0,1}(\underline{\theta})$, $M_{1,2}(\underline{\theta})$, and $M_{0,2}(\theta_1, \underline{\theta})$, respectively.

Our assumption that $K\bar{\theta}\underline{\theta} < P$ implies that no loan can be assured of full repayment until after θ_2 is realized, so all loans are always risky. Furthermore, if either θ_1 or θ_2 equals $\underline{\theta}$ the bank becomes the full owner of the returns from the collateral. So the amount of riskless debt a loan can support is always equal to $\delta\underline{\theta}$ times the current value of the collateral. Finally, our assumption that $K\underline{\theta} < 1$ assures that $\underline{M}_{0,1} = \underline{M}_{1,2} = \delta K\underline{\theta} < 1$ and that $\underline{M}_{0,2} < M_{0,1}$.

Market Capital Requirements

For simplicity, our discrete-time model assumes the bank can sell loans only (immediately) after a shock to their value. So, absent deposit insurance, the most the bank can borrow *risklessly* is $\underline{M}_{i,i+1}$ at $t = i$ per newly-made loan, plus $\underline{M}_{0,2}(\theta_1)$ at $t = 1$ per *old* loan (that is, a $t = 0$ loan that the bank will hold for a second period). So at $t = 0$, guaranteeing $t = 1$ solvency implies $D_0 + U_0 \leq L_0 \underline{M}_{0,1}$. At $t = 1$, guaranteeing $t = 2$ solvency requires $D_1 + U_1 \leq L_0^R \underline{M}_{0,2}(\theta_1) + L_1 \underline{M}_{1,2}$. We call these conditions the Market Capital Requirements (MCRs)

It is common for constraints to be quoted as the minimum fractions of equity required rather than the maximum amounts of debt. We define the fractions of incremental loans' values that must be financed by equity to meet the market capital constraints as $m \equiv 1 - \underline{M}_{0,1} = 1 - \underline{M}_{1,2}$ for new loans, and $m_1(\theta_1) \equiv 1 - \underline{M}_{0,2}(\theta_1)/M_{0,1}(\theta_1)$ for old loans. We will generally suppress θ_1 and refer to m and m_1 as the market capital charges or haircuts on new and old loans. We define $\bar{m}_1 \equiv m_1(\underline{\theta})$.

The MCRs at $t = 0$ and $t = 1$ can therefore be rewritten as $D_0 + U_0 \leq L_0(1 - m)$ and $D_1 + U_1 \leq L_0^R M_{0,1}(1 - m_1) + L_1(1 - m)$.

Regulatory Asset Values

We define $B_{i,j}$ as the book (i.e., regulatory) value of a $t = i$ loan at $t = j$, after θ_j is realized (for $j > 0$). All new loans are marked at cost, so $B_{i,i} = 1$. However, book values adjust by only the fraction $\alpha_j \leq 1$ of the

change in the loan's value after θ_j is realized, $B_{i,j} = (1 - \alpha_j)B_{i,j-1} + \alpha_j M_{i,j}$ (for $j > i$). That is, a loan's book value is a weighted average of its previous period's book value and its current market value.²³ Thus $(1 - \alpha_j)$ determines *regulatory forbearance* at $t = j$.

Through most of our analysis we will assume that the α_j are constants, and known by all agents at $t = 0$, but Section 5 will consider the possibility of regulatory intervention to alter beliefs about α_2 .

We write $\underline{B}_{0,1}$, $\underline{B}_{1,2}$, and $\underline{B}_{0,2}(\theta_1)$ for $B_{0,1}(\underline{\theta})$, $B_{1,2}(\underline{\theta})$, and $B_{0,2}(\theta_1, \underline{\theta})$, respectively. (We will omit the dependences of $\underline{M}_{0,2}$, $\underline{B}_{0,2}$, $M_{i,j}$ and $B_{i,j}$ on their arguments, when convenient.)

Regulatory Capital Requirements

Even if a bank is fully insured it will be limited by regulators as to the amount of debt its assets may support. We assume that banks are allowed to have debts equal to no more than $(1 - m_r)$ of their book (or regulatory) value. To make clear that our results are not due to low capital charges on new loans, we assume $m_r = m$. So at $t = 0$ the *Regulatory Capital Requirement* (RCR) limits debt to $U_0 + D_0 \leq (1 - m_r)L_0 = (1 - m)L_0$, the same as the MCR. However, at $t = 1$ the RCR becomes $D_1 + U_1 \leq L_0^R B_{0,1}(1 - m) + L_1(1 - m)$. If $B_{0,1} \leq M_{0,1}$ and $m_1 > m$ the regulatory capital constraint allows more debt than the market constraint.²⁴ Because debt limits are the additive inverse of equity requirements, the regulatory constraint requires less equity in these circumstances.

Regulatory Capital

The bank's *regulatory capital* equals the book value of assets less debt. In particular, its regulatory capital is $B_{0,1}L_0 - (D_0 + U_0)$ immediately after θ_1 is realized at $t = 1$, and is $(B_{0,2}L_0^R + B_{1,2}L_1) - (D_1 + U_1)$ immediately after θ_2 is realized at $t = 2$. A bank whose regulatory capital becomes negative

²³ Actual practice is probably to mark (even) more generously than we assume. Perhaps closer to what is done in practice, would be to set $B_{i,j} = (1 + r)\delta(1 - \alpha_j)B_{i,j-1} + \alpha_j M_{i,j}$ (for $j > i$). That is, book value would be a weighted average of expected value and market value. This would allow the possibility of banks reporting regulatory profits when they have market value losses, as occurred in the first halves of both 2008 and 2020. In our approach when banks suffer losses they report smaller losses but not profits.

Even more realistic might be to allow banks to mark immediately to market when an asset rises in value (since it is likely there are actions they can take to achieve that marking). This would make no important difference to our results, since we will focus on behaviour in bad times.

²⁴ That is, regulators use market capital requirements for new loans, but fail to increase the risk weights for riskier old loans.

goes into immediate receivership.²⁵

3 Preliminary Analysis

3.1 Management Costs and Bank Efficiency

It is immediate that the market haircuts for new loans, and for old loans at $t = 1$, are $m = 1 - \delta K \underline{\theta}$ and $m_1 = 1 - \delta^2 K \theta_1 \underline{\theta} / M_{0,1}(\theta_1)$, respectively. Furthermore, m_1 is (weakly) decreasing in θ_1 , since $M_{0,1} = \delta E\{\min(K\theta_1\theta_2, P)\} / (1+r)$ increases (weakly) less than in proportion to θ_1 as θ_1 increases. So $m_1(\theta_1)$ is highest at $\bar{m}_1 = m_1(\underline{\theta})$. Also, for all $\theta_1 \leq P/(K\bar{\theta})$, $M_{0,1}(\theta_1) = \delta K \theta_1$ and $m_1(\theta_1) = \bar{m}_1 = 1 - \delta \underline{\theta}$. So clearly $m < \bar{m}_1$.²⁶

The bank's cost of managing loans (the fraction $1 - \delta$ of the loan per period) means that its per-period expected return from holding a loan is $\delta(1+r) - 1$. Absent any regulation or deposit insurance, the bank's cost of capital for holding a new loan for one period is rm : it pays 0 on $(1-m)$ of riskless debt, and r on m of equity. Likewise, its cost of capital for holding an old loan at $t = 1$ is $rm_1(\theta_1)$. So, an uninsured bank's expected economic profits from holding a loan for one period are the savings from its being able to obtain cheap financing minus its cost of managing loans. The case of interest is

$$r(1-m) - (1-\delta)(1+r) > 0 > r(1-\bar{m}_1) - (1-\delta)(1+r) \quad (\text{A1})$$

We will make Assumption (A1) throughout, to ensure that new loans can finance enough riskless debt to be profitable, but that, at least if $\theta_1 \leq P/(K\bar{\theta})$ (so $m_1 = \bar{m}_1$), old loans will be unprofitable to retain unless they are subsidized by the regulatory and deposit-insurance system.

Note that the expected one period excess return to shareholders from holding an additional loan if equity is determined by the RCR and we otherwise ignore the effect of insurance is

$$r(1-m)B_{i,j} - (1-\delta)(1+r)M_{i,j} \geq 0$$

by (A1), if $B_{i,j} \geq M_{i,j}$.

²⁵It is not possible for a firm with negative regulatory capital to have positive capital on a market-value basis.

²⁶That is, for new loans the value of collateral exceeds the value of a loan but when $\theta_1 = \underline{\theta}$ the value of an old loan just equals the value of collateral. The extra collateral backing a new loan means that it is less risky and faces a smaller maximum loss over the next period.

3.2 Investment Activity at $t = 0$

The additional equity required to meet the RCR when one new loan is added to the bank's portfolio is the market haircut, m . Because $\delta(1+r) - 1 > rm$, it is profitable in expectation for the bank to make a new loan at $t = 0$, finance it with m in equity and $1 - m$ in safe debt, and sell it at $t = 1$.

Because the RCR for a portfolio of new loans is the MCR, and meeting the MCR guarantees solvency in the next period, the bank will always be solvent at $t = 1$ even if $\theta_1 = \underline{\theta}$. So it does not matter whether the debt issued at $t = 0$ is insured or not, and the option to retain the loan after θ_1 is realized only increases the attractiveness of making the initial loan. It follows that:

Proposition 1 *Regardless of D_0 , the bank will make all the loans available to it at $t = 0$, i.e., choose $L_0 = N_0$.*

3.3 Bank's Investment Problem at $t = 1$

Since the bank will necessarily be solvent at $t = 1$, it always has the option to stay in business until $t = 2$, and since the government covers all the bank's losses if its assets at that time, $M_{0,2}L_0^R + M_{1,2}L_1$, are worth less than its debts, $D_1 + U_1$, we have that,

at $t = 1$, the bank maximises

$$[E\{\max(0, M_{0,2}L_0^R + M_{1,2}L_1 - D_1 - U_1)\} - Q_1(1+r)]$$

by choosing

$$L_0^R, L_1, D_1, U_1, \text{ and } Q_1 \text{ (as functions of } L_0, \theta_1 \text{ and } N_1),$$

subject to the feasibility constraint

$$0 \leq L_0^R \leq L_0 \text{ and } 0 \leq L_1 \leq N_1,$$

the balance sheet constraint

$$M_{0,1}L_0^R + L_1 = D_1 + U_1 + Q_1,$$

the regulatory capital requirement

$$(1 - m)(B_{0,1}L_0^R + L_1) \geq D_1 + U_1 \quad (\text{RCR})$$

the constraint on the amount of insured debt available, $D_1 \leq \bar{D}$,

and the *pseudo-liquidity constraint* (PLC) that U_1 be riskless if $U_1 > 0$.

The constraint that U_1 be riskless requires that the bank has enough equity that it will with certainty be able to pay it off at $t = 2$, while retaining

assets with a book value at least equal to D_1 . Let $L_0^U \in [0, L_0^R]$ and $L_1^U \in [0, L_1]$ be the numbers of $t = 0$ and $t = 1$ loans, respectively, required to pay off U_1 if $\theta_2 = \underline{\theta}$. So

$$L_0^U \underline{M}_{0,2} + L_1^U \underline{M}_{1,2} = U_1 \quad (1)$$

Our main interest will be in the case that $M_{0,1} \leq 1$, so $M_{0,1}/B_{0,1} \leq 1$, in which case our model's assumption implies the $t = 1$ loans will be sold in preference to the $t = 0$ loans, i.e., L_0^U will be chosen as small as possible.

The PLC is that if $U_1 > 0$, then

$$(L_0^R - L_0^U) \underline{B}_{0,2} + (L_1 - L_1^U) \underline{B}_{1,2} \geq D_1 \quad (\text{PLC})$$

Selling assets at market value reduces a bank's regulatory capital if it previously benefited from regulatory forbearance, so sales to pay off uninsured debt may force a bank to recognize its insolvency and go out of business. So although banks in our model can sell assets at market value, this constraint looks similar to a liquidity constraint in a bank-run model in which withdrawals that force asset sales for less than fair value can cause bankruptcy.

3.4 Incentives on Portfolios Created by Deposit Insurance

Combining two insured portfolios reduces the deposit-insurer's losses—and so hurts shareholders—if one portfolio returns more than its associated debt while the other returns less. But it otherwise has no effect on the deposit insurer. So it is immediate that:

Lemma 1 (*Insurance disincentive to diversify*): *An additional asset is worth more to shareholders when held in a stand-alone bank than if combined with the bank's existing assets, if the total capital requirement is the same either way.*

Moreover if, for any given resolution of uncertainty, additional loans return more than their associated debt while the existing loans return less than their associated debt, then the marginal surplus from an added new loan goes first to the deposit insurer, and only goes to the bank after the deficit on the original portfolio is covered. So, for any realization of uncertainty, shareholders lose weakly less on the margin because of insurance as the new portfolio is increased, for any given size of the original portfolio. Since, absent insurance, any asset financed with a constant proportion of debt yields a linear return, we have:

Lemma 2 (*Marginal profitability of homogeneous assets is increasing*): *The marginal profitability of adding a homogeneous asset increases in the ratio of the quantity of the asset added to the size of the existing portfolio, assuming the capital requirement is proportional to the amount added.*

So the marginal profitability from making (retaining) additional new (old) loans, each with a constant marginal capital requirement, is increasing in the quantity of new (old) loans.

