Optimal Delegation, Limited Awareness and Financial Intermediation*

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Abstract

We study the delegation problem between an investor and a financial intermediary, who not only has better information about the performance of the different investments but also has superior awareness of the available investment opportunities. The intermediary decides which of the feasible investments to reveal and which ones to hide. We show that the intermediary finds it optimal to make the investor aware of investment opportunities at the extremes, e.g. very risky and very safe projects, but leaves the investor unaware of intermediate options. We further study the role of competition between intermediaries and allow for investors with different levels of awareness to coexist in the same market. Self-reported data from customers in the Italian retail investment sector support the key predictions of the model: the menus offered to less knowledgable investors contain few products, most of them are nevertheless perceived to be at the extremes.

JEL Codes: D82, D83, G24.

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1 Introduction

One of the many striking features of the recent financial crisis was the extreme exposure of investors to risk. Both investment and commercial banks had been selling excessively risky assets to investors, sometimes hiding some of the asset characteristics and safer options (e.g., Gerardi et al., 2008). At the same time, despite the impressive amount of new financial instruments and the rapidly changing financial world, since the 1950s a large fraction (of approx. 33% in the US) of investment demand has remained on 'safe' assets (e.g., Gordon et al. (2008) and Garcia (2012)).

Most of financial investments are intermediated by professionals. Financial intermediaries are non-neutral brokers and, as such, direct, influence, and distort the demand of assets in the economy. For instance, investment bankers commonly underwrite transactions of newly issued securities, whereby they raise investment capital from investors on behalf of corporations and governments both for equity and debt capital. Financial intermediaries may also operate on the supply side of the asset market (unloading). Such practices give rise to conflicts of interest, sometimes leading to investments that are not necessarily in the best interest of the client.

At the same time, investors differ widely in their financial literacy. They not only face limits in their ability to assess the profitability of particular investments, but often also have limited awareness of the available investment opportunities and must therefore rely on professional advice. For example, Guiso and Jappelli (2005) document the lack of awareness of financial assets among the 1995 and 1998 waves of the survey of Italian households (SHIW).[1] Although almost 95% of respondents in the dataset are aware of checking accounts and almost 90% are aware of saving accounts, only 65% of potential investors are aware of stocks and only 30% of investment accounts; mutual funds and corporate bonds are known by only 50% of the sample. Less than 30% of respondents are simultaneously aware of stocks, mutual funds and investment accounts.[2]

[1] The surveys are representative samples of the Italian resident population, covering 8,135 households in 1995 and 7,147 households in 1998.

[2] The share of wealth in the hand of unaware agents is also substantial. The share of wealth owned by

This paper studies the implications of such limitations by incorporating unawareness into the canonical delegation model. Specifically, we consider the problem of an investor (the principal, she) who wants to invest her savings and delegates the task of picking the right project to a financial intermediary (the agent, he). The intermediary has private information about the payoffs of each investment opportunity and the investor's problem is to determine a set of projects from which the intermediary can choose (see for example Alonso and Matouschek, 2008). We depart from this traditional framework of optimal delegation by considering a situation where the intermediary not only has private information about the suitability of the different investments, but also about the set of investment opportunities that are actually available to the investor. This second dimension of asymmetry is captured by the assumption that the investor is only *partially aware of the feasible investment projects*. Before the delegation stage the intermediary has the possibility to enrich the investor's awareness by revealing additional investment opportunities. We are interested in the questions of whether the intermediary expands the investor's awareness, which projects the intermediary reveals, and what the properties of the realized investment projects are.

We address these questions in an environment with a continuum of states and a continuum of investment projects, some of which the investor is aware of. The intermediary's and investor's preferences are represented by quadratic loss functions with differing bliss points for the two agents. We view the bliss point to be the investment opportunity which generates the best combination between risk, illiquidity, and return as a function of the state. The divergence between the investor's and intermediary's bliss point can be interpreted as financial professionals being less risk averse, having limited liability, having different liquidity needs, etc. When deciding on the set of projects from which the intermediary can select, the investor then faces the usual tradeoff between granting flexibility to the intermediary so he can react to his private information and precluding the intermediary from exploiting his bias.

households that are not aware of corporate bonds is approximately 20%, and so is the share owned by those unaware of mutual funds. In 1995, the households that were unaware of investment accounts owned 40% of the total wealth (the fraction decreased to 32.5% in 1998).

We show that the consideration of unawareness has important implications for delegation and investment. In the benchmark case of full awareness the optimal delegation set for the investor is an interval. The investor effectively imposes a cap: if, for instance, the intermediary has strong preferences for risky investments, the investor places an upper bound on the riskiness of projects from which the intermediary can select. For the case of partial awareness, our main result then shows that generically the intermediary leaves the investor unaware of an interval of intermediate investments.[3] In other words, the intermediary makes the investor *aware of investment opportunities at the extremes* (e.g. very safe and very risky projects). The awareness gap is chosen in a way such that the investor - who still cares about the intermediary's information - finds it optimal to permit projects at both extremes. Thus, by leaving the investor unaware of intermediate investment options, the intermediary is able to select projects that would be precluded if the investor was fully aware.

We incorporate our baseline model into a search environment with multiple investors and intermediaries in order to study the effect of competition on the awareness of investors. We show that for intermediate degrees of competition intermediaries either reveal everything or leave investors unaware of a significant number of intermediate investment options. We thus obtain polarization in the market: some banks fully disclose all investment opportunities, others try to obtain a higher profit by offering only extremes. We further show that unawareness is exacerbated in times of economic downturns and we study the effect of heterogeneity among investors. With regard to the latter, we find that in a market where investors differ in their initial awareness, there are situations where a larger share of sophisticated, fully aware investors leads to less disclosure in equilibrium. Hence, the presence of sophisticated investors in the market can impose a negative externality on those with limited awareness.

In the empirical section, we report the regression results based on self-reported data collected through an online survey we constructed. The data consists in approximately 1,400 investors reporting their experience in the retail investment sector during the last decade. We regress both the number of products offered and a measure of perceived 'extremeness'

[3] More precisely, this always happens unless the investor is initially aware of the project at the optimal upper bound under full awareness.

in the menu of products the investor received from the financial intermediary on an index of knowledge. The index is based on 17 questions eliciting investors' knowledge of the financial market and of the products available in the market. Consistent with the theory, our knowledge index is positively associated with the number of products offered and negatively associated with our measure of extremeness of the offered menu. These findings are robust to introducing several controls, including proxies for the naivity of the investor, his/her wealth, income, education, and his/her self-reported propensity to take risk or to invest in long-term maturity assets.

Finally, we discuss the policy implications of our findings. Clearly, promoting financial literacy among investors improves their welfare in our model. Interestingly though, our results show that it is not necessary to educate investors about all possible investment projects. Instead, we observe that making investors aware of *only few or even only one intermediate project* may give intermediaries incentives to reveal several other intermediate projects as well. We therefore have an interesting complementarity between the regulator and the market, suggesting a surprisingly simple, yet powerful, policy intervention. Moreover, in terms of optimal financial literacy policies, we find that a soft training provided to a relatively large fraction of individuals is typically more effective than training intensively a small fraction of potential investors. This is so because partially trained investors are more attractive to intermediaries than fully sophisticated ones and therefore induce intermediaries to compete against each other more fiercely. The competitive effect leads to a shift in the equilibrium awareness distribution, ultimately generating positive spillovers on the investors that remain unaware.

The paper makes two main contributions. First, it makes a methodological contribution in that we introduce limited awareness into the classic model of delegation. The potential applications are much broader than the financial market and include, for instance, organizational economics, political economy, etc. Second, the stated framework is able to generate predictions on the equilibrium portfolio differentiation, in particular on the demand of safe and risky assets, as a function of the knowledge of the investor and the nature of misalignment between the investor and the financial intermediary.

Related Literature: This paper is first of all related to the literature on optimal delegation. Starting with Holmström (1984), who first defines the delegation problem and provides conditions for the existence of its solution, this literature, which includes Melamud and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), Armstrong and Vickers (2010) and Amador and Bagwell (2013), Halac and Yared (2017), and others, studies optimal delegation problems in environments of increasing generality. None of them consider limited awareness in this framework. Szalay (2005) considers a delegation problem where the agent has costly access to valuable information. He shows that - in order to motivate information acquisition - it might be optimal to remove intermediate actions from the delegation set, which are by assumption optimal when the information is poor.

Furthermore, our work is related to a relatively small literature on contract theory and unawareness. The application of the concept of unawareness to contracting problems is still at its beginnings. In contrast to our setting, existing work considers contracting problems where contingent transfers are feasible and where the agent is unaware - either of possible actions (Von Thadden and Zhao, 2012 and 2014) or of possible states (Zhao, 2011; Filiz-Ozbay, 2012; Auster, 2013) - while the principal is fully aware.

This paper is also related to the literature on financial intermediation which sees banks as 'efficient brokers' who reduce transaction and information costs. The information based brokerage role of financial intermediaries has been studied by many authors starting from Leland and Pyle (1977), Ramakrishan and Thakor (1984), Diamond (1984) and Allen (1990). Empirical evidence on financial intermediarys' misbehaviour from the US retail investment market include Mullainathan et al. (2012) and Woodward and Hall (2012). Evidence from the UK includes Halan and Sane (2016). Recently, Guiso et al. (2017) and Foa' et al. (2017) use administrative data from the Italian Credit Register and Survey on Loan Interest Rates and document that Italian Banks provide distorted advice at the moment of counselling households between fixed and adjustable rate mortgages. Our empirical investigation is based on the theory we develop in this paper, we hence focus on different aspects of the bank-investor relationship from those emphasized in these works. In particular, we study the relationship between the richness and extremity of the menu of products offered by the financial intermediary and the knowledge of the investor about financial products.

Finally, our paper is related to the literature on financial literacy. We already mentioned the work by Guiso and Jappelli (2005). Virtually all other works study the implications of financial literacy on portfolio diversification or on market participation (e.g., van Rooij et al. (2011) and Guiso and Jappelli (2008)) of the investors. We analyse the behaviour of banks when facing investors of different levels of financial literacy.

The paper is organized as follows. The next section presents the delegation model; in Section 3 we characterize the monopolistic solution when the intermediary can perfectly discriminate among investors; in Section 4 we study the effect of competition between financial intermediaries, heterogeneity of investors, and how awareness changes with the aggregate state of the economy. Section 5 is devoted to the empirical analysis and Section 6 concludes with a few policy implications.

2 Environment

Although we present an abstract model of delegation, for concreteness of the exposition, we retain the financial market terminology which is associated to our leading application.

There is an investor (she) who acts as the principal and a financial intermediary (he) who acts as the agent. The intermediary has access to a set of investment projects $Y^A = [y_{min}, y_{max}]$, the return to which depends on the state of the world. Let $\Theta = [0, 1]$ be the set of states and let $F(\theta)$ denote the cumulative distribution function on Θ, assumed to be twice differentiable on the support.[4] Both the investor and the intermediary have von-Neumann-Morgenstern utility functions that take the quadratic form

$$u(y, \theta) = -(y - \theta)^2 \quad \text{and} \quad v(y, \theta) = -(y - (\theta - \beta))^2.$$

[4] For $\theta = 0$ and $\theta = 1$, this condition holds for, respectively, the right and left derivative.

The intermediary's preferred policy is $y = \theta$, while the investor's preferred policy is $y = \theta - \beta$. We assume $\beta > 0$, hence the intermediary has an upward bias of size β.[5] We can, for instance, interpret low values of y in $[y_{min}, y_{max}]$ as relatively safe investments and high values of y as relatively risky ones.

As in the canonical delegation problem, we assume that the intermediary is informed about the state of the world θ, while the investor is not. We rule out monetary transfers and assume that the intermediary's participation constraint is always satisfied. The contracting problem of the investor then reduces to the decision over the set of projects from which the intermediary can choose.[6]

In contrast to the standard model of optimal delegation, we assume that the investor is not aware of all investment opportunities in Y^A but only of a closed subset $Y^P \subseteq Y^A$. Hence, unawareness in our framework does not take the form of unforeseen contingencies but concerns the set of available actions: while the investor knows Θ, she has an incomplete understanding of the set of investment options that are out there.[7] We assume that the investor's unawareness restricts the language with which she can write a contract. In particular, we make the assumption that the investor can only include projects into the contract that she can name explicitly. This implies that her delegation set must be a subset of her awareness set.[8] Thus, the larger the investor's awareness set is, the richer is the set of contracts she can write.

In contrast to the investor, the intermediary is fully aware. Before the investor makes her delegation choice and the intermediary observes the state of the world,[9] the intermediary

[5]We can interpret β itself as the result of a contracting problem that generates β as the minimal level of conflict of interest between the investor and the intermediary.

[6]Formally, the investor commits to a mechanism that specifies the project which will be implemented as a function of the intermediary's message. Alonso and Matouschek (2008) show that this contracting problem is equivalent to delegating a set of projects $D \subseteq Y^A$ from which the investor can choose freely after observing the state of the world. Their argument continues to hold in our setting.

[7]Karni and Vierø (2015 and 2017) formalize this idea in a decision theoretic model that not only allows for unawareness of contingencies and outcomes but also of acts.

[8]Another option would be to specify those investment projects in the contract that the intermediary is not allowed to take. It can be shown that in this case the intermediary will never have incentives to reveal any projects to the investor.

[9]If the latter assumption is dropped, signalling becomes an issue and multiple equilibria will typically exist. Note, however, that the equilibrium we characterize continues to exist, even when the intermediary

can make the investor aware of additional projects. The investor fully understands the investment opportunities that are revealed to her and accordingly updates her awareness to the union of whatever she knew initially and what the intermediary reveals. Given her updated awareness, the investor determines a delegation set. Finally the intermediary learns the state of the world and chooses a project from those permitted by the investor. The timing of the game can be summarized as follows:

1. The investor's initial awareness Y^P is realized and observed by all parties.
2. The intermediary reveals a set of projects $X \subseteq Y^A$ and the investor updates her awareness to $Y \equiv Y^P \cup X$.
3. Given Y, the investor chooses a delegation set $D \in \mathcal{D}(Y)$, where $\mathcal{D}(Y)$ is the collection of closed subsets of Y.[10]
4. The intermediary observes the state of world θ and chooses an action from set D.
5. Payoffs are realized.

The assumption that the investor's awareness imposess restrictions on the delegation sets that are available to her will play an important role in the analysis that follows. One might argue that, even if the investor is aware of a strict subset of Y^A, she could still understand that Y^A is an interval. The investor might then attempt to include projects outside her awareness, maybe through a description of the properties of such projects. However, as we argue in the discussion below, our analysis applies when Y^A is an arbitrary subset of $\mathbb{R}$, e.g. a finite set, so that *a priori* there is no specific structure of Y^A that might be commonly known.

At this stage we do not need to make any explicit assumption on whether or not the investor is aware of her unawareness. The investor might take the world at face value or she might understand that there exist investment projects outside her awareness. Since she cannot include such projects in the delegation set, awareness of their possible existence nei-

learns θ before revealing additional projects to the investor.

[10] As discussed in Alonso and Matouscheck, the restriction to closed sets is without loss of generality.

ther affects her expected payoff nor optimization problem.[11] As well, within the constraints of her awareness, the investor is perfectly rational: she anticipates correctly the expected payoff associated to each feasible delegation set and will not be surprised ex-post. Using the formalization of Heifetz et al. (2013), the Online Appendix shows how our framework can be described formally as a generalized extensive form game with unawareness of actions.

Remark: *There is an alternative reading of our model that does not involve unawareness. Instead, we can think of a situation where the intermediary, rather than just advising the investor, actually provides access to the different investment options, e.g. by finding a counterparty. The intermediary thus decides on the set of investment projects he makes available to the investor and, as before, the investor delegates a subset of those investment options to the intermediary. By deciding on which investments to make accessible, the intermediary is given commitment power not to implement certain projects. Since such commitment limits the investor's choice over available contracts, we are ultimately faced with a game of double delegation between intermediary and investor. Under this alternative reading, the predictions are more sensitive to the timing of the game and some of our results in the competition part will change.*

3 Equilibrium Analysis

We will now proceed with the analysis of the awareness and delegation sets that obtain in equilibrium. We will start our analysis by first describing the benchmark case of full awareness and then turn to the subject of our interest: the case of partial awareness. Before entering the equilibrium analysis, it is useful to mention that optimal awareness sets and optimal delegation sets will typically not be unique since different awareness sets may induce the same delegation set and different delegation sets may induce the same implemented actions for each state of the world. In what follows, we will assume that, if the investor

[11] In a dynamic environment, where the investor has ways to expand her awareness set (for example, by using a costly technology or by sampling alternative intermediaries), the initial awareness of being partially aware and the confidence about the value of new information might matter (Karni and Vierø, 2017).

is indifferent between two delegation sets D and D' such that $D' \subset D$, she chooses the larger set D. Similarly, if the intermediary is indifferent between two revelation strategies that yield awareness sets Y and Y' such that $Y' \subset Y$, we will assume that he expands the investor's awareness to Y. That is, we will consider the sets that yield maximal awareness and maximal discretion.

Throughout the analysis, we will adopt a couple of regularity conditions on the distribution that are common in the delegation literature. Furthermore, we will assume that in each state of the world both the investor's and the intermediary's ideal projects are available.[12]

Assumption 1. *$f'(\theta)\beta + f(\theta) > 0$ for all $\theta \in (0,1)$; and $\mathbb{E}[\theta - \beta] > 0$.*[13]

Assumption 2. *$y_{min} < -\beta$ and $y_{max} > 1$.*

3.1 Full Awareness

For the specification $Y^P = Y^A$, the existing literature shows that if the density function $f(\theta) \equiv F'(\theta)$ satisfies the first regularity condition in Assumption 1, the optimal delegation set is an interval of the form $[y_{min}, \hat{y}]$ (Martimort and Semenov, 2006, and Alonso and Matouschek, 2008). The second condition in Assumption 1 guarantees that $\hat{y} > 0$ and solves:

$$\hat{y} = \mathbb{E}[\theta - \beta | \theta \geq \hat{y}]. \tag{1}$$

Otherwise, the optimal delegation set is $[y_{min}, \mathbb{E}[\theta - \beta]]$. In this case however, the intermediary will choose the upper bound of the set for all θ, so it is effectively the singleton $\{\mathbb{E}[\theta - \beta]\}$. Hence, delegation is valuable if and only if $\mathbb{E}[\theta - \beta] > 0$. In that case, the intermediary chooses his preferred project $y = \theta$ for all $\theta < \hat{y}$ and the project $\hat{y}$ in all remaining states. Moreover, under the same assumptions, we can show that $\hat{y}$ decreases with the bias β.

Proposition 1. *Under Assumptions 1 and 2 the maximizing solution to the optimal delegation problem with full awareness is an interval of the form $[y_{min}, \hat{y}]$ where the (unique) upper*

[12] The sole purpose of the latter assumption is to reduce the number of cases we need to distinguish.
[13] All expectations are taken with respect to F.

bound $\hat{y} \in (0,1)$ *solves equation* (1).

