FOOTNOTES

SOCIAL RISK AND FINANCIAL MARKETS*

by

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p. 361, last para. 1. This formulation (which assumes individuals are von Neumann-Morgenstern expected utility maximizers) does not allow states to be identified by any important personal characteristics (e.g. an individual's death—which would cause him to forego any claims for that contingency no matter how well society is endowed).

p. 361, last para. 2. In a one period model as above, $1 for certain would be a convenient numéraire, so prices could be normalized so that $\sum P_i = 1$. In a two (or more) period model, the time period in which the claims paid off would be important. If all of the claims pay off in the second period, and the prices are in terms of $1 for certain in the first period, $\sum P_i$ would equal $1/(1+r)$, where $r$ is the riskless one period rate of return.

p. 361, last para. 3. Risk discounts should not be confused with time discounts; comparison should be made between claims, contingent or certain, in the same time period. Risk premia or discounts can be expressed equivalently in terms of prices (as above) or rates of return. Obviously, if the price of a claim embodies a risk discount, the expected rate of return on an investment in the claim would embody a risk premium.

*This paper attempts to apply the analysis and various "well-known" results from the theory of portfolio choice and market equilibrium under risk to the topic of this session. In particular, we owe an intellectual debt to Sharp's seminal article on the valuation of risk and the more recent work of Cass and Stiglitz on separability.
4. For a demonstration that the risk premium goes to zero, see Arrow and Lind.

5. The supply of the $i^{th}$ contingent commodity after creation of bonds $(\gamma_i)$ is given by

$$\gamma_i = \left( \gamma_i - \frac{E\gamma S_i}{R_i} \right) \geq 0 \quad i = 1, S-1$$

The supply of bonds is of course

$$\gamma_B = \frac{E S \gamma}{R_B} = E S \gamma \sum_{i=1}^{S} \frac{R_i}{R_B}$$

6. Since this system of equations can be written in the form

$$A[\gamma] = -\frac{U_1}{U_2} \left[ R_i - R_B \right]$$

where $A$ is nonsingular.

7. A discussion of the necessary and sufficient conditions for this type of separation is provided by Cass and Stiglitz.

8. Arrow and Lind recognize that the business cycle may cause some correlation between the returns from public investment and other components of national income. They argue (p. 373), however, that the business cycle should be ignored as a source of risk; rather we should "assume that stabilization policies are successful [at maintaining full employment]." With the current state of political economy it would be foolish for either firms, or the government, to assume that the returns on its investments will, with certainty, be those which would be expected at full employment.
9. In the absence of social risk, the existence of private risk premia and risk-taking in the real world would be indication of the failure of financial (and insurance) markets to provide opportunities for the efficient diversification of private risk. Private risk premia, in that case, are irrelevant for evaluating government projects unless they happen to distribute returns to individuals in the same way as some particular private investment. If a government project has returns distributed randomly, or uniformly, across individuals, and it does not contribute to social risk, then no premium should be required. In general, distribution effects become an important part of the problem of evaluation in the presence of market imperfections.

We would like to comment on one of the cases Arrow and Lind discuss where there are significant risks associated with government investment. Consider a project where (possibly large) benefits (uncorrelated with other income) accrue to a small group of individuals and costs are borne publicly. Arrow and Lind suggest that a risk discount rate should be used on the benefits but that a certainty discount rate should be used on the costs incurred by the general public. They find it "somewhat ironic" (p. 378) that this procedure qualifies fewer projects than the usual procedure of using a risk discount rate for both benefits and costs. We're not certain whether or not it's ironic, but we are convinced it's good economics. The essence of the project is a proposal to transfer income from the general public to a subgroup. In a state preference context it can easily be shown that the proposed project involves transferring dollars from states with average expected utility per dollar for the general public to states which would, after project benefits were included, exhibit relatively low expected utility per dollar for the recipients. Since we usually presume it is possible to transfer certain dollars from the general public directly to a
subgroup without incurring risk costs, it would be inefficient to effect a transfer which was worth fewer dollars to the recipients than the dollar costs to the donors. Suppose government officials proposed to take one dollar from all taxpayers, package these dollars into several large prizes and then pass out (free) lottery tickets to a subgroup of the public. Risk averse recipients would score this program below a straight income redistribution program. This does not mean the subgroup wouldn’t prefer the handout to nothing. Rather it would be an inefficient technique for increasing the expected utility of the recipients, and we should, if we are interested in efficiency, expect our discounting techniques to disapprove of such arrangements.

In the same vein many analysts who have considered uncertainties in the costs of weapons systems have been puzzled about the necessity to use a lower than riskless discount rate on costs. Looked at in the state preference framework, high costs would mean lower than normal endowments in "unlucky" states. Hence the marginal utility for losses would be great and their costs high; a lower discount rate is appropriate.

p. 365, 2nd para.