It follows immediately that:

Lemma 3 (*All or nothing portfolio additions*): *If a bank is given the opportunity to add up to some amount of a homogeneous asset, it will either reject the opportunity or accept it in full, assuming the capital requirement is proportional to the amount added.*

Lemma 1 is related to “debt overhang”, which is commonly thought of as a disincentive for risky firms to raise money for profitable new investments if doing so will benefit existing debtholders. In our model, there is a symmetric disincentive to finance new opportunities and old ones. That is, the deposit insurance system can reverse the normal banking incentive to pool assets to reduce risk: shares in a combined bank are an option on a portfolio of assets, while shares in separated banks are a portfolio of options.

In our simple model, assets’ returns are perfectly correlated. In a richer model, uninsured banks would gain from pooling old and new assets because doing so would increase the capacity to issue riskless debt. The exact opposite occurs in a regulatory system where capital charges are determined by risk-weighted assets (or a leverage limit) and at least some capital charges are reduced by forbearance.

Finally, deposit insurance incentivizes banks to add assets that can be financed with relatively little equity compared to the market haircut, and discourages investment in assets that require equity at least equal to the market haircut.

Lemma 4 (*Insurance incentive to invest in undercapitalized loans*): *If old loans are each financed with less than their market capital charge, m_1 , and new loans are each financed with their market capital charge, m , then the value to shareholders of deposit insurance is strictly increasing in the number of old loans retained and is weakly decreasing in the number of new loans made.*

Proof: see Appendix.

3.5 Loan Value and Riskiness

The volatility of a loan's market capital charge may be substantially higher than the volatility of its value.

In particular, if a loan's market value falls, the amount of one-period riskless debt it can support must fall *more than proportionally* to the loan's value. The reason is that if the value of the loan's collateral remains constant, then the loan's value (strictly) rises, both because its payoff is discounted by less and because there is less risk left in the collateral's ultimate value. So if the loan's value falls, its collateral's value must also fall. Moreover, the loan's value falls less than proportionally to the collateral's value, because the loan is a senior claim. And the amount of debt that can be supported is proportional to the value of the loan's collateral. So:

Lemma 5 (*If loan values do not rise, then market capital charges increase*):
If $M_{0,1} \leq 1$, then $m_1 > m$.

Proof: see Appendix.

The example below, uses a standard binomial model of the evolution of the collateral's value to demonstrate that the volatility of the amount of debt that can be supported may be much greater than that of the loan's value. (Some readers may prefer to look at the simpler example in Appendix A, which is less realistic, but correspondingly requires much less computation.)

We also use the example below to show that our simplifying assumption that there are lowest-possible realisations of θ_1 and θ_2 (i.e., $\underline{\theta}$) that make the debt completely riskless, is unimportant—a model in which θ_1 and θ_2 could be arbitrarily low, but investors are willing to accept a very tiny risk of loss, would yield the same results.

Example: Assume $K = 1.03$, and that $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are each distributed as the product of ten independent binomial draws, each of which raises the collateral's value by 1% with probability .6 and reduces its value by 1% with probability .4. The expected return per draw is therefore .2%, so $(1 + r) = 1.002^{10} = 1.0202$. We require $E\{\min(K\tilde{\theta}_1\tilde{\theta}_2, P)\} = (1 + r)^2 = 1.0408$, so (tedious) calculation reveals the promised repayment, P , must be 1.0504. For simplicity, assume $\delta = 1$. The amount of riskless debt the loan can support is $(.99^{10})K = .9315$, and $m = 1 - .9315 = .0685$.

Assume that θ_1 is low but not the worst possible— say 3 positive draws and 7 negative draws, so the collateral’s value falls to .9891. The value of the loan, $M_{0,1}$, falls to .9880. The amount of riskless debt the loan can support is reduced to $(.99^{10}).9891 = .8945$. So the amount of debt the loan can support has fallen by three times as much, $.9315 - .8945 = .037$, as the value of the loan has fallen, $1 - .9880 = .012$.

It would perhaps be more realistic for lenders to accept a small premium in return for taking some very small risk,²⁷ so only protecting against, e.g. the possibility of 9 of the first 10 draws being bad (rather than 10 out of 10).²⁸ In this case, supportable debt would fall from about .95 to .9126 after the same low θ_1 (3 good and 7 bad draws)—again about three times as much as the value of the loan has fallen.

Note that whilst the ultimate payoff on the loan is a function of the product of a series of i.i.d. random draws (either θ_1 and θ_2 or the 20 binomial draws), so all the draws affect the final value equally, the importance of the later draws is a function of the early realizations. If the early draws are good then the value of collateral grows, the loan becomes safer, and its value becomes less sensitive to a bad draw. If the early draws are bad, as in the example and as we assumed in the text ($M_{0,1} \leq 1$), then the loan becomes more sensitive to the later random draw(s) and therefore becomes riskier, so requiring more capital.²⁹

Observe also that it follows from the discussion above that if the collateral’s value falls only slightly, then the value of the loan *rises*, but the amount of debt supported *falls*, requiring the bank to raise additional equity. The example in Appendix A provides a very simple, but extreme example of this.

²⁷An adjustment of this sort would be necessary to allow “riskless” lending if we were to model asset prices as moving continuously and so having a very small chance of falling close to zero in any given period.

²⁸This would create about a 1 in 600 chance of loss for depositors.

²⁹As an analogy consider the value of a European equity put option when the stock price follows geometric Brownian motion. *Ex ante* each day’s share price movement is equally important in determining the value of the option but *ex post* the final days’ movements will have almost no effect on the value of the put if the share price is well above the strike price at the time and will have an approximate dollar-for-dollar effect if the share price is well below the strike price.

4 Investment Incentives at $t = 1$, in Bad Times

Our focus is on banks' responses to stressed conditions, so we henceforth assume $M_{0,1} \leq 1$. This implies that the return earned by the bank (net of management fees) was less than the riskless rate between $t = 0$ and $t = 1$. We will see that some banks, such as those with unlimited deposit insurance, will be constrained by the RCR while others, with more limited deposit insurance, will be constrained by the PLC.

4.1 Banks with No Deposit Insurance

An unregulated bank with no deposit insurance is constrained by the MCR, so it follows immediately from Lemma 5 that

Proposition 2a *If $M_{0,1} \leq 1$, an uninsured bank can create strictly less riskless debt per dollar of market value of old loans than per dollar of market value of new loans.*

Since there are no insurance distortions—all profits and losses go to the shareholders in full,

Proposition 2b *If $M_{0,1} \leq 1$, an uninsured bank's expected rate of return from making an additional new loan strictly exceeds its expected rate of return from retaining an additional old loan, for any L_0^R and L_1 .*

and also, using (A1):

Proposition 2c *If $M_{0,1} \leq 1$, an uninsured bank will choose $L_1 = N_1$. It will choose $L_0^R = N_0$ if $rm_1(\theta_1) \leq \delta(1+r) - 1$, and $L_0^R = 0$ otherwise.³⁰*

4.2 Banks with Unlimited Deposit Insurance

A bank that never uses uninsured debt need only meet the RCR. Deposit insurance and regulatory-capital regulation mean it prefers to hold old loans than to make new loans after a bad shock ($M_{0,1} \leq 1$), because old loans both require less equity and are also advantaged by the insurance system:

³⁰An uninsured bank that just satisfies the MCR will choose $U_1 = L_0^R \underline{M}_{0,2} + L_1 \underline{M}_{1,2}$ (see Section 2). The bank would then need to set $L_0^U = L_0^R$ and $L_1^U = L_1$ to pay off U_1 in the worst $t = 2$ state, so the PLC is satisfied with equality (since $D_1 = 0$ for this bank). That is, since an uninsured bank must satisfy the MCR, it automatically satisfies the PLC.

Marking old loans at book value rather than (the lower) market value, means that the capital (i.e., market capital, not regulatory capital) that the bank has to hold decreases more than proportionately to the market value of the loan ($B_{0,1} \geq M_{0,1}$).

Moreover, the bank's capital requirement as a percentage of book value is unchanged, even though the loan has become riskier. So the riskless debt that the bank can create decreases less than proportionately to any decline in market value ($\frac{(1-m)B_{0,1}}{M_{0,1}} > 1 - m$). So, by contrast with Proposition 2a, we have:

Proposition 3a *If $M_{0,1} \leq 1$, a fully-insured bank can create weakly more riskless debt per dollar of market value of old loans than per dollar of market value of new loans.*

Proof: see Appendix.

In addition, insurance for banks holding just old loans subsidizes shareholders by covering shortfalls when $M_{0,2} < (1 - m)B_{0,1}$. But in those states any new loans will have a value net of borrowing of $M_{1,2} - (1 - m) \geq 0$, and that residual will reduce or eliminate insurer losses. So the deposit insurance system makes new loans less profitable because they reduce the expected transfer from taxpayers. By contrast, regardless of L_1 , increasing L_0^R (weakly) adds to insurer losses, which benefits shareholders. So, by contrast with Proposition 2b, we have:

Proposition 3b *If $M_{0,1} \leq 1$, a fully-insured bank's expected rate of return from retaining an additional old loan strictly exceeds the expected rate of return from making an additional new loan, for any L_0^R and L_1 .*

Proof: see Appendix.

The biases in favour of old loans mean that the bank always retains all its old loans, but will not necessarily make new loans, at $t = 1$:

Proposition 3c *If $M_{0,1} \leq 1$, a fully-insured bank will choose $L_0^R = N_0$ and either $L_1 = 0$ or $L_1 = N_1$. The bank may choose $L_1 = 0$ in some cases, for any δ , even if $\alpha_1 = 1$.*

Proof: see Appendix.

Even if $\alpha_1 = 1$ so that the bank is required to mark to market, it has valuable insurance on its old loans, and so may forgo the new loans, because

$m < m_1$. Moreover, it is easy to see that the incentives for retaining bad old loans and rejecting good new loans are the strongest in bad times (when θ_1 is lowest so both (m_1/m) and $(B_{0,1}/M_{0,1})$ are highest, and when N_1 is lowest relative to N_0). So the riskier a loan is, and the less profitable it is for an uninsured bank to hold, the higher the expected rate of return to shareholders from retaining the loan. Even if it were profitable for an uninsured bank to keep all its old loans, a bank that could not sell stock and so had to compensate for losses by reducing its balance sheet would have precisely the wrong incentives about what to include in its portfolio.

Finally, though regulatory capital requirements based on $\alpha_1 < 1$ and $m < m_1$ make new investments less attractive at $t = 1$, they also make new two-period investments more attractive at $t = 0$, because of the option value of retaining them at $t = 1$.

4.3 Partially-Insured Banks

If a bank has a combination of enough insurance and the assurance of enough regulatory forbearance in the future, then it can operate as if it had unlimited insurance, even while taking on some uninsured debt. We will see that insurance and expected future regulatory forbearance are substitutes for the bank; the more regulatory forbearance it anticipates, the less insured debt it needs in order to be able to operate as if it has unlimited insurance, unconstrained by the PLC.

For any given bank portfolio, we can determine whether the PLC or RCR allows less debt by subtracting the debt the RCR allows the bank — the left hand side of (RCR) — from the debt the PLC allows — the left hand side of (1) plus the left hand side of (PLC). This yields

$$\begin{aligned} & (L_0^R - L_0^U)(\underline{B}_{0,2} - (1 - m)B_{0,1}) + L_0^U(\underline{M}_{0,2} - (1 - m)B_{0,1}) \quad (2) \\ & + (L_1 - L_1^U)(\underline{B}_{1,2} - (1 - m)) + L_1^U(\underline{M}_{1,2} - (1 - m)) \end{aligned}$$

So if we substitute in the portfolio that a fully insured bank, unconstrained by the PLC, would choose ($L_0^R = L_0$ and $L_1 = 0$ or N_1) then the PLC is binding if and only if the PLC reduces the debt the bank can issue, that is, (2) is strictly negative.

The four terms of (2) represent the amounts of regulatory capital that the bank will have from old and new loans if $\theta_2 = \underline{\theta}$, in each case separated into those loans that will have to be sold to repay the uninsured debt and those that would be retained until after the uninsured debt is repaid. The

fourth term, regulatory capital from new loans that will have to be sold to repay U_1 , is zero because $\underline{M}_{1,2} = (1 - m)$.

4.3.1 Partially-Insured Banks limited only by the RCR

It is easy to see that $\underline{B}_{0,2}$ is decreasing in α_2 , and that if α_2 is low enough then $\underline{B}_{0,2} > (1 - m)B_{0,1}$.³¹ In this case a bank that has enough insured debt to finance all its old loans, that is, $\bar{D} \geq L_0B_{0,1}(1 - m)$, will be no more constrained than a fully insured bank. In (2) the first term will be positive and, because there is enough insured debt that $L_0^U = 0$, the second term will be zero. Finally, since $\underline{B}_{1,2} \geq (1 - m)$, if there are any new loans that can be financed with insured debt the third term in (2) will be positive. So (2) will be positive and in meeting the RCR the bank automatically satisfies the PLC.