Moreover, under the same assumptions, if we let $\hat{y}(\beta)$ *be the cap in the optimal delegation set when principal's preferences parameter is* $\beta \in (0, \mathbb{E}[\theta])$ *then* $\hat{y}(\cdot)$ *is decreasing and continuously differentiable.*

Proof. See Appendix A.1. □

To gain some intuition for why the optimal delegation set takes this form, it is useful to describe the investor's tradeoff in more detail. We can first explain why the optimal delegation set does not have gaps. For a graphical illustration see Figure 7 in Appendix A.1. Consider two projects y_1 and y_2 with $y_1 < y_2$. If all projects in the interval $[y_1, y_2]$ belong to the delegation set and the realized state θ falls into that interval, the intermediary chooses his ideal project $y = \theta$. On the other hand, if the investor excludes projects (y_1, y_2) from the delegation set, the intermediary cannot take his preferred project but chooses the one closest to his bliss point. Hence, in states below the midpoint $\frac{y_1+y_2}{2}$ the intermediary chooses y_1, while in states above the midpoint he chooses y_2. Given that the investor's ideal project lies strictly below the ideal project of the intermediary, this implies that in states below $\frac{y_1+y_2}{2}$, the implemented project moves closer to the investor's bliss point, whereas in states above $\frac{y_1+y_2}{2}$ it moves further away. Since the cost of moving away from the bliss point is convexly increasing in the distance, the investor's loss outweighs the gain, as long as the probability weight attached to the states below $\frac{y_1+y_2}{2}$ is not too large. The first condition in Assumption 1 assures that this is indeed the case.

The investor's problem then reduces to finding the optimal upper and lower bound of the delegation interval. Since the intermediary is upward biased it is never optimal to reduce the intermediary's flexibility from below, so the optimal lower bound is y_{min}. The optimal upper bound is determined by condition (1). Notice that, conditional on the state being greater than y, the investor's preferred constant project is $\mathbb{E}[\theta - \beta | \theta > y]$. Condition (1) says that the optimal threshold $\hat{y}$ is such that the project the intermediary implements in all states above the threshold is exactly the investor's constant preferred project in those states.

3.2 Partial Awareness: Main Result

Our main result shows that, maintaining the regularity condition on the state distribution, it is strictly optimal for the intermediary to leave the investor partially unaware if and only if the investor is initially unaware of the project at the optimal threshold under full awareness, $\hat{y}$. In that case, the intermediary optimally reveals projects at the extremes but leaves the investor unaware of intermediate projects.[14]

Proposition 2. *Let Assumptions 1 and 2 be satisfied.*

- *If $\hat{y} \in Y^P$, the investor becomes fully aware and the optimal delegation set is $[y_{min}, \hat{y}]$.*
- *If $\hat{y} \notin Y^P$, the investor remains unaware of projects in $(\hat{y} - \Delta, \hat{y} + \Delta)$ for some $\Delta > 0$ and the optimal delegation set is $[y_{min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}$.*

Proposition 2 shows that whether the investor is made aware of all projects by the intermediary is determined only by her awareness of $\hat{y}$, the optimal cap under full awareness. If she is unaware of $\hat{y}$, the intermediary optimally leaves the investor unaware of an interval of projects around $\hat{y}$. As we will show, this makes it optimal for the investor to choose a delegation set that includes a project to the right of $\hat{y}$. By leaving the investor unaware of intermediate projects, the intermediary thus incentivizes the investor to permit investment projects that the intermediary is biased towards and that would be precluded under full awareness. As a result, the equilibrium delegation set is no longer an interval, illustrated in Figure 1.

The statement of the proposition will be proven in a series of lemmas in the remainder of this section. We proceed recursively by first considering the investor's delegation choice for a given awareness set Y. With the solution to this problem, we then turn to the intermediary's problem of choosing the optimal awareness set.

[14] In order to avoid notational complications, in what follows, we assume y_{min} is 'small enough'.

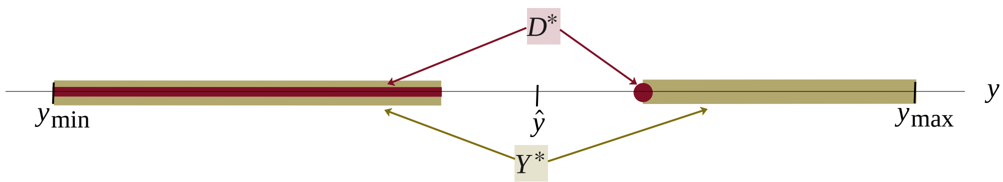

Figure 1: Equilibrium awareness and delegation. The yellow area represents a typical example of equilibrium awareness set Y^* when the investor has limited awareness. The red subset represents the resulting optimal delegation set. When $\hat{y} \notin Y^P$, the investor will be kept unaware of an interval of projects around $\hat{y}$, the cap in the optimal delegation set under full awareness. This way the intermediary will be allowed to choose the project represented by the red bullet to the right of $\hat{y}$, a project that would be excluded from the delegation set under full awareness.

3.3 Delegation Choice

Let $D^*(Y)$ denote the optimal delegation set when the awareness set is Y. Alonso and Matouschek (2008) derive conditions under which, in the benchmark case of full awareness, the optimal delegation set is an interval and therefore has no gaps. They show that, provided these conditions are satisfied, whenever the investor includes two distinct projects in the delegation set, it is strictly optimal to include all projects that lie in between. As we show in the Appendix, their argument perfectly generalizes to generic sets Y that may be non-connected. In our setting Alonso and Matouschek's (2008) conditions correspond to Assumption 1. We thus obtain the following result.

Lemma 3 (Alonso and Matouschek, 2008)**.** *Let Assumption 1 be satisfied and consider* $y_1, y_2 \in Y$ *with* $y_1 < y_2$*. If* $y_1, y_2 \in D^*(Y)$*, then all* $y \in (Y \cap (y_1, y_2))$ *belong to* $D^*(Y)$*.*

Proof. See Appendix A.2. □

Lemma 3 implies that the optimal delegation set $D^*(Y)$ has no "holes" with respect to Y. With this, we again only need to find the optimal lower and the upper bound of the delegation set. The following lemma shows that, as in the case of full awareness, the investor never finds it optimal to restrict the intermediary's choice from below.

Lemma 4. *The optimal delegation set satisfies* $\min D^*(Y) = \min Y$*.*

Proof. See Appendix A.2. □

The intuition for Lemma 4 is simple. Since the intermediary is biased upwards, whenever he prefers $\min Y$ over some other project in the delegation set, so does the investor. Thus, in those states where the intermediary chooses $\min Y$, the investor strictly prefers $\min Y$ over any other project the intermediary can select. As a result, the investor optimally includes $\min Y$ into the delegation set.

We turn next to the optimal upper bound of the delegation set. Here we can show that - thanks to the symmetry properties of the quadratic functions - the largest element of Y included in the delegation set is the project closest to the optimal upper bound under full awareness, $\hat{y}$. This project may be smaller or greater than $\hat{y}$.

Lemma 5. *Let Assumption 1 be satisfied. The optimal delegation set is such that*

$$\max D^*(Y) = \arg\min_{y \in Y} |y - \hat{y}|.$$

Proof. See Appendix A.2. □

The fact that the optimal upper bound is the project in Y that is closest to $\hat{y}$ has two implications: first, the optimal delegation set includes all projects belonging to Y that are weakly smaller that $\hat{y}$; second, it includes at most one project strictly greater than $\hat{y}$. The optimal delegation set under partial awareness can be seen as the closest approximation of $[y_{min}, \hat{y}]$, i.e. the optimal interval under full awareness, that is available to the investor given her restricted awareness. This approximation includes an element $y > \hat{y}$ if and only if y is closer to $\hat{y}$ than any element of Y smaller than $\hat{y}$. This is illustrated in Figure 2.

3.4 Awareness Choice

We can now turn our attention to the intermediary's optimal strategy of expanding the investor's awareness. As a first observation, notice that if the investor is aware of the threshold project $\hat{y}$, the intermediary optimally reveals all other projects. Since there is no

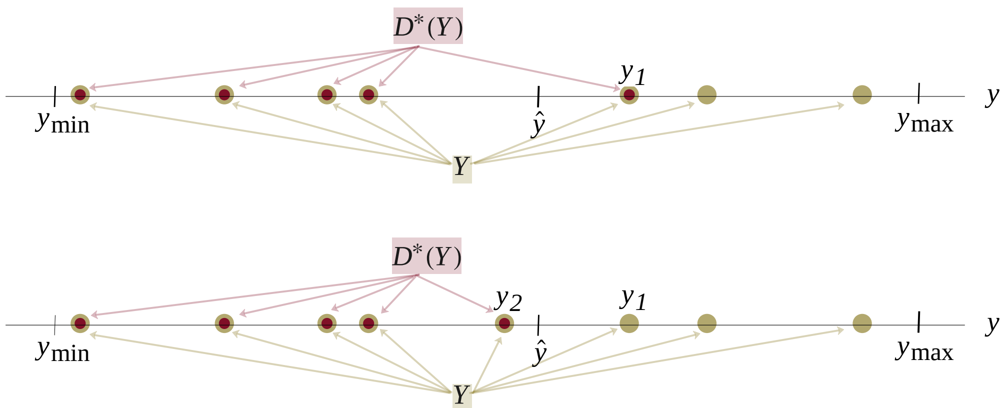

Figure 2: Optimal delegation set $D^*(Y)$. The figures represent two examples of the investor's awareness set Y. In both figures, the yellow bullets represent the set Y while the red bullets represent the resulting optimal delegation set $D^*(Y)$. In the upper figure, the investor includes project y_1 in the delegation set as it is the closest project to $\hat{y}$. In the lower figure, the investor is aware of project y_2 as well and, for this reason, she excludes project y_1 from $D^*(Y)$. Differently put, the awareness of project y_2 by the investor 'crowds out' project y_1 from the resulting delegation set.

project closer to $\hat{y}$ than $\hat{y}$ itself, the upper bound of the optimal delegation set will always be $\hat{y}$. Disclosing projects above $\hat{y}$ is thereby irrelevant; the investor will never allow the intermediary to implement any of them. On the other hand, revealing projects below the threshold $\hat{y}$ is strictly optimal since they will be included in the optimal delegation set, therefore expanding the intermediary's choice.

Starting now from an arbitrary set Y^P, the above argument implies that the optimal awareness set Y^* is such that the upper bound of the corresponding delegation set $D^*(Y^*)$ is at least $\hat{y}$. Moreover, the only reason for the intermediary to leave the investor unaware of certain projects is to induce the investor to permit some project strictly greater $\hat{y}$. By Lemma 5 this is optimal for the investor if and only if the investor is not aware of any project closer to $\hat{y}$. Letting $\hat{y} + \Delta, \Delta \geq 0$ denote the upper bound of the induced delegation set, we thus require $(\hat{y} - \Delta, \hat{y} + \Delta) \cap Y = \emptyset$. At the same time, revealing projects below $\hat{y} - \Delta$ and above $\hat{y} + \Delta$ either does not affect the induced delegation set or strictly expands it. It follows that the optimal awareness set is of the form

$$Y^* = [y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}],$$

with the corresponding delegation set

$$D^*(Y^*) = [y_{min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}.$$

The intermediary is thus permitted to choose from an interval of projects strictly to the left of the full awareness threshold $\hat{y}$ and one project to the right. Given such delegation set, the intermediary's optimal policy is as follows. In states below $\hat{y} - \Delta$ the intermediary uses his flexibility and implements his preferred project $y = \theta$. In states above $\hat{y} - \Delta$ the preferred project is not available, so the intermediary chooses the one closest to his bliss point. For states in the interval $(\hat{y} - \Delta, \hat{y})$ this is the project $\{\hat{y} - \Delta\}$, for the remaining states it is $\{\hat{y} + \Delta\}$. The intermediary's optimal policy can thus be sumarized by

$$y^*(\theta; \Delta) = \begin{cases} \theta & \text{if} \quad \theta \leq \hat{y} - \Delta \\ \hat{y} - \Delta & \text{if} \quad \hat{y} - \Delta < \theta < \hat{y} \\ \hat{y} + \Delta & \text{if} \quad \theta \geq \hat{y}. \end{cases}$$

Taken together, the previous analysis provides us with a very simple description of the class of delegation and awareness sets that are candidates for an equilibrium in our environment: when deciding which projects to reveal to the investor, the intermediary implicitly chooses an awareness gap, parametrized by Δ.

To complete the proof of Proposition 2 it remains to show that whenever a gap is feasible, it is also optimal. We can find the optimal awareness gap by considering the intermediary's reduced form problem of choosing Δ. The feasible values of Δ are determined by the initial level of awareness of the investor Y^P. In particular, the implementable values of Δ are weakly smaller than $\bar{\Delta}(Y^P) := \min_{y \in Y^P} |y - \hat{y}|$, the distance between the project in the investor's awareness closest to $\hat{y}$ and $\hat{y}$. For each Δ in that set, the intermediary then anticipates the investor's optimal delegation choice and his own optimal policy. Substituting $y^*(\theta; \Delta)$ into the intermediary's expected payoff, his optimization problem amounts to

$$\max_{\Delta\in[0,\bar{\Delta}(Y^P)]} -\int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y}-\Delta-\theta)^2 dF(\theta) - \int_{\hat{y}}^{1} (\hat{y}+\Delta-\theta)^2 dF(\theta). \tag{2}$$

The following proposition characterizes the solution of this problem.

Proposition 6. *Let Assumptions 1 and 2 be satisfied. The solution of problem* (2) *is given by Min*$[\bar{\Delta}(Y^P), \Delta^*]$*, where* $\Delta^* > 0$ *solves*

$$\int_{\hat{y}+\Delta^*}^{1} [\theta-(y+\Delta^*)]\, dF(\theta) = \int_{\hat{y}-\Delta^*}^{\hat{y}} [\theta-(y-\Delta^*)]\, dF(\theta) - \int_{\hat{y}}^{\hat{y}+\Delta^*} [\theta-(y+\Delta^*)]\, dF(\theta). \tag{3}$$

Proof. See Appendix A.5. □

The proof of Proposition 6 shows that the intermediary's payoff as a function of Δ is strictly concave and attains its maximum at Δ^*, as determined by (3). In (3), the left hand side represents the gain from increasing Δ while the right hand side represents the cost of such change. For a graphical illustration see Figure 8 in Appendix A.5. From $\hat{y}+\Delta^*$ the intermediary gains as the new pooling project is uniformly closer to his ideal point. The marginal cost of increasing Δ is the utility loss in the states $[\hat{y}-\Delta^*, \hat{y}+\Delta^*]$, where the intermediary moves away from his ideal action.

The proposition states that the unconstrained solution Δ^* is strictly positive. This can be easily understood by considering the net effect of increasing the gap at $\Delta = 0$. At $\Delta = 0$, the right hand side equals zero: the marginal cost of moving away from the bliss point at the bliss point is zero. The left had side instead clearly takes a positive value for each Δ. At all these states, he cap $\hat{y}+\Delta$ forces the intermediary to take an action that is too low from his point of view. By increasing the cap he can increase the implemented action in these states, thereby moving closer to his bliss point.

Proposition 7. *Let* $\Delta^*(\beta)$ *be the unrestricted solution to problem* (2) *as in Proposition 6 when principal's preferences parameter is* $\beta \geq 0$*. Then* $\Delta^*(\cdot)$ *is an increasing function.*

Proof. See Appendix A.6. □

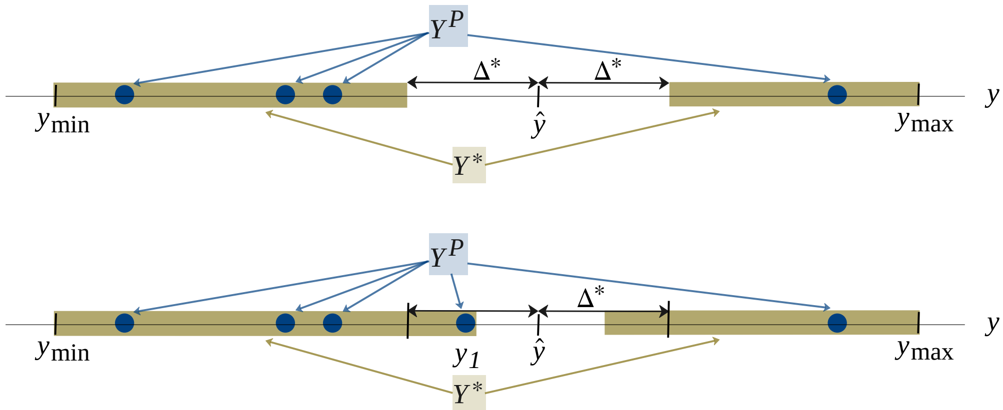

Figure 3: Optimal awareness set Y^*. The figures represent two examples of the investor's initial awareness set Y^P and associated awareness sets $Y^* = Y^P \cup X^*$ after including the information X^* received from the intermediary. In both figures, the blue bullets represent the set Y^P while the yellow set represents the resulting optimal awareness set Y^*. In the upper figure, the intermediary keeps the investor unaware of the interval $(\hat{y} - \Delta^*, \hat{y} + \Delta^*)$. In the lower figure, the investor is also aware of project y_1 and, for this reason, the intermediary finds it optimal to increase the investor's awareness.

The proposition shows an intuitive result: that the larger the divergence between the investor's and the intermediary's preferred investment is, the more investment projects the intermediary wants to hide from the investor. The solution Δ^* is implemented whenever the investor's initial awareness does not constrain the intermediary in his choice of the gap. If, however, the investor is aware of some project in the interval $(\hat{y} - \Delta^*, \hat{y} + \Delta^*)$, the intermediary's optimal strategy is to simply choose the largest feasible gap, as shown in Figure 3. The following example illustrates the findings for the uniform distribution.

Example: *Suppose $f(\theta) = 1$. The optimal threshold in the benchmark case of full awareness is given by $\hat{y} = 1 - 2\beta$ and the interior solution for the optimal awareness set, characterized by condition (3), is $\Delta^* = 2\beta\left(\sqrt{2} - 1\right)$. If $2\beta\left(\sqrt{2} - 1\right) \le \bar{\Delta}(Y^P)$, the intermediary leaves the investor unaware of all projects in the interval $\left(1 - 2\beta\sqrt{2}, 1 + 2\beta\left(\sqrt{2} - 2\right)\right)$. The resulting equilibrium awareness and delegation sets are then given by*

$$\begin{aligned} Y^* &= [y_{min}, 1 - 2\beta\sqrt{2}] \cup [1 + 2\beta\left(\sqrt{2} - 2\right), y_{max}], \\ D^*(Y^*) &= [y_{min}, 1 - 2\beta\sqrt{2}] \cup \{1 + 2\beta\left(\sqrt{2} - 2\right)\}. \end{aligned}$$

If $2\beta\left(\sqrt{2}-1\right) \leq \bar{\Delta}(Y^P)$ *is not satisfied, the intermediary is restricted by the investor's initial awareness and therefore leaves her unaware of all projects in the smaller interval* $\left(1-2\beta-\bar{\Delta}(Y^P), 1-2\beta+\bar{\Delta}(Y^P)\right)$.

