THE BEST AND WORST OF ALL POSSIBLE WORLDS: SOME CRUDE EVALUATIONS

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Abstract

The 2×2 matrix game plays a central role in the teaching and exposition of game theory. It is also the source of much experimentation and research in political science, social psychology, biology and other disciplines. This brief paper is addressed to answering one intuitively simple question without going

into the many subtle qualifications that are there. How efficient is the non-cooperative equilibrium? This is part of a series of several papers that address many of the qualifications concerning the uses of the 2 X 2 matrix games.
JEL Classifications: C63, C72, D61
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1 The 2×2 Matrix Games with Cardinal Payoffs

In the folklore and elementary education of game theory the 2×2 matrix game has played a special role. Several of these games bear special names such as The Prisoner's Dilemma, The Stag Hunt, and the Battle of the Sexes. There are only 144 strategically different 2×2 games with strictly ordinal preferences. We are often interested in considering related games with cardinal preferences and a moment's consideration shows that there is an indefinite number of these games. Many applications make it desirable to examine a large but finite set of 2×2 games with specific numbers of highly different sizes. In a different paper we suggest how to do this [4]. This paper utilizes all the 144 games listed in Appendix 2 as they serve to give a sufficiently exhaustive coverage of all 2×2 games to be able to get a feeling for the efficiency of noncooperative and some other behaviors without delving into a more refined apparatus as we do in [11].

2 Outcome Sets

The 2×2 matrix game with strictly ordinal payoffs may be cardinalized by $4 > 3 > 2 > 1$. We may study the strictly ordinal set of games utilizing just the symbols $1, 2, 3, 4$ [1].

Two of the top desirable properties of a good society are efficiency and fairness, they can only be defined after the assumptions concerning preferences have been made.

2.1 Fairness and efficiency

Fairness cannot be defined before the initial conditions are spelled out. The full specification of initial conditions requires the assumptions made about innate property rights of individuals. Symmetry involves the consideration of intrinsic property rights. These are discussed elsewhere[3], [6],[1],[5],. Here we concentrate only on efficiency,which amounts to considering how close actual outcomes are to a jointly maximal payoff.

[1] or a, b, c, d or any other icons.

3 Behavior and Structure

We consider and contrast several other behaviors beyond noncooperative behavior. Metaphorically we consider four player types described as: The individualist; the pessimist; the optimist or idealist; and the know-nothing, zero-intelligence or fool or entropic player.

The **Entropic Player** : The Entropic player is the player that always acts or chooses a strategy uniformly at random[2].

MaxMax($\mathbf{P}_1$+$\mathbf{P}_2$): The MaxMax Player is the player who always chooses the strategy for which the maximum social welfare is achieved. Call her **MaxMax** or the **Utopian Player.** she acts as if she will do the right thing and knows the other will do so[3]. The player makes one simple inference about what the the other player will do.

Max$_s$Min$_{\bar{s}}$P$_1$: is defensive he assumes the other side is out to damage him. We may call the players **MaxMin** or pessimistic.

A N**oncooperative Equilibrium player** or **NEP** assumes that she faces a player motivated like herself. This behavior can be described by two parallel maximizing equations [4].

We consider a pair of noncooperative players playing each other and also contrast their performance with three other pairs of player types playing four specially named games, We then consider all 144 games in aggregate and broken into four structural categories now noted.

We know that in the initial resource distribution we may break all 144 games into four natural categories

Joint maximum	Frequency
8	36
7	60
6	42
5	6

Table 1
Frequency of maximal wealth

The 36 games with a joint wealth of 8 each can be called 'Games of coordination'. There is a single cell with value $(4, 4)$ that is a natural point of attraction.

The 6 games with joint maximum equal to 5 are games of pure opposition. There is no potential opportunity for individual gain from collaboration. The games with a joint maximum of 6 or 7 are mixed motive games where gains can be made by

[2] He is a syntactic agent whose distinguishes structure but is unable to interpret content or the semantics.

[3] If there is more than one joint maximum a selection rule is needed.

[4] As is noted in [4] there are several tehnical details that must be considered concerning how to handle multiple equilibria. We need to obtain higher and lower bounds on eficiency,

coordination and collaboration. We note that the modal wealth of the four types is 7.