If α_2 is higher, so $\underline{B}_{0,2} < B_{0,1}(1 - m)$, each old loan may produce negative net regulatory capital (of $\underline{B}_{0,2} - (1 - m)B_{0,1}$) at $t = 2$. But if $\bar{D} > L_0B_{0,1}(1 - m)$ the bank will have $\bar{D} - L_0B_{0,1}(1 - m) > 0$ of insured debt left after financing all its old loans, and each remaining dollar of insured debt can finance $\frac{1}{1 - m}$ new loans, each of which will generate at least $\underline{B}_{1,2} - (1 - m)$ of net regulatory capital.

So, if $L_0(\underline{B}_{0,2} - (1 - m)B_{0,1})$ (the first term of (2) when $L_0^R = L_0$ and $L_0^U = 0$) plus $\frac{1}{1 - m}(\bar{D} - L_0B_{0,1}(1 - m))(\underline{B}_{1,2} - (1 - m))$ (the third term of (2) when $L_1 - L_1^U = \frac{1}{1 - m}(\bar{D} - L_0B_{0,1}(1 - m))$) exceeds 0, which simplifies to $\bar{D} \geq L_0 \left[\frac{(1 - m)(B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2})}{\underline{B}_{1,2} - (1 - m)} \right]$, the net regulatory capital from insured loans, old and new combined, will be at least zero at $t = 2$. Since with $L_0^U = 0$ the remaining terms of (2), will also be zero, the bank will then only be constrained by its regulatory requirement, exactly as if it were fully insured. So:

Proposition 4 *If $M_{0,1} \leq 1$, $\bar{D} \geq L_0 \left[\frac{(1 - m)(B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2})}{\underline{B}_{1,2} - (1 - m)} \right]$, and $\underline{B}_{0,2} < B_{0,1}(1 - m)$, a partially-insured bank will behave exactly as if it were fully insured bank (i.e., as described in Proposition 3c).³²*

Proof: see Appendix.

³¹ $\underline{B}_{0,2} = (1 - \alpha_2)B_{0,1} + \alpha_2M_{0,1}(1 - m_1)$, and when $M_{0,1} \leq 1$ we have $B_{0,1} \geq M_{0,1} > M_{0,1}(1 - m_1)$, so $\underline{B}_{0,2}$ is decreasing in α_2 . When $\alpha_2 = 0$, we have $\underline{B}_{0,2} = B_{0,1} > (1 - m)B_{0,1}$, and when $\alpha_2 = 1$, we have $\underline{B}_{0,2} = M_{0,1}(1 - m_1) < M_{0,1}(1 - m) < (1 - m)B_{0,1}$.

³² Since our model assumes $L_0 > 0$, there is no ambiguity when the denominator of the fraction is 0.

“Virtual Insurance”

More dramatically, even a bank that requires uninsured debt to finance some of its *old* loans may be able to act like a bank with unlimited insurance, if it is assured of sufficient regulatory forbearance. If α_2 is low enough that $\underline{B}_{0,2} > (1 - m)B_{0,1}$, then old loans financed with insured debt will always generate strictly positive regulatory capital, and the bank can take advantage of this slack to finance some additional old loans with uninsured debt at only the regulatory capital rate. We call uninsured debt that is used in this way *virtually insured*, because it is financing loans on the same terms as if it were insured, and the bank can treat it exactly as if it were fully guaranteed by the deposit insurer.

How much virtually-insured debt can the bank issue? Assume it issues an amount V , such that the bank’s insured and virtually-insured debt ($\bar{D} + V$, in total) exactly finances all the retained old loans. Since, then, $(L_1 - L_1^U) = 0$, (2) holds with equality if the first two terms in (2) exactly cancel. The first term, which represents the regulatory capital net of RCR borrowing of old loans that will not have to be sold to pay off uninsured debt, is $(\bar{D}/\underline{B}_{0,2})(\underline{B}_{0,2} - (1 - m)B_{0,1})$ since the number of unsold old loans after the worst possible shock must be at least $(\bar{D}/\underline{B}_{0,2})$ to avoid a default on the uninsured debt. The second term, which represents the proceeds from old loans sold to pay uninsured debt minus the amount borrowed against those loans, is $(V/\underline{M}_{0,2})(\underline{M}_{0,2} - (1 - m)B_{0,1})$ since the total revenue from the sold loans must be V . Setting the sum of the two terms equal to zero, and solving, yields $V = \bar{D} \frac{\underline{M}_{0,2}(\underline{B}_{0,2} - (1 - m)B_{0,1})}{\underline{B}_{0,2}((1 - m)B_{0,1} - \underline{M}_{0,2})}$.

It is easy to see that the maximum feasible amount of virtual insured debt is therefore $\max\{0, \bar{V}\}$, in which $\bar{V} = \bar{D} \frac{\underline{M}_{0,2}(\underline{B}_{0,2} - (1 - m)B_{0,1})}{\underline{B}_{0,2}((1 - m)B_{0,1} - \underline{M}_{0,2})}$. So if $L_0(1 - m)B_{0,1} \leq \bar{D} + \bar{V}$ the bank can finance all its old loans at the RCR and, since even uninsured new loans can be financed at the RCR, such a bank faces no more constraints than a fully insured bank:

Proposition 5 *If $M_{0,1} \leq 1$, $\underline{B}_{0,2} \geq (1 - m)B_{0,1}$, and $L_0 \leq (\bar{D} + \bar{V})/((1 - m)B_{0,1})$, where $\bar{V} = \bar{D} \frac{\underline{M}_{0,2}(\underline{B}_{0,2} - (1 - m)B_{0,1})}{\underline{B}_{0,2}((1 - m)B_{0,1} - \underline{M}_{0,2})}$, a partially-insured bank will behave exactly as if it were a fully-insured bank (i.e., as described in Proposition 3c).*

Proof: see Appendix.

Importantly, \bar{V} may represent a substantial increase in effective debt insurance—it is possible that \bar{V} is substantially larger than \bar{D} .³³

Moreover, a bank with $\bar{D} + \bar{V} = T$ and $\bar{V} \geq 0$ will be at least as well off, and sometimes better off, than a bank with $\bar{D} = T$ but with $\bar{V} < 0$. In both cases the bank could finance the same number of old loans at the regulatory rate. However, the bank with $\bar{V} \geq 0$ could then finance any additional assets at the market haircut. By contrast, the bank with $\bar{V} < 0$ would have to raise additional capital before it would be able to raise any uninsured debt, as we show in the next subsection.

4.3.2 Partially-Insured Banks limited by the PLC

The previous subsection hints at the two kinds of cases where the PLC binds. The first is straightforward: If $\bar{V} \geq 0$ but $\bar{V} + \bar{D} < L_0(1 - m)B_{0,1}$ then the second term in (2) will outweigh the first. In this case the bank cannot fund all its old loans with its insured and virtually-insured debt, so it must either dispose of its additional old loans, or fund them at the market rate of $M_{0,2} = (1 - m_1)M_{0,1}$ debt per loan.³⁴

The more interesting case is when $\bar{V} < 0$ and, even if the bank has enough insurance to finance all of its old loans (so $L_0^U = 0$ and the second term in (2) is zero), any regulatory surplus that would be generated by new insured loans, the third term in (2), would not fully compensate for the first term deficit.³⁵

In this case, the bank will have three options for meeting the PLC. The first would be to simply limit its size to the point where it could finance its remaining portfolio without uninsured debt. Recall from Section 3.1

³³ \bar{V}/\bar{D} is maximised at $M_{0,1} = 1$ and $\alpha_2 = 0$ where $\bar{V} = \bar{D} \left(\frac{(1-m_1)m}{m_1-m} \right)$ so, if also e.g., $m = .10$ and $m_1 = .15$, then $\bar{V} = 1.7\bar{D}$.

³⁴Note that the bank would prioritize new loans over old loans at the market rate because both (i) the new loans have a lower weighted average cost of capital than the old loans (because $m < m_1$ when $M_{0,1} \leq 1$), and (ii) the old loans' greater equity financing make them weakly more susceptible to having returns extracted by the insurer should the bank fail. (See Lemma 6 in the Appendix.)

Because new uninsured loans do not affect whether the PLC binds (because the fourth term in (2) is zero), but do reduce the likelihood that an old loan's net returns would be taken by the insurer, they weakly increase the attractiveness of uninsured old loans (although the old loans would never be retained if they would be unprofitable on a stand-alone basis). So in this case the bank may make all its new loans but not all its old uninsured loans, but not the other way around.

³⁵Of course, if \bar{D} is large enough to finance all new and old loans the bank need not meet the PLC even if (2) would be negative. However, we think of such a bank as having effectively unlimited insured deposits.

that, ignoring insurance effects, the expected economic profits from retaining an old loan and a new loan are $r(1 - m)B_{0,1} - (1 - \delta)(1 + r)M_{0,1}$ and $r(1 - m) - (1 - \delta)(1 + r)$ per dollar of debt, respectively. Since $B_{0,1} \geq M_{0,1}$ when $M_{0,1} \leq 1$, the former are weakly more profitable per dollar of debt, ignoring insurance, and Lemma 4 shows that insurance effects strictly increase the profitability of old loans and decrease the profitability of new loans. So if the bank limits itself to only assets it can finance at the RCR, it will most profitably stuff its portfolio with as many old loans as possible.

Moreover, even if the bank has some insured finance capacity left after retaining all its old loans, in this case the PLC may even discourage it from holding new loans that could be financed with *insured* deposits. The reason is because of the increasing returns from holding homogeneous assets financed at a constant rate (Lemma 2) a bank that would have made all of its new loans if financed at a constant rate of m in equity per loan might choose to make none, if it can only make some limited (insured) new loans at that rate.

A second option for the bank is to satisfy the PLC by holding more equity than required by the RCR. Pursuing this approach means that, holding constant its use of insured deposits, the bank would have to finance some loans entirely with equity, before it could borrow any uninsured money.³⁶ In this case the bank would make all of its new loans, and perhaps also retain any of its old ones that could not be financed with uninsured debt.³⁷ If both this bank and a fully insured bank would choose to make all new loans and retain all old ones, the deficit in (i.e., the negative value of) (2) would exactly equal the amount of extra equity that the bank would hold to meet the PLC.

A final option for meeting the PLC is to sell some old loans. Reducing L_0^R both reduces the deficit in the first term of (2) and increases the surplus in the second term by increasing the number of new loans that can be financed with insured debt. However, the bank will not eliminate old loans entirely. Once L_0^R is low enough that the sum of the first two terms in (2) is positive

³⁶For example, say a bank wanted to retain all its old loans using insured debt, some new loans using insured debt, and its remaining new loans using uninsured debt. However, say at those quantities (2) had a value of $-Z$. Then the first $Z/\underline{M}_{1,2} = Z/(1 - m)$ new loans not using insured debt would have to be financed entirely with equity, so assuring that the PLC would be met, before the remaining new loans could be financed at the market haircut.

³⁷Lemma 6 in the Appendix shows new loans are preferred to old ones at the MCR. If the bank was not going to make these additional new loans (which we know from Lemma 2 would include all remaining new loans), there would be no point to paying the costs of meeting the PLC to make uninsured borrowing feasible.

the bank need only meet the RCR to satisfy uninsured creditors that they will be paid before any bank failure, and old loans financed at the RCR are always profitable.³⁸

Proposition 6 : *If $M_{0,1} \leq 1$, and $\alpha_2 < 1$, a partially or fully insured bank will always choose $L_0^R > 0$.*

Proof: see Appendix.

The case in which the bank restricts itself to using only insured debt is highly inefficient: the bank “gambles for resurrection” by retaining old, under-capitalized risky investments and by shunning new investments that would require more equity. It is being constrained by the behavior of uninsured creditors, not because of a genuine liquidity crisis—all assets can be sold at fair value, and so can additional shares—but because of a PLC that is tougher than the RCR but easier than the MCR. In this case, the bank is deterred from making new loans in two ways. First, before the bank could take on any uninsured debt it would have to pay the discrete costs of meeting the PLC (by either selling some old loans or adding some new loans financed entirely by equity, as discussed above). Second, even if it paid those costs it would still face an insurance disincentive to making new loans.

The model thus suggests an alternative, or at least a complement, to a traditional liquidity story. Regardless of whether banks can’t, or simply won’t, repair their balance sheets after a negative shock, policies like virtually insuring more bank debt by reducing α_2 and increasing deposit insurance can sometimes become second-best options. However, the net effects of such policies are complex: they may cause banks that were already

³⁸ Lemma 2 guarantees that the bank would not choose an interior strategy that would be a linear combination of the “add equity” and the “reduce old loans” strategies. (Moving between the second strategy and the first corresponds to adding old loans at a constant capital cost, since the number of new loans is held constant – they are merely being displaced from insured to uninsured debt.) So it would either make the minimum number of old loans or the maximum that it could fund with insured debt at $\underline{B}_{0,2}$ of debt per old loan.

The marginal equity cost per old loan up to the minimum number is $(M_{0,1} - (1-m)B_{0,1})$.