One might wonder whether the investor, even if unaware of an interval $(\hat{y}-\Delta, \hat{y}+\Delta)$, could replicate investment projects in that interval by diversifying her investment across different projects within her awareness. For instance, if y captures the riskiness of the different investment options, an investor who is aware of $\hat{y}-\Delta$ and $\hat{y}+\Delta$ might generate intermediate levels of risk by investing parts of her savings in $\hat{y}-\Delta$ and others in $\hat{y}+\Delta$. Notice, however, that the Markowitz frontier describing the efficient combinations in the risk-return spectrum is typically viewed to be concave, implying that the expected return associated to the investment generated as a convex combination of $\hat{y}+\Delta$ and $\hat{y}-\Delta$ will fall below the efficient frontier. Moreover, portfolios of projects are often "lumpy," which further imposes restrictions on investors' abilities to diversify. For example, many securities and funds, in particular mutual funds, require a sizeable minimum investment. Finally, riskiness is just one of many interpretations applicable to our model. The value of y might, for instance, capture the term to maturity of the investment. Clearly, splitting up the investment into short and long term funds will not replicate an investment option with an intermediate maturity date. A similar argument can be made for liquidity or other characteristics of the assets (such as the skewness or higher moments in the return distribution). Thus, in most situations unawareness will impose important restrictions on the payoffs an investor can achieve.

3.5 Discussion

Set of feasible projects. So far we have assumed that the set of available projects is given by the interval $Y^A = [y_{min}, y_{max}]$. In some applications it may be more natural to assume that there is a finite set of feasible projects from which the investor can choose. In light of some of the previous lemmas, it is straight forward to extend our analysis to more general sets of projects. To see this, assume Y^A is an arbitrary closed subset of $\mathbb{R}$ and define

$\hat{y}(Y) \equiv \arg\min_{y \in Y} |y - \hat{y}|$ as the element of $Y \subseteq Y^A$ closest to $\hat{y}$. Our Lemmas (3-5) then imply that, given awareness Y, the optimal delegation set is

$$D^*(Y) = \{y \in Y : y \leq \hat{y}(Y)\}.$$

With regard to the optimal awareness set, it is easy to see that if the intermediary reveals some $y \in Y^A$, he also reveals all those projects that have a greater distance to $\hat{y}$ than y: their inclusion will weakly expand the intermediary's choice set. This implies that the optimal awareness set can again be described by a gap Δ and takes the form

$$Y^* = \{y \in Y^A : |y - \hat{y}| \geq \Delta\} \quad \text{with} \quad 0 \leq \Delta \leq \bar{\Delta}(Y^P),$$

where $\bar{\Delta}(Y^P) = \min_{y \in Y^P} |y - \hat{y}|$, as above. Whether or not the intermediary reveals all feasible projects to the investor depends on the particular form of Y^A and the investor's initial awareness Y^P. A necessary condition for partial unawareness is that there exists some $y \in Y^A$ such that $y > \hat{y}(Y^A)$ and $|y - \hat{y}| \leq \bar{\Delta}(Y^P)$. That is, there exists some project strictly greater than the optimal threshold under full awareness that is implementable given the constraints imposed by the investor's initial awareness. These conditions become sufficient if we add, for instance, the assumption that the distance $|y - \hat{y}|$ is weakly smaller than Δ^*, as determined by (3).

Quadratic loss preferences and constant bias. The utility functions we consider seem rather special.[15] It should be noted, however, that the main result of our model - the fact that the intermediary has an incentive to leave the investor unaware of a set of projects around the optimal threshold under full awareness - remains valid more generally: as long as the investor's and intermediary's preferences are represented by smooth, single-peaked utility functions that have the property that the ideal project is strictly monotonic in the realized state of the world, the intermediary's incentives to leave an awareness gap are much the same

[15] The optimal delegation literature provides conditions that make interval delegation optimal for a considerably larger class of environments. For the most general treatment see Amador and Bagwell (2013).

as in the baseline model. More specifically, imposing an appropriate regularity condition on the state distribution, if the intermediary is upward biased, the optimal delegation set under full awareness will again be an interval with some upper bound $\hat{y}$.[16] Since the investor cares about the intermediary's information, the intermediary can then find some awareness gap around $\hat{y}$ (not necessarily symmetric) such that the investor optimally permits a project greater than $\hat{y}$, provided that $\hat{y} \notin Y^P$. Under the assumption that the intermediary's utility function is differentiable, we can then replicate the argument following Proposition 6: given that the marginal cost of moving away from the bliss point at the bliss point is equal zero, the net benefit of introducing a marginal gap around $\hat{y}$ will be strictly positive.

To see more concretely how our results generalize to a larger set of models, suppose the intermediary's and investor's preferences are as follows:[17]

$$u(y,\theta) = y\theta - C(y) \quad \text{and} \quad v(y,\theta) = y(\theta-\beta) - C(y),$$

with $C(\cdot)$ strictly convex and twice differentiable. For $C(y) = \frac{1}{2}y^2$ we are back to the quadratic case. It is easy to show that - under the same Assumption 1 - the optimal delegation set is an interval of the form $[y_{min}, \hat{y}]$, where $\hat{y} = g(\hat{\theta})$, $\hat{\theta}$ solves $\hat{\theta} = \mathbb{E}[\theta - \beta | \theta \geq \hat{\theta}]$, and g is such that $C'(g(\theta)) = \theta$ for all θ. In this case, when the investor has limited awareness, the optimal unawareness interval of Lemma 5 might be non-symmetric around $\hat{y}$. Following the same line of proof of the Lemma in Appendix A.4, we can easily show that the class of sets of projects the investor is left unaware of can be described by the following intervals

$$[\hat{y}-\Delta_1, \hat{y}+\Delta_2], \qquad \text{where} \qquad \frac{C(\hat{y}+\Delta_2) - C(\hat{y}-\Delta_1)}{\Delta_1+\Delta_2} = C'(\hat{y}).$$

[16]For some sufficient conditions see Matouschek and Alonso (2008) and Amador and Bagwell (2013).

[17]Monotone increasing transformations of u and affine transformations of v can easily be allowed as well without any additional complication. State dependent bias can also be introduced with the preferences:

$$u(y,\theta) = y\theta - C(y) \quad \text{and} \quad v(y,\theta) = y(\theta - \beta(\theta)) - C(y),$$

with $\beta'(\theta) \in [0,1]$. In this case, the analog to the first condition in Assumption 1, which implies interval delegation, is

$$\beta(\theta)f'(\theta) + (1-\beta'(\theta))f(\theta) > 0.$$

The right had condition above delivers a unique Δ_1 for each Δ_2 and vice versa. The intermediary optimization problem can therefore be stated as a *function of only one variable* and the solution satisfies the analog to condition (3) in Proposition 6.

Contingent transfers. The optimal delegation problem differs from the usual contract design problem in that message-contingent transfers are not feasible. We conjecture that, even if they are in fact available, unawareness would still matter. Morgan and Krishna (2008) analyze our setting with full awareness for the case where the investor can offer message-contingent transfers t. They assume that the intermediary is protected by limited liability so that $t \geq 0$ and that preferences of both contracting parties are quasi-linear. In this case the intermediary's and investor's payoff function are, respectively, given by[18]

$$u(y, \theta) = -(y - \theta)^2 + t \quad \text{and} \quad v(y, \theta) = -(y - (\theta - \beta))^2 - t.$$

Morgan and Krishna (2008) show that under the optimal contract the implemented project y is non-decreasing in the realized state θ and constant on some interval $[z, 1]$. As before, z can be interpreted as a cap above which no project is permitted. Transfers, on the other hand, are non-increasing in θ and equal to zero on the interval $[z, 1]$ (see Morgan and Krishna (2008), Proposition 1). For the case when f is the uniform distribution they show that, provided the bias is not too large, there is an interval of low states, $[0, z')$, bounded away from z, where the investor pays a positive transfer and the implemented project lies strictly between the investor's and the intermediary's preferred project. As θ increases, the implemented project in this region increasingly tilts in favor of the intermediary, until it reaches the intermediary's ideal point and the transfer is zero. In the region $[z', z]$ the intermediary simply chooses his preferred project without receiving any transfers, while in the region $[z, 1]$ the intermediary optimally selects z. Apart from the the first region, the optimal contract thus replicates the allocation we obtain in our framework under full awareness. This in turn implies that the motivation for the agent to keep an awareness gap, in this case around z, is still present: by

[18] More precisely, Morgan and Krishna (2008) assume that the investor's bliss point is θ and the intermediary's bliss point is $\theta + \beta$. In what follows, we adapt their results accordingly.

leaving the investor unaware of an interval of projects around z, the intermediary induces the investor to permit a project strictly greater than z, exactly as in the case when message-contingent transfers are not feasible.

4 Market Competition and Investors Heterogeneity

The previous section characterized the equilibrium in an environment where the intermediary is a monopolist and, by implication, fully determines the investor's awareness. In reality, investors can seek consult from multiple financial professionals, possibly to expand their choice between different investment options. To capture the interaction between multiple intermediaries, we adopt a simple model of imperfect competition that is based on the work of Burdett and Judd (1983) and has been recently employed by Lester et al. (2017). Considering a model of imperfect competition accounts for some important features of the financial market, especially over-the-counter trading. It further allows us to consider the effect of the degree of competition on the awareness of investors and the composition of financial products traded in the market. When considering the case of heterogeneous investors, an added bonus of this model of competition is that financial intermediaries post their menus based on the ex-ante composition of investors, that is, the equilibrium we derive below is robust to allowing for the awareness set Y^p to be private information to the investor.

Environment: Following the approach of Lester et al. (2017), we assume that there are two intermediaries and a unit measure of investors.[19] Intermediaries have no capacity constraints and can therefore contract with many investors. There is a friction in that investors do not necessarily have access to both intermediaries. In particular, a fraction of investors is matched with one intermediary, while the remaining investors are matched with both. We refer to investors that have access to only one intermediary as captive. Whether an investor is captive or non-captive is not observable to the intermediaries. Instead, from the viewpoint of an intermediary, conditional on meeting a particular investor, the investor is non-captive

[19] As Lester et al. (2017) show, the restriction to two intermediaries can be easily relaxed.

with some probability, denoted by π. The parameter π can then be viewed as a measure of competitiveness in the market: if $\pi = 0$ we are back in the monopoly case; if $\pi = 1$ intermediaries engage in Bertrand competition.

To simplify the analysis, we first assume that investors are initially unaware of all projects.[20] Upon meeting an investor, intermediaries disclose a set of investment projects, as before. If an investor meets with two intermediaries, her updated awareness set is the union of the projects that are revealed by either intermediary. The investor then decides to which of the intermediaries to delegate her investment. In principle, after updating her awareness, the investor is indifferent between both intermediaries. To make competition matter, we assume that the investor chooses the intermediary that reveals more investment opportunities. More specifically, if the set revealed by the first intermediary is a strict subset of the set revealed by the second, the investor chooses the second, and viceversa. If both intermediaries choose the same awareness set or if the awareness sets cannot be ordered, the investor chooses either intermediary with equal probability. Once an investor selects an intermediary, the interactions unfolds exactly as in the monopoly case. For a given investor, the timing can be summarized as follows:

1. The investor privately observes whether she meets with one or two intermediaries.
2. Each intermediary reveals a set of projects and the investor updates her awareness to the union of both sets.
3. The investor chooses an intermediary and a delegation set.
4. The selected intermediary observes the state of world and chooses an action from the delegation set.
5. Payoffs are realized.

The assumption that an investor delegates to the intermediary that reveals more investment opportunities implies that intermediaries optimally choose awareness sets of the form $[y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$, $\Delta \geq 0$.[21] Intermediaries, therefore, compete over awareness gaps,

[20] What matters is wether the investor is aware of projects in the maximal unawareness set $(\hat{y} - \Delta^*, \hat{y} - \Delta^*)$.
[21] For a formal argument see the proof of Proposition 8.

parameterized by Δ: a smaller value of Δ increases an intermediary's chances of attracting an investor.

Payoffs. An intermediary's expected payoff depends on the probability of being selected by an investor, which in turn depends on the strategy of the other intermediary. Define $H(\Delta)$ as the probability with which the other intermediary chooses an awareness gap smaller than Δ. Upon meeting an investor, the probability of the investor selecting the competitor is then equal to the product of the probability that the investor meets the other intermediary and $H(\Delta)$. Letting $U(\Delta)$ denote the expected payoff conditional on being selected by an investor with awareness parameterized by Δ and $\bar{U}$ denote the outside option, an intermediary's expected payoff is therefore given by

$$(1 - \pi H(\Delta))U(\Delta) + \pi H(\Delta)\overline{U}. \tag{4}$$

We can then use the intermediary's optimal policy $y^*(\theta; \Delta)$, as specified in the previous section, in order to define the conditional expected payoff $U(\Delta)$:

$$U(\Delta) \equiv -\frac{1}{2}\int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 f(\theta)\mathrm{d}\theta - \frac{1}{2}\int_{\hat{y}}^{1} (\hat{y} + \Delta - \theta)^2 f(\theta)\mathrm{d}\theta. \tag{5}$$

From the analysis in Section 3 we know that $U(\Delta)$ is strictly concave and attains its maximum at Δ^*, as determined by (3). We assume $U(\Delta^*) > \bar{U}$ so that intermediation is profitable, at least for some values of Δ.

Equilibrium: A *symmetric* equilibrium in this environment is a cumulative distribution function $H^*(\cdot)$ such that intermediaries are indifferent between all elements in the support of H^* and weakly prefer those over any other values of Δ. The next proposition describes the equilibrium distribution H^* in our environment. While the appendix provides a complete characterization, we focus here on some key features of the equilibrium.

Proposition 8. *Let Assumptions 1 and 2 be satisfied and let $\underline{\pi}, \overline{\pi} \in [0, 1]$. There exists an equilibrium, characterized by H^*, with the following properties:*

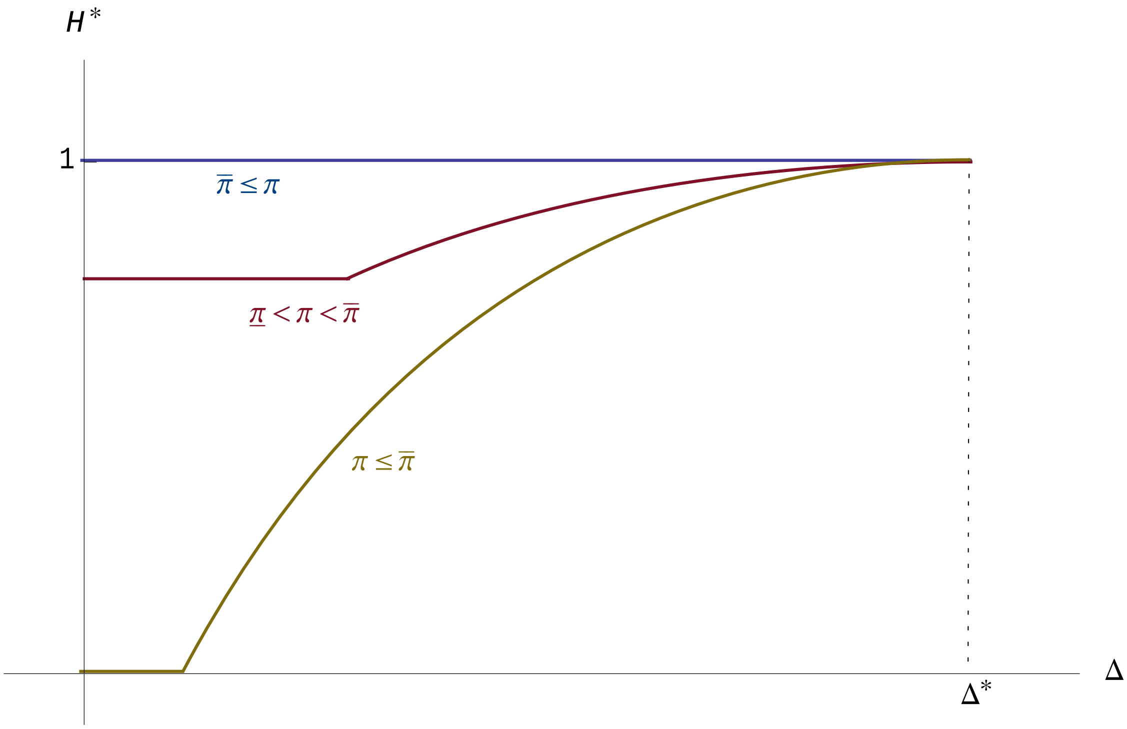

Figure 4: Equilibrium distribution H^* under imperfect competition. The figure displays three possible equilibrium distributions H^* over awareness gaps Δ assuming that the investor is initially not aware of any project ($Y^P = \emptyset$) and that $\overline{U} < U(0)$. As the probability of meeting a second intermediary π increases, competition gets tougher and the equilibrium distribution H^* becomes more concentrated towards small awareness gaps. For $\pi \geq \bar{\pi}$, market competition generates full awareness. If $\overline{U} > U(0)$, the support of H^* only includes strictly positive Δ for each π.

- *if* $\pi \leq \underline{\pi}$*, the support of* H^* *is* $[\Delta', \Delta^*]$ *for some* $\Delta' \geq 0$*;*
- *if* $\underline{\pi} < \pi \leq \overline{\pi}$*, the support of* H^* *is* $\{0\} \cup [\Delta', \Delta^*]$ *for some* $\Delta' > 0$*;*
- *if* $\overline{\pi} < \pi$*, the support of* H^* *is* $\{0\}$*.*

We have $\underline{\pi} = \overline{\pi} = 1$ *if* $U(0) \leq \overline{U}$ *and* $0 < \underline{\pi} < \overline{\pi} < 1$ *otherwise.*

Proof. See Appendix A.7. □

Proposition 8 shows that, if $U(0) > \overline{U}$ so that intermediation is profitable even when investors are fully aware, there are three parameter regions to be distinguished. If the degree of competition is sufficiently small, then intermediaries choose awareness gaps parameterized by values of Δ in the interval $[\Delta', \Delta^*]$. As we show in the proof of Proposition 8, the lower bound of this interval is strictly decreasing in the competition parameter π. At the threshold $\underline{\pi}$, we have $\Delta' = 0$, implying that intermediaries randomize over all values of Δ in the interval $[0, \Delta^*]$. When π increases further we observe polarization: there is a strictly positive probability that intermediaries disclose everything ($\Delta = 0$), while otherwise they leave investors unaware of a significant part of the available investment opportunities. The probability that intermediaries reveal everything increases in the parameter π, up to the point where this probability is one, which happens at the second threshold $\overline{\pi}$. For all values of π greater than this threshold, investors are made fully aware in equilibrium. Figure 4 depicts the equilibrium distribution for the different regimes of π. When $U(0) < \overline{U}$ only the first parameter region exists, that is, $\Delta = 0$ is never offered for any π.

Taken together, we find that competition promotes awareness. In fact we can show that equilibrium awareness is stochastically monotonic in π.

