REAL TIME MONITORING OF ASSET MARKETS: BUBBLES AND CRISES

By

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Real Time Monitoring of Asset Markets: Bubbles and Crises*

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Abstract

While each financial crisis has its own characteristics there is now widespread recognition that crises arising from sources such as financial speculation and excessive credit creation do inflict harm on the real economy. Detecting speculative market conditions and ballooning credit risk in real time is therefore of prime importance in the complex exercises of market surveillance, risk management, and policy action. This chapter provides an R implementation of the popular real-time monitoring strategy proposed by Phillips, Shi and Yu (2015a,b;PSY), along with a new bootstrap procedure designed to mitigate the potential impact of heteroskedasticity and to effect family-wise size control in recursive testing algorithms. This methodology has been shown effective for bubble and crisis detection (PSY, 2015a,b; Phillips and Shi, 2017) and is now widely used by academic researchers, central bank economists, and fiscal regulators. We illustrate the effectiveness of this procedure with applications to the S&P financial market and the European sovereign debt sector. These applications are implemented using the *psymonitor* R package (Phillips et al., 2018) developed in conjunction with this chapter.

Keywords: Bubbles, crises, real-time detection, recursive evolving test.

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1 Introduction

Speculative behavior and crises in the financial system can inflict serious harm on the real economy. Central banks, regulators, and policy makers therefore seek effective early warning devices of such episodes to assist in maintaining economic and financial stability. To meet the need for ongoing market surveillance, the recent literature on bubble detection has focused on real-time monitoring techniques rather than ex-post identification strategies which were emphasized in earlier research (see Gürkaynak (2008) for a review).

A practical real-time bubble detection method was proposed by Phillips, Shi and Yu (2015a,b; PSY hereafter) and has now been successfully employed as an early warning alert system for exuberance in a wide variety of financial, commodity, and real estate markets. For many of these diverse applications readers may usefully refer to the following papers: Bohl (2003); Etienne et al. (2014a,b); Gutierrez (2012); Pavlidis et al. (2016); Adämmer and Bohl (2015); Figuerola-Ferretti et al. (2015, 2016); Caspi et al. (2015); Caspi (2016); Shi et al. (2016); Phillips and Yu (2011, 2013); Greenaway-McGrevy and Phillips (2016); Hu and Oxley (2017a,b,c, 2018a,b). The potential of the PSY method has been recognised by central bank economists and fiscal regulators, as well as more widely in the financial industry and financial press. It is now employed by the Federal Reserve Bank of Dallas, providing an exuberance indicator for 23 international housing markets.[1] Researchers from many central banks, including the Hong Kong Monetary Authority (Yiu and Jin, 2013), the Central Bank of Colombia (Amador-Torres et al., 2018; Gomez-Gonzalez et al., 2018), and Bank of Israel (Caspi, 2016), have applied the PSY test to study real estate bubbles in their respective economies.

The PSY procedure serves as an early warning device for crises, as indicated in Phillips and Shi (2017). This capability has been noted in the many recent studies considering stock prices and exchange rates and other financial time series. See, Phillips et al. (2015a); Phillips and Shi (2017, 2018); Shi (2017); Deng et al. (2017); Yiu and Jin (2013); Fantazzini (2016); Hu and Oxley (2017b), among others.

The PSY procedure employs the augmented Dickey-Fuller (ADF) model specification and a recursive evolving algorithm. The recursive evolving algorithm relies only on historical information and permits a time-varying model structure. The method has general applications in regression. When

[1] See https://www.dallasfed.org/institute/houseprice.

applied to ADF regressions, the recursive evolving algorithm fixes the end point on the observation of interest and searches for the optimal starting point. As such, it minimizes the impact of previous episodes on the current identification and is less sensitive to the random choice of sample starting point. In effect, the method selects the most appropriate initialization for conducting a regression fit with given data, as considered in early work on econometric model determination (Phillips, 1996).

In detecting change and for bubble identification the recursive evolving algorithm has been shown to outperform the forward recursive algorithm (Phillips et al., 2011), the rolling window approach (Shi, 2007; Chong and Hurn, 2017), and the cusum monitoring strategy (Homm and Breitung, 2012). Unlike regime switching methods (Hall et al., 1999; Shi et al., 2016), it is a real-time procedure and easy to implement in practical work.

The identification of bubbles is based on their defining time series characteristics. During the expansionary phase of a bubble, asset prices follow a mildly explosive process as opposed to the martingale behavior that is typical during normal market conditions. In the event of a crisis or rapidly escalating credit risk, asset price (and hence bond yield) dynamics typically switch to a random drift martingale often accompanied by a large negative shock or a sequence of negative shocks. The PSY procedure which provides a joint test for the drift and the autoregressive coefficients of the ADF model is capable of detecting both bubbles and crises. The approach also delivers a mechanism for date stamping the origination and termination of bubbles. Consistency of the estimated bubble origination and termination dates was established in PSY (2015b) and Phillips and Shi (2018) under various data generating processes; and Phillips and Shi (2017)) proved consistency of the estimated switch date for crises.