The marginal equity cost per old loan beyond the minimum number is the capital requirement for an old loan $(M_{0,1} - \underline{B}_{0,2})$ plus the opportunity cost of not being able to finance new loans with insured debt (which is $(\underline{B}_{1,2} - (1-m))$ on the $(\underline{B}_{0,2}/\underline{B}_{1,2})$ new loans displaced by each old loan) which equals $(M_{0,1} - \underline{B}_{0,2}) + (\underline{B}_{1,2} - (1-m))(\underline{B}_{0,2}/\underline{B}_{1,2}) = M_{0,1} - (1-m)(\underline{B}_{0,2}/\underline{B}_{1,2})$. It is not hard to check that, for all $\alpha_2 < 1$, this is less than the cost per old loan of the market requirement, $M_{0,1} - \underline{M}_{0,2}$ (with equality when $\alpha_2 = 1$).

retaining their old loans to make more new ones than they would have made with less (official and virtual) insurance. But, they may simply allow banks to avoid raising new equity and/or avoid selling toxic assets, further discouraging new loans. These issues will resurface in our discussion of stress tests in the next section.

5 Regulatory Intervention and Stress Tests

So far we have assumed all agents know α_1 and α_2 at $t = 0$. In a more realistic model, the bank and potential investors will assess probability distributions for α_j prior to $t = j$. It is easy to see that beliefs about α_1 and α_2 do not affect any $t = 0$ actions, and we assume all agents observe α_1 before any $t = 1$ actions are taken. However, beliefs about α_2 do affect $t = 1$ behaviour.

Assume that at $t = 1$ agents believe the regulator's choice of α_2 at $t = 2$ is distributed with strictly positive density on $[\alpha_2, \bar{\alpha}_2]$

Since uninsured debt must be completely riskless, the amount the bank can issue at $t = 1$ therefore depends on $\bar{\alpha}_2$ (as well as on α_1) but cannot otherwise depend on anyone's beliefs about α_2 . Moreover, all loans mature at $t = 2$, so it is easy to see that the actual value of α_2 has no effect on the bank's returns.³⁹ So:

Proposition 7 *At $t = 1$, the bank (and uninsured depositors) act as if the regulator will choose $\alpha_2 = \bar{\alpha}_2$ at $t = 2$.*

It follows that all our previous results hold after substituting $\bar{\alpha}_2$ for α_2 (including in $B_{0,2}$, $B_{1,2}$, etc.).

5.1 Stress Tests

During the 1933 Bank Holiday 18,000 banks were audited. 14,000 were certified as viable and allowed to reopen, of which 6,000 eventually received capital infusions.⁴⁰ Importantly, the Emergency Banking Act allowed the Fed to issue enough currency to meet all demand from the reopened banks. In his first Fireside Chat, President Roosevelt reported that "The new currency is being sent out by the Bureau of Engraving and Printing to every

³⁹Note the contrast between α_1 and α_2 . At $t = 0$, all new loans are made regardless of beliefs about α_1 , but the distribution of α_1 affects the expected value of the bank's equity. At $t = 1$, investment decisions and share value are both affected by $\bar{\alpha}_2$ but otherwise not by the distribution of α_2 .

⁴⁰Federal Reserve (1937).

part of the country." Depositors did not have to actually trust the one week audits, only that if a bank was certified for reopening it would be able to access enough money to pay off all creditors. The plan was immediately successful in assuring depositors, even though (limited) formal deposit insurance did not come until much later in the year.

The original 2009 Stress Tests followed a similar model. As in 1933, some banks required capital infusions, but the tests then assured creditors that regardless of whether a bank was really solvent it had enough book capital to be supported by regulators for the period covered by the test. The tests did not seem to measure whether a bank was economically solvent (see next subsection). As Geithner (2014) contends, the tests provided transparency. But the transparency was about the regulatory, not economic, valuation of assets.

That is, the results of a test at $t = 1$ tell investors that contingent on a bank's investment and financing plan (i.e., L_0^R, L_1, D_1 and U_1) then even if the economy experiences some awful shock ($\theta_2 = \underline{\theta}$) the regulator will use a low enough value of α_2 , that the bank will have enough regulatory capital to stay in business past the point where all uninsured debt has been paid off, regardless of whether the bank will subsequently have to be liquidated at a loss to the deposit insurer.

In terms of our model, since $\underline{\theta}$ is common knowledge, the report of the banks' plans and stress test results reveals that $\alpha_2 \leq \alpha_2^S$ (with $\alpha_2^S \leq \bar{\alpha}_2$). Assuming banks are allowed to adjust plans in response to the revelation of α_2^S , the tests substitute

$$(L_0^R - L_0^U)B_{0,2}^S + (L_1 - L_1^U)B_{1,2}^S \geq D_1 \quad (\text{STRESS})$$

for (PLC), where $B_{i,2}^S = (1 - \alpha_2^S)B_{i,1} + \alpha_2^S M_{i,2}$, that is, $B_{0,2}^S$ and $B_{1,2}^S$ are the values of $B_{0,2}$ and $B_{1,2}$ when these are calculated using $\alpha_2 = \alpha_2^S$ instead of $\alpha_2 = \bar{\alpha}_2$ (cf. Proposition 7). Thus if $U_1 > 0$, the stress test is, in effect, a PLC imposed by regulators to give the uninsured creditors the guarantee they need. (STRESS) is weaker than (PLC), but if a bank faces a stress test it is subject to (STRESS) even if it chooses $U_1 = 0$. Observe that since fixing $\alpha_2^S < \bar{\alpha}_2$ simply eliminates the upper tail of the distribution for α_2 , it is good news for the banks' shareholders even if α_2 's expected value had been less than α_2^S prior to the stress tests.

As in Section 4, it is useful to consider banks that were unconstrained by the PLC prior to the stress tests separately from those which were previously constrained by the PLC.

Banks previously unconstrained by the PLC

As shown in section 4, a bank will be unconstrained by the PLC if it is either fully insured, or if it is partially insured but meets the conditions of Proposition 4 and/or 5. For these banks, the stress tests are either positive, or neutral, for efficiency.

Proposition 5 covered banks that both have $\bar{V} \geq 0$ and have sufficient insured or virtually insured debt to finance all their old loans. With $\bar{V} \geq 0$, the RCR is a tougher requirement than the PLC when applied to assets financed with insured debt: recall that the PLC limits borrowing on insured debt to $\underline{B}_{0,2}$ whilst the RCR limits borrowing to $(1 - m)B_{0,1}$ and $\bar{V} \geq 0$ implies $\underline{B}_{0,2} \geq (1 - m)B_{0,1}$. Furthermore, substituting α_2^S for $\bar{\alpha}_2$ increases \bar{V} by effectively relaxing the PLC. So in these cases (RCR) will continue to be the binding constraint and the stress test will have no effect.

Proposition 4 covers the case where $\bar{V} < 0$ but in maximizing subject to just the regulatory capital constraint the bank chooses to make both its old loans and all its new loans, and the minimum regulatory capital that will be generated by the new loans financed with insured debt equals or exceeds $|\bar{V}|$. In this case as well, the stress test simply guarantees the bank more $t = 2$ regulatory capital (through the substitution of α_2^S for $\bar{\alpha}_2$) and so again only (RCR) binds and the test is ineffective.

Finally, though, there are fully or partially insured banks that choose to make either old loans only, or both new and old loans, and that would end up with negative regulatory capital when $\theta_2 = \underline{\theta}$ at $t = 2$. In these cases, if the bank would still end up with negative regulatory capital after the substitution of α_2^S for $\bar{\alpha}_2$ then (STRESS) binds. The bank will have to find a way to eliminate the deficit, which it may do by adding equity beyond the RCR, or by continuing to meet the RCR with equality but reducing the number of old loans financed with insured debt and/or adding new loans financed with insured debt to meet (STRESS)

In this last case the test promotes efficiency and benefits taxpayers by increasing the marginal capital requirements for any number of old loans, while increasing the profitability of new loans. This is both because of an insurance effect and because, in some cases when (STRESS) is binding, adding a new loan may only require adding $\alpha_2^S m$ in equity instead of at least $\bar{\alpha}_2 m$. The extreme case is where $\alpha_2^S = \bar{\alpha}_2 = 1$ and the stress test effectively requires even a fully insured bank to behave as an uninsured bank.

Banks previously limited by the PLC

For partially insured banks that were previously limited by the PLC, the effect of replacing (PLC) with (STRESS) is more complicated, and ambigu-

ous for efficiency. Because the test applies even if the bank chooses $U_1 = 0$ there is no longer a discontinuity in capital requirements when virtual insurance is negative and the bank considers whether to start issuing uninsured debt. Increasing the equity needed to maintain old insured loans, and eliminating the discontinuity, improves the incentive for a partially insured bank that would have foregone uninsured debt, and so also have foregone at least some of its new lending opportunities, to make all of its new loans. The addition of the equity to back insured old loans, plus the addition of the new loans, also make it cheaper to retain any remaining old loans with uninsured debt because of the insurance effect, further improving efficiency.

On the other hand, the tests can reduce the incentive to make new loans. If, for example, virtual insurance would be positive without the stress test then the replacement of (PLC) with (STRESS) acts simply as a reduction in α_2 . The test then increases the number of old loans that the bank can retain using insured or virtually insured debt. This benefits shareholders but the relaxation of capital requirements on old loans reduces the marginal incentive to make new loans.

Differential capital requirements

Finally, while the tests create a PLC substitute that applies to both fully and partially insured banks, this does not mean that it makes the capital requirements the same for both types of banks. A partially insured bank will still need to hold some of its assets using the market capital requirement to pay off U_1 , as reflected in (1). So, if we compare two banks which have the same assets but different amounts of insurance the one with more insurance will be assumed to lose less for regulatory purposes in any adverse scenario and so will be required to hold less equity to pass the test.

5.2 The 2009 Tests: Expanding Virtual Insurance

As discussed above, the initial (2009) stress tests were introduced together with promises that all banks would be required to raise enough capital, or would be given enough capital, to pass. To this day, the stress tests are explicitly about the regulatory value of bank assets.⁴¹ However, it seems

⁴¹As three senior officials wrote, "...by its design, [the stress tests] pulled back on the narrative that banks should be held to a mark-to-market standard....[W]e reinforced what some Treasury staff called the theory of special bank relativity — that banks exist through time and shouldn't be judged simply by how they stand at a certain point in time. Banks are designed to be "unstable" in this way" (Clark, Kabacher, and Sachs (2020), p.269.

As described in the tests themselves, "... revenues and losses from traditional banking are generally measured using the accrual method." Federal Reserve (2021), p. 10.

fairly clear that solvency was not required for a passing grade in 2009.

Compare the financial condition of Lehman Brothers when it failed in September 2008 with that of the largest US commercial banks at the time of the stress tests. Ben Bernanke (2009) estimated that Lehman legally could not be bailed out because “they were insolvent and had a thirty-to-forty-billion-dollar hole in their capital structure”.⁴² At the same time, Lehman had higher Tier 1 and Tier 1 plus Tier 2 capital ratios than any of the large banks (or the commercial banking sector as a whole) reported in either the third or fourth quarter of 2008.⁴³ Moreover, as a fair value firm, Lehman’s accounting position did not comparably benefit from hold to maturity accounting, and its fair value marks appeared to be substantially more conservative than those the commercial banks used when they were required to use fair value.⁴⁴ Finally, market conditions as represented by either the S&P 500 or the share prices of the largest banks were notably worse at the time the stress test results were announced compared to the week after the Lehman bankruptcy. (The situation in the U.K. seemed no better.⁴⁵)

The announcement of the tests and the accompanying promises of no more large failures for some time to come, caused U.S. bank stocks to rally. The early May announcement of the results asked less of the banks than many analysts anticipated; nevertheless bank CDS prices rose as higher than expected forbearance (i.e., low α_2^S) pushed any prospective bank failures farther away.⁴⁶ Whilst the response to these announcements could be

⁴²See Financial Crisis Inquiry Commission (2009), p. 29. Indeed, Lehman’s unsecured creditors took haircuts of well over \$100 billion on their unsecured loans (though some creditors may have profited more from the reorganisation than if Lehman had continued as an ongoing firm).

⁴³The same was true in the first quarter of 2009, with the exception of Citibank, whose ratios improved after receiving substantial government funds.

⁴⁴See Goldman Sachs (2008). For example, Lehman marked its commercial real estate at 85% of par, vs. 95-96% for Bank of America and Citibank, its subprime CDOs at 29% of par vs. 44-46% for Bank of America and Citibank, and its alt-A portfolio at 39% of par vs. 80% for Citibank (no data for Bank of America). AIG (2008) provides examples of different marks for commercial and investment banks for the same securities; the commercial banks’ marks are always higher.

⁴⁵The shares of Barclays, Lloyds, and Royal Bank of Scotland fell by more than 95% top to bottom, putting them in the same league as Citigroup and Bank of America. (Ratios of equity to assets fell below 1% at even the US banks which, because of different accounting rules, looked relatively less levered than they would have under European rules.)