Corollary 9. *Let Assumption 1 and 2 be satisfies and let $0 \leq \pi < \pi' \leq 1$. The equilibrium distribution $H^*(\,\cdot\,; \pi)$ first-order stochastically dominates $H^*(\,\cdot\,; \pi')$.*

Proof. See Appendix A.8. □

In the proposed environment the disclosure of available investment opportunities is an instrument to compete for costumers. An interesting question is how the extent to which investors are left unaware of certain investment opportunities varies over the business cycle. In our framework the state of the economy might be captured by the profitability of investments: when the economy is doing well, financial market investments yield particularly high returns, some of which are appropriated by the financial intermediaries. In our model we thus interpret good times as an upward shift of $U(\Delta)$ relative to $\overline{U}$. In Corollary 10 we show that as the difference between $U(\Delta)$ and $\overline{U}$ increases, the equilibrium distribution H^* shifts towards smaller values of Δ, as illustrated in Figure 5. That is, when the gains from intermediation increase, investors become aware of more investment opportunities in equilibrium. Intuitively, when the value of attracting an investor becomes larger, competition for investors increases and this results in smaller awareness gaps. Viceversa, when times are bad

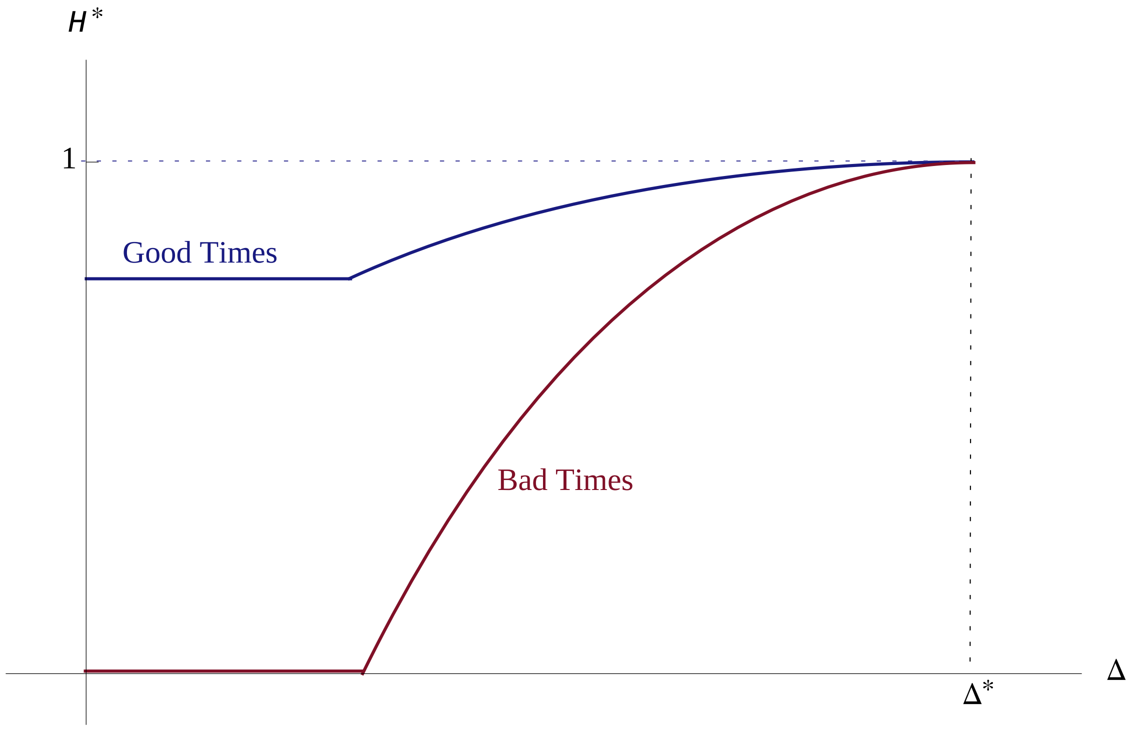

Figure 5: Equilibrium awareness in good and bad times. The figure displays two possible equilibrium distributions H^*, showing that awareness is procyclical. When the average profits per customer increase competition becomes tougher, increasing investors' awareness.

and gains from intermediation are small, intermediaries worry less about loosing investors to competitors and hence behave more predatory. Our model therefore suggests that in bad times we will observe more banks taking advantage of costumers by hiding certain investment opportunities than in good times.

Corollary 10. *Let Assumption 1 and 2 be satisfies and let* $\overline{U}' < \overline{U}$*. The equilibrium distribution* $H^*(\cdot\,;\overline{U}')$ *first-order stochastically dominates* $H^*(\cdot\,;\overline{U})$*.*

Proof. See Appendix A.9. □

4.1 Heterogenous Investors

Thus far we have assumed that investors are equally sophisticated, in particular, that they are all completely unaware.[22] In reality, investors vary widely in their financial literacy, which gives rise to the question of how intermediaries optimally act when they face heterogenous investors and whether the presence of more sophisticated investors is beneficial or detrimental to the welfare of the less sophisticated ones.

To address these questions in the simplest fashion, we assume that there are two types of investors, more sophisticated ones that are aware of all investment opportunities and less sophisticated ones that are aware of none. We denote the fraction of unaware investors by μ and assume that each investor is privately informed about her type. Upon meeting an investor, an intermediary is then confronted with two unknowns. He does not know whether the investor has access to the second intermediary and he does not know the investor's level of awareness. If the investor is aware, she delegates the optimal full awareness interval $[0, \hat{y}]$, no matter what the intermediary reveals. Nevertheless, she still rewards an intermediary for disclosure by choosing the one that reveals more. If the investor is unaware, everything remains as above.

As before, intermediaries compete over awareness gaps, parametrized by Δ. An intermediary's expected payoff as a function of Δ is now given by

$$(1 - \pi H(\Delta))[\mu U(\Delta) + (1 - \mu)U(0)] + \pi H(\Delta)\overline{U} \tag{6}$$

Following the steps of the equilibrium construction above, we can characterize the equilibrium for this environment. As we show in the Appendix, the equilibrium properties described in Proposition 8 continue to hold for the case when investors are heterogenous.[23] Of course, the equilibrium distribution H^* will now depend on μ. The following proposition shows that, provided intermediaries can make positive profits with aware investors, an increase in the

[22] The extension to the case where investors are homogenous but aware of some projects is straight forward. We just replace Δ^* by $\min\{\Delta^*, \bar{\Delta}(Y^P)\}$.

[23] For details see the proof of Proposition 11.

fraction of those investors leads intermediaries to disclose more investment opportunities in equilibrium.

Proposition 11. *Let Assumption 1 and 2 be satisfies. Assume* $\overline{U} \leq U(0)$ *and let* $0 \leq \mu < \mu' \leq 1$*. The equilibrium distribution* $H^*(\,\cdot\,;\mu)$ *is first-order stochastically dominated by* $H^*(\,\cdot\,;\mu')$*.*

Proof. See Appendix A.10. □

The result in Proposition 11 is intuitive. Whenever an intermediary meets an investor who is aware of all investment opportunities, revealing additional projects does not affect the delegation set the investor chooses but increases the probability with which the intermediary is selected. The assumption $\overline{U} \leq U(0)$ implies that intermediaries make weakly positive profits with fully aware investors, thus, conditional on meeting such investor, it is optimal to reveal everything. By implication, the larger the probability an intermediary attaches to that event is, the more attractive disclosing additional projects becomes. The presence of *more sophisticated investors in the market consequently leads to more disclosure* and thereby benefits the unaware ones.

Suppose now that intermediaries can make positive profits with unaware investors but not with those that are fully aware. If intermediaries can reject the latter investors, the equilibrium is as if they did not exist and so the previous analysis applies. There are, however, situations where it is reasonable to assume that intermediaries cannot avoid negative profits with some types of investors. For example, advising and setting up a contract may imply certain fixed costs. If the investor's type is initially unknown and if the expected profits with fully aware investors do not compensate these costs, intermediaries make losses with such investors. As long as these losses are compensated by the profits with other investors, intermediaries may still find it worthwhile to enter the market.

In our framework this situation is captured by the specification $U(0) < \bar{U} < \mu U(\Delta^*) + (1-\mu)U(0)$ and the assumption that intermediaries cannot reject any delegation sets. In contrast to the previous case, intermediaries are then no longer interested in attracting fully aware investors but would rather have them go to competitors. This brings about a new interesting equilibrium property.

Proposition 12. *Let Assumption 1 and 2 be satisfies. Assume* $U(0) < \overline{U} < \mu U(\Delta^*) + (1-\mu)U(0)$ *and let* $0 \leq \mu < \mu' \leq 1$*. The equilibrium distribution* $H^*(\,\cdot\,;\mu)$ *first-order stochastically dominates* $H^*(\,\cdot\,;\mu')$*.*

Proof. See Appendix A.10. □

Proposition 12 shows that *a larger share of sophisticated investors leads to more unawareness among the other investors.* Given $U(0) < \underline{U}$, revealing all investment opportunities cannot be optimal for intermediaries. Indeed, regardless of how large π is, there is no 'full awareness' equilibrium. Instead, intermediaries randomize across an interval of values of Δ bounded away from zero. The losses they make with sophisticates can thereby be compensated with the profits they make with unaware investors. The larger the share of sophisticated investors is, the larger this compensation has to be. Hence, as μ decreases, the equilibrium distribution shifts towards higher values of Δ. The presence of sophisticated investors in the market thus reduces awareness and, by implication, welfare of the unsophisticated ones. The feature that there is unawareness in equilibrium - no matter how intense competition is - with cross-subsidization towards sophisticated investors is reminiscent of the shrouding equilibrium in Gabaix and Laibson (2006). In their work firms hide costly add-ons, which in equilibrium will be purchased by naive customers only.[24]

[24]In contrast to our model, Gabaix and Laibson (2006) consider a market with perfect competition where firms compete over prices and customers have access to all firms.

5 Empirical Analysis

In this section, we aim at bringing some empirical support to the model of delegation with limited awareness using our main application: financial intermediation.

5.1 Data

The data is based on a 30-40 minutes survey we administered online to Italian retail investors. The survey enquires about their experience with the financial intermediary at the moment of taking the investment decision, which occurred between 2007 and 2017.[25] On top of demographics (such as wealth, income, sex, age, education, occupation, etc.) we elicited some proxies for the investor's cognitive abilities, tastes, and other behavioural factors. The survey also contains several questions regarding the knowledge of the investor about the financial sector and the products available in the market.

Some of the main descriptive statistics are summarised in Table 1. The first column reports the statistics for the full sample. The second column displays the same summary statistics of the data restricting the sample to investors that reported having chosen the financial intermediary because of its geographical proximity or because it was the institution where s/he usually conducted other financial transactions. We consider the latter as a more 'exogenous' choice of the intermediary. The last column displays the summary statistics for the sample restricted to investors that declared to have received important shocks (such as divorce, layoff, acquisition of new house, etc ..) that might have triggered the choice to invest or borrow in the first place. We indicate them as 'non-economic triggers' and the main aim here is to exclude investment decisions made mainly for speculative motives.[26] Most variables and statistics are self-explanatory. To construct the dummy variable 'sophistication' the respondent were asked to state wether a discount of 10% over a 600 euro TV was larger,

[25] An english translation of the complete survey is reported in the Online Appendix.

[26] Investors subject to 'non-economic triggers' are detected using question S1.10 (*'Did any of the following events happen in the 3-4 months prior to arranging or making your investment?'*), where we excludes from the sample all investors that either did not select any of the listed triggers or indicated triggers S1.10.07 (changed job) or S1.10.12 (changed account provider).

equal, of smaller than a 55 euro discount. The discrete variable ‘risk propensity’ is obtained from the respondent’s reported attitude towards taking risk in exchange for higher returns; the variable ’long term propensity’ measures the attitude towards investing a larger fraction of wealth in long term versus short term products. Both these variables take increasing numerical integer values starting from the value of 1 for the entries ’No Risk’ and ’Only Short-Term products’, respectively. We also asked the respondents how carefully they compiled the MiFiD form. Italian commercial banks are required to propose such questionnaire to potential investors in order to asses some of the client’s characteristics (such as his/her propensity to take risk) and, in principle, should ‘modulate’ the offer in line with the investor’s preferences. An accurately compiled MiFiD might also give important information to the intermediary about the level of knowledge of the investor. The only notable difference regards the distribution of the risk propensity for the ‘exogenous choice’ subsample. The full sample and the ‘non-economic triggers’ subsample have similar distributions of such index. Within the ‘exogenous choice’ subsample instead, the proportion of individuals classified into ‘high’ or ‘very high’ risk propensity is much larger compared to the other sample selections, while a much smaller fraction of individual declare to be unwilling to take any risk. As well, the individuals in the ‘exogenous choice’ subsample are on average wealthier and earn more compared to the other samples.

The first panel in Figure 9 in Appendix B reports the distribution of products acquired in our sample. The two most popular products in the sample are mortgages and deposit accounts, followed by personal loans and investment in stocks and shares. These 4 products alone, cover almost 60% of investors.

Dependent variable I: ‘Number of Products’ The first dependent variable we analyse provides a basic measure of the richness of the menu offered by the intermediaries. In question S1.3, the survey contains a question that elicit an ordered categorical variable on the number of products offered. The distribution of this variable is reported in the sixth entry of Table 1. It includes the following bins: 1 to 5; 5 to 20; 20 to 100; 100 and more.

Dependent variable II: 'Extremeness of the Offer' The variable that aims at measuring the extremeness in the menu of products offered by the intermediary to the investor measures extremeness as perceived by the investor. The variable ranges between 1 and 10 and is the linear aggregation of two discrete variables obtained from questions asking the respondent to indicate how much s/he agrees with the indicated statement. The two statements with associated potential answers are reported below.

I was offered very few products with an intermediate levels of risk and return; I would have liked to see more products of this kind.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
O	O	O	O	O

I believe that the intermediary or online interface offered me only "extreme" products: either standard/safe products or very risky/exotic products.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
O	O	O	O	O

Knowledge index At the beginning of Appendix B, we list the 17 dummy variables used to construct our index of investor's knowledge of available financial products. In the index, all dummies have equal weight. In the total sample, the index ranges between 0 and 16 with a mean of 5.98 and a standard deviation of 3.25. We divided the variables constituting our index into 5 main categories. In the first category - indicated in the appendix with (P) - we find dummies associated to the investor's 'perception' about his/her knowledge. The second set of variable are indicated with (U) and refer to the investor's self-reported 'understanding' of the financial products; while the variables indicated with (S) capture knowledge obtained from more intensive market 'search' activities. The last two block of variables refer to harder information. The block of variables indicated with (B) refers to the investor's 'background' relevant for financial decisions, while the dummies (T) are obtained by directly 'testing' the knowledge of the investor: they are based on multiple choice questions with only one correct answer out of four.

Table 1: **Main Descriptive Statistics**

Variable		Full Sample N = 1362	Exogenous Intermed. Choice N = 443	Non-Econ. Triggers N=698
Gender (S0.2)	Male	47.06	46.95	47.13
	Female	52.94	53.05	52.87
Education (Distribution %) (S2.3)	Elementary	0.22	0	0.14
	Middle School	6.09	4.97	5.87
	High School	51.25	51.69	49.00
	University	33.26	36.12	34.10
	Master/PhD	9.18	7.22	10.89
Sophistication (S2.12)	Naive	21.15	23.48	23.78
	Sophisticated	78.85	76.52	76.22
Risk Propensity (Distribution %) (S2.28)	No Risk	37.96	5.64	32.66
	Middle Risk	32.42	35.67	31.66
	High Risk	15.35	31.38	20.20
	Very High	6.61	17.83	8.60
	No Answer	7.64	9.48	6.88
Long Term Propensity (Distribution %) (S2.22)	Only Short-Term prod.	13.58	16.25	14.33
	Mostly Short-Term	29.00	30.93	29.94
	Half-Wealth Short-Term	25.26	23.25	26.79
	Mostly Long Term	11.89	12.87	10.74
	Only Long term	2.57	3.16	3.01
	No Answer	17.69	13.54	15.19
Number of Products Offered (Distribution %) (S1.3)	1-5 Products	92.44	93.45	91.69
	5-20 Products	5.87	5.19	6.59
	20-100 Products	0.66	0.45	1.00
	100+ Products	1.03	0.90	0.72
Financial Wealth (Distribution %) (S2.20)	No Wealth	21.29	18.74	20.06
	< 20,000 Euro	18.72	17.83	21.63
	20,000 - 50,000 Euro	16.81	17.16	17.19
	50,000 - 150,000 Euro	16.15	17.38	16.62
	150,000 - 300,000 Euro	8.44	10.16	9.31
	> 300,000 Euro	2.94	4.06	2.87
	No Answer	15.64	14.67	12.32
Year of Purchase (Distribution %) (S1.1a)	2017	19.16	23.93	16.05
	2016	12.70	15.35	12.46
	2015	14.54	15.35	16.33
	2014	13.80	12.64	13.75
	2013	7.20	6.32	7.31
	2012	8.88	8.35	9.89
	2011	3.96	2.03	4.30
	2010	4.41	2.48	4.73
	2009	3.89	3.61	4.58
	2008	1.84	0.68	1.86
	2007	9.62	9.26	8.74
Age (S0.1)	Mean	45.61	46.00	43.82
	Std. Dev.	11.03	11.14	10.29
	Min; Max	26; 90	26; 80	26;75
Income (in Euro) (S0.3.1)	Median	32,000	34,000	30,000
	Std. Dev.	42,730.14	35,072.14	51,673.74
	Min; Max	0; 1,000,000	800; 350,000	0;1,000,000
	No Answ. (%)	25.77	20.3	17.77

Table 2: Number of Products Offered: Full Sample

	(1) No Control Poisson	(2) With Controls Poisson	(3) Controls & Products Poisson	(4) No Controls Ord. Probit	(5) With Controls Ord. Probit	(6) Controls & Products Ord. Probit
KnowIndex_total (Std. Deviations)	0.305***	0.230***	0.189***	0.239***	0.212**	0.186**
	(0.0118)	(0.0152)	(0.0154)	(0.0485)	(0.0660)	(0.0679)
Sophisticated Respondent	0.0167	0.0453	-0.0552	0.0627	0.189	0.123
	(0.0310)	(0.0423)	(0.0430)	(0.130)	(0.186)	(0.192)
Female Respondent		-0.226***	-0.196***		-0.160	-0.142
		(0.0354)	(0.0355)		(0.144)	(0.149)
Financial Wealth (Std. Dev.)		0.102***	0.0855***		0.0630	0.0557
		(0.0181)	(0.0186)		(0.0791)	(0.0840)
Risk Propensity (Std. Dev.)		0.224***	0.185***		0.263***	0.244***
		(0.0159)	(0.0163)		(0.0676)	(0.0710)
Long Term Propensity (Std. Dev.)		0.0631***	0.0729***		0.00752	0.00484
		(0.0153)	(0.0156)		(0.0671)	(0.0702)
MiFiD Responsiveness (Std. Dev.)		0.0513**	0.0419*		-0.0406	-0.0554
		(0.0176)	(0.0176)		(0.0708)	(0.0726)
Additional Socio-Economic Controls		YES	YES		YES	YES
Year of Purchase		YES	YES		YES	YES
Product Purchased		NO	YES		NO	YES
_cons	1.534***	-0.781*	0.203			
	(0.0277)	(0.321)	(0.344)			
cut1						
_cons				1.525***	3.913**	3.326*
				(0.117)	(1.379)	(1.550)
cut2						
_cons				2.238***	4.741***	4.190**
				(0.136)	(1.387)	(1.556)
cut3						
_cons				2.445***	5.089***	4.551**
				(0.149)	(1.393)	(1.560)
N	1362	868†	868†	1362	868†	868†
adj. R^2						

Standard errors in parentheses

$^{+}p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

5.2 Empirical Findings

Table 2 reports the results of Poisson and ordered Probit regressions where the dependent variable is the increasing category of the number of products offered to the investor.[27] In the first three columns we report the results for the Poisson regressions, while the results of the ordered Probit are reported in the last three columns. Consistently with the main idea of the paper, an investor with higher knowledge receives on average a richer menu. This remains true, even after controlling for a number of variables including the year of purchase, the risk propensity of the investor, the propensity to invest in long term assets, a proxy for his/her level of naivete, and the product acquired. As well, male, richer, and more risk loving investors tend to receive a richer menu of products. The results are confirmed in Tables 4, 5, 6 and 7 in Appendix B. Table 4 presents the results of the same exercises as in Table 2 with errors clustered at product level. Although, as expected, in most cases the significance is reduced compared to the baseline specification, it is never lost. In Table 5 we use a 'hard' version of the knowledge index. The hard version of the index is constituted by dummies only belonging to the (B) and (T) blocks, that is, variables that tend to provide harder information on investor's knowledge.[28] Table 6 reports the results for the sample restricted to investors that made an 'exogenous choice' of intermediary as described above. In Table 7, the sample is restricted to investors who had 'non-economic triggers' to investment. By conditioning to the different samples we find no noticeable differences in the empirical results.