Harvey et al. (2016) showed that the presence of heteroscedasticity can affect the performance of the forward recursive method of Phillips et al. (2011) and can lead to severe size distortions in testing. The same fragility to heteroskedasticity is expected for the PSY procedure. Several methods have been proposed to overcome this problem (Harvey et al., 2016, forthcoming, 2018). The wild bootstrap approach proposed by Harvey et al. (2016) has been found to have satisfactory asymptotic and finite sample performance. But an additional issue arises from the sequential nature of recursive hypothesis testing. It is well known that the probability of making false positive conclusions rises with the number of hypotheses tested, a phenomenon that is sometimes referred to as the *multiplicity* or *family-wise size control* issue in testing. This problem is common to all recursive testing procedures. In this chapter we propose a new bootstrap procedure that simultaneously addresses both heteroskedasticity and multiplicity issues in testing.

The PSY procedure is now a standard item in the econometric toolkit. Matlab, Eviews, and R software programs are available for practical implementation.[2] This chapter illustrates implementation of the methodology with a new R package that incorporate the bootstrap procedure for dealing with heteroskedasticity and multiplicity in recursive testing. With this software we apply the procedure to S&P 500 stock market data to detect both bubble and crisis episodes in the stock market and to the European sovereign debt market to detect episodes of escalating credit risk. The new R package (Phillips et al., 2018) is named *psymonitor* and can be installed with the following command sequence:

```
install.packages("psymonitor")
library(psymonitor)
```

The rest of the chapter is organized as follows. Section 2 introduces the PSY procedure. The rationale and limiting properties of the PSY procedure for bubble identification (crisis detection) are described and illustrated in Section 3 (Section 4). Section 5 introduces the new bootstrap procedure for accommodating heteroskedasticity and addressing multiplicity issues. Empirical applications to the S&P 500 market and the European sovereign market are given in Sections 6 and 7. Section 8 concludes.

2 The PSY Procedure

The PSY procedure was originally designed to identify and date stamp explosive periods in asset prices. Subsequent research (Phillips and Shi, 2017) has shown that the method has detective power against both speculative bubbles and market collapses, including flash crashes. The method is based on an ADF model specification for the fitted regression equation but uses flexible window widths in its implementation to take time-varying dynamics and structural breaks into consideration.

2.1 The Augmented Dickey-Fuller test

It is well known in the unit root literature that the limit distribution of the ADF statistic depends on both the null hypothesis and the precise regression

[2] See the website `https://sites.google.com/site/shupingshi/home/codes` for the Matlab codes, the *Rtadf* Eviews Addin (Caspi, 2017), and the *MultipleBubbles* (Araujo et al., 2018) and *exuber* (Vasilopoulos et al., 2018) packages in R.

model specification.[3] Appropriate choices of both therefore have a material impact in practical implementation.

The null hypothesis (H_0) of the PSY test captures normal market behaviors and states that asset prices follow a martingale process with a mild drift function such that (Phillips et al., 2014)

$$y_t = g_T + y_{t-1} + u_t, \tag{1}$$

where $g_T = kT^{-\gamma}$ (with constant k, $\gamma > 1/2$, and sample size T) captures any mild drift that may be present in prices but which is of smaller order than the martingale component and is therefore asymptotically negligible.

The regression model chosen for the PSY procedure is

$$\Delta y_t = \mu + \rho y_{t-1} + \sum_{j=1}^{p} \phi_j \Delta y_{t-j} + v_t, \tag{2}$$

where for implementation purposes the regression error v_t is assumed to satisfy $v_t \overset{i.i.d}{\sim} (0, \sigma^2)$. The p lag terms of Δy_t are included to take care of potential serial correlation. The lag order p is often selected by information criteria. The regression model includes an intercept but no time trend and nests the null hypothesis as a special case with $\mu = g_T$ and $\rho = 0$. The ADF statistic is simply the t-ratio of the least squares estimate of the coefficient ρ.

The *i.i.d* error condition may be replaced with a martingale difference sequence (mds) condition in (2). More general specifications on the error u_t in the generating mechanism (1), such as those in Assumption 1 below, may be employed and are accommodated by allowing the regression lag order $p \to \infty$ as $T \to \infty$ in (2). Nonparametric adjustments for serial correlation may also be used, such as those developed in Phillips (1987) and Phillips and Perron (1988).

Assumption 1 *The error term u_t is allowed to be serial correlated such that*

$$u_t = \psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j},$$

where $\sum_{j=0}^{\infty} j|\varphi_j| < \infty$ and ε_t is an mds satisfying:

(i) ε_t is strongly uniformly integrable with a dominating random variable η that satisifies $\mathbb{E}\left(\eta^2 \ln^+ |\eta|\right) < \infty$;

[3] See Hamilton (1994) for a textbook discussion and Phillips et al. (2014) for details in the context of bubble testing with localized drift specifications.