⁴⁶See “Fed Sees Up to \$599 Billion in Bank Losses: Worst Case Capital Shortfall of \$75 Billion at 10 Banks is Less Than Many Feared; Some Shares Rise on Hopes Crisis is Easing”, by David Enrich, Robin Sidel, and Deborah Solomon, *Wall Street Journal*, May 8, 2009.

interpreted as an indication that the banks were much stronger than anyone had imagined, our model suggests an alternative explanation.

Say that the tests occurred at a time when regulatory capital significantly exceeded the market value of tangible equity (e.g., $\theta_1 = \underline{\theta}$). At the same time "volatile liabilities", somewhat similar conceptually to U_1 , were $2/3$ as large as deposits in 2007.⁴⁷ Holders of uninsured debt were uncertain about what regulators would choose for α_2 and so, without assurance that the banks would not be shut down, the banks would lose their uninsured funding. This would have forced banks to sell assets at market values well below regulatory value.

In the model, even with $\theta_1 = \underline{\theta}$, the sale of an asset at $t = 1$ weakly relaxes the bank's RCR because the sale price always yields at least as much as needed to repay the debt associated with the asset (that is, $M_{0,1} \geq (1 - m)$). So if capital requirements were raised to market levels banks could survive by selling assets and shares. However, as we discuss in Section 6.3, the 2008-9 situation was likely even worse for many banks.⁴⁸

By choosing a low value for α_2^S , regulators allowed banks to pass the tests while adding little new capital.⁴⁹ The commitment to low α_2 likely created large amounts of virtual insurance,⁵⁰ allowing banks to borrow large amounts of technically uninsured debt that was *de facto* insured. So uninsured lenders were then likely able to "front run" the regulators for the foreseeable future, and get their money out even if the bank would collapse but for forbearance and insurance.

The theoretical ambiguity of the stress tests' impact on partially insured banks constrained by the PLC, discussed in section 5.1, can be seen in the

⁴⁷There are some volatile liabilities, like FHLB repayments due within a year, that are *de facto* insured and so likely to be refinancable even if the bank is in dire straits.

⁴⁸Assume we extended the model to allow for the possibility that $M_{0,1} < (1 - m)B_{0,1}$. Then selling an old loan at $t = 1$ would require the bank to raise additional equity to continue to meet its RCR if it wished to continue to $t = 2$, even though the sale would reduce economic risk. Similarly, if $M_{0,1} < \underline{B}_{0,2}^S$ then the sale of old loans that would have been financed with insured debt would raise a bank's (STRESS) capital requirement even while reducing risk.

⁴⁹In Europe, the initial 2010 tests deemed that no privately controlled bank needed additional capital. A bank 77% owned by the Greek government, several cajas controlled by regional governments in Spain, and one German bank already in receivership were told to raise small amounts of capital. The 2011 tests were tougher but were met with skepticism because of the limited reserving against sovereign defaults. For test results see Committee of European Banking Supervisors (2010, 2011).

⁵⁰The stress tests projected that worst case losses would leave the banks with more than enough regulatory capital, implying that they had positive virtual insurance at least after the tests.

2009 tests. (It seems clear that banks were constrained by the PLC prior to the tests and other bailout programs, since banks that easily satisfied the RCR were facing increasing difficulty in raising uninsured debt.) The expansion of virtual insurance made it much easier for banks to retain existing “toxic assets” that they would have had to sell if forced to meet market capital requirements. However, the impact on banks’ incentives to make new loans was mixed. Being able to retain more old loans increased the insurance disincentive to make new loans, and the tests allowed banks to pass without raising much new equity that would have balanced this. On the other hand, banks that would have been constrained by the PLC did gain access to uninsured (but virtually insured) debt which could be used to make new loans. Any lack of new lending could have been due either to a low N_1 or to an insurance disincentive to raise new equity for these investments.⁵¹

5.3 The 2019-2021 Tests: Different Treatment for Commercial and Investment Banks

The June 2021 US stress tests make clear that regulators are still committing to a low enough α_2 to create substantial virtual insurance. They project that if the stock market falls by 55%, residential real estate by 23.5% and commercial real estate by 40% that the banks will only be charged with regulatory capital losses of 1% of assets, with no effect on banks’ risk as measured by the ratio of risk-weighted assets to assets.^{52,53}

But perhaps the most important change in U.S. commercial bank balance sheets since the Financial Crisis has been the move towards practically unrunnable balance sheets. Table 3 shows that domestic deposits, roughly analogous to D in our model, were enough to finance less than 90% of net

⁵¹As Ivashina and Sharfstein (2010) showed, new lending started to collapse in early 2008, presumably responding to economic losses even as regulatory capital remained stable—consistent with the implication of our solvency model that forbearance creates an insurance disincentive to make new investments.

⁵²See Federal Reserve Board of Governors (2021). Tables 3 and 6 indicate losses of \$151 billion comprising net income before taxes and all other comprehensive income losses. Total assets are \$17 trillion, from multiplying risk-weighted assets (Table 3) times the Tier 1 capital ratio divided by the Tier 1 leverage ratio (also Table 3) at both the start and end of the tests. Capital ratios fall by more than 1 percent after allowing for projected distributions.

⁵³The U.K. Stress Tests (Bank of England, 2019b) do reflect increases in the ratio of risk-weighted assets to assets for most banks. However, the market value of equity in U.K. banks is consistently below book value, suggesting that asset values may be overstated even in good times; see Table 1.

loans and leases in 2007, but over 125% in 2019 and 150% in 2020. By contrast, volatile liabilities less cash and due from depository institutions, roughly analogous to U , fell from 32% of assets in 2007 to 6% in 2019. Easy monetary policy in response to the pandemic actually pushed aggregate cash and deposits above volatile liabilities for banks as a whole in 2020.⁵⁴ At the individual bank level, Bank of America and Wells Fargo were practically fully insured by 2019.^{55,56}

For banks that have become fully insured, our model suggests a useful role for the stress tests, as we discussed in Section 5.1. Since regulators seem to find it difficult to force banks to raise risk capital in a downturn, a test that requires additional regulatory capital, either through (STRESS) or supplements such as the Stress Capital Buffer⁵⁷ is a second-best option.

That said, the U.S. tests have two features which are consistent with our model, but inconsistent with them being a good measure of capital adequacy.

First, none of the U.S. banks in Table 1 was projected to increase its ratio of risk-weighted assets to assets during the test period. This is roughly consistent with assuming m is constant in determining RCRs, rather than increasing to m_1 after a negative shock (Table 2), despite significant bank stock volatility (Table 1).⁵⁸ By contrast, under the U.K. tests, most banks' ratios of risk-weighted assets to assets do increase at the stress low point.⁵⁹ (The U.K. must contend with a different issue: its tests use book values that are well above stock market values (Table 1).)⁶⁰

⁵⁴Uninsured domestic time deposits count as volatile liabilities.

⁵⁵The least insured commercial bank, Citi, moved from $U = 2.5D$ to $U < D$ in 2019 and $U = .6D$ in 2020 by this measure.

⁵⁶Insurance has also increased in the U.K. See note 3.

⁵⁷Large U.S. banks now face a Stress Capital Buffer of at least 2.5% and, if considered a global systemically important bank, at least another 1%. U.K. banks face similar surcharges.

⁵⁸The same is true for the U.S. operations of the three banks that are included in the U.K. tests. A low m might perhaps reflect an assumption that banks will alter the composition of new investments to offset the increased risk in retained loans, but an expressed goal of the tests is to assure that banks have enough capital to continue normal lending.

⁵⁹With the use of accounting standard IFRS 9 in place of IAS 39, U.K. banks recognize more losses as expected rather than as incurred, and so earlier within the test period. The U.K. banks use lower ratios of risk-weighted assets to assets than the U.S. banks (ranging from 29 percent at the start of the latest tests to a projected 38 percent at the nadir), but the absolute numbers are difficult to compare because of different accounting standards.

⁶⁰Acharya et al (2014) claim that bank risk weights are too low and presents as evidence that bank equity values are considerably more sensitive to adverse economic conditions than the estimates of the stress tests. They focus significantly on macroprudential risks

Second, the U.S. tests treat different banks differently: a bank's losses in an extremely adverse scenario (e.g., $\theta_2 = \underline{\theta}$) should theoretically be (roughly) proportional to its risk-weighted assets. That is the argument for making capital requirements proportional to RWA. But the 2021 U.S. stress tests estimate that Bank of America's Tier 1 capital ratios will fall by one-third of Goldman Sachs'.⁶¹ More generally, the large commercial banks had substantially lower projected losses than the investment banks.⁶² These differences in expected losses are consistent with the implication of our model that firms with lots of uninsured debt, like the investment banks, will be required to hold more equity against assets with the same risks.⁶³

6 Distinguishing Solvency from Liquidity Problems

Many of the phenomena that are commonly cited as caused by liquidity problems might instead reflect solvency issues. This section gives four examples.

6.1 Deposit Runs

We showed in Section 3 that market haircuts can rise substantially as the value of collateral falls, even though the underlying riskiness of the collateral

as a cause of understated loss reporting.

⁶¹Bank of America's Tier 1 capital and common equity fall from 13.5% to 11.5%, and 11.9% to 9.9% of risk-weighted assets respectively; Goldman Sachs' ratios fall from 16.7% to 10.8%, and 14.7% to 8.8%. Moreover, these declines include reductions for expected capital distributions, so the disparity in net losses is even greater than 3 to 1.

⁶²The falls in Tier 1 common equity as percentages of RWA for the six largest U.S. banks are: Bank of America 2.0%, JP Morgan, 2.4%, Citibank 2.7%, Wells Fargo 2.8%, Morgan Stanley 4.7%, and Goldman Sachs 5.9%. Goldman and Morgan are susceptible to trading losses that may not be a major issue for most other banks, but the trading assets should be reflected in risk-weighted assets.

⁶³Regulators might perhaps give a different explanation of Morgan Stanley's and Goldman Sachs' high projected losses: because these banks use fair-value accounting, their accounting losses will be closer to their market losses than if they used hold-to-maturity accounting as the commercial banks do. This accounting explanation would raise the question of why the former investment banks choose to be fair value firms, but our model suggests an explanation for this: because these banks have little insured debt, they have little to gain from regulatory accounting; and since they use long-term debt as a substitute for insured deposits, they may benefit from the assurance that fair value accounting provides to creditors that they will raise enough equity when needed to keep this debt safe.

remains the same as the loan ages (we assumed $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are distributed identically).

In our example in Section 3, an uninsured bank would have to raise equity from $.9880 - .9315 = .0565$ per loan to $.9880 - .8945 = .0935$. It could do this by increasing shares outstanding in the ratio $935/565$ (about 65%),⁶⁴ but it would otherwise be forced by creditor withdrawals to sell $(935-565)/935$ (about 40%) of loans.⁶⁵ This decline in the bank's ability to borrow risklessly might be interpreted as a liquidity run, but it is clear that the issue here is one of solvency.⁶⁶

So in times like 2007-9 when loan collateral was declining in value, solvency has to be considered a serious alternative to an exogenous liquidity run.

It seems at least possible that the much discussed 2007-8 "bank run" in the asset-backed commercial paper (ABCP) market, which fell from \$1,179 billion in mid-2007 to \$840 billion at the end of 2007 and \$704 billion by the end of 2008,⁶⁷ was a solvency run. Commercial banks' uninsured off-balance-sheet subsidiaries accounted for 74% of borrowing in this "shadow banking" market.⁶⁸ The run (essentially a decline in 7 to 30 day uninsured lending using subprime residential mortgages as collateral) corresponded to a drop in the prices ("low θ_1 ") in the lower end of the real-estate market.⁶⁹ Since the market for using risky mortgages to back very short term commercial paper has now essentially disappeared, the run was perhaps more of a fundamentals or solvency run, consistent with the increased market haircuts that arise in our model during a downturn, than with a temporary liquidity panic against

⁶⁴Share sales would be a bit lower if bank shares can sell for more than tangible equity, which would be true with a low enough δ .

⁶⁵If creditors demanded less than absolute safety to hold debt, capital requirements would be lower, but would rise by similar absolute amounts as collateral values fell, so the market would demand the banks sell a higher percentage of their assets or new stock. In our section 3 example, if investors "only" demanded that their claims be protected against 9 in 10 bad draws, initial capital requirements would be 4.97% instead of 6.85%. However, if the first period realized 7 bad draws the bank would have to reduce its borrowing by 3.77% of the initial value, versus 3.70% when investors demanded full protection. So the bank would have to raise a similar amount of equity from a smaller base, roughly doubling its share count, or sell half its assets.

⁶⁶Moreover, if the bank did benefit from forbearance, and raised no equity, its chance of becoming insolvent would be only 1 in 80, so it might well be argued ex post that the forbearance was the correct response to a liquidity run.

⁶⁷See Krishnamurthy *et al* (2014) and Bernanke (2015). Anderson and Gascon (2009) present evidence that as the market deteriorated most ABCP switched to one day maturity.

⁶⁸See Acharya *et al* (2013), table 2. Investment banks accounted for 1% of this market.

⁶⁹The Case Shiller house price index fell by 10% in the second half of 2008 and "low tier" indexes (calculated by metropolitan area) generally fell considerably more.

well-collateralized debt.