Tables 3 reports the results of our main regressions, where the dependent variable is the perceived extremeness of the offer. In all regressions, a higher level of reported knowledge is associated to menus with a lower level of perceived 'extremeness'. The coefficient associated to the total knowledge index is remarkably stable across specifications, including the last column where we control for the type of product purchased. Since we normalized both the dependent variable and the index, a coefficient of -0.25 means that an increase in the

[27] In the Poisson regressions, we associated to each category the number of products resulting from the arithmetic average of the extremes.

[28] Only in the very last column of Table 5 the coefficient associated to (the hard version of) the knowledge index looses significance. While keeping the right sign, in that case, the p-value is 11.8%.

Table 3: Extremeness of Offer (Std. Deviations)

	(1) No Controls	(2) With Controls	(3) Controls & Years	(4) Controls & Products	(5) All
KnowIndex_total (Std. Deviations)	-0.264***	-0.262***	-0.251***	-0.266***	-0.257***
	(0.0261)	(0.0326)	(0.0323)	(0.0324)	(0.0321)
Sophisticated Respondent	-0.256***	-0.207*	-0.198*	-0.190*	-0.186*
	(0.0639)	(0.0853)	(0.0844)	(0.0854)	(0.0848)
Female Respondent		-0.100	-0.0989	-0.105	-0.104
		(0.0718)	(0.0711)	(0.0710)	(0.0706)
Financial Wealth (Std. Dev.)		-0.0463	-0.0314	-0.0286	-0.0170
		(0.0361)	(0.0359)	(0.0359)	(0.0358)
Risk Propensity (Std. Dev.)		0.252***	0.243***	0.249***	0.238***
		(0.0335)	(0.0332)	(0.0336)	(0.0334)
Long Term Propensity (Std. Dev.)		-0.0356	-0.0392	-0.0204	-0.0252
		(0.0329)	(0.0325)	(0.0327)	(0.0324)
MiFiD Responsiveness (Std. Dev.)		0.00627	0.00723	0.0183	0.0197
		(0.0344)	(0.0340)	(0.0341)	(0.0338)
Additional Socio-Economic Controls	NO	YES	YES	YES	YES
Product Purchased	NO	NO	NO	YES	YES
Year of Purchase: 2017			-0.183		-0.121
			(0.133)		(0.136)
Year of Purchase: 2016			0.196		0.192
			(0.145)		(0.146)
Year of Purchase: 2015			0.212		0.208
			(0.143)		(0.145)
Year of Purchase: 2014			0.406**		0.397**
			(0.142)		(0.143)
Year of Purchase: 2013			0.0634		0.0707
			(0.166)		(0.166)
Year of Purchase: 2012			0.0462		0.0209
			(0.152)		(0.152)
Year of Purchase: 2011			-0.0408		-0.0133
			(0.201)		(0.201)
Year of Purchase: 2010			-0.177		-0.167
			(0.196)		(0.195)
Year of Purchase: 2009			0.144		0.119
			(0.205)		(0.205)
Year of Purchase: 2008			-0.169		-0.183
			(0.266)		(0.265)
_cons	0.202***	0.944	0.608	0.444	0.188
	(0.0566)	(0.656)	(0.668)	(0.707)	(0.721)
N	1362	868†	868†	868†	868†
adj. R^2	0.086	0.180	0.206	0.207	0.226

Standard errors in parentheses

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

knowledge index by one standard deviation reduces the level of extremeness by one fourth of its standard deviation. Furthermore, in all specifications, risk propensity is positively correlated with offer extremeness, while female and more sophisticated investors receive less extreme menus.

In our regressions, in addition to the ones explicitly shown in the tables, we have some recurrent socio-demographic and economic control variables. In particular: age of the respondent and age squared, neither of which is significant; education of the respondent, always negatively correlated with extremeness but not always significantly so; reported income, which is never significant; a series of dummies on the occupation of the respondent, the majority of which result not to be significant. When introducing time dummies we find that, in the year 2014, investment retailers increased the extremeness of their menus.[29]

In Tables 8, 9, 10 and 11 in Appendix B we performed the same robustness exercises as we did for the regressions with the number of offered products. Table 8 reports the results for the errors clustered at the product level and we have no noticeable difference to report compared to the baseline regressions. Table 9 considers the 'hard' version of the index. Although the coefficient decreases compared to the total index and is a bit less stable across specifications, it always remains significantly larger than zero with a p-value of 1% or lower. In Tables 10 and 11 we restrict the sample according to the 'exogenous choice' of the intermediary (Table 10) and the presence of 'non-speculative triggers' to investment (Table 11). Our result are again fully confirmed for the different sample selections.

6 Policy Implications and Conclusion

We studied the delegation problem between a financial intermediary and an investor with limited awareness of the available investment opportunities. We showed that the intermedi-

[29]This year the BCE started the quantity easing program, drastically reducing the spread between Italian and German bonds. Moreover, in Italy, in July 2014 the taxation of capital gains increased from 20% to 26%. In January 2015, the taxation on pension funds almost doubled, from 11% to 20%. These changes (or their expectation of such changes) might have reduced expected net returns, perhaps increasing the investors' propensity towards more risky and less traditional (and/or less liquid) investments.

ary finds it optimal to make the investor aware of investment opportunities at the extremes, but leaves her unaware of intermediate options. We further showed that competition between intermediaries increases awareness in the market and found that the coexistence of investors with different levels of awareness might generate positive or negative externalities on the other agents, depending on the profitability of intermediation with very sophisticated investors. We collected self-reported data from customers in the Italian retail investment sector and found results in line with the predictions of the theoretical model. The menus offered to less knowledgable investors contained fewer products than those offered to sophisticated investors. At the same time, agents with a lower knowledge index perceived more strongly that their menu contained products at the extremes.

An achievement of this paper is to formulate a flexible model of financial intermediation with limited awareness and derive a number of properties of the optimal solution. Despite the potential complexity of the resulting double delegation problem, the solution found is remarkably simple and can easily be embedded into more complex frameworks. As well, the key insights of the theory seems to be robust to a number of possible extensions and not rejected by the data.

The paper has implications for bank regulation and brokerage practices. Assuming that small investors are those more likely to have limited awareness, our results show that, when interacting with such investors, financial professionals may have incentives to eliminate certain investment opportunities so as to induce investments they prefer, such as risky assets. Of course educating investors about available investment options, thereby expanding Y^P, benefits them in our environment. In reality, however, promoting full awareness in that way might not always be feasible or might be very expensive.

Our model suggests that there could exist a much simpler and equally effective intervention. We have seen that what determines the final awareness of an investor is not the number of investment products of which she is initially aware but rather how close to the optimal cap under full awareness these products are. In our stylized model, it is sufficient that the investor is aware of $\hat{y}$ and the intermediary will make him fully aware. This in turn

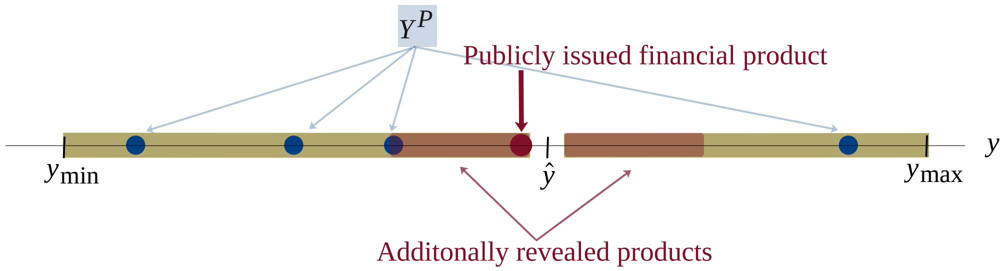

Figure 6: Policy intervention and equilibrium awareness. The yellow area at the extremes represents the equilibrium awareness set when the investor has initial awareness set Y^P represented by the blue bullets. The red subset is added to the yellow set whenever the investor is also aware of the publicly issued financial product indicated by the red bullet near $\hat{y}$.

implies that all a regulator must do is promoting awareness of exactly that product, e.g. by issuing and publicly propagating a financial product with the characteristics of $\hat{y}$. An intermediary will then find it in his best interest to educate the investor about the remaining investment opportunities. It is not crucial however, that the regulator promotes exactly $\hat{y}$. As long as the issued product is relatively close, the set of investment opportunities of which the investor remains unaware is very small, as we illustrate in Figure 6. Our findings thus point to an important complementarity between the regulator and private actors, suggesting that a relatively simple policy intervention can lead banks to reveal investment opportunities more suitable to the needs of investors. The results of our paper further indicate, that when full awareness for the whole population is unfeasible, the optimal financial training policy is often characterized by a widespread moderate training as opposed to an intensive training policy to a small fraction of potential investors. In particular, consider the equilibrium with competition under the assumption $U(0) < \overline{U}$ (second part of Section 4.1). In this case trained individuals remain attractive to intermediaries only if they retain some unawareness. Provided that this is the case, competition for (partially) trained individuals generates positive spillovers for those that remain unaware, along the lines of Proposition 11.[30] In contrast, an intensive training policy that only affects a fraction of individuals (with, say, full awareness after training) generates a negative market externality on investors with limited awareness.

[30]Let Δ^{tr} be the awareness gap of the trained individuals. The role of $U(0)$ in Proposition 11 is taken by the profit associated to Δ^{tr}. The key requirement for positive spillovers is hence $U(\Delta^{tr}) > \overline{U}$.

References

[1] Allen, F., 1990. The market for information and the origin of financial intermediation. *Journal of Financial Intermediation*, 1, 3-30.

[2] Alonso, R. and Matouschek, N., 2008. Optimal Delegation. *The Review of Economic Studies*, 75 (1), 259-293.

[3] Amador, M. and Bagwell, K., 2013. The Theory of Optimal Delegation with an Application to Tariff Caps. *Econometrica*, 81(4), 1541-1599.

[4] Armstrong, M. and Vickers, J., 2010. A Model of Delegated Project Choice. *Econometrica*, 78 (1), 213-244.

[5] Auster, S. (2013). Asymmetric awareness and moral hazard, *Games and Economic Behavior*, 82, 503-521.

[6] Burdett, K., and K. Judd, 1983. Equilibrium Price Dispersion. *Econometrica*, 51(4), 955-969.

[7] Diamond, D. W., 1984. Financial Intermediation and Delegated Monitoring. *Review of Economic Studies*, LI, 393-414.

[8] Filiz-Ozbay, E., 2012. Incorporating unawareness into contract theory, *Games and Economic Behavior*, 76, 181-194.

[9] Foà, G., L. Gambacorta, L. Guiso, and P. Mistrulli, 2017. The Supply Side of Household Finance. Working Paper.

[10] Garcia, C., 2012. The Puzzling, Perpetual Constancy of Safe Asset Demand, *FTAlphaville*, January 23rd.

[11] Gerardi, K., Lehnert, A., Sherlund, M. S., and Willen, P., 2008. Making Sense of the Financial Crisis, *Brooking Papers on Economic Activity*, Fall, 69-145.

[12] Gorton, G., Lewellen, S., and Metrick A., 2012. The Safe-Assets Share, *American Economic Review Papers & Proceedings.* 102(3): 101-106.

[13] Guiso, L. and T. Jappelli, 2005. Awareness and Stock Market Participation, *Review of Finance*, 9: 1-31.

[14] Guiso, L. and T. Jappelli, 2008. Financial Literacy and Portfolio Diversification, *mimeo.*

[15] Guiso, L., A., Pozzi, A. Tsoy, L. Gambacorta, and P. Mistrulli, 2017. Distorted Advice in Financial Markets: Evidence from the Mortgage Market, Working Paper.

[16] Halac, M., and P. Yared, 2017. Commitment vs. Flexibility with Costly Verification, *mimeo*, Columbia Business School.

[17] Halan, M. and R. Sane, 2016. Misled and mis-sold: Financial misbehaviour in retail banks, Working Paper.

[18] Heifetz, A., Meier M., Schipper, B., 2013. Dynamic Unawareness and Rationalizable Behavior, Games and Economic Behavior 81 (2013), 50-68.

[19] Holmström, B., 1977. *On Incentives and Control in Organizations*, Ph.D. Thesis, Stanford University.

[20] Karni, E., and M. L. Vierø, 2013. Reverse Bayesianism. *American Economic Review*, 103(7): 2790-2810.

[21] Karni, E., and M. L. Vierø, 2017. Awareness of Unawareness: A Theory of Decision Making in the Face of Ignorance. *Journal of Economic Theory,* 168:301-32.

[22] Krishna, V. and Morgan, J., 2008. Contracting for Information under Imperfect Commitment. *RAND Journal of Economics*, 39 (4), 905-925.

[23] Leland, H., and D. Pyle, 1977. Information asymmetries, financial structure, and financial intermediation. *Journal of Finance*, 32, 371-387.

[24] Lester, B., Shourideh, A., Venkateswaran, V., and Ariel Zetlin-Jones, 2017. Screening and Adverse Selection in Frictional Markets, Working Paper.

[25] Markowitz, H.M., 1952. Portfolio Selection, *The Journal of Finance*, 7 (1), 77-91.

[26] Markowitz, H.M., 1959. Portfolio Selection: Efficient Diversification of Investments, New York: John Wiley & Sons.

[27] Martimort, D. and Semenov, A., 2006. Continuity in mechanism design without transfers. *Economics Letters*, 93 (2), 182-189.

[28] Melumad, N.D. and Shibano, T., 1991. Communication in settings with no transfers. *RAND Journal of Economics*, 22, 173-198.

[29] Mullainathan, S., M. Noeth, and A. Schoar, 2012. The Market for Advice: An Audit Study. *NBER working paper*, 17929.

[30] Ramakrishan, R. T., and A. V. Thakor, 1984. Information reliability and a theory of financial intermediation. *Review of Economic Studies*, 51, 415-432.

[31] Szalay, D., 2005. The economics of clear advice and extreme options, *Review of Economic Studies*, 72(4), 1173-1198.

[32] Tobin, J., 1958. Liquidity Preference as Behavior Towards Risk, *Review of Economic Studies*, 25, 65-86.

[33] van Rooij, M., Lusardi A., and Alessie R., 2011. Financial literacy and stock market participation, *Journal of Financial Economics,* 101(2), 449-472.

[34] von Thadden, E.-L. and Zhao X. J., 2012. Incentives for unaware agents, *Review of Economic Studies*, 79, 1151-1174.

[35] von Thadden, E.-L. and Zhao X. J., 2014. Multitask agency with unawareness, *Theory and Decisions*, 77(2), 197-222.

[36] Woodward, S. and R. Hall, 2012. Diagnosing Consumer Confusion and Sub-Optimal Shopping Effort: Theory and Mortgage-Market Evidence. *The American Economic Review,* 102(7): 3249-3276.

[37] Zhao, X.J., 2011. Framing contingencies in contracts, *Mathematical Social Sciences*, 61, 31-40.

Appendix

A Proofs

For the proofs of the following results it is useful to introduce the terms

$$T(y) := F(y)\left(y - \mathbb{E}[\theta - \beta|\theta \leq y]\right),$$

and

$$S(y) := (1 - F(y))\left(y - \mathbb{E}[\theta - \beta|\theta \geq y]\right),$$

in the literature referred to as, respectively, backward bias and forward bias (see Alonso and Matouschek, 2008). By Assumption 1 we have

$$T''(y) = \beta f'(y) + f(y) > 0 \quad \text{and} \quad S''(y) = -(\beta f'(y) + f(y)) < 0 \quad \text{for all } y \in [0, 1]$$

Note first that - since $\beta > 0$ - we have $T(y) \geq 0$ for all $y \in [y_{min}, y_{max}]$ and $T(y) > 0$ for $y \geq 0$. The variable S may change sign. Noticing however, that $S(\hat{y}) = S(1) = 0$, strict concavity of S implies that $S(y) > 0$ for all $y \in (\hat{y}, 1)$.

A.1 Proof of Proposition 1

Alonso and Matouschek (2008) show that under Assumption 1 the class of delegation sets are interval of the form $[y_{min}, \bar{y}]$. We do not repeat their proof here. In Figure 7 we provide a graphical intuitive explanation for the result. In Lemma 3, we show this result even allowing for partial awareness.