(ii) $T^{-1}\sum_{t=1}^{T}\mathbb{E}\left(\varepsilon_t^2|\mathcal{F}_{t-1}\right)\rightarrow_{a.s.}\sigma^2$, *where* $\mathcal{F}_t=\sigma\{\varepsilon_t,\varepsilon_{t-1}...\}$ *is the natural filtration.*

Under Assumption 1 ε_t is potentially conditionally heteroskedastic, as for instance under stable ARCH or GARCH errors. The partial sums of ε_t satisfy the functional law (Phillips and Solo, 1992)

$$T^{-1/2}\sum_{t=1}^{\lfloor Tr\rfloor}\varepsilon_t\Rightarrow\sigma W(r), \tag{3}$$

where W is standard Brownian motion, $\Rightarrow$ signifies weak convergence on the Skorohod space $D[0,1]$, and $\lfloor . \rfloor$ signifies the integer part of the argument.

Under the null hypothesis (1), Assumption 1, and regression (2) with side conditions that ensure $p\rightarrow\infty$, the ADF statistic has a limit distribution given by (Phillips et al., 2014)

$$ADF\Longrightarrow\frac{\frac{1}{2}\left[W(1)^2-1\right]-W(1)\int_0^1W(s)ds}{\left[\int_0^1W(s)^2ds-\left(\int_0^1W(s)ds\right)^2\right]^{1/2}},$$

where $\Longrightarrow$ denotes convergence in distribution on $\mathbb{R}$.

2.2 The Recursive Evolving Algorithm

The recursive evolving algorithm of PSY enables real-time identification of bubbles and crises while allowing for the presence of multiple structural breaks within the sample period. PSY (2015a,b) show that this algorithm is superior to the forward expanding and rolling window algorithms in bubble identification, especially when the sample period contains multiple bubbles.

For the convenience of exposition, we use the standard 'fraction of the total sample' notation for observations. Thus if $t=\lfloor Tr\rfloor$ is the integer part of Tr, then observation t is represented fractionally as observation r and then the total sample runs over values of r from 0 to 1. Suppose the observation of interest is $r^{\dagger}$. The PSY procedure calculates the ADF statistic recursively from a backward expanding sample sequence. Let r_1 and r_2 be the start and end points of the regression sample. The ADF statistic calculated from this sample is denoted by $ADF_{r_1}^{r_2}$. We fix the end point of all samples on the observation of interest such that $r_2=r^{\dagger}$ and allow the start point r_1 to vary within its feasible range, i.e. $[0,r^{\dagger}-r_0]$, where r_0 is the minimum window required to initiate the regression. The recommended setting of r_0

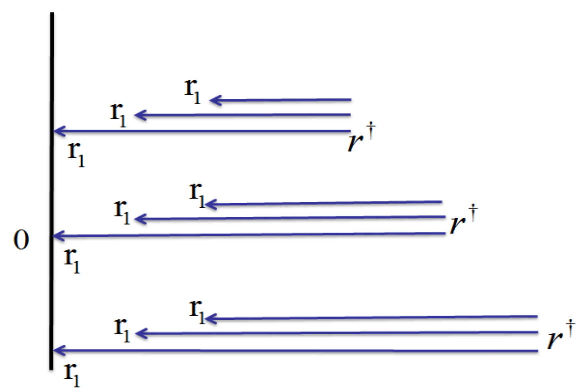
for practical implementation is $r_0 = 0.01 + 1.8/\sqrt{T}$. The PSY statistic is the supremum taken over the values of all the ADF statistics in the entire recursion, which is represented mathematically as

$$PSY_{r^\dagger}(r_0) = \sup_{r_1 \in [0, r^\dagger - r_0], r_2 = r^\dagger} \left\{ ADF_{r_1}^{r_2} \right\}.$$

The supremum enables the selection of the 'optimal' starting point of the regression in the sense of providing the largest ADF statistic.

The PSY test can be conducted for each individual observation of interest ranging from r_0 to 1, i.e. for $r^\dagger \in [r_0, 1]$. The recursive calculation evolves as the observation of interest moves forward and therefore the procedure is called a recursive evolving algorithm. See Figure 1 for a graphical illustration of the algorithm. The corresponding PSY statistic sequence is $\{PSY_{r^\dagger}(r_0)\}_{r^\dagger \in [r_0, 1]}$.

Figure 1: The recursive evolving algorithm with $r_1 \in [0, r^\dagger - r_0]$ and $r_2 = r^\dagger$.



Calculation of the PSY statistic sequence can be achieved with the command *PSY* contained in the *psymonitor* R package. This routine requires the input of data (y), a minimum window size (*swindow0*), and a choice of information criterion for the lag order selection (*IC* and *adflag*). The syntax of the call is

```
PSY(y,swindow0,IC,adflag).
```

IC has value 0 when a fixed lag order of *adflag* is used, 1 for use of an AIC lag order selector, and 2 for a BIC order selector. In the latter two cases, a maximum lag order *adflag* is employed in the information criteria.