If the underlying tail risk in collateral values increases, as it might have done in 2020, rational investors would require banks to adjust their balance sheets to include even more risk capital.

6.2 Mean Reversion

Bankers often argue that (whether because of liquidity, or other, problems) assets cannot be sold at fair value. When the market price of a loan falls after a low θ_1 , they have strong incentives to claim that this is just a “temporary impairment”, that the expectation is that the loan will ultimately be repaid in full, and that regulators should therefore choose $\alpha_1 < 1$. And their arguments are made superficially more plausible by the fact that they are usually right, *ex post*. While $\theta_1/\delta < 1$ reduces the value of loan collateral and worsens the tail risk (because if $M_{0,1}(\theta_1) < 1$, then $m_1(\theta_1) > m$), see Lemma 4, the probability of ultimate full repayment may remain high.

The problem is that the high probability of “mean reversion” is balanced by a small probability of a much larger loss. In the example of Section 3, the bank would be more likely than not to end with a profit on the loan even after a realization of θ_1 in the bottom 6% of its distribution, and in most other cases losses would be small.⁷⁰ But the market will demand large increases in capital precisely because creditors demand that the probability of default be remote if they are to accept low yields.

So regulators are making *two* mistakes if they accept the temporary impairment argument. They need to respond to both $M_{0,1} < 1$ (the decline in long-term value of the asset) *and*, possibly more important, $m_1 > m$ (the increase in risk).

However, the stress tests, which generally begin with a dramatic fall in asset values followed by an upturn,⁷¹ and which recognize book rather than market losses, implicitly assume some reversion. And the Last Taxi Fable, that is often used to argue for relaxed capital requirements after a bad shock,

⁷⁰Note that if we introduced bankruptcy costs into our model, so that defaulting loans would on average have a lower recovery value, these effects would be increased: for any given market value of bank loans both the probability of the loans ultimately being profitable and the market capital requirement would be higher.

⁷¹See e.g. Bank of England (2019b) and Federal Reserve (2021).

also relies on assuming mean reversion.^{72,73}

One prediction of a solvency model is that haircuts or capital requirements rise most for assets whose tail risk increase the most. This is consistent with the 2008 changes in the haircuts of the Boston Federal Home Loan Bank. The FHLBs, owned by commercial banks, effectively insured by the US government, and designed to make collateralized loans to commercial banks on favorable terms, are unlikely to have raised collateral requirements for liquidity reasons. In fact, they reduced the haircuts on short-term (less than three years) US Government securities by 40%, while raising requirements on various mortgage-backed securities by 75-150% and no longer accepting various second liens as collateral.⁷⁴

6.3 (Reverse) Fire Sales

Liquidity models of banking crises emphasise the possibility that solvent banks may be unable to sell assets except at “fire sale” prices. But in the financial crisis, banks refused to sell "toxic assets" to the government at fair value or even above.⁷⁵ That is why the TARP program had to pivot from purchasing toxic assets⁷⁶ to providing equity.⁷⁷

⁷²The Fable analogizes capital requirements to a taxi company which is required to always have a taxi waiting at the train station—so the last taxi is rendered useless. A more accurate analogy is that two trains arrive some time apart, and the company is required to have enough cabs on hand to meet all demand at the station with high probability. When a train arrives, all cabs are available, but as cabs are taken the company arranges for new cabs to join the queue before the next train arrives. By contrast, the bankers’ fable corresponds to assuming that total demand across two trains is fixed (mean reversion), and a foolish contract requires that some cabs wait for the second train.

⁷³The pandemic-related 2020 financial markets experienced mean reversion: but those who argue that the banks surviving this crisis proves they were adequately capitalized are implicitly arguing that it was reasonable to assume, after observing θ_1 , that conditions could not get worse.

⁷⁴See Trent and Crispin (2008).

⁷⁵A contemporaneous report noted "Risk-taking institutional investors, ... have refused to pay more than about 30 cents on the dollar for many bundles of mortgages But banks holding those mortgages, not wanting to book huge losses on their holdings, have often refused to sell for less than 60 cents on the dollar." (<https://www.nytimes.com/2009/03/21/business/21bank.html>.)

⁷⁶As Edward Lazear, the 2006-9 Chair of the Council of Economic Advisors, has explained "In 2008, the Troubled Asset Relief Program, as the name implies, was initially pitched as a program to buy up toxic assets. But it quickly became obvious that financial firms did not want to sell assets that would force them to mark-to-market and reduce the stated value of their portfolios." (Lazear, 2020).

⁷⁷The regulatory capital system can also discourage banks from buying assets they think are cheap, harming seller liquidity. For example, in the FCIC transcript of a July

A solvency model explains such "reverse fire sales". As an example, in 2008 Citibank marked its alt-A assets at 80 cents on the dollar while Lehman marked its assets at 39 cents. It seems reasonable to assume that Lehman did not understate the true value of its assets, so if Citibank could have sold out at fair value it likely would not have received more than 39 cents.⁷⁸ The consequences for its RCR would have been disastrous. If the alt-A assets faced a 12% capital charge⁷⁹ then Citi's alt-A assets would count for the equivalent of $80(1 - .12) \approx 70$ cents of cash. So selling out at 39 cents would leave the bank with an equity hole of $70 - 39 = 31$ cents for each 39 cents actually raised. The bank would be required to raise more equity even though it had actually reduced its riskiness through the sale.⁸⁰

The implication is that, after an extended market decline met with inadequate response, many banks were arguably in worse shape than if facing $\theta_1 = \underline{\theta}$ in our model. In our model, a bank can always sell assets at $t = 1$ for at least the amount of associated debt (70 cents in the example), and so can still operate efficiently without a bailout, if it sells inefficient assets and raises new equity. When problems fester beyond that point, asset sales tighten rather than relax the regulatory capital problem, and selling shares may be impossible. This is because, if the bank's value is less than its short term debt, then new shareholders will be unwilling to invest, even if given 100% ownership, unless the post-recapitalization bank receives insurance worth at least as much as the pre-recapitalization deficit.⁸¹ So the bank will

30, 2007 telephone call (available at http://fcic-static.law.stanford.edu/cdn_media/fcic-testimony/2010-0701-AIG-Goldman-supporting-docs.pdf), AIG trader Andrew Forster tells a colleague that he would not buy bonds at 90 cents on the dollar because "If we start buying [more of the] bonds ... then any accountant is going to turn around and say, well, John, you know you traded at 90, you must be able to mark your [existing] bonds then." See also Goldman Sachs (2009).

⁷⁸See Goldman Sachs (2008).

⁷⁹Under Basel III past-due non-guaranteed real estate exposures carry a risk weight of 150%. If assets with a 100% risk weight have a capital requirement of 8% then these assets would have a 12% requirement.

⁸⁰Also consistent with our analysis, the asset sales that commercial banks did make seemed to focus on assets with market values in excess of regulatory values. For example, in 2010-2011 Bank of America sold \$50 billion in assets at a net gain of \$11 billion for the purpose of boosting its regulatory capital (Bank of America 2012), p.3 "Since the beginning of 2010, Bank of America has completed more than 20 non-core asset sales as part of an overall strategy to streamline the Company and focus on serving its three core customer groups individuals, companies and institutional investors. These actions have generated more than \$50 billion in liquidity and \$11 billion in Tier 1 regulatory capital. At the same time, the non-core asset sales have reduced risk-weighted assets by nearly \$58 billion and have made Bank of America more streamlined."

⁸¹For example, if the bank had a value (including intangible assets such as the ability

have no option but to liquidate or reorganise, just as an insolvent non-bank in need of new funding.

To the extent that their long-term debt shares some of the characteristics of insured deposits in our model, investment banks may have also had some incentive to overstate valuations⁸² – but less than the commercial banks. Developing a reputation for marking assets accurately would be of value to an investment bank wanting to sell longer term debt, as a way of assuring creditors that the bank would repair its balance sheet when it faced losses. So Merrill Lynch chose to sell its subprime CDOs, while Wachovia held on.

6.4 Bank Failures

If regulatory values and capital requirements lag behind market values, taxpayers can be exposed. In particular, if $M_{0,1} \leq 1$ and $\bar{\alpha}_2$ is sufficiently small, then $(1 - m)\underline{B}_{0,2} > \underline{M}_{0,2}$. So banks that depend mostly on government-guaranteed debt can lose more than 100% of their equity, on a market value basis, before being closed, even though “Prompt Corrective Action” theoretically closes banks even before their regulatory capital falls to zero. But the U.S. banks that went bust in the crisis in 2008-10 imposed losses on the FDIC of 14% of deposits, even though they averaged approximately zero regulatory capital when they failed. Moreover, small banks, though not too big to fail, are more heavily subsidized than larger banks, because they tend to have less uninsured debt and, in recent years, lower capital requirements. Indeed the FDIC’s losses exceeded 20% of deposits in more than two-thirds of cases.⁸³ It seems unlikely that these banks simply suffered from liquidity problems.

7 Self-Repairing Balance Sheets

Because bank creditors benefit from both formal and virtual insurance, regulators cannot rely on the market to set appropriate capital requirements.

to make profitable new loans but excluding deposit insurance) of 90 and debts of 100 then new shareholders, even if given full ownership of the bank, would only be willing to do so if the value of deposit insurance after the recapitalization was at least 10.

⁸²See Morgan Stanley (2007, 2008). The latter estimated that, as of May 2018, broker-dealers who had taken write-downs of \$230 billion would have to take another \$90-\$180 billion.

⁸³In only 7 cases out of 414 failures did resolution leave the FDIC without losses; in just 47 cases losses were less than 10% of deposits, while in 281 cases losses exceeded 20%. See Federal Deposit Insurance Corporation (2020). For perspective, a bank needed only 6% (positive) regulatory capital to be rated as “well-capitalized”.

for either fully- or partially-insured banks. This creates a daunting problem:

In non-crisis times regulators do not know whether bank shares are too high, implying a good time to raise equity to protect against a coming decline, or too low, in which case debt might be an appropriate source of new funds. But when a crisis arises governments may find themselves unable to require much new private sector investment: shareholders will not voluntarily raise new equity in a solvency crisis because much of the value will go to creditors, and a liquidity crisis is defined by the unavailability of private risk capital.⁸⁴

So the ideal would be to have banks issue securities which are equity when, in retrospect, they were issued when share prices were too high but which are debt otherwise. This would privatize much of the risk currently borne by taxpayers.

Taxpayer risk and moral hazard could be eliminated for retail deposits and other fully insured claims if those obligations were backed by narrow collateral, as though in a government money market fund.⁸⁵ For other debt, a creditor's recourse could be limited to a fixed number of shares of stock, the number tied to the share price on the date the loan was made and to the amount of promised repayment. Specifically, replacing most unsecured debt with "Equity Recourse Notes" (ERNs), achieves the hindsight we require. ERNs are a form of debt whose currently-due payments convert into equity if the issuer suffers a substantial decline in share price; that is, they are a kind of "smart coco" (contingent convertible security).⁸⁶ If all debt except retail deposits were replaced by ERNs, banks would automatically be fully subject to the discipline of the market.⁸⁷

⁸⁴During the financial crisis there was tremendous pressure to move further away from fair value losses. See, e.g., House of Representatives (2009) when members of Congress pressed for accounting changes that reduced the regulatory capital losses the Federal Home Loan Bank of Atlanta had to recognize for regulatory purposes from \$89.7 million to \$44,000 (against fair value losses of \$2.8 billion). See Acharya et al. (2019) for evidence on the lack of equity raises during the financial crisis. (U.S. banks did sell stock to escape TARP's restrictions on executive compensation.)

While dividends (Europe) and buybacks (U.S.) were curtailed in 2020, firms sold little if any stock, either when share prices were high or low.

⁸⁵Advocates of narrow banking are historically as diverse as Milton Friedman and James Tobin. In the Financial Crisis, Mervyn King and Lawrence Kotlikoff were two prominent advocates for the money market fund approach.

⁸⁶ERNs are distinct from traditional cocos in several important ways, including that they convert based on market prices and only one payment at a time, so they avoid the significant incentive problems that bedevil existing cocos (see Bulow and Klemperer, 2015).

⁸⁷Advocates for more market-based capital requirements include King (2016), Sarin and

An ERN can be understood as a bundle of zero coupon bonds, with the bank having a European put option to issue shares in lieu of cash for each payment.⁸⁸ An investor who desires a completely riskless claim can get one by buying an offsetting put option in the market. But the "property rights" in the loss stay in the private sector.

For example, if an ERN promised a payment (interest or principle) of \$1 on a specific day, and the bank's share price was $\$S$ on the day the bond was issued, the bank could pay either \$1 in cash or E/S shares (for some $E > 1$; the maximum value of E would be set by regulation⁸⁹). So if the bank's share price fell below $\$S/E$ it would be cheaper for the bank to pay in shares, and in this case it would be required to do so. We provide an example of how ERNs would work within our model in Appendix B.