3.1 Efficiency and behavior

The work presented here on all possible worlds is made under a considerable heap of assumptions with no comment on the trade-offs between efficiency and symmetry. This problem is considered elsewhere. Furthermore we consider only 'pure populations' where agents are matched only against agents of the same type. Except for one illustrative example, players of different types being matched against each other are considered elsewhere [?], [12]. The mixed, more complex approach is congenial with evolutionary game theory[2].[5]

3.1.1 Inference and pure believers

We carry out our evaluation for the pairs of four types on four specific named games that we call prisoner's dilemma, stag hunt, battle of the sexes and best of all possible 2×2 worlds and then on all of the 144 games categorized into several segments as noted below. In doing so contrast between structure and behavior emerge as do contrasts among different structures and behaviors.

The first game is the prisoner's dilemma

	1	2
1	$3, 3$	$1, 4$
2	$4, 1$	$2, 2$

Table 2
Prisoner's dilemma

Two numbers are given in each cell in the table below. They are the payoff to the row player , then column player.

	P D
NCE,NCE	$(2, 2)$
ENT,ENT	$(2.5, 2.5)$
MAXMAX,MAXMAX	$(3, 3)$
MAXMIN,MAXMIN	$(2, 2)$

Table 3
PD games
Table 3

[5] There are several excellent texts on evolutionary game theory such as [13] that approach game dynamics in terms of competition among species rather than conscious optimization.

We see immediately Maxmax$\succ$Entropy$\succ$Maxmin$\sim$NCE. Thus do-gooders are first with 6, next random with 5, followed by the cautious and individualist with 4 in this structurally worst 2×2 world.

The second game is the stag hunt

	1	2
1	4, 4	1, 3
2	3, 1	2, 2

Table 4

Stag hunt

There is a coordination problem with the NCE for both the battle of the sexes and stag hunt. As they have two NCEs and the battle of the sexes has two JM as well, for each some form of tie-breaking rule is required. This is not a problem in pure logic. It requires an extra assumption. The difficulty can be avoided here by defining and considering three types of NCE players $\mathbf{NCE}_1$, $\mathbf{NCE}_2$, and $\mathbf{NCE}_3$, one for each equilibrium outcome and similarly $\mathbf{JM}_1$ and $\mathbf{JM}_2$ for different joint maxima. Reporting for the illustrative calculations both the higher and lower NCE we note

	S H
NCE_1,NCE_1	(4, 4)
NCE_2,NCE_2	(2, 2)
ENT,ENT	(2.5, 2.5)
MAXMAX,MAXMAX	(4, 4)
MAXMIN,MAXMIN	(2, 2)
ENT,ENT	(2.5, 2.5)
MAXMAX,MAXMAX	(4, 4)
MAXMIN,MAXMIN	(2, 2)

Table 5

	1	2
1	4, 4	1, 2
2	2, 1	3, 3

Table 6

Stag hunt with the NCE players NCE_1, NCE_2.

We see immediately Individualist type 1 and idealists are tied at first, fools or entropic players are second and last are individualists type 2 and the cautious. Maxmax$\sim NCE_1$ $\succ$Entropy$\succ$Maxmin$\sim NCE_2$.

The third game is the battle of the sexes, again there is a problem with two PSNE

	1	2
1	4, 3	1, 2
2	2, 1	3, 4

Table 7

Battle of the sexes

	BS: mean
NCE_1,NCE_1	$(4,3)$
NCE_2,NCE_2	$(3,4)$
ENT,ENT	$(2.5,2.5)$
$MAXMAX_1$,$MAXMAX_1$	$(4,3)$
$MAXMAX_2$,$MAXMAX_2$	$(3,4)$
MAXMIN,MAXMIN	$(3,4)$

Table 8
Battle of the sexes

The do-gooders, individualists and cautious are all optimal and fools are last. $\text{Maxmax}_1 \sim \text{Maxmax}_2 \sim \text{NCE}_1 \sim \text{Maxmin} \sim \text{NCE}_2 \succ \text{Entropy}$.