10. There are a number of possible objections to this procedure: (1) It uses variance in annual, rather than "permanent," income. Only under rather special assumptions is variance in annual income a good proxy for variance in permanent income. (2) In removing a trend to take out the effect of labor force growth (presumably computations should be per capita) and anticipated growth in per capita income, we ignore the uncertainty associated with the growth rate. It can be argued that uncertainty in the growth rate is itself one of the major sources of risk, private and social. (3) The data period (1947-1965) excludes two events, the depression and WWII which would be major sources of variation.
11. The numbers should also be adjusted for taxes and transfers. This is not likely to have a substantial effect on corporate profits, but might substantially decrease the coefficient of variation for wages and salaries.

12. Tobin demonstrates this possibility for the quadratic utility function. Cass and Stiglitz provide the most complete discussion available of the circumstances in which such separation is possible.

13. If the requisite quantity of bonds are created, then the Modigliani-Miller theorem, stating that the value of a firm is independent of its debt structure, would hold. If for some reason (transaction costs, bankruptcy costs, manager's desire for control) the required quantity of bonds is not created, the "market value" of the firm (not counting these costs) would increase with an increase in the debt-equity ratio.

14. The qualification "it is in his interest" is important, for the ability to violate the rules or misrepresent the state of nature without detection is not by itself enough to remove the possibility of a mutually beneficial contract. Analytically, how can one tell whether or not a given moral hazard distortion will prevent an otherwise attractive contract from being consummated? To keep matters simple, suppose a risk-averse individual will, in the absence of a risk-spreading contract, earn $A$ with the probability $\pi$ and $B$ with probability $(1-\pi)$. Also suppose that if he enters into a contract for a certain income, the individual will reduce his effort so that his wage will be $(A-s)$ with probability $\pi$, and $(B-b)$ with probability $(1-\pi)$. Finally, suppose there's no social risk so that a contract
can be written which does not include a risk premium. We should compute
the expected utility of income with no contract, \( \pi U(A) + (1-\pi)U(B) \), and
compare it with the expected utility of the actuarial value of the con-
tact in the presence of the moral hazard distortion, \( C[\pi(A-a) + (1-\pi)(B-b)] \).

This calculation reveals a similarity between moral hazard and trans-
actions costs. The costs of moral hazard distortion \( \pi a + (1-\pi)b \) can
be thought of as a loading charge. Moreover, the presence of either cost
means the gains from trades are reduced, but given this constraint, the
market will achieve a second-best solution, undertaking only those contracts
which have sufficient surplus to cover the extra costs. The results would
be similar if the moral hazard takes the form of changing the probabilities
of \( A \) and \( B \).

Since transactions costs and moral hazard are important in limiting
the number of contracts, a natural question arises as to what circumstances
will provide the largest margins for absorbing loading charges. As Pauly
(p. 524) has pointed out in the context of insurance, contracts will be
more likely the greater the risk aversion to the loss. He has also asserted
that insurance is more likely to be purchased, the lower the probability
of a given size loss. This second assertion would appear to be incorrect.

Risk-averse individuals would purchase fair insurance; we have only
to determine whether the loading charge wipes out the expected utility surplus.
The loading charge is subtracted from the expected value of the laborer's
income to obtain the certain payment option. The certain payment will be
selected only if its utility is at least as great as the expected utility
of the laborer's income gamble. Of course, the largest loading charge con-
sistent with purchase of the insurance, will be obtained for the value of
π which maximizes the horizontal distance between the utility function and the line segment between the points \([A, U(A)]\) and \([B, U(B)]\). The exact value of \(\pi\), the probability of loss, depends on the shape of the utility function, but for risk-averse individuals, it will exceed one-half.

We make use of the expected utility apparatus to make one more point; namely, that unemployment compared to sharing the work is an inefficient arrangement for absorbing fluctuations in aggregate demand. Unemployment has the effect of imposing larger losses with a lower probability instead of smaller losses with a higher probability. If individuals are risk-averse, the large loss lottery is, for the same expected value, the inferior option.

p. 369, 2nd para. 15. There is at least one not so minor problem with this idea. Whenever an individual wishes to take an asset position with possibilities of capital loss and he has insufficient wealth to cover such eventualities, he may find it difficult to arrange a contract. Since economists' median income in 1985 is a short sale, the market will be concerned about collateral. How can an economist sell some of his future income without putting up assets which would guarantee that the contract can be fulfilled? Part of the collateral of course would be the median incomes of other workers which the economist buys. And we always have the possibility of margin calls if, for example, the market value of economists' median becomes greatly in excess of the value of the other income securities the economist holds in his portfolio. Perhaps a limit should be placed on the percentage of economists' median income that could be sold to keep the probability high that the economist could deliver out of his own income even in the event that such income turned out to be lower than anticipated.