Under the null hypothesis of normal market conditions and the conditions described earlier, the PSY statistic has the following limit distribution (PSY, 2015a)

$$\sup_{r_1 \in [0, r^\dagger - r_0], r_2 = r^\dagger} \left\{ \frac{\frac{1}{2} r_w \left[W(r_2)^2 - W(r_1)^2 - r_w \right] - \int_{r_1}^{r_2} W(s) ds \left[W(r_2) - W(r_1) \right]}{r_w^{1/2} \left[r_w \int_{r_1}^{r_2} W(s)^2 ds - \left(\int_{r_1}^{r_2} W(s) ds \right)^2 \right]^{1/2}} \right\}, \tag{4}$$

where $r_w = r_2 - r_1$.

The origination of a bubble or crisis episode is taken to be where the PSY test statistic first exceeds its critical value – a first stopping time for this episode. Likewise, the termination date is taken to be where the supremum test statistic subsequently falls below its critical value – a second stopping time for this episode. Suppose the sample contains only one episode originating at r_e and finishing at r_f. The estimated origination and termination dates (denoted by $\hat{r}_e$ and $\hat{r}_f$) are then given by the stopping times

$$\hat{r}_e = \inf_{r^\dagger \in [r_0, 1]} \left\{ r^\dagger : PSY_{r^\dagger}(r_0) > cv_{r^\dagger}(\beta_T) \right\}, \tag{5}$$

$$\hat{r}_f = \inf_{r^\dagger \in [\hat{r}_e, 1]} \left\{ r^\dagger : PSY_{r^\dagger}(r_0) < cv_{r^\dagger}(\beta_T) \right\}, \tag{6}$$

where $cv_{r^\dagger}(\beta_T)$ is the $100(1 - \beta_T)$ critical value (quantile of the distribution) of the $PSY_{r^\dagger}(r_0)$ statistic. The notation for test size β_T being sample size dependent allows for the property that $\beta_T \to 0$ as $T \to \infty$. This property in turn leads to $cv_{r^\dagger}(\beta_T) \to \infty$ under the null hypothesis, thereby ensuring that the probability of falsely detecting the presence of a bubble under the null passes to zero in large samples.

Estimation of the origination and termination dates are achieved by the *locate* function in the R package

```
locate(ind,date),
```

where *ind* is the vector of PSY indicators taking value one when the test statistic is above the critical value and zero otherwise and *date* is the vector of calendar dates associated with the observation.

3 The PSY Test for Bubble Identification

3.1 The Rationale

To illustrate the idea of bubble identification, consider the present value asset price formula

$$P_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+r_f} \right)^i \mathbb{E}_t \left(D_{t+i} \right) + B_t, \tag{7}$$

where P_t is the price of the asset, D_t is the payoff received from the asset, r_f is the risk-free interest rate, $\mathbb{E}_t(\cdot)$ is the conditional expectation operator given information to time t, and B_t is the bubble component. The bubble component satisfies the submartingale property (Diba and Grossman, 1988)

$$\mathbb{E}_t \left(B_{t+1} \right) = \left(1 + r_f \right) B_t. \tag{8}$$

In the absence of a bubble, the degree of nonstationarity of the asset price is controlled entirely by the dividend series and hence is believed from empirical evidence to be at most I(1). On the other hand, asset prices will be explosive in the presence of a bubble component in formula (7) whenever the initialization $B_0 > 0$ in (8).

Asset price dynamics over the expansionary phase of a bubble period may be modelled in terms of a mildly explosive process (Phillips and Yu, 2009; Phillips et al., 2011; Phillips and Magdalinos, 2007) of the form

$$\log P_t = \delta_T \log P_{t-1} + u_t, \tag{9}$$

where the autoregressive coefficient $\delta_T = 1 + cT^{-\eta}$ mildly exceeds unity (with $c > 0$ and $\eta \in (0,1)$) and yet still lies in its general vicinity. Detection of a bubble process in the data is therefore equivalent to distinguishing a martingale process of asset prices from a mildly explosive process. This can be achieved by the PSY procedure with null and alternative hypotheses specified as

$$\begin{aligned} H_0 &: \mu = g_T \text{ and } \rho = 0, \\ H_A &: \mu = 0 \text{ and } \rho > 0. \end{aligned}$$

3.2 Consistency

The data generating process (9) assumes the presence of an expansionary bubble over the entire sample period. In practice, bubbles exist only over subperiods and involve periods of collapse or contraction as well as expansion, thereby justifying the terminology. A given sample of data may include only martingale behavior, a single bubble episode set amidst martingale behavior on either side, or a sequence multiple bubble episodes interspersed amidst normal martingale behavior. An important task in the real-time dating literature is to demonstrate consistency of the estimated origination and termination dates of such bubble episodes.