The fourth game is the best of all possible worlds game

	1	2
1	$4,4$	$3,2$
2	$2,3$	$1,1$

Table 9
Best of all possible 2×2 worlds

We see immediately that the individualists, do-gooders and conservatives are equal and first and fools last in a best of all possible 2×2 worlds

	BOPW
NCE,NCE	$(4,4)$
ENT,ENT	$(2.5,2.5)$
MAXMAX,MAXMAX	$(4,4)$
MINMAX,MINMAX	$(4,4)$

Table 10

	1	2
1	$4,4$	$3,2$
2	$2,3$	$1,1$

Table 14

Indvidualists, cautious and do-gooders are first and fools last in the best of all possible 2×2 worlds.

Maxmax$\sim$NCE$\sim$Maxmin$\succ$Entropy

The summation of rankings is

Games	Domination
PD	JM$\succ$ENT$\succ$NC$\succ$CAUT
Stag hunt	JM$\sim$$NCE_1$ $\succ$ENT$\succ$CAUT$\sim$$NCE_2$
BS	JM_1 $\sim$$JM_2$ $\sim$$NCE_1$ $\sim$CAUT$\sim$$NCE_2$ $\succ$ENT
BOPW	JM$\sim$NCE$\sim$CAUT$\succ$ENT

Table 15

PD = Prisoner's dilemma; BS = Battle of sexes; BOPW = Best of all possible worlds

3.1.2 Some lessons on structure and behavior

The efficiency indices for the four games above appear to show

- In the best of all possible worlds structure guides all pairs of players to Optimality except for the fools.
- In the PD or worst of all possible worlds the idealists do well (but it is knife-edged as noted below), the ignorant are next best and the 'rational' and cautious actors have the worst outcome. Structure goes against individualistic behavior.
- When there is the possibility of more than one equilibrium the problems of coordination become critical. If all are behaviorally cooperative a convention is still needed for coordination

We enlarge our evaluation for the combinations of four types of players matched against all types just for the PD game to show the new phenomena that appear with heterogeneous players [6].

	P D
NCE,NCE	$(2, 2)$
NCE,ENT	$(3, 1.5)$
NCE,MAXMAX	$(4, 1)$
NCE,MAXMIN	$(2; 2)$
ENT,ENT	$(2.5, 2.5)$
ENT,MAXMAX	$(3.5, 2)$
ENT,MINMAX	$(1.5, 3)$
MAXMAX,MAXMAX	$(3, 3)$
MAXMAX,MAXMIN	$(1, 4)$
MAXMIN,MAXMIN	$(2, 2)$

Table 16

We see immediately the NCE versus ENT or JM or NCE or MinMax has an average yield of $((2 + 3 + 4 + 2) /4) = 11/4$, ENT gives $((1.5 + 2.5 + 3.5 + 1.5) /4) = 9/4$, JM or MaxMax gives $((1 + 2 + 3 + 1) /4) = 7/4$ and MinMax or cautious yields

[6] In a one shot game each individual does not have the opportunity to learn about the player type she faces. A reasonable prior is that it is someone similar; alternatively one might wish to contrast true belevers with agnostics where all have the common belief that the competitor could, with equal probability be any of the four types (note [1]).

$(2+2+4+2) = 10/4$. Thus selfish is first with 2.75, cautious next with 2.5, followed by random with 2.25 and do-gooders last with 1.75 in this structurally worst 2×2 world.

We note that depending on structure and the behavior of others, fools may gain or lose from the damage of their blunders Here the fools hurt themselves against realists or pessimists, but gain considerably if they play against idealists or do-gooders.

3.1.3 A note on viability and environment

The purpose of these tedious but simple calculations (at least to us) is to show both the gains and the dangers from going between the descriptive words and their simple mathematical representations. If not interpreted too literally they can show when fools hurt themselves more or less than they hurt others. They show where the idealists shine in the best of all possible worlds but can be taken advantage of otherwise. In even these few instances there appear to be examples of 'anything goes'. In the context of one period with no social learning the individualistic and the cautious (maxmin) behaviors appear as the most viable. In Section 3.2.3 and Appendix 1 we make this more precise over all 2×2 games.

The four games were structurally highly different and we could safely observe that the viability of the different agents varied with the environment, and we considered different types of agent matched against each other.