Since the bank can pay in a fixed number of shares, ERNs need not be considered debt for capital requirement purposes. This can be seen by noting that a promised ERN payment is mathematically equivalent to the bank issuing a zero exercise price warrant but retaining a call option that allows it to repurchase the shares that the warrant holder would receive.⁹⁰

Many banks regularly argue that they have "fortress balance sheets" with more than enough risk capital. Our system would allow these banks to substitute ERNs for equity, and if bond buyers agreed that the prospect of the bank's stock falling by much more than $(E - 1)/E$ was remote then the risk premium for ERNs would be low. However, if in retrospect the bank was much weaker than expected, the loss would be retained in the private sector.

Note that ERNs are counter-cyclical, because new ERNs issued in bad times would effectively be senior to those issued in good times (because of a lower conversion price). Banks would therefore be incentivised to raise new risk capital in bad times – the opposite of the normal "debt overhang" situ-

Summers (2016), Fuster and Vickrey (2018), Vickers (2019), Bulow and Klemperer (2013), and Bulow, Goldfield and Klemperer (2013).

⁸⁸The bank would be required to exercise the put option if the share price was below the exercise price; it would have the option of exercising if the share price was above the exercise price. While we would not expect a bank to exercise when the share price is high, it might do so if, e.g., it faced a liquidity crunch.

⁸⁹In Bulow and Klemperer (2015) we suggested $E = 4$ so bondholders would be paid in cash unless shares fell by 75% or more; approximately the amount the best capitalized banks' shares fell in 2008-9.

⁹⁰An analogy is to a sovereign borrower who borrows in dollars, but retains an option to repay in its own currency at an exchange rate less favorable than when the debt is issued. If the country can print its own currency, it can always repay the debt, albeit at the cost of expanding the domestic money supply. With ERNs, the company's shares are its currency.

ation in which selling equity in bad times transfers wealth from shareholders to creditors and insurers.

Importantly, because ERNs automatically repair a bank's balance sheet as needed (so that bankruptcy is always avoided) regulators need only stay the course and not take any extraordinary actions to keep the banks going. Instead of regulators trying with limited success to get banks to issue equity when the banks claim the equity markets are closed for secondary offerings, shares would automatically be issued and the bank managers would have to convince the regulators to let them buy shares back.

Finally, if share prices fell too far because of a liquidity crisis, ERNs would help by guaranteeing the effective sale of bank shares at a sale price that would exceed the current "fire sale" market price, and avoid any bankruptcy trigger. Central bankers would still have the option of intervening when they believed the system faced a liquidity crisis, but would no longer be compelled to use taxpayer money to bail out insolvent banks.

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Table 1
Bank Stock Volatilities and Price/Book Ratios

Bank	Volatility	P/B	P/B	P/B
Bank		4.25.21	12.31.20	3.31.20
Barclays	42.2%	.47	.38	.25
HSBC	24.8%	.60	.56	.62
Lloyds	42.5%	.61	.52	.45
RBS	45.8%	.50	.47	.31
Santander	37.9%	.61	.53	.35
Std. Chartered	32.7%	.41	.41	.34
Bank of America	33.1%	1.35	1.07	.76
Citigroup	36.4%	.82	.73	.50
Goldman Sachs	32.4%	1.18	1.11	.67
JP Morgan	28.7%	1.82	1.61	1.17
Morgan Stanley	33.0%	1.64	1.55	.73
Wells Fargo	35.1%	1.09	.78	.71

Weekly share prices from Yahoo Finance adjusted close, 4.25.2016 - 4.19.2021. Weekly percentage change calculated, then excel VAR function applied for weekly variance. Annual standard deviation calculated by taking the square root of weekly variance and by taking the square root of weekly variance and multiplying by the square root of 365.25/7. Price/Book ratios are from Yahoo Finance, 25 April 2021.

Table 2

Regulatory Recognition of Risks During Most Recent Stress Tests

Bank	RWA	A	Ratio	RWA-S	A-S	Ratio-S	m_1/m
Barclays	312	999	31.2%	394	1148	34.3%	110%
HSBC	865	2413	35.8%	1189	2292	51.9%	145%
Lloyds	206	663	31.1%	240	626	38.3%	123%
Nationwide Bldg.	33	232	14.2%	77	228	33.8%	237%
RBS	189	560	33.8%	254	631	40.3%	119%
Santander	79	276	28.6%	75	282	26.6%	93%
Std. Chartered	37	741	5.0%	27	698	3.9%	77%
Bank of America	1480	2699	54.8%	1467	2721	53.9%	98%
Citigroup	1222	2262	54.0%	1206	2264	53.3%	99%
Goldman Sachs	554	1143	48.5%	550	1142	48.1%	99%
J.P. Morgan	1561	3344	46.7%	1540	3372	45.7%	98%
Morgan Stanley	453	1046	43.3%	445	1063	41.9%	97%
Wells Fargo	1194	1913	62.4%	1184	1913	61.9%	99%
Barclays US	86	172	50.0%	85	174	48.9%	98%
HSBC North Am.	115	245	47.0%	110	246	44.7%	95%
Santander US	120	142	79.0%	119	153	77.8%	98%

Sources: Bank of England (2019b), and Federal Reserve (2021).

Santander USA data is from the 2020 tests (it was untested in 2021).

RWA is Risk-Weighted Assets; A is assets.

RWA-S and A-S are projected RWA and A in the stress test.

Ratio is RWA/A; Ratio-S is RWA-S/A-S. m_1/m is Ratio-S/Ratio.**Table 3**

Aggregate Statistics for US Banks, Selected Years, % of Assets

	2020	2019	2009	2008	2007
(1) Cash + Due from Depository Inst.	14.57	8.94	8.02	7.98	4.02
(2) Net Loans + Leases	48.56	55.75	53.90	55.63	59.87
(3) Deposits held in Dom. Offices	74.44	70.90	58.81	54.16	53.04
(4) Total Risk-Weighted Assets	58.26	70.11	72.75	72.41	75.26
(5) Volatile Liabilities	12.04	14.88	29.42	34.69	36.02
(6) Tier 1 + Tier 2 Capital	9.40	10.27	10.39	9.25	9.62
(7) Line (5) - Line (1)	-2.17	5.94	21.40	26.71	32.00
(8) Line (6) ÷ Line (4) (%)	16.13	14.65	14.28	12.77	12.78

Source: FDIC Statistics of Depository Institutions, end of year data.

Appendix A

Example referred to in Section 3.5; this is also an elaboration of the example referred to in the Introduction

Assume $K = 1.20$, $P = 1.05$, $r = .021$, and (for simplicity) $\delta = 1$. The distributions of $\tilde{\theta}_i$ are such that $\theta_i = 1.05$ with probability .77, $\theta = .95$ with probability .20, and $\theta_i = \underline{\theta} = .75$ with probability .03. So $m = 1 - \delta K \underline{\theta} = .1$.

If $\theta_1 = .95$, then the collateral is worth $\delta K \theta_1 = 1.14$ at $t = 1$, so the loan will be paid off in full unless $\theta_2 = \underline{\theta}$, and $M_{0,1} = (.97(1.05) + .03(1.14)(.75))/1.021 = 1.0227$, so $M_{0,1} > 1$. Indeed the value of the bank's claim has risen by more than the expected market return ($M_{0,1} > 1 + r$). However, $\delta K \theta_1 \underline{\theta} = 1.14(.75) = .855$. Therefore $m_1 = 1 - .855/1.0227 = .1688$, so $m_1 > m$. To make uninsured investors safe, debt must be reduced from .90 to .855, requiring the bank to raise an additional .045 in equity.

So, even though equity has risen by 22.7% (from .10 to .1227 per loan) the bank must raise an additional .045 per loan, requiring either "dilution" of $(.045/ (.045 + .1227)) = 26.8\%$, or the sale of that percentage of the bank's assets.

Appendix B

Example of use of ERNs in the model

Suppose, in our model, that a bank that makes L_0 loans at $t = 0$ can take no further actions before the loans are repaid at $t = 2$. Then the amount that it can borrow safely (with either two-period debt or one-period debt that investors can safely roll over) is reduced to $L_0 \delta^2 K \underline{\theta}^2$, from the $L_0 \delta K \underline{\theta}$ it could borrow in our basic model. However, the bank can achieve the equivalent of borrowing the additional $L_0 \delta K \underline{\theta} - L_0 \delta^2 K \underline{\theta}^2$, by issuing ERNs that convert from debt into equity after any sufficiently bad shock at $t = 1$, thus automatically replenishing the equity as required.⁹¹

For example, Bulow and Klemperer (2015) suggested ERNs that would be convertible into shares at 25% of the share price on the issue date, chosen because the share price of the strongest banks fell by around 75% during the financial crisis. Say that $L_0 = 3000$, $K = 1.2$, and $\delta \underline{\theta} = .75$. Then the bank could borrow $L_0 \delta^2 K \underline{\theta}^2 = 2025$ on a collateralized basis at $t = 0$. The bank could issue equity of 400 and ERNs, due at $t = 1$, with a face value of

⁹¹In a more realistic setting, ERNs would be long-term bonds with repayments spread over many periods.

575. If $\theta_1 = \underline{\theta}$, the firm's value would fall from $L_0 = 3000$ to $L_0 K \delta \underline{\theta} = 2700$, so the value of the equity would fall from 400 to 100. Since the share price would then have fallen 75%, the ERNs could then be paid in shares, and their holders would receive equity worth their initial investment of 575. So at $t = 1$, the bank would have $100 + 575 = 675$ in equity, and 2025 in debt, exactly as in our basic model, after $\theta_1 = \underline{\theta}$. Furthermore, the ERNs would have generated a riskless return, assuming that any shares received could be sold at fair value.

Banks could choose any combination of ERNs and stock they thought would maximize their value; in the context of our model this would involve the lowest amount of equity that assured ERNholders that they would not suffer a loss even if $\theta_1 = \underline{\theta}$, so long as securities could still be sold for fair value (as in the example above).

If the bank's shares fell to less than 25% of the price on the day the ERNs were issued, then ERNholders would own a fixed fraction of the bank's equity (in the example $575/675$), regardless of whether this lower price was due to a liquidity shock that caused shares to sell for less than fair value, or because fair value had fallen by more than 75% of the initial share value. Regardless of what happened at $t = 1$ the bank would not go bankrupt, and it would end with a limited number of shares outstanding at a positive price per share (so long as $\theta_1 > 0$ or $\theta_1 = 0$ but $N_1 > 0$).

Appendix C

Lemmas and proofs omitted from main text

Lemma 0 $P > P'(1 + r) > P'/\delta$

Proof of Lemma 0

The second inequality follows straightforwardly from rearranging (A1).

For the first, note that one-period and two-period loans must earn the same expected rate of return, that is, $E\{\min(K\tilde{\theta}_2, P')\} = E\{\min(K\tilde{\theta}_1\tilde{\theta}_2, P)\} / (1 + r) = 1 + r$.

Assume, for contradiction, that $P/(1 + r) \leq P'$. Then conditional on any θ_2 :

either (i) $\min(K\theta_2, P') = P'$. Our model assumed $K\bar{\theta} < P$ so, for all θ_2 , we have $K\theta_1\theta_2 < P$ for at least some θ_1 , so $E\{\min(K\theta_1\theta_2, P)\} / (1 + r) < P/(1 + r)$. So in this case, if $P/(1 + r) \leq P'$, the expected return

on one-period loans is strictly higher than on two-period loans. Moreover, $E\{K\tilde{\theta}_2\} = K(1+r) > (1+r) = E\{\min(K\tilde{\theta}_2, P')\}$, so $K\theta_2 > P'$ for at least some θ_2 . So case (i) applies for at least some θ_2 .

or (ii) $\min(K\theta_2, P') = K\theta_2$. But $E\{\tilde{\theta}_1\} = (1+r)$ so $E\{\min(K\tilde{\theta}_1\theta_2, P)\}/(1+r) \leq K\theta_2$. So in this case, the expected return on one-period loans is at least as high as on two-period loans.

So we have a contradiction. \square

Proof of Lemma 4

The payoff from deposit insurance equals the maximum of zero and the difference between the bank's borrowing and the value of its assets at $t = 2$, that is, $\max\{0, L_0^R((1 - \hat{m})M_{0,1} - M_{0,2}) + L_1((1 - m) - M_{1,2})\}$, if old loans are financed with a haircut of \hat{m} in equity. Since $(1 - m) - M_{1,2} \leq 0$, an additional new loan weakly reduces this value.

Assume $\hat{m} < m_1$. If $\theta_2 = \underline{\theta}$, then $L_1((1 - m) - M_{1,2}) = L_1((1 - m) - \underline{M}_{1,2}) = 0$, so the insurer must pay $L_0^R((1 - \hat{m})M_{0,1} - \underline{M}_{0,2})$, which is strictly positive, and so also increasing in L_0^R . Moreover, if $\theta_2 > \underline{\theta}$ but $(1 - \hat{m})M_{0,1} - M_{0,2} > 0$, then a small increase in L_0^R will either leave the insurer's payment at 0 or increase it by $(1 - \hat{m})M_{0,1} - M_{0,2}$ per old loan. Finally, an additional old loan can never reduce the value of insurance, because if $(1 - \hat{m})M_{0,1} - M_{0,2} \leq 0$, then the value of insurance is 0, independent of the number of old loans. So the expected cost to the insurer, and therefore the expected benefit to shareholders from insurance, is strictly increasing in L_0^R .⁹² \square

Proof of Lemma 5

Since any draw of θ_1 or θ_2 equal to $\underline{\theta}$ gives the bank economic ownership of the collateral (because $K\underline{\theta} < P$), the maximum amount of riskless debt that an uninsured bank can issue is always $\delta\underline{\theta}$ times the current value of the collateral. If $\theta_1 = 1/\delta$, both new and old loans support $\delta K\underline{\theta} = 1 - m$ in debt. New loans have a market value of $E\{\min(K\tilde{\theta}_2, P')\}/(1+r) = 1$, while old loans have a market value of $M_{0,1} = \delta E\{\min((K/\delta)\theta_2, P)\}/(1+r)$.