The simplest example is a sample which contains a single bubble expansionary episode which does not terminate or collapse within the sample period. Specifically, asset prices follow a small drift martingale as in (1) before period $\tau_e = \lfloor r_e T \rfloor$ and then switch to a mildly explosive process as in (9), viz.,

$$\log P_t = \begin{cases} g_T + \log P_{t-1} + u_t & \text{if } t < \tau_e \\ \delta_T \log P_{t-1} + u_t & \text{if } t \geq \tau_e \end{cases}. \tag{10}$$

This DGP can be extended to include the bubble collapse dynamics. Various patterns of collapse have been considered in the literature. Suppose the end date of the bubble episode is $\tau_f = \lfloor r_f T \rfloor$. Phillips et al. (2011) proposed an abrupt bubble collapse pattern where asset prices return immediately to the level before the bubble origination allowing for a stationary perturbation, so that

$$\log P_{\tau_f} = \log P_{\tau_e - 1} + O_p(1). \tag{11}$$

Phillips and Shi (2018) recommended a mildly integrated reversion pattern for observations in the collapsing regime in which prices follow the mechanism

$$\log P_t = \gamma_T \log P_{t-1} + u_t, \tag{12}$$

where the autoregressive coefficient $\gamma_T = 1 - c_1 T^{-\beta}$ is smaller than unity ($c_1 < 0$ and $\beta \in (0, 1)$). By varying the value of β, this process can generate abrupt, randomly disturbed, or smooth patterns of collapse behavior.

Under the data generating process (10), PSY (2015b) show that the PSY test statistic has order of magnitude $O_p(1)$ if the observation of interest $r^\dagger$ falls in the normal regime and diverges to positive infinity at the rate $O_p(T^{1/2}\delta_T^{\tau^\dagger - \tau_e})$ with $\tau^\dagger = \lfloor r^\dagger T \rfloor$ if the observation lies in the bubble regime. For observations in the collapse regime, the PSY statistic diverges to negative infinity at the rate $O_p(T^{(1-\eta)/2})$ when the bubble collapses in the fashion of (11) or $O_p\left(T^{\omega(\eta,\beta)}\right)$[4] when the collapse process is (12). It

[4] $\omega(.)$ is a linear function of the arguments.

transpires that under the condition that test size $\beta_T \to 0$ as $T \to \infty$ and

$$\frac{1}{cv_{r^\dagger}(\beta_T)} + \frac{cv_{r^\dagger}(\beta_T)}{T^{1/2}\delta_T^{\tau^\dagger - \tau_e}} \to 0,$$

we have the consistency of the estimated bubble origination and termination dates, i.e., $\hat{r}_e \to r_e$ and $\hat{r}_f \to r_f$.

The process can be generalized to allow for the presence of multiple bubbles. Consistency of the estimated bubble origination and termination dates in the presence of multiple bubbles was shown by PSY (2015b) for the DGP with the abrupt collapsing pattern (11) and by Phillips and Shi (2018) with the mildly integrated reverting pattern (12).

4 The PSY Test for Crisis Identification

4.1 The Rationale

Market crashes are defined as a discontinuity in asset prices that is characterized by large downward movements (Gennotte and Leland, 1990; Barlevy and Veronesi, 2003). The dynamics of asset prices during crisis periods may be modeled as a random drift martingale process (Phillips and Shi, 2018)

$$\log P_t = -L_t + \log P_{t-1} + u_t, \tag{13}$$

in which L_t is a random sequence independent of u_t. The sequence L_t produces a random drift in the observed price process and L_t may take various forms, which lead to a corresponding variety of collapse mechanism. The simple process used in Phillips and Shi (2018) follows an asymmetric scaled uniform distribution such that

$$L_t = Lb_t, \qquad b_t \overset{iid}{\sim} U[-\epsilon, 1], \qquad 0 < \epsilon < 1.$$

where L is a positive scale quantity measuring shock intensity and b_t is uniform on an interval ranging from a (usually small) negative value $-\epsilon$ to unity. The mean of the drift term takes a negative value (i.e., $-(1-\epsilon)L/2$) and hence the process exhibits an overall downward trend. The magnitude of this downward trend depends on the values of the scalar parameters L and ϵ.

Suppose P_t is the price of a stock and the logarithmic dividend is a martingale with drift generated as

$$\log D_t = \alpha + \log D_{t-1} + v_t, \tag{14}$$

where α is a constant and the v_t are mds innovations. Under the price process (13), the logarithmic price-dividend ratio also follows a random drift martingale process of the form

$$\log P_t/D_t = -L_t^* + \log P_{t-1}/D_{t-1} + \varepsilon_t^*, \tag{15}$$

where $L_t^* = Lb_t^*$ with $b_t^* \sim_{iid} U[-\epsilon + \alpha/L, 1 + \alpha/L]$ and $\varepsilon_t^* = \varepsilon_t - v_t$.

Suppose P_t is the price of a τ-period discount bond. The relationship between the continuously compounded zero-coupon nominal yield to maturity (z_t) and the bond price is

$$z_t = -\frac{\log P_t}{\tau}.$$

Bond yields often serve as a proxy for credit risk. A loan default or other credit events may trigger a sharp decline in bond prices and hence a fast expansion in bond yields. Under the assumption of a bond price crash (13), bond yields follow

$$z_t = \frac{1}{\tau}L_t + z_{t-1} - \frac{u_t}{\tau}. \tag{16}$$

The drift term has a positive mean of $(1 - \epsilon) L/(2\tau)$, implying an overall upward trend in the dynamics.