Given we can concentrate on intervals of the mentioned class, the relevant objective function is as follows:

$$-\int_{y_{min}}^{\bar{y}} (\beta)^2\, dF(\theta) - \int_{\bar{y}}^{1} (\bar{y} - (\theta - \beta))^2\, dF(\theta).$$

Note that the objective function is continuous and the set Y is compact so a maximum exists. The derivative of the objective function with respect to $\bar{y}$ is:

$$-\bar{y}(1 - F(\bar{y})) + \int_{\bar{y}}^{1} (\theta - \beta) dF(\theta). \tag{7}$$

Equation (1) is a simple rearrangement of the first order condition for the principal. The condition $\mathbb{E}[\theta - \beta] > 0$ implies that at the point $\bar{y} = 0$ the derivative of the objective function is positive. It is

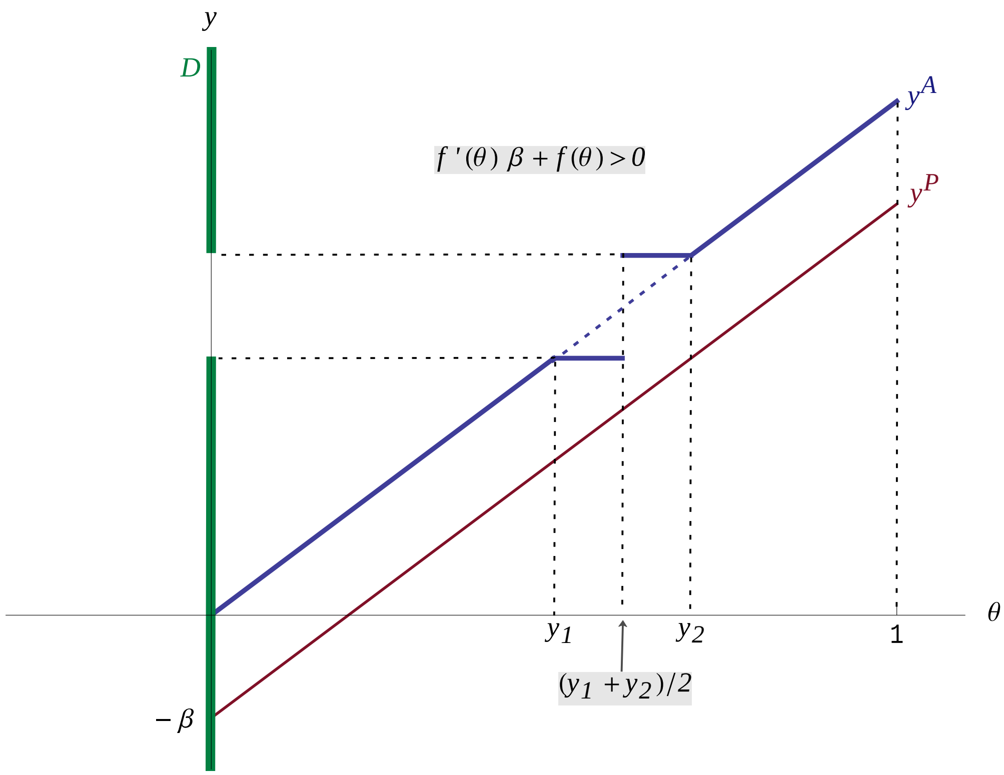

Figure 7: In the horizontal axis the figure reports the set of states, while in the vertical axIs it reports the projects. For each θ, the blue 45-degree line represents the preferred project for the intermediary while the red line reports the preferred projects for the investor. The green set in the vertical axis represents an example of delegation set. Consider two projects y_1 and y_2 with $y_1 < y_2$. If all projects in the interval $[y_1, y_2]$ belong to the delegation set and the realized state θ falls into that interval, the intermediary chooses his ideal project $y = \theta$. This is represented by the dotted line. If the investor excludes projects (y_1, y_2) from the delegation set, the intermediary cannot take his preferred project but chooses the one closest to his bliss point. Hence, in states below the midpoint $\frac{y_1+y_2}{2}$ the intermediary chooses y_1, while in states above the midpoint he chooses y_2. Given that the investor's ideal project lies strictly below the ideal project of the intermediary, this implies that in states below $\frac{y_1+y_2}{2}$, the implemented project moves closer to the investor's bliss point, whereas in states above $\frac{y_1+y_2}{2}$ it moves further away. Since the cost of moving away from the bliss point is convexly increasing in the distance, the investor's loss outweighs the gain, as long as the probability weight attached to the states below $\frac{y_1+y_2}{2}$ is not too large. The first condition in Assumption 1 - $\beta f'(\theta) + f(\theta) > 0$ - assures that this is indeed the case.

easy to see that for $\bar{y} < 0$ the derivative of the objective function is larger that that at $\bar{y} = 0$ so no $\bar{y} \in [y_{min}, 0)$ can be a solution to the first order condition. The second derivative of the objective function is:

$$\beta f(\bar{y}) - (1 - F(\bar{y})).$$

Assumption 1 implies that the third derivative of the objective function is positive, implying strict convexity of the first derivative. This implies that the first derivative can cross zero at most at another point. It can be checked by direct inspection of (7) that the other point where the first order condition is satisfied is $\bar{y} = 1$. Since $\hat{y} < 1$, by convexity of the first derivative, it must be that for $\bar{y} < 1$ and close enough to 1 the first derivative is negative. So $\bar{y} = 1$ cannot be a maximum. What remains is the interior $\hat{y}$ as claimed.

We now shown the monotonicity of $\hat{y}$ in β. Since a maximum exists, it must be that the following second order necessary condition is satisfied at $\hat{y}(\beta)$ (note we now write explicitly the dependence of $\hat{y}$ on β):

$$\beta f(\hat{y}(\beta)) - (1 - F(\hat{y}(\beta))) \leq 0. \tag{8}$$

By the strict convexity of the first derivative and the fact that it equals zero from below at $\bar{y} = 1$ it must be at $\hat{y}$ the first derivative crosses zero from above at it cannot be flat around $\hat{y}$. Condition (8) must hence be satisfied with strict inequality. We can hence use the implicit function theorem to (7) (Assumption 1 implies that the cumulate F is $\mathcal{C}^1$) to show that $\hat{y}(\beta)$ admits a derivative at each β, which equals:

$$\hat{y}'(\beta) = -\frac{1 - F(\hat{y}(\beta)}{\beta f(\hat{y}(\beta)) - (1 - F(\hat{y}(\beta)))} < 0,$$

where we used the necessary second order condition (8) with strict inequality. Continuous differentiability is guaranteed by the implicit function theorem and can be checked directly in the above expression.

A.2 Proof of Lemma 3

Suppose not and let $y \in Y$ be such that $y \notin D^*(Y)$ and $D^*(Y) \cap [y_{min}, y] \neq \emptyset$, $D^*(Y) \cap [y, y_{max}] \neq \emptyset$. Further, let y^- be the largest element of $D^*(Y)$ strictly smaller than y and let y^+ be the smallest element of $D^*(Y)$ strictly greater than y, that is $y^- = \max\{y' \in D^*(Y) : y' < y\}$ and $y^- = \min\{y' \in D^*(Y) : y' > y\}$. Define $s := \frac{y^- + y^+}{2}$ to be the state at which the intermediary is indifferent between choosing project y^- and project y^+, and similarly define $r := \frac{y + y^-}{2}$ and $t := \frac{y^+ + y}{2}$ as the states in which the intermediary is indifferent, respectively, between choosing y^- and y and between y^+ and y.

Following Alonso and Matouschek (2008), we can write the change in the investor's expected payoff when including project y into the delegation set. The intermediary changes his choice of project only in states $[r, t]$. In states $[r, s]$ he switches from y^- to y, while in the remaining states

$(s,t]$ he switches from y^+ to y. The change in the investor's expected payoff is thus given by

$$\begin{aligned} & -\int_r^t (y-\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_r^s (y^- -\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_s^t (y^+ -\theta+\beta)^2 f(\theta)\mathrm{d}\theta, \\ = \;& 2(y-y^-)\underbrace{F(r)\left[r-\mathbb{E}\left[\theta-\beta|\theta\leq r\right]\right]}_{=T(r)} + 2(y^+-y)\underbrace{F(t)\left[t-\mathbb{E}\left[\theta-\beta|\theta\leq t\right]\right]}_{=T(t)} \\ & -2(y^+-y^-)\underbrace{F(s)\left[s-\mathbb{E}\left[\theta-\beta|\theta\leq s\right]\right]}_{=T(s)}. \end{aligned}$$

Letting $y=\lambda y^+ + (1-\lambda)y^-$ for some $\lambda\in(0,1)$ so that $y-y^-=\lambda(y^+-y^-)$, $y^+-y=(1-\lambda)(y^+-y^-)$ and $s=\lambda r+(1-\lambda)t$, the payoff difference can be written as

$$2(y^+-y^-)\left[\lambda T(r)+(1-\lambda)T(t)-T(\lambda r+(1-\lambda)t)\right].$$

From the strict convexity of T, it then follows that the payoff difference is strictly positive. A contradiction. □

A.3 Proof of Lemma 4

Consider delegation set D with $\min D(Y) > \min Y$. Letting $y=\min Y$ and $\underline{y}=\min D(\widehat{y})$, the state at which the intermediary is indifferent between the two projects is given by $s:=(y+\underline{y})/2$. If the investor includes y in the delegation set, the intermediary switches from $\underline{y}$ to y in all states $\theta\leq s$. The investor's change in expected payoff when including y is hence given by

$$\begin{aligned} & -\int_0^s (y-\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_0^s (\underline{y}-\theta+\beta)^2 f(\theta)\mathrm{d}\theta, \\ = \;& \int_0^s \left[(\underline{y}-y)(\underline{y}+y)-2(\underline{y}-y)(\theta-\beta)\right] f(\theta)\mathrm{d}\theta, \\ = \;& 2(\underline{y}-y)T(s), \end{aligned}$$

which is strictly positive. Including y in the delegation set therefore strictly increases the investor's payoff, which implies $\min D^*(Y)=\min Y$. □

A.4 Proof of Lemma 5

Consider delegation set D and suppose $\max D < \max Y$. Let $\overline{y}=\max D$ and consider project $y>\overline{y}, y\in Y$. Let $t=\frac{y+\overline{y}}{2}$ denote the state at which the intermediary is indifferent between the two projects. The change in the investor's payoff when including project y is given by

$$\begin{aligned} & -\int_t^1 (y-\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_t^1 (\overline{y}-\theta+\beta)^2 f(\theta)\mathrm{d}\theta, \\ = & -\int_t^1 \left[(y-\overline{y})(y+\overline{y}) - 2(y-\overline{y})(\theta-\beta)\right] f(\theta)\mathrm{d}\theta, \\ = & -2(y-\overline{y})S\left(t\right). \end{aligned}$$

This change is weakly positive if and only if $S(t) \leq 0$ and hence if and only if $t \leq \hat{y}$. Since t is the midpoint of $\overline{y}$ and y, this condition holds if and only if the distance between $\overline{y}$ and $\hat{y}$ is weakly greater than the distance between y and $\hat{y}$, i.e. $|\overline{y} - \hat{y}| \geq |y - \hat{y}|$. □

A.5 Proof of Proposition 6

Let the intermediary's payoff as a function of Δ be defined by (recall that for $\theta \leq \hat{y}$ the payoff equal ds zero):

$$U(\Delta) = -\int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y}-\Delta-\theta)^2 f(\theta)\mathrm{d}\theta - \int_{\hat{y}}^{1} (\hat{y}+\Delta-\theta)^2 f(\theta)\mathrm{d}\theta.$$

The first and second derivative of $U(\Delta)$ are

$$\frac{\mathrm{d}U(\Delta)}{\mathrm{d}\Delta} = 2\int_{\hat{y}-\Delta}^{\hat{y}} \left[\hat{y}-\Delta-\theta\right] f(\theta)\mathrm{d}\theta - 2\int_{\hat{y}}^{1} \left[\hat{y}+\Delta-\theta\right] f(\theta)\mathrm{d}\theta, \tag{9}$$

$$\frac{\mathrm{d}^2U(\Delta)}{\mathrm{d}\Delta^2} = -2[1-F(\hat{y}-\Delta)] < 0. \tag{10}$$

The function $U(\Delta)$ is strictly concave in Δ and hence has a unique solution on $[0, \bar{\Delta}(Y)]$. The interior solution of the intermediary's optimization problem, Δ^*, is characterized by the first-order condition that equalizes the expression (9) to zero. The expression in (3) is obtained after simple rearrangements of the terms in (9).

A.6 Proof of Proposition 7

First, let us write the intermediary's payoff as a function of Δ and the parameter β:

$$U(\Delta;\beta) = -\int_{\hat{y}(\beta)-\Delta}^{\hat{y}} (\hat{y}(\beta)-\Delta-\theta)^2 f(\theta)\mathrm{d}\theta - \int_{\hat{y}(\beta)}^{1} (\hat{y}(\beta)+\Delta-\theta)^2 f(\theta)\mathrm{d}\theta.$$

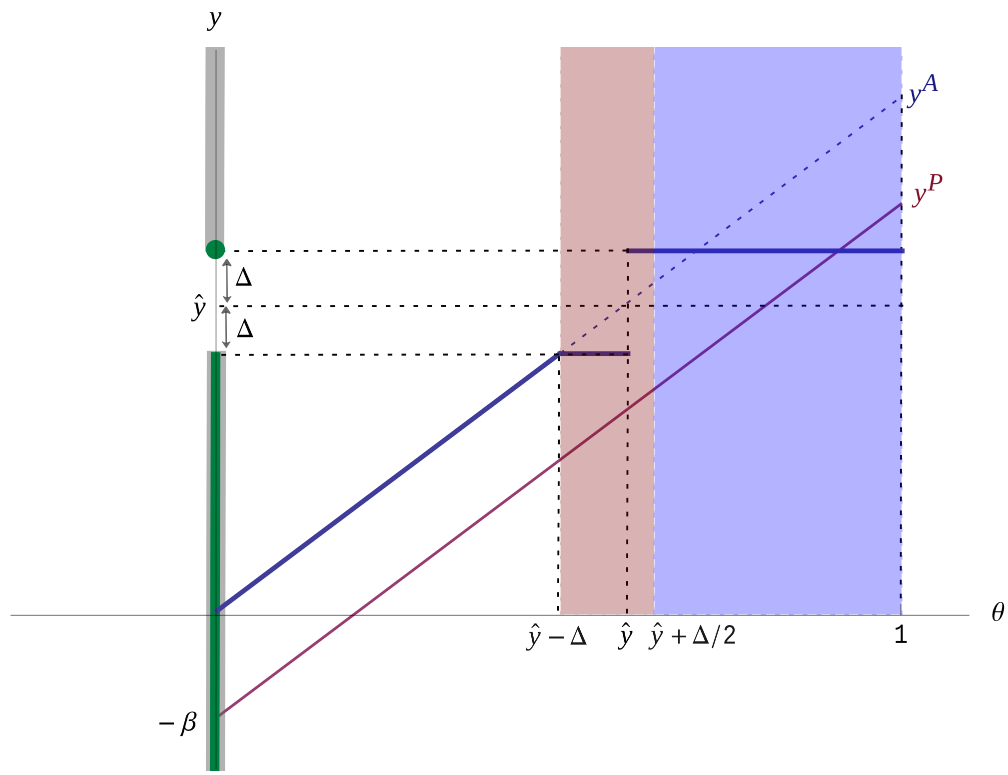

Figure 8: In the horizontal axis the figure reports the set of states, while in the vertical axis it reports the feasible projects. The green set in the vertical axis represents a typical equilibrium delegation set. For each θ, the blue 45-degree line represents the preferred project for the intermediary while the red line reports the preferred projects for the investor. For states from $\hat{y}+\frac{\Delta}{2}$ the intermediary gains as the new pooling project is uniformly closer to his ideal point compared to $\hat{y}$. This is represented by the blue area in the figure. The cost of increasing the gap is the utility loss in the states $\left[\hat{y}-\Delta,\hat{y}+\frac{\Delta}{2}\right]$, where the intermediary moves away from his ideal action. The states implying a loss compared to $\hat{y}$ are represented by the red area. Note that for $\Delta\approx 0$ the red area vanishes. Since at $\hat{y}$ the intermediary chooses his bliss point action, by increasing the gap he enjoys first order gains while losses are zero to the first order. The unrestricted optimal Δ^* equalizes marginal gains with marginal losses maximizing the overall net gain.

Recall, the solution to the problem solves

$$\int_{\hat{y}(\beta)-\Delta^*}^{\hat{y}(\beta)}\left[\hat{y}(\beta)-\Delta^*-\theta\right]f(\theta)\mathrm{d}\theta-\int_{\hat{y(\beta)}}^{1}\left[\hat{y}(\beta)+\Delta^*-\theta\right]f(\theta)\mathrm{d}\theta=0.$$

Since F is $\mathcal{C}^1$ by Assumption 1, $\hat{y}(\beta)$ is $\mathcal{C}^1$ from 1, and $U''_{\Delta,\Delta}<0$, the conditions for applying the implicit function theorem are satisfied. There is hence a function $\Delta^*(\beta)$ describing the unrestricted solution for the intermediary that solves the first order condition: $U'_{\Delta}(\Delta^*(\beta);\beta)=0$, which becomes an identify when seen as a function of β, and:

$$\Delta^{*\prime}(\beta) = -\frac{U''_{\Delta,\beta}(\Delta^*(\beta);\beta)}{U''_{\Delta,\Delta}(\Delta^*(\beta);\beta)}.$$

The statement in the proposition will hence be shown if we can prove that $U''_{\Delta,\beta}(\Delta^*(\beta);\beta) > 0$. Differentiating the expression of the first order condition (3) with respect to β keeping Δ^* as fixed, after some rearrangement, delivers:

$$U''_{\Delta,\beta}(\Delta^*(\beta);\beta) = -\hat{y}'(\beta)\left[1 + F\left(\hat{y}(\beta) - \Delta^*(\beta)\right) - 2F(\hat{y}(\beta))\right].$$

Since $\hat{y}'(\beta) < 0$ from Proposition 1, we would be done if $1 + F\left(\hat{y}(\beta) - \Delta^*(\beta)\right) - 2F(\hat{y}(\beta)) > 0$. Note that this inequality can be equivalently written as:

$$2(1 - F(\hat{y}(\beta))) > 1 - F\left(\hat{y}(\beta) - \Delta^*(\beta)\right).$$

Now, if we use $\hat{y}(\beta) = \mathbb{E}[\theta - \beta|\theta \geq \hat{y}(\beta)]$ and rearrange the first order condition, we obtain:

$$[1 - F(\hat{y}(\beta) - \Delta^*(\beta))]\left[\mathbb{E}[\theta|\theta \geq \hat{y}(\beta) - \Delta^*(\beta)] - (\hat{y}(\beta) - \Delta^*(\beta))\right] = 2[1 - F(\hat{y}(\beta))]\beta.$$

Since $\hat{y}(\beta) - \Delta^*(\beta) < \hat{y}(\beta)$ from the definition of $\hat{y}(\beta)$, it must be that

$$\mathbb{E}[\theta|\theta \geq \hat{y}(\beta) - \Delta^*(\beta)] - (\hat{y}(\beta) - \Delta^*(\beta)) > \beta,$$

which implies $(1 - F(\hat{y}(\beta) - \Delta^*)\beta))) < 2(1 - F(\hat{y}(\beta)))$ as desired.