⁹² Adding old loans always strictly increases the value of insurance—it suffices that there is positive probability that $\theta_2 < \underline{\theta} + \epsilon$ for all $\epsilon > 0$. But adding new loans may only weakly decrease the value of insurance if there is a discrete distribution of θ_2 —the reason is that insurance does not affect the value of new loans when $\theta_2 = \underline{\theta}$ and, with a discrete distribution of $\tilde{\theta}_2$, the marginal insurance cost of a new loan may be zero if the second-lowest state of θ_2 and/or N_1 is high enough that the insurance won't pay in the second-lowest state of θ_2 .

But from Lemma 0, $\delta P > P'$, so $M_{0,1} > 1$. (The inequality is strict because (i) $K\theta_2 < P'$ for at least some θ_2 , by the assumption of our model that $K\underline{\theta} < 1$, and (ii) $E\{\min(K\tilde{\theta}_2, P')\}/(1+r) = 1$, but also $E\{K\tilde{\theta}_2\}/(1+r) = K > 1$, so $K\theta_2 > P'$ for at least some θ_2 .) Therefore if $\theta_1 = 1/\delta$, then $m_1 = 1 - \delta K\underline{\theta}/M_{0,1} > 1 - \delta K\underline{\theta} = m$. Furthermore, lower values of θ_1 proportionally reduce the debt an old loan can support, proportionally reduce its market value if θ_1 is low enough that the bank is already *de facto* owner of the collateral, and strictly less than proportionally reduce its market value otherwise), and so weakly increase the capital requirement, m_1 , on the old loans. And $M_{0,1}$ is strictly increasing in θ_1 , since $K\theta_1\underline{\theta} < K\theta\underline{\theta} < P$. So if $M_{0,1} \leq 1$, then $m_1 > m$ and also $\theta_1 < 1/\delta$. \square

Lemma 6 *If $M_{0,1} \leq 1$ a bank may make new loans financed with m in equity and not old loans financed with m_1 in equity, but not the opposite.*

Proof of Lemma 6

For each uninsured new loan the bank adds at $t = 1$, it will receive a gross return of $\delta \min\{K\theta_2, P'\}$ against which it will have debt of $\delta\underline{\theta}K$. So each uninsured new loan returns $\min\{K\theta_2, P'\}/\underline{\theta}K$ per dollar of debt supported. Each uninsured old loan returns $\delta^2 \min\{K\theta_1\theta_2, P\}$ and supports debt of $\delta^2\theta_1\underline{\theta}K$, so returns $\min\{K\theta_1\theta_2, P\}/\theta_1\underline{\theta}K$ per dollar of debt supported. The proof of Lemma 5 shows that if $M_{0,1} \leq 1$ then $1/\theta_1 > \delta$, so by Lemma 0 $P/\theta_1 > P'$, so the gross return from adding enough old loans to support a dollar of debt weakly exceeds the gross return from adding new loans to support a dollar of debt. But this means that, per dollar of debt, for the old loans (i) a weakly greater amount will be available to the deposit insurer to offset any losses on the insured loans, and (ii) more equity is being used. Since the bank loses $1 - \delta$ in expected present value per dollar of loans financed with equity (it earns a positive expected present value return of $\delta - (1/(1+r))$ per dollar of loans financed with debt), both (i) and (ii) make incremental old loans less profitable (i.e, they have lower net returns), per dollar of debt, than incremental new loans. So the net returns (per dollar of debt, or per loan, or per dollar of equity) may be positive for the new loans and negative for the old loans but not the other way around. \square

Proof of Proposition 3a

The regulatory capital that the bank has to have to retain an old loan at $t = 1$ is $mB_{0,1}$, so the amount that it is permitted to borrow against the

loan is $(1 - m)B_{0,1}$, or $(1 - m)(B_{0,1}/M_{0,1})$ per dollar of its market value, which equals or exceeds $(1 - m)$ since $B_{0,1} = (1 - \alpha_1 + \alpha_1 M_{0,1}) \geq M_{0,1}$ when $\alpha_1 \leq 1$.⁹³□

Proof of Proposition 3b

Since all assets earn the same expected rate of return but old loans require weakly less equity (Proposition 3a), old loans earn a weakly higher expected rate of return than new loans ignoring insurance effects. Lemma 4 shows that insurance effects strictly increase the difference in expected rate of return.□

Proof of Proposition 3c

Since all assets earn the same expected rate of return but old loans require weakly less equity (Proposition 3a), old loans earn a weakly higher expected rate of return than new loans ignoring insurance effects. Since new loans are profitable on a stand-alone basis (assumption A1), and the insurance effects of adding old loans always benefit the bank (Lemma 4), the fully-insured bank will always retain all its old loans, that is, $L_0^R = N_0$. From Lemma 3, either $L_1 = 0$ or $L_1 = N_1$.

Consider a θ_1 and a distribution of θ_2 such that there is positive probability that $\theta_2 > \underline{\theta}$ and $M_{0,2}(\theta_2) < (1 - m)B_{0,1}$. Then, for these θ_2 , the old loans lose more than the market value of the equity that needs to be held against them, so the deposit insurer would benefit from the (strictly positive) returns on any new loans. If $rm = \delta(1 + r) - 1$ the bank's expected returns from the new loans, ignoring the effects of deposit insurance, would equal its cost of capital. So if r is sufficiently close to (but, to satisfy (A1), greater than) $(1 - \delta)/(\delta - m)$, the bank chooses $L_1 = 0$.

On the other hand, the new loans' expected returns strictly exceed the cost of capital, and the maximum loss per old loan is bounded, so if N_0 is sufficiently small relative to N_1 , the bank chooses $L_1 = N_1$.□

Proof of Proposition 4

First, note that if $\underline{B}_{0,2} < B_{0,1}(1 - m)$ and $\bar{D} \geq L_0 \left[\frac{(1 - m)(B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2})}{\underline{B}_{1,2} - (1 - m)} \right]$, then $\bar{D} > L_0 B_{0,1}(1 - m)$ (because $\underline{B}_{0,2} < B_{0,1}(1 - m) \implies B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2} >$

⁹³The fully-insured bank can create *strictly* more riskless debt per dollar of market value of old loans than per dollar of market value of new loans if $M_{0,1} < 1$ and $\alpha_1 < 1$

$B_{0,1}\underline{B}_{1,2} - (1 - m)B_{0,1} \implies \frac{(1-m)(B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2})}{\underline{B}_{1,2} - (1-m)} > (1 - m)B_{0,1}$. So there is enough insured debt to finance all old loans.

If $L_1(1 - m) \leq (\bar{D} - L_0B_{0,1}(1 - m))$, the bank can also finance all new loans with insured debt, so the bank is only bound by its RCR.

If, instead, $L_1(1 - m) \geq (\bar{D} - L_0B_{0,1}(1 - m)) > 0$ then the bank can finance $(\bar{D} - L_0B_{0,1}(1 - m)) / (1 - m)$ new loans with insured debt, and each would generate $\underline{B}_{1,2} - (1 - m)$ in net regulatory capital, creating a total of $\frac{(\underline{B}_{1,2} - (1 - m))(\bar{D} - L_0B_{0,1}(1 - m))}{1 - m}$, if $\theta_2 = \underline{\theta}$. So if $\frac{(\underline{B}_{1,2} - (1 - m))(\bar{D} - L_0B_{0,1}(1 - m))}{1 - m} + L_0(\underline{B}_{0,2} - (1 - m)B_{0,1}) \geq 0$, that is, $\bar{D} \geq L_0 \left[\frac{(1-m)(B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2})}{\underline{B}_{1,2} - (1-m)} \right]$, then the bank will be assured of having non-negative regulatory capital at $t = 2$ from its assets backed by insured debt. Furthermore, if apart from assets backed by insured debt the bank only has new loans, and if the regulatory requirement for those uninsured new loans (limiting the bank to $1 - m$ of debt per loan) is met, the PLC will also be met, since the uninsured loans can be sold for at least $\underline{M}_{0,2} = 1 - m$ at $t = 2$. So if $\bar{D} \geq L_0 \left[\frac{(1-m)(B_{0,1}\underline{B}_{1,2} - \underline{B}_{0,2})}{\underline{B}_{1,2} - (1-m)} \right]$, the bank is only bound by its RCR. \square

Proof of Proposition 5

For a fully insured bank only the RCR, binds, since $U_1 = 0$. A bank that is not fully insured must also meet the PLC, when it chooses $U_1 = 0$. So its optimization problem will be the same as the fully-insured bank if (PLC) is satisfied whenever (RCR) is satisfied.

If $\bar{D} \leq L_0(1 - m)B_{0,1}$, then $L_1 = L_1^U$, so when the RCR and PLC both hold with equality we can write (RCR), (1) and (PLC) as

$$(1 - m)(B_{0,1}L_0^R + L_1) = \bar{D} + U_1 \quad (\text{a1})$$

$$L_1\underline{M}_{1,2} + L_0^U\underline{M}_{0,2} = U_1 \quad (\text{a2})$$

$$(L_0^R - L_0^U)\underline{B}_{0,2} = \bar{D} \quad (\text{a3})$$

Rewriting (a1) as

$$\bar{D} + (U_1 - L_1(1 - m)) = (L_0^R - L_0^U)(1 - m)B_{0,1} + L_0^U(1 - m)B_{0,1} \quad (\text{a4})$$

and substituting $(L_0^R - L_0^U)(1 - m)B_{0,1} = \bar{D} \frac{(1-m)B_{0,1}}{\underline{B}_{0,2}}$ from (a3), $L_0^U(1 - m)B_{0,1} = (U_1 - L_1\underline{M}_{1,2}) \frac{(1-m)B_{0,1}}{\underline{M}_{0,2}}$ from (a2), and $\underline{M}_{1,2} = (1 - m)$ into (a4), gives

$$\bar{D} + (U_1 - L_1(1 - m)) = \bar{D} \frac{(1 - m)B_{0,1}}{\underline{B}_{0,2}} + (U_1 - L_1(1 - m)) \frac{(1 - m)B_{0,1}}{\underline{M}_{0,2}}$$

which simplifies to $U_1 = \overline{D} \frac{M_{0,2}(\underline{B}_{0,2} - (1-m)B_{0,1})}{\underline{B}_{0,2}((1-m)B_{0,1} - M_{0,2})} + L_1(1-m) = \overline{V} + L_1(1-m)$, with $\overline{V} \geq 0$, since we assumed $\underline{B}_{0,2} - (1-m)B_{0,1} \geq 0$.

So if all the old loans can be financed with a debt of exactly $\overline{D} + \overline{V}$, satisfying (RCR) exactly satisfies (PLC).

It is straightforward that if the old loans can be financed with an amount of debt $\in [\overline{D}, \overline{D} + \overline{V})$, the same amount of "excess regulatory capital" is created by the old loans financed with insured debt (at the regulatory capital rate) but less is eaten up by the old loans financed with uninsured debt (at the regulatory capital rate) so there is then slack in (PLC). Moreover, if all the old loans can be financed with an amount of debt less than \overline{D} , there is still positive "excess regulatory capital" created by (all) the loans financed with insured debt (at the regulatory capital rate), and none of it is eaten up by the (new) loans financed with uninsured debt (at the regulatory capital rate) so there is again slack in (PLC).

So if $L_0 B_{0,1}(1-m) \leq \overline{D} + \overline{V}$ and $\underline{B}_{0,2} \geq (1-m)B_{0,1}$ a partly insured bank is constrained only by (RCR) so behaves exactly as a fully insured bank. \square

Proof of Proposition 6

The bank will not choose $L_0^R = L_1 = 0$ since, by Proposition 2c, even an uninsured bank would make some loans. So assume, for contradiction, it makes only new loans. Assume it funds these loans with $D_1 \leq \overline{D}$ of insured debt (and possibly some uninsured debt). Then in the worst $t = 2$ state it will have $(\underline{B}_{1,2} - (1-m))D_1/(1-m)$ of regulatory capital, which is strictly positive if $\alpha_2 < 1$. So the bank could instead use some insured debt to finance old loans at the regulatory capital rate without violating its PLC, and without changing the number of new loans financed. Displacing new loans to uninsured debt (if necessary) without violating the PLC has no insurance effects, since all new loans can be sold at $t = 2$ for at least the debt supporting them, however they are financed. The additional old loans would increase the bank's profits, because old loans financed at the regulatory capital rate are always profitable. So the bank will always choose $L_0^R > 0$. \square