In the setting of this model detecting financial crises or ballooning credit risk is equivalent to distinguishing a martingale process with a small drift (null) from a random-drift martingale process (alternative). The null and alternative hypotheses of the PSY test for crises may now be formulated in terms of the fitted ADF regression equation (2) as follows

$$\begin{aligned} H_0 &: \mu = g_T \text{ and } \rho = 0 \\ H_{1,crash} &: \mu = K \text{ and } \rho = 0. \end{aligned}$$

where K is the expected value of the random drift process L_t and g_T is an asymptotically negligible deterministic drift as in (1).

4.2 Consistency

The specification (13) can be modified to switch on or off depending on the financial environment. The data generating process for asset prices considered in Phillips and Shi (2017) is

$$\log P_t = \begin{cases} g_T + \log P_{t-1} + u_t & \text{if } t < \tau_e \\ -L_t + \log P_{t-1} + u_t & \text{if } t \geq \tau_e \end{cases}. \tag{17}$$

The origination of the event is denoted again by τ_e. Suppose that P_t is the price of a discount bond. It is straightforward to show that the bond yield z_t follows a stochastic process that switches between a martingale with a small deterministic drift and a martingale with a positively scaled random drift

$$z_t = \begin{cases} -\frac{1}{\tau} g_T + z_{t-1} - \frac{u_t}{\tau} & \text{if } t < \tau_e \\ \frac{1}{\tau} L_t + z_{t-1} - \frac{u_t}{\tau} & \text{if } t \geq \tau_e \end{cases}. \tag{18}$$

Phillips and Shi (2017) show that under the DGP (17), the PSY test statistic diverges to positive infinity at the rate $O_p(T^{1/2})$ as the test proceeds from the normal regime to the crash regime. It follows that the PSY procedure can consistently estimate the break date τ_e when

$$\frac{1}{cv_{r^\dagger}(\beta_T)} + \frac{cv_{r^\dagger}(\beta_T)}{T^{1/2}} \to 0.$$

5 A New Composite Bootstrap

The bootstrap procedure described here combines the two procedures of Harvey et al. (2016) and Shi et al. (2018). It is designed to mitigate the potential influence of unconditional heteroskedasticity and to address the multiplicity issue in recursive testing. Let $\tau_0 = \lfloor Tr_0 \rfloor$ and τ_b be the number of observations in the window over which size is to be controlled.

Step 1: Using the full sample period, estimate the regression model (2) under the imposition of the null hypothesis of $\rho = 0$ and obtain the estimated residual e_t.

Step 2: For a sample size $\tau_0 + \tau_b - 1$, generate a bootstrap sample given by

$$\Delta y_t^b = \sum_{j=1}^{p} \hat{\phi}_j \Delta y_{t-j}^b + e_t^b \tag{19}$$

with initial values $y_i^b = y_i$ with $i = 1, \ldots, j+1$, and where the $\hat{\phi}_j$ are the OLS estimates obtained in the fitted regression from Step 1. The residuals $e_t^b = w_t e_l$ where w_t is randomly drawn from the standard normal distribution and e_l is randomly drawn with replacement from the estimated residuals e_t.

Step 3: Using the bootstrapped series, compute the PSY test statistic sequence $\{PSY_t^b\}_{t=\tau_0}^{\tau_0+\tau_b-1}$ and the maximum value of this test statistic sequence, giving

$$\mathcal{M}_t^b = \max_{t \in [\tau_0, \tau_0 + \tau_b - 1]} \left(PSY_t^b \right).$$

Step 4: Repeat Steps 2-3 for $B = 999$ times.

Step 5: The critical value of the PSY procedure is now given by the 95% percentiles of the $\left\{ \mathcal{M}_t^b \right\}_{b=1}^B$ sequence.

Step 2 of this iteration implements a wild bootstrap to address heteroskedasticity; and Steps 3-5 of the iteration replicate the PSY recursive test sequence and create critical values that account for multiplicity in the test sequence recursion.

The bootstrap procedure can be implemented with the following call syntax in R with the package

```
cvPSYwmboot(y,swindow0,IC,adflag,Tb,nboot,nCores),
```

where the argument Tb corresponds to τ_b, *nboot* is the number of bootstrap repetitions, and $nCores$ is the number of cores used for the calculation. The other arguments are the same as those described earlier.