A.7 Proof of Proposition 8

We will first show that choosing an awareness set of the form $[y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$ is always optimal. In general, an intermediary may be faced with a distribution of arbitrary competing awareness sets. Let $\mathcal{Y}$ denote the support of this distribution with typical element Y. Suppose the intermediary's best response to this distribution is an awareness set $\tilde{Y}$, where $\tilde{Y}$ is not of the form $[y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$. Define $\tilde{\Delta}$ to be the smallest value of Δ such that $\tilde{Y} \subseteq [y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$. Clearly, the awareness set $[y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}]$ yields a weakly larger probability for the intermediary to attract an investor. Consider then the intermediary's expected payoff conditional on attracting an investor who also meets the other intermediary. When the other intermediary reveals $Y \in \mathcal{Y}$, the induced delegation sets from revealing, respectively, $\tilde{Y}$

and $[y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}]$ are

$$D^*(\tilde{Y} \cup Y) \quad \text{and} \quad D^*([y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}] \cup Y).$$

In order for $D^*(\tilde{Y} \cup Y)$ to yield a strictly higher payoff (for the intermediary) than $D^*([y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}] \cup Y)$ it must be that there exists some element $y \in D^*(\tilde{Y} \cup Y)$ that does not belong to $D^*([y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}] \cup Y)$. Suppose this is the case. By Lemma 5 we know that $D^*([y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}] \cup Y)$ includes all projects in the interval $[y_{min}, \hat{y} - \tilde{\Delta}]$ and all projects in Y weakly smaller than $\hat{y}$. We therefore have $y > \hat{y}$. Lemma 5 further tells us that the optimal delegation set includes at most one project strictly greater than $\hat{y}$. Next, by definition of $\tilde{\Delta}$, the set $\tilde{Y}$ includes a project whose distance to $\hat{y}$ is $\tilde{\Delta}$. This implies that the largest project in $D^*(\tilde{Y} \cup Y)$ is weakly smaller than $\hat{y} + \tilde{\Delta}$, so we have $y \leq \hat{y} + \tilde{\Delta}$. Also, since y belongs to $D^*(\tilde{Y} \cup Y)$, we know that there is no project in Y strictly closer to $\hat{y}$ than y. However, the facts that the distance between y and $\hat{y}$ is smaller than $\tilde{\Delta}$ and that there is no project in Y that is closer to $\hat{y}$ than y imply that y must also belong to $D^*([y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}] \cup Y)$. A contradiction. Hence, the awareness set $[y_{min}, \hat{y} - \tilde{\Delta}] \cup [\hat{y} + \tilde{\Delta}, y_{max}]$ yields a weakly larger expected payoff than $\tilde{Y}$. Choosing an awareness set of the form $[y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$ for some $\Delta \geq 0$ therefore constitutes a best response.

Given the argument above, a symmetric equilibrium can be described by a distribution function over awareness gaps parameterized by Δ, denoted by H^*. Standard arguments imply that the equilibrium distribution H^* cannot have any mass points, except possibly at $\Delta = 0$ (see Burdett and Judd, 1983). Moreover, since an intermediary's payoff conditional on being chosen, $U(\Delta)$, is strictly decreasing in Δ for $\Delta > \Delta^*$, the support of H^* is a subset of $[0, \Delta^*]$: offering any $\Delta > \Delta^*$ yields a weakly lower probability of being chosen by an investor and a strictly lower conditional payoff. Finally, if there is some $\Delta' > 0$ that belongs to the support of H^*, all values in the interval $[\Delta', \Delta^*]$ belong to the support as well. Suppose instead there exists an interval $(\Delta_1, \Delta_2) \subset [\Delta', \Delta^*]$ such that Δ_1 belongs to the support of H^* and the values of Δ in the interval (Δ_1, Δ_2) do not. Then choosing an awareness gap parametrized by $\Delta \in (\Delta_1, \Delta_2)$ would yield the same probability of being selected by the investor as Δ_1 but a strictly higher conditional payoff and would, hence, be profitable for the intermediary.

i.) Consider first the possibility where $H^*(0) = 1$ so that both intermediaries reveal all available projects with probability one. Since $\Delta = 0$ maximizes the investors' payoff, any deviating offer will only be accepted if the investor is captive. The best deviating offer is thus characterized by Δ^*. This deviation is not profitable if

$$\pi \bar{U} + (1 - \pi) U(\Delta^*) \leq \left(1 - \frac{1}{2}\pi\right) U(0) + \frac{1}{2}\pi \bar{U}. \tag{11}$$

If condition (11) is satisfied, the described equilibrium exists. It cannot be satisfied when $U(0) < \bar{U}$. On the other hand, when $U(0) = \bar{U}$, condition (11) is satisfied for $\pi = 1$ and when $U(0) > U(1)$ it is satisfied for a continuum of values of π sufficiently large.

ii.) Consider now the possibility where intermediaries choose a non-zero gap with probability one ($H^*(0) = 0$). In that case the support of H^* is an interval $[\Delta', \Delta]$. Indifference requires that an intermediary's expected payoff is constant across the values of Δ in the interval. Differentiation of (4) yields the following first order conditions:

$$\begin{aligned} (1-\pi H(\Delta))U'(\Delta) &= \pi H'(\Delta)(U(\Delta)-\bar{U}) \quad \text{for} \quad \Delta > 0; \\ (1-\pi H(\Delta))U'(\Delta) &\geq \pi H'(\Delta)(U(\Delta)-\bar{U}) \quad \text{for} \quad \Delta = 0. \end{aligned}$$

Let $\hat{H}(\Delta)$ be the solution of the differential equation defined by the first order condition with border condition $\hat{H}(\Delta^*) = 1$. We obtain

$$\hat{H}(\Delta) = \frac{U(\Delta) - [\pi\bar{U} + (1-\pi)U(\Delta^*)]}{\pi[U(\Delta) - \bar{U}]}. \tag{12}$$

We further need $\hat{H}(\Delta') = 0$ for some $\Delta' \geq 0$. Since $\hat{H}(\Delta)$ strictly increases in Δ, this requires

$$U(0) \leq \pi\bar{U} + (1-\pi)U(\Delta^*) \tag{13}$$

This inequality is satisfied for a continuum of values of π sufficiently small and it is satisfied for all values of π if and only if $U(0) \leq \bar{U}$. If (13) holds, the equilibrium distribution is given by

$$H^*(\Delta) = \begin{cases} 0 & \text{if} \quad \Delta < \Delta' \\ \frac{U(\Delta)-[\pi\bar{U}+(1-\pi)U(\Delta^*)]}{\pi[U(\Delta)-\bar{U}]} & \text{if} \quad \Delta' \leq \Delta < \Delta^* \\ 1 & \text{if} \quad \Delta^* \leq \Delta \end{cases}$$

with Δ' such that $U(\Delta') = \pi\bar{U} + (1-\pi)U(\Delta^*)$. Deviating to some Δ strictly smaller than Δ' yields the same probability of attracting the investor as Δ' but a strictly lower conditional payoff and is hence not profitable.

iii.) Finally, we consider an equilibrium where $H^*(0) \in (0, 1)$. We can first show that the intermediaries' strategy cannot have a mass point at zero and at the same time positive density arbitrarily close to zero. This follows from the fact that an intermediary's probability of being selected by the investor drops discontinuously at $\Delta = 0$, given that there is a strictly positive probability that the other intermediary chooses $\Delta = 0$. The support of $H^*(\Delta)$ thus has a gap and is so given by $\{0\} \cup [\Delta', \Delta^*]$ for some Δ' strictly positive. On the interval $[\Delta', \Delta^*]$ indifference requires $H^*(\Delta) = \hat{H}(\Delta)$ as before. Moreover, the intermediary must be indifferent between offering Δ' and no gap, that is:

$$\left(1 - \pi\hat{H}(\Delta')\right) U(\Delta') + \pi\hat{H}(\Delta')\bar{U} = \left(1 - \pi\frac{1}{2}\hat{H}(\Delta')\right) U(0) + \pi\frac{1}{2}\hat{H}(\Delta')\bar{U}. \tag{14}$$

The left hand side is equal to $\pi\bar{U} + (1-\pi)U(\Delta^*)$ and thus constant in Δ', while the right hand side is strictly decreasing in Δ'. At $\Delta' = 0$ the left hand side is strictly larger than the right hand side when condition (13) is violated and at $\Delta' = \Delta^*$ the left hand side is strictly smaller than the right hand side when condition (17) is violated. Hence, whenever neither of the above equilibria exists, condition (14) has a unique solution on $(0, \Delta^*)$. In that case the equilibrium is characterized by

$$H^*(\Delta) = \begin{cases} 0 & \text{if} \quad \Delta < 0 \\ \hat{H}(\Delta') & \text{if} \quad 0 \leq \Delta < \Delta' \\ \frac{U(\Delta) - [\pi\bar{U} + (1-\pi)U(\Delta^*)]}{\pi[U(\Delta) - \bar{U}]} & \text{if} \quad \Delta' \leq \Delta < \Delta^* \\ 1 & \text{if} \quad \Delta^* \leq \Delta \end{cases}$$

where Δ' is defined by (14).

A.8 Proof of Proposition 9

To prove the statement we will show that both the function $\hat{H}(\Delta)$ as well as the mass at zero, $H^*(0)$, are increasing in π. The first property can be seen by taking the first derivative of $\hat{H}$ with respect to π at any $\Delta \in (\Delta', \Delta^*)$. Using (12) we obtain:

$$\frac{\partial \hat{H}(\Delta; \pi)}{\partial \pi} = \frac{U(\Delta^*) - U(\Delta)}{\pi^2[U(\Delta) - \bar{U}]}. \tag{15}$$

Notice that for all values of Δ in the randomization interval $[\Delta', \Delta^*]$ the expected payoff for the intermediary is equal to the payoff at Δ^* and thus equal to $\pi\bar{U} + (1-\pi)U(\Delta^*)$. Since this payoff is strictly greater than $\bar{U}$ for all $\pi < 1$, we have $U(\Delta) > \bar{U}$ for all $\Delta \in [\Delta', \Delta^*]$,[31] so the derivative is positive.

Next we want to show that $H^*(0)$ is increasing in π. Notice that $H^*(0) = \hat{H}(\Delta')$, where Δ' denotes again the lower bound of the randomization interval. Given the previous result, it suffices

[31] For $\pi = 1$, the equilibrium distribution function H^* becomes a step function with a single step at $\Delta' = U^{-1}(\bar{U})$ if $U(0) < \bar{U}$ and at zero otherwise.

to show that Δ' is increasing in π for all $\pi \in (\underline{\pi}, \overline{\pi})$. Solving (14) for $U(\Delta')$ we obtain

$$U(\Delta') = \frac{\frac{1}{2}(\pi\overline{U} + (1-\pi)U(\Delta^*))(U(0) + \overline{U}) - U(0)\overline{U}}{\pi\overline{U} + (1-\pi)U(\Delta^*) - \frac{1}{2}(\overline{U} + U(0))}.$$

The first derivative with respect to π is given by

$$\frac{\partial U(\Delta')}{\partial \pi} = \frac{\frac{1}{4}(U(\Delta^*) - \overline{U})(U(0) - \overline{U})^2}{(\pi\overline{U} + (1-\pi)U(\Delta^*) - \frac{1}{2}(\overline{U} + U(0)))^2} > 0.$$

Since U strictly increases in Δ on $[0, \Delta^*]$, it follows that Δ' strictly increases in π.

A.9 Proof of Proposition 10

To prove the statement we will show that both the function $\hat{H}(\Delta)$ as well as the mass at zero, $H^*(0)$, are decreasing in $\overline{U}$. The first property can be seen by taking the first derivative of $\hat{H}$ with respect to $\overline{U}$ at any $\Delta \in (\Delta', \Delta^*)$. Using (12) we obtain:

$$\frac{\partial \hat{H}(\Delta; \overline{U})}{\partial \overline{U}} = -\frac{1}{U(\Delta) - \overline{U}} - \frac{(U(\Delta^*) - U(\Delta))\left(U(\Delta^*) - \overline{U}\right)}{\pi[U(\Delta) - \bar{U}]^2}. \tag{16}$$

Notice again that for all values of Δ in the randomization interval (Δ', Δ^*) the expected payoff for the intermediary is equal to the payoff at Δ^* and thus equal to $\pi\bar{U} + (1-\pi)U(\Delta^*)$. Since this payoff is strictly greater than $\bar{U}$ for all $\pi < 1$, we have $U(\Delta) > \bar{U}$ for all $\Delta \in (\Delta', \Delta^*)$. As well, we obviously have $U(\Delta^*) > U(\Delta) > \overline{U}$, so the derivative is negative.

Next we want to show that $H^*(0; \overline{U})$ is decreasing in $\overline{U}$. Notice that $H^*(0) = \hat{H}(\Delta')$, where Δ' denotes again the lower bound of the randomization interval. Given the previous result, it suffices to show that Δ' is decreasing in $\overline{U}$ for all $\pi \in (\underline{\pi}, \overline{\pi})$. Rearranging (14) we obtain

$$(1 - \pi\hat{H}(0; \overline{U}))(U(\Delta') - \overline{U}) = (1-\pi)U(\Delta^*).$$

If $\overline{U}$ increases either $\hat{H}(0; \overline{U})$ decreases or $U(\Delta')$ increases, or both. If $\hat{H}(0; \overline{U})$ decreases we are done. Otherwise, since U increases in Δ on $[0, \Delta^*]$, it must be that Δ' increases. But then again from the previous result it follows that $H^*(0; \overline{U})$ decreases in $\overline{U}$.

A.10 Proof of Proposition 11

The construction of the equilibrium is analogous to the one in Section A.7.

i.) $H^*(0) = 1$: The equilibrium where both intermediaries choose $\Delta = 0$ exists if a deviation to

$\Delta = \Delta^*$ is not profitable. This requires

$$\pi\bar{U} + (1-\pi)[\mu U(\Delta^*) + (1-\mu)U(0)] \leq \left(1 - \frac{1}{2}\pi\right) U(0) + \frac{1}{2}\pi\bar{U}. \tag{17}$$

ii.) $H^*(0) = 1$: We can first derive the function $\hat{H}(\Delta)$ for the specification with heterogenous investors. Differentiating the intermediaries' expected payoff in (6) yields

$$\begin{aligned}
(1-\pi H(\Delta))\mu U'(\Delta) &= \pi H'(\Delta)(\mu U(\Delta) + (1-\mu)U(0) - \bar{U}) \quad \text{for} \quad \Delta > 0; \\
(1-\pi H(\Delta))\mu U'(\Delta) &\geq \pi H'(\Delta)(\mu U(\Delta) + (1-\mu)U(0) - \bar{U}) \quad \text{for} \quad \Delta = 0.
\end{aligned}$$

With the condition $\hat{H}(\Delta^*) = 1$ we obtain

$$\hat{H}(\Delta) = \frac{\mu U(\Delta) - [\pi\bar{U} + (1-\pi)\mu U(\Delta^*) - \pi(1-\mu)U(0)]}{\pi[\mu U(\Delta) + (1-\mu)U(0) - \bar{U}]}.$$

Provided $\hat{H}(0) \leq 0$, or equivalently

$$(1-\pi)\mu(U(\Delta^*) - U(0)) \geq \pi(U(0) - \bar{U}), \tag{18}$$

following the argument in Section A.7, iii.), there exists an equilibrium described by

$$H^*(\Delta) = \begin{cases} 0 & \text{if} \quad \Delta < \Delta', \\ \hat{H}(\Delta) & \text{if} \quad \Delta' \leq \Delta < \Delta^*, \\ 1 & \text{if} \quad \Delta^* \leq \Delta, \end{cases} \tag{19}$$

where Δ' is such that $\mu U(\Delta') = [\pi\bar{U} + (1-\pi)\mu U(\Delta^*) - \pi(1-\mu)U(0)]$.

iii.) $H^*(0) \in (0,1)$: The support of H^* is given by $\{0\} \cup [\Delta', \Delta^*]$ for some $\Delta' > 0$. As before, we set $H^*(\Delta) = \hat{H}(\Delta)$ for all $\Delta \in [\Delta', \Delta^*]$. Indifference between Δ' and $\Delta = 0$ then requires

$$\left(1 - \pi\hat{H}(\Delta')\right)[\mu U(\Delta') + (1-\mu)U(0)] + \pi\hat{H}(\Delta')\bar{U} = \left(1 - \pi\frac{1}{2}\hat{H}(\Delta')\right) U(0) + \pi\frac{1}{2}\hat{H}(\Delta')\bar{U} \tag{20}$$

By the same argument as in Section A.7, condition (20) has a unique solution in $(0, \Delta^*)$ if and only if neither of the above equilibria exists. Just notice that the left hand side of (20) is equal to $\pi\bar{U} + (1-\pi)[\mu U(\Delta^*) + (1-\mu)U(0)]$. We thus have

$$H^*(\Delta) = \begin{cases} 0 & \text{if} \quad \Delta < 0, \\ \hat{H}(\Delta') & \text{if} \quad 0 \leq \Delta < \Delta', \\ \hat{H}(\Delta) & \text{if} \quad \Delta' \leq \Delta < \Delta^*, \\ 1 & \text{if} \quad \Delta^* \leq \Delta, \end{cases} \tag{21}$$

with Δ' defined by (20).

To show the statement of the proposition we will show that both the function $\hat{H}(\Delta)$ as well as the mass at zero, $H^*(0)$, are decreasing in μ. The first property can be seen by taking the first derivative of $\hat{H}$ with respect to μ:

$$\frac{\partial \hat{H}(\Delta;\mu)}{\partial \mu} = -\frac{(1-\pi)(U(\Delta^*) - U(\Delta))(U(0) - \bar{U})}{\pi[\mu U(\Delta) + (1-\mu)U(0) - \bar{U}]^2}. \tag{22}$$

Under the assumed condition $\bar{U} \leq U(0)$, the above term is weakly negative for all $\Delta \in [\Delta', \Delta^*]$.

We show next that $H^*(0) = \hat{H}(\Delta')$ is decreasing in μ. Given the above result, it suffices to show that Δ' as determined by (20) is decreasing in μ. This directly follows from the fact that the left-hand side of (20) increases in μ, whereas the right-hand side does not depend on μ. As a result, the solution of (20) decreases as μ increases.

□

A.11 Proof of Proposition 12

Under the assumption $U(0) < \bar{U}$, condition (18) is always satisfied, so the equilibrium distribution H^* is described by (19). We then need to show that $\hat{H}$ shifts upwards as μ increases. This follows directly from (22). □

B Additional Tables and Knowledge Index Variables

In this section, we report some of the the descriptive statistics and a few robustness checks. Before that, we list the 17 variables constituting our 'knowledge index' with a brief description. For each dummy we also indicate the location of the question within the survey.[32]

(P) S1.13: investor reports to be well informed on financial products

(P) S1.44.9: investor says s/he knew what product was best for her/him

(P) S1.44.10: investor reports to knew well the products available on the market

(P) S2.11.15: investor likes to be informed on every option before taking any decision

(P) S1.45.8: investor says s/he did not only consider types of product suggested by others

(U) S1.44.12: investor had no troubles understanding what the products were

(U) S1.44.13: investor understood the terminology in products' description

(U) S1.44.16: investor understood the information on the products offered

(U) S1.52: investor understood all the aspects of the operation

(S) S1.15: investor visited 3 or more institutions

(S) S1.44.6: investor has been searching with care before choosing

(S) S1.47.1: investor independently (from the intermediary) collected information

(B) S1.12.8: investor is a professional in financial sector

(B) S1.12.9: investor attended courses on financial sector

(T) S2.14: investor knows which financial instruments is less liquid

(T) S2.15: investor knows that the return of a financial instrument in foreign currency depends on the exchange rate

(T) S2.16: investor knows what is a derivative product

[32] An english translation of the complete survey is reported in the online appendix.

Figure 9: The figure summarises the average investors' characteristics by products. The first panel reports the distribution of products acquired in our sample. The second panel reports the average index of risk aversion of the investors who purchased the products indicated in the first panel. The index has mean 1.898 and standard deviation 0.924. The third panel reports the average values of the index measuring the attitude of the investor towards investing in long term products. The index has mean 2.524 and standard deviation 1.028. The last panel reports the average values of our knowledge index. The index has mean 5.98 and standard deviation 3.25.