In our last set of comparisons we consider the whole set of $144 - 6$ games that can be considered as societies with joint gain available

We limit ourselves to two crude measures. The first uses the ratio of the joint outcome achieved to the joint maximum that is feasible.

$$Index_1 = \frac{OUTCOME}{JM}$$

The second uses as a basis a zero point of worst joint sum feasible and takes the ratio

$$Index_2 = \frac{OUTCOME - WORST}{JM - WORST}$$

We do not dwell on the many index construction problems here[7]. We nevertheless make a case for crude estimates of the gap between different behaviors and an ideal collaboration. Appendix 1 gives the calculations for all 144 games for joint maximum, the best and worst noncooperative equilibria, the maxmin or pessimist players and the entropy players.

[7] We discuss better measures involving both efficiency and symmetry elsewhere [4] that provide simple upper and lower bounds and note a somewhat more sophisticated index involving in its determination of a zero point the Nash axioms for noncomparable but measurable utility.

3.2 Efficiency Measures in All 2×2 games

The summary of the final calculations given in Appendix 1 are presented here broken into the four natural classes then breaking them into the three classes $n = 8, 7, 6$ and 5.

3.2.1 Games of coordination: $n = 8$

There are 36 games of coordination where each pair has the opportunity to select an outcome of $(4, 4)$.

	JM	NCEH	NCEL	MXMN	ENT	WORST
Total	288	288	264	224	180	144
Eff_1	1	1	.92	.78	.63	.50
Eff_2	1	1	.83	.56	.25	0

Table 17
Games of coordination

The structure offers a natural signal for optimal coordination, but with Index_1 on the average the worst noncoperative equilibrium misses by 8%. The cautious leave 22% and the fools 37%.

The Index_2 is possibly more reasonable inasmuch as it is anchored on the gains above the worst outcome. On the average the worst noncooperative equilibrium misses by 17%. The cautious leave 44% and the fools 75%.

3.2.2 Mixed motive games 1: $n = 7$

There are 60 mixed motive games with $n = 7$. These are the modal structure.

	JM	NCEH	NCEL	MXMN	ENT	WORST
Total	420	388	372	368	300	176
Eff_1	1	.92	.89	.88	.71	.42
Eff_2	1	87	.80	.79	.51	0

Table 18
Mixed motive games 1

The structure highlights the need for collaboration in face of no universal easy coordination in the structure. Index_1 on the average has the best noncooperative equilibrium miss by 8% the worst noncooperative equilibrium misses by 11%. The cautious leave 12% on the table and the fools 29%.

The Index_2 yields for the best noncooperative equilibrium a miss of 13% the average worst noncooperative equilibrium misses by 20%. The cautious leave 21% and the fools 49%.

3.2.3 Mixed motive games 2: $n = 6$

There are 42 mixed motive games with $n = 6$. They have less fat to fight over than those previously noted.

	JM	NCEH	NCEL	MXMN	ENT	WORST
Total	252	236	236	236	210	144
Eff_1	1	.94	.94	.94	.83	.57
Eff_2	1	.85	.85	.85	.61	0

Table 19
Mixed motive games 2

$Index_1$ on the average has the best noncooperative equilibrium, the worst noncooperative equilibrium and the cautious all leave 7% on the table and the fools 17%.The $Index_2$ yields the same for best noncooperative equilibrium worst noncooperative equilibrium and the cautious all leave 15% and the fools 39%

3.2.4 Games of pure opposition: $n = 5$

There are only 6 mixed motive games with $n = 5$. They have a pure opposition of interests as is noted in Table 20

	JM	NCEH	NCEL	MXMN	ENT	WORST
Total	30	30	30	30	30	30
Eff_1	1	1	1	1	1	1
Eff_2	1	1	1	1	1	1

Table 20

Paradoxically the handful of constant sum games with a joint maximum of 5 have every outcome as Pareto optimal thus to include them in a measure primarily aimed at considering joint gains in a society is misleading. The intent operators **NCE, MaxMax, Maxmin** all collapse to yielding the same behavior in a two person Hobbsian constant sum world.

3.2.5 All mixed motive games: $Index_2$

Table 21 displays the values of the second index over all games with mixed motives

	JM	NCEH	NCEL	MXMN	ENT	WORST
n=7	1	.87	.8	.79	.51	0
n=6	1	.85	.85	.85	.61	0
	1	.86	.82	.81	.55	0

Table 21

The bounds on the average efficiency of purely individualistic behavior appear to be between at least 14% and at most 18% with purely defensive behavior coming in close at 19%

3.2.6 Fools matter

Looking over the various games and solutions damage done by the ignorant varies. We may regard all non learning purely syntactic players as being societally tone deaf.