6 Empirical Applications with R

6.1 Example 1: The S&P 500 Market

The S&P 500 stock market has been a central focus of attention in global financial markets due to the size of this market and its impact on other financial markets. As an illustration of the methods discussed in this chapter, we conduct a pseudo real-time monitoring exercise for bubbles and crises in this market with the PSY strategy. The sample period runs from January 1973 to July 2018, downloaded monthly from *Datastream International*. The price-dividend ratios are computed as the inverse of dividend yields. The first step is to import the data to R, using the following code:

```
sp500 <- read.csv("sp500.csv")
date <- as.Date(sp500[,1],"%d/%m/%Y")
dy <- sp500[,2]
pd <- 1/dy
```

In the presence of a speculative bubble, asset prices characteristically deviate in an explosive way from fundamentals, representing exuberance in the speculative behavior driving the market. In the present case, this deviation implies that the log price-dividend ratio is expected to follow an explosive process over the expansive phase of the bubble. But during crisis periods, the price-dividend ratio is expected to follow a random (downward) drift martingale process, in contrast to a small (local to zero) constant drift martingale process that typically applies under normal market conditions. According to the theory detailed in Section 3 and 4, we expect to witness rejection of the null hypothesis in the PSY test empirical outcomes during both bubble and crisis periods.

Figure 2 plots the price-to-dividend ratio of the S&P 500 index. We observe a dramatic increase in the data series in the late 1990s, followed by a rapid fall in the early 2000s. The market experienced another episode of slump in late 2008. With a training period of 47 observations, we start the pseudo real-time monitoring exercise from November 1976 onwards. The PSY test statistics are compared with the 95% bootstrapped critical value. The empirical size is controlled over a two-year period, i.e., by taking $\tau_b = 24$. The lag order is selected by BIC with a maximum lag order of 6, applied to each subsample. The PSY statistic sequence and the corresponding bootstrap critical values can be calculated as follows in R.

```
y<-pd
obs<-length(y)
r0<-0.01+1.8/sqrt(obs)
swindow0<-floor(r0*obs)
dim<-obs-swindow0+1

IC<-2
adflag<-6
yr<-2
Tb<-12*yr+swindow0-1
nboot<-999
nCore<-2

bsadf<-PSY(y,swindow0,IC,adflag)
quantilesBsadf<-cvPSYwmboot(y,swindow0,IC,adflag,Tb,nboot,nCore)
```

The identified origination and termination dates can be calculated and viewed with the following commands.

```
date<-date[swindow0:obs]
quantile95<-quantilesBsadf%*%matrix(1,nrow=1,ncol=dim)
ind95<-(bsadf>t(quantile95[2,]))*1

OT <-locate(ind95,date)
BCdates<-disp(OT,obs)
print(BCdates)
```

where the last two command syntax print the dates on the screen with the first (second) column being the origination (termination) date. The outputs are

```
       start        end
1 1986-05-30 1986-06-30
2 1987-07-31 1987-08-31
3 1996-01-31 1996-01-31
4 1996-05-31 1996-05-31
5 1996-11-29 1997-02-28
6 1997-04-30 1998-07-31
7 1998-09-30 2000-10-31
8 2000-12-29 2001-01-31
9 2008-10-31 2009-02-27
```

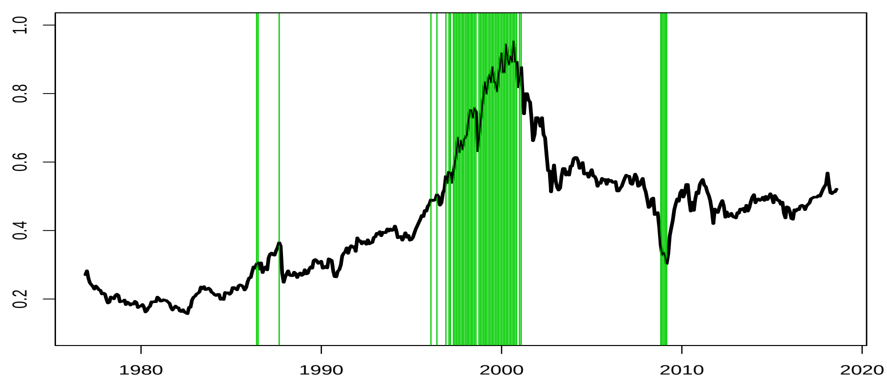
The identified periods are shaded in green in Figure 2. As is evident in the figure, the procedure detects two bubble episode and one crisis episode. The first bubble episode only lasts for three months (1986M05-M06 and 1987M08) and occurred before the Black Monday crash on October 1987. The second bubble episode is the well-known dot-com bubble, starting from January 1996 and terminating in October 2000 (with several breaks in between). For the dot-com bubble episode the identified starting date for market exuberance occurs well before the speech of the former chairman of the Federal Reserve Bank Alan Greenspan in December 1996 where the now famous question 'how do we know when irrational exuberance has unduly escalated asset values' was posed to the audience and financial world. The identified subprime mortgage crisis starts in October 2008, which is one month after the collapse of Lehman Brothers, and terminates in February 2009.

The codes for generating the plot and shaded overlays in the figure are as follows.

```
plot(date,y[swindow0:obs],xlim=c(min(date),max(date)),ylim=c(0.1,1),
xlab='',ylab='',type='l',lwd=3)
```

```
for(i in 1:length(date)){
        if (ind95[i]==1){abline(v=date[i],col=3)}}
points(date,y[swindow0:obs],type='l')
box(lty=1)
dev.off()
```

Figure 2: Bubble and crisis periods in the S&P 500 stock market. The solid line is the price-to-dividend ratio and the shaded areas are the periods where the PSY statistic exceeds its 95% bootstrapped critical value.