Table 4: Number of Products Offered: Full Sample - Cluster(Product)

	(1) No Control Poisson	(2) With Controls Poisson	(3) Controls & Products Poisson	(4) No Controls Ord. Probit	(5) With Controls Ord. Probit	(6) Controls & Products Ord. Probit
KnowIndex_total (Std. Deviations)	0.305*	0.230**	0.189**	0.239*	0.212*	0.186^{+}
	(0.145)	(0.0700)	(0.0674)	(0.112)	(0.0992)	(0.104)
Sophisticated Respondent	0.0167	0.0453	-0.0552	0.0627	0.189	0.123
	(0.163)	(0.228)	(0.224)	(0.150)	(0.241)	(0.244)
Female Respondent		-0.226	-0.196		-0.160	-0.142
		(0.151)	(0.153)		(0.176)	(0.185)
Financial Wealth (Std. Dev.)		0.102*	0.0855*		0.0630	0.0557
		(0.0466)	(0.0419)		(0.0642)	(0.0666)
Risk Propensity (Std. Dev.)		0.224**	0.185***		0.263***	0.244***
		(0.0717)	(0.0536)		(0.0447)	(0.0446)
Long Term Propensity (Std. Dev.)		0.0631	0.0729		0.00752	0.00484
		(0.0577)	(0.0584)		(0.0578)	(0.0627)
MiFiD Responsiveness (Std. Dev.)		0.0513	0.0419		-0.0406	-0.0554
		(0.0689)	(0.0691)		(0.0595)	(0.0619)
Additional Socio-Economic Controls		YES	YES		YES	YES
Year of Purchase		YES	YES		YES	YES
Product Purchased		NO	YES		NO	YES
_cons	1.534***	-0.781	0.203			
	(0.0976)	(1.057)	(1.113)			
cut1						
_cons				1.525***	3.913***	3.326**
				(0.108)	(1.122)	(1.147)
cut2						
_cons				2.238***	4.741***	4.190***
				(0.126)	(1.121)	(1.145)
cut3						
_cons				2.445***	5.089***	4.551***
				(0.107)	(1.134)	(1.149)
N	1362	868†	868†	1362	868†	868†
adj. R^2						

Standard errors in parentheses

$^{+}p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 5: Number of Products Offered: Hard Index

	(1) No Control Poisson	(2) With Controls Poisson	(3) Controls & Products Poisson	(4) No Controls (Ord. Probit)	(5) With Controls (Ord. Probit)	(6) Controls & Products (Ord. Probit)
KnowIndex_hard (Std. Deviations)	0.234***	0.201***	0.157***	0.156**	0.151*	0.115
	(0.0121)	(0.0161)	(0.0164)	(0.0493)	(0.0704)	(0.0737)
Sophisticated Respondent	-0.00154	0.0401	-0.0586	0.0581	0.173	0.110
	(0.0314)	(0.0423)	(0.0431)	(0.130)	(0.184)	(0.191)
Female Respondent		-0.207***	-0.189***		-0.149	-0.137
		(0.0354)	(0.0355)		(0.144)	(0.148)
Financial Wealth		0.111***	0.0912***		0.0729	0.0600
(Std. Dev.)		(0.0180)	(0.0185)		(0.0785)	(0.0835)
Risk Propensity (Std. Dev.)		0.228***	0.189***		0.261***	0.243***
		(0.0158)	(0.0162)		(0.0673)	(0.0707)
Long Term Propensity		0.0485**	0.0654***		0.00263	0.00244
(Std. Dev.)		(0.0155)	(0.0158)		(0.0674)	(0.0703)
MiFiD Responsiveness		0.0511**	0.0452*		-0.0352	-0.0473
(Std. Dev.)		(0.0182)	(0.0181)		(0.0727)	(0.0746)
Additional Socio-Economic Controls		YES	YES		YES	YES
Year of Purchase		YES	YES		YES	YES
Product Purchased		NO	YES		NO	YES
_cons	1.569***	-0.723*	0.413			
	(0.0277)	(0.319)	(0.342)			
cut1						
_cons				1.498***	3.713**	2.957
				(0.116)	(1.364)	(1.529)
cut2						
_cons				2.198***	4.529***	3.808*
				(0.134)	(1.371)	(1.534)
cut3						
_cons				2.398***	4.873***	4.165**
				(0.146)	(1.377)	(1.538)
N	1362	868†	868†	1362	868†	868†
adj. R^2						

Standard errors in parentheses

$^{+}p < 0.1$, $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 6: Number of Products Offered: Sample Restricted to Exogenous Choice of Intermediary

	(1) No Control Poisson	(2) With Controls Poisson	(3) Controls & Products Poisson	(4) No Controls Ord. Probit	(5) With Controls Ord. Probit	(6) Controls & Products Ord. Probit
KnowIndex_total (Std. Deviations)	0.293***	0.338***	0.215***	0.334***	0.552**	0.525**
	(0.0225)	(0.0303)	(0.0330)	(0.0953)	(0.177)	(0.197)
Sophisticated Respondent	-0.185***	0.168*	-0.0920	0.0779	0.878+	0.615
	(0.0504)	(0.0738)	(0.0794)	(0.234)	(0.492)	(0.636)
Female Respondent		-0.114+	-0.108+		-0.268	-0.245
		(0.0626)	(0.0642)		(0.301)	(0.321)
Financial Wealth (Std. Dev.)		0.214***	0.198***		0.427*	0.461*
		(0.0342)	(0.0358)		(0.189)	(0.212)
Risk Propensity (Std. Dev.)		0.162***	0.0902**		0.342*	0.269
		(0.0269)	(0.0285)		(0.137)	(0.153)
Long Term Propensity (Std. Dev.)		-0.141***	-0.0854**		-0.236	-0.217
		(0.0289)	(0.0295)		(0.147)	(0.157)
MiFiD Responsiveness (Std. Dev.)		0.0666+	0.0725+		0.0449	0.0948
		(0.0367)	(0.0387)		(0.189)	(0.206)
Additional Socio-Economic Controls		YES	YES		YES	YES
Year of Purchase		YES	YES		YES	YES
Product Purchased		NO	YES		NO	YES
_cons	1.623***	0.188	2.821***			
	(0.0435)	(0.569)	(0.743)			
cut1						
_cons				1.660***	4.871	-12.29
				(0.211)	(3.079)	(2817.2)
cut2						
_cons				2.396***	5.813	-11.21
				(0.250)	(3.093)	(2817.2)
cut3						
_cons				2.558***	6.109*	-10.82
				(0.267)	(3.102)	(2817.2)
N	443	304†	304†	443	304†	304†
adj. R^2						

Standard errors in parentheses

$^{+}p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 7: Number of Products Offered: Sample Restricted to non-economic triggers to invest/borrow

	(1) No Control Poisson	(2) With Controls Poisson	(3) Controls & Products Poisson	(4) No Controls Ord. Probit	(5) With Controls Ord. Probit	(6) Controls & Products Ord. Probit
KnowIndex_total (Std. Deviations)	0.259***	0.269***	0.229***	0.168*	0.194*	0.160+
	(0.0173)	(0.0216)	(0.0219)	(0.0693)	(0.0921)	(0.0970)
Sophisticated Respondent	-0.00354	-0.0989+	-0.179***	0.158	0.163	0.111
	(0.0411)	(0.0529)	(0.0540)	(0.171)	(0.230)	(0.242)
Female Respondent		-0.0551	-0.0160		-0.0152	0.0581
		(0.0470)	(0.0475)		(0.189)	(0.201)
Financial Wealth		0.0930***	0.0636*		0.115	0.123
(Std. Dev.)		(0.0246)	(0.0257)		(0.108)	(0.117)
Risk Propensity (Std. Dev.)		0.132***	0.116***		0.173+	0.185+
		(0.0218)	(0.0223)		(0.0927)	(0.0987)
Long Term Propensity		0.00758	0.0266		-0.0381	-0.0125
(Std. Dev.)		(0.0209)	(0.0214)		(0.0948)	(0.102)
MiFiD Responsiveness		-0.0587**	-0.0852***		-0.184*	-0.223*
(Std. Dev.)		(0.0225)	(0.0229)		(0.0914)	(0.0953)
Additional Socio-Economic Controls		YES	YES		YES	YES
Year of Purchase		YES	YES		YES	YES
Product Purchased		NO	YES		NO	YES
_cons	1.570***	-0.582	0.736			
	(0.0359)	(0.440)	(0.467)			
cut1						
_cons				1.518***	2.692	1.436
				(0.153)	(1.819)	(2.031)
cut2						
_cons				2.263***	3.538	2.349
				(0.180)	(1.827)	(2.036)
cut3						
_cons				2.611***	4.095*	2.941
				(0.214)	(1.841)	(2.046)
N	698	475	475	698	475	475
adj. R^2						

Standard errors in parentheses

$^{+}p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 8: Extremeness of Offer (Std. Deviations) - Cluster(Product)

	(1) No Controls	(2) With Controls	(3) Controls & Years	(4) Controls & Products	(5) All
KnowIndex_total (Std. Deviations)	-0.264***	-0.262***	-0.251***	-0.266***	-0.257***
	(0.0272)	(0.0344)	(0.0306)	(0.0356)	(0.0333)
Sophisticated Respondent	-0.256**	-0.207*	-0.198*	-0.190*	-0.186*
	(0.0753)	(0.0745)	(0.0648)	(0.0759)	(0.0698)
Female Respondent		-0.100+	-0.0989*	-0.105+	-0.104*
		(0.0460)	(0.0379)	(0.0488)	(0.0418)
Financial Wealth (Std. Dev.)		-0.0463	-0.0314	-0.0286	-0.0170
		(0.0387)	(0.0342)	(0.0375)	(0.0331)
Risk Propensity (Std. Dev.)		0.252***	0.243***	0.249***	0.238***
		(0.0194)	(0.0207)	(0.0191)	(0.0204)
Long Term Propensity (Std. Dev.)		-0.0356	-0.0392	-0.0204	-0.0252
		(0.0261)	(0.0242)	(0.0285)	(0.0268)
MiFiD Responsiveness (Std. Dev.)		0.00627	0.00723	0.0183	0.0197
		(0.0358)	(0.0381)	(0.0367)	(0.0384)
Additional Socio-Economic Controls	NO	YES	YES	YES	YES
Product Purchased	NO	NO	NO	YES	YES
Year of Purchase: 2017			-0.183		-0.121
			(0.144)		(0.101)
Year of Purchase: 2016			0.196		0.192
			(0.146)		(0.126)
Year of Purchase: 2015			0.212		0.208
			(0.123)		(0.114)
Year of Purchase: 2014			0.406**		0.397**
			(0.117)		(0.0986)
Year of Purchase: 2013			0.0634		0.0707
			(0.139)		(0.148)
Year of Purchase: 2012			0.0462		0.0209
			(0.177)		(0.172)
Year of Purchase: 2011			-0.0408		-0.0133
			(0.271)		(0.270)
Year of Purchase: 2010			-0.177		-0.167
			(0.112)		(0.121)
Year of Purchase: 2009			0.144		0.119
			(0.199)		(0.184)
Year of Purchase: 2008			-0.169		-0.183
			(0.310)		(0.313)
_cons	0.202**	0.944	0.608	0.444	0.188
	(0.0650)	(0.606)	(0.649)	(0.631)	(0.687)
N	1362	868†	868†	868†	868†
adj. R^2	0.086	0.180	0.206	0.207	0.226

Standard errors in parentheses

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 9: Extremeness of Offer: Hard Index (Std. Deviation)

	(1) No Controls	(2) With Controls	(3) Controls & Years	(4) Controls & Products	(5) All
KnowIndex_hard (Std. Deviations)	-0.152***	-0.194***	-0.174***	-0.183***	-0.169***
	(0.0270)	(0.0350)	(0.0348)	(0.0352)	(0.0351)
Sophisticated Respondent	-0.263***	-0.184*	-0.176*	-0.173*	-0.169+
	(0.0662)	(0.0872)	(0.0863)	(0.0875)	(0.0869)
Female Respondent		-0.139+	-0.134+	-0.143+	-0.139+
		(0.0736)	(0.0730)	(0.0730)	(0.0726)
Financial Wealth (Std. Dev.)		-0.0559 (0.0368)	-0.0396 (0.0367)	-0.0393 (0.0368)	-0.0261 (0.0368)
Risk Propensity (Std. Dev.)		0.254***	0.246***	0.249***	0.240***
		(0.0341)	(0.0338)	(0.0344)	(0.0342)
Long Term Propensity (Std. Dev.)		-0.0201 (0.0336)	-0.0258 (0.0332)	-0.00815 (0.0335)	-0.0144 (0.0332)
MiFiD Responsiveness (Std. Dev.)		0.00615 (0.0352)	0.00479 (0.0348)	0.0152 (0.0350)	0.0150 (0.0347)
Additional Socio-Economic Controls	NO	YES	YES	YES	YES
Product Purchased	NO	NO	NO	YES	YES
Year of Purchase: 2017			-0.199		-0.152
			(0.136)		(0.139)
Year of Purchase: 2016			0.174		0.164
			(0.148)		(0.149)
Year of Purchase: 2015			0.165		0.157
			(0.146)		(0.149)
Year of Purchase: 2014			0.405**		0.394**
			(0.145)		(0.147)
Year of Purchase: 2013			0.0615		0.0634
			(0.169)		(0.170)
Year of Purchase: 2012			0.0128		-0.0155
			(0.155)		(0.156)
Year of Purchase: 2011			-0.0384		-0.0138
			(0.205)		(0.206)
Year of Purchase: 2010			-0.170		-0.163
			(0.200)		(0.200)
Year of Purchase: 2009			0.0771		0.0474
			(0.209)		(0.209)
Year of Purchase: 2008			-0.190		-0.209
			(0.272)		(0.271)
_cons	0.207***	1.018	0.699	0.636	0.383
	(0.0585)	(0.668)	(0.682)	(0.724)	(0.738)
N	1362	868†	868†	868†	868†
adj. R^2	0.039	0.148	0.174	0.169	0.189

Standard errors in parentheses

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 10: Extremeness of Offer : Exogenous Selection (Std. Deviations)

	(1) No Controls	(2) With Controls	(3) Controls & Years	(4) Controls & Products	(5) All
KnowIndex_total (Std. Deviations)	-0.188***	-0.215***	-0.181**	-0.208***	-0.181**
	(0.0488)	(0.0552)	(0.0559)	(0.0542)	(0.0554)
Sophisticated Respondent	-0.513***	-0.363**	-0.322**	-0.299*	-0.264*
	(0.109)	(0.124)	(0.124)	(0.122)	(0.124)
Female Respondent		-0.270*	-0.262*	-0.186	-0.184
		(0.113)	(0.115)	(0.111)	(0.113)
Financial Wealth (Std. Dev.)		-0.0637	-0.0471	-0.0271	-0.0154
		(0.0543)	(0.0546)	(0.0539)	(0.0545)
Risk Propensity (Std. Dev.)		0.236***	0.240***	0.228***	0.225***
		(0.0505)	(0.0507)	(0.0505)	(0.0511)
Long Term Propensity (Std. Dev.)		-0.114*	-0.112*	-0.0950	-0.0954*
		(0.0501)	(0.0498)	(0.0483)	(0.0484)
MiFiD Responsiveness (Std. Dev.)		0.102+	0.106+	0.105+	0.108+
		(0.0589)	(0.0590)	(0.0573)	(0.0579)
Additional Socio-Economic Controls	NO	YES	YES	YES	YES
Product Purchased	NO	NO	NO	YES	YES
Year of Purchase: 2017			-0.0719		0.0528
			(0.213)		(0.214)
Year of Purchase: 2016			0.337		0.357
			(0.220)		(0.217)
Year of Purchase: 2015			0.498*		0.514*
			(0.227)		(0.225)
Year of Purchase: 2014			0.343		0.331
			(0.234)		(0.231)
Year of Purchase: 2013			0.282		0.193
			(0.271)		(0.269)
Year of Purchase: 2012			0.157		0.106
			(0.251)		(0.247)
Year of Purchase: 2011			0.246		0.341
			(0.404)		(0.395)
Year of Purchase: 2010			0.155		0.152
			(0.353)		(0.348)
Year of Purchase: 2009			0.246		0.205
			(0.323)		(0.315)
Year of Purchase: 2008			-0.293		-0.0404
			(0.693)		(0.673)
_cons	0.481***	0.103	-0.364	-2.232+	-2.419+
	(0.0954)	(1.104)	(1.120)	(1.257)	(1.265)
N	443	362†	362†	362†	362†
adj. R^2	0.078	0.196	0.211	0.265	0.270

Standard errors in parentheses

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.

Table 11: Extremeness of Offer : Trigger Selection (Std. Deviations)

	(1) No Controls	(2) With Controls	(3) Controls & Years	(4) Controls & Products	(5) All
KnowIndex_total (Std. Deviations)	-0.208***	-0.226***	-0.217***	-0.224***	-0.216***
	(0.0390)	(0.0439)	(0.0437)	(0.0436)	(0.0434)
Sophisticated Respondent	-0.269**	-0.191+	-0.207*	-0.150	-0.160
	(0.0874)	(0.0990)	(0.0995)	(0.0988)	(0.0995)
Female Respondent		-0.0697	-0.0676	-0.0910	-0.0929
		(0.0917)	(0.0917)	(0.0915)	(0.0916)
Financial Wealth		-0.0111	-0.00778	-0.00906	-0.00448
(Std. Dev.)		(0.0445)	(0.0445)	(0.0446)	(0.0446)
Risk Propensity (Std. Dev.)		0.295***	0.277***	0.291***	0.270***
		(0.0420)	(0.0421)	(0.0423)	(0.0426)
Long Term Propensity		0.0264	0.0350	0.0322	0.0372
(Std. Dev.)		(0.0418)	(0.0417)	(0.0419)	(0.0417)
MiFiD Responsiveness		-0.0135	-0.0144	-0.0310	-0.0305
(Std. Dev.)		(0.0477)	(0.0474)	(0.0475)	(0.0472)
Additional Socio-Economic Controls	NO	YES	YES	YES	YES
Product Purchased	NO	NO	NO	YES	YES
Year of Purchase: 2017			-0.0414		0.0546
			(0.183)		(0.188)
Year of Purchase: 2016			0.398*		0.445*
			(0.191)		(0.196)
Year of Purchase: 2015			0.372*		0.444*
			(0.184)		(0.190)
Year of Purchase: 2014			0.320+		0.381*
			(0.186)		(0.191)
Year of Purchase: 2013			0.0540		0.0907
			(0.210)		(0.214)
Year of Purchase: 2012			0.112		0.194
			(0.196)		(0.201)
Year of Purchase: 2011			-0.0354		0.0258
			(0.256)		(0.262)
Year of Purchase: 2010			-0.0582		-0.00759
			(0.254)		(0.256)
Year of Purchase: 2009			0.285		0.280
			(0.259)		(0.260)
Year of Purchase: 2008			-0.220		-0.200
			(0.352)		(0.352)
_cons	0.345***	0.586	0.329	1.136	0.0599
	(0.0764)	(0.870)	(0.896)	(0.919)	(0.933)
N	698	545†	545†	545†	545†
adj. R^2	0.054	0.163	0.179	0.185	0.200

Standard errors in parentheses

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

† sample decreases since some individuals refused report some relevant information.