6.2 Example 2: Credit Risk in the European Sovereign Sector

The European sovereign debt sector experienced an extremely turbulent period over the last decade, which caused significant harm to the real economy (Acharya et al., 2018) and led to an unprecedented level of unemployment (Karafolas and Alexandrakis, 2015). The PSY detection algorithm can serve as a useful early warning mechanism for escalating credit risk, which is acknowledged as a leading indicator of financial and economic crises, and thereby enable timely policy action and effective risk management to avert more serious economic damage. To show the potential efficacy of this early warning system, we conduct a pseudo monitoring exercise of credit risk in the European sovereign sector.

Credit risk in the European sovereign sector is proxied by an index constructed as a GDP weighted 10-year government bond yield of the GIIPS

(Greece, Ireland, Italy, Portugal, and Spain) countries.[5] The PSY strategy is applied to the spread between the GIIPS bond yield index and the 10-year government bond yield of Germany (used as a proxy for a prevailing benchmark of economic fundamentals). The sample data runs from June 1997 to June 2016 and was downloaded from *Datastream International*. The GDP data are downloaded quarterly and converted to a monthly frequency by assuming a constant value within each quarter.

```
data <- read.csv("spread.csv")
date <- as.Date(data[,1],"%d/%m/%Y")
spread <- data[,2]
y<-spread
```

Figure 3 plots the bond yield spread over the sample period. The bond yield index experienced a rapid and substantial rise between 2008-2009. It continued to mount to historical highs from 2010 onwards and peaked in June 2012. The bond yield index has dropped since then and becomes relatively stable over the last two years. The codes for implementing the PSY procedure are identical to those for Example 1. The estimated start and end dates of the crisis episodes are displayed below.

```
       start        end
1 2008-03-23 2008-03-23
2 2008-10-23 2009-03-23
3 2010-05-23 2012-08-23
```

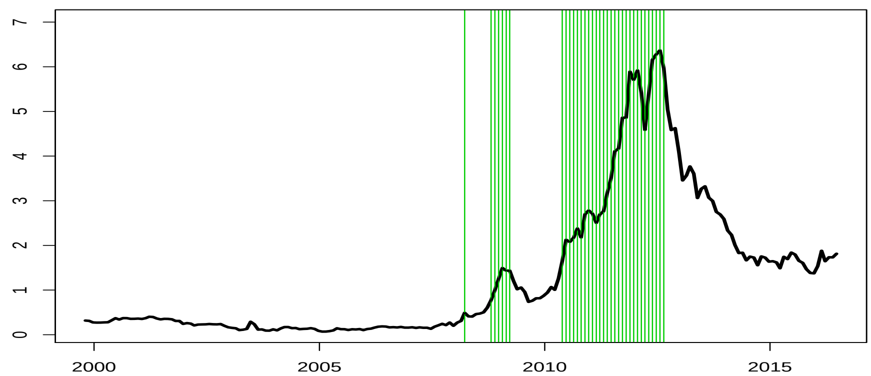
The shaded areas in Figure 3 are the identified periods of crisis obtained using the 95% bootstrap critical values. The first alarm signal of risk appeared in March 2008 and lasts for one month. The alarm was triggered again after the collapse of Lehman Brothers in October 2008 and turns off in March 2009. The stress indicator switched on again in May 2010 and lasted until August 2012.

7 Conclusion

The recursive evolving test algorithm proposed by Phillips, Shi and Yu (2015a,b) provides a real-time empirical device for detecting speculative bubbles, crises, and ballooning credit risks that can foreshadow impending damage to the real economy. The multi-functionality and the real-time

[5]These are the five EU countries that were unable to refinance their government debt or to bail out banks on their own during the debt crisis.

Figure 3: Crisis episodes in the European sovereign sector. The solid line is the 10-year government bond yield spread between the GIIPS countries and Germany and the shaded areas are the periods where the PSY statistic exceeds its 95% bootstrap critical value.



features of this algorithm assist policymakers in market surveillance and investors in risk management. The approach has enjoyed widespread use in academic circles and among central bank economists.

This chapter overviews the main features of the PSY approach and details a new combined bootstrap procedure for dealing with both heteroskedasticity and multiplicity issues in recursive inference methods. A new R package *psymonitor* (complete with the combined bootstrap procedure) is provided for convenient implementation of the methods. For empirical illustration of the use of the R codes, the procedures are applied to the S&P 500 stock market for the detection of bubbles and crises and to the European sovereign debt sector for detection of ballooning credit risks. We hope that the R package and these illustrations[6] will assist in making these methods widely available to empirical researchers, industry economists, and policy makers.

[6]For more illustrations, see `https://itamarcaspi.github.io/psymonitor/`.

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