The other players may or may not be concerned with how much the ignorant damage themselves, but it is easy to produce games where the fools damage others as well as themselves. The clever game theorist, can easily cook up examples where it pays the cunning to play the fool, but often fools are fools.

3.2.7 Being Nice matters in context

In various outcomes here always being intrinsically cooperative does not pay. When the structure is as the best of all possible worlds joint maximality is coaxed out of all syntactic,non-malicious player types modeled here.

In philosophical writings we have the realism and skepticism of Hobbes and Voltaire contrasting with the fuzzy-headedness mythology of the original perfect primitive world of the trendy salon speaker Rousseau. The more realistic view appears to be that of Hume where the social individual is cooperative but no fool. In order to start to do justice to such a player we would need at least two plays where learning can begin. It is here where a tit-for-tat player can be considered. The one ply does not permit flexibility but the two ply opens up a manageable set of minimal learning possibilities

4 Costs, Coordination and Cooperation

- Civilization, culture, society and law move on broader and slower time scales than everyday life and almost all of the individual consumer and worker economic and political activities.
- Efficiency and symmetry are critical features of everyday life.
- The construction of indices are critical for the measurements of deviations from efficiency and symmetric treatment of individuals
- Threats play a key role in considering what the zero point should be on any index understanding the implications of comparable utility is merited first.

We do not live in the utopian best of all possible worlds and do not live in purely dystopian structures. We have rich or not so rich mixed motive structures. The models of the noncooperative and cautious or pessimistic agents are metaphors for decentralized behavior. But the behavior is within the structure of the rules of the game. The departure from Optimality appears to range from a low of around 15 to 20% that can be considered as the potential gain available from coordination and cooperation.

5 References

References

[1] Harsanyi, J. C. 1963. A simplified bargaining model for the n-person cooperative game. International Economic Review 4:194–220.

[2] Maynard Smith, J. 1982 Evolution and the Theory of Games Cambridge: Cambridge University Press.

[3] Nash, J. F. 1953. Two-person cooperative games. Econometrica 21 (1):128–140.

[4] Powers, M. and M. Shubik. The value of government. Manuscript in preparation.

[5] J.A. Rawls 1971Theory of Justice Belknap, Cambridge

[6] Shapley, L. S. A value for n-person games in: H. Kuhn, A.W. Tucker (Eds.), *Contributions to the Theory of Games, vol. 2,* Princeton University Press, Princeton, NJ (1953), pp. 307–317

[7] Shubik, M. 2008.A note on fairness, power, property and behind the veil. *Economics Letters* 98, 1: 29–30

[8] Shubik,M. "A Web Gaming Facility for Research and Teaching" (May 2012) CFDP 1860 and "What Is a Solution to a Matrix Game" (July 2012) CFDP 1866.

[9] Shubik, M., J. Faber, D. Li. The many worlds of the 2×2 games with heterogeneous agents. Manuscript in preparation.

[10] Shubik, M and E. Smith 2016 The Guidance of a Enterprise Economy (with D Eric Smith) Cambridge, MA; UK: MIT Press,

[11] Shubik M, G. .Powers and Wen

[12] Shubik M, G. .Powers

[13] Weibull J. 1995 *Evolutionary Game Theory* Cambridge, MA: M.I.T. Press: ISBN 0-262-23181-6; Paper: ISBN 0-262-73121-5

6 Appendix 1 Indices

36 GAMES OF COORDINATION

	JM	NCEH	NCEL	MAXM	ENT	WORST
3	8	8	8	8	5	2
7	8	8	5	8	5	3
8	8	8	8	8	5	3
15	8	8	8	8	5	4
19	8	8	8	8	5	2
20	8	8	8	8	5	2
26	8	8	8	8	5	4
28	8	8	8	8	5	3
29	8	8	8	8	5	3
32	8	8	8	8	5	3
41	8	8	4	4	5	4
42	8	8	8	5	5	3
48	8	8	8	6	5	3
54	8	8	8	6	5	2
55	8	8	5	5	5	2
61	8	8	5	5	5	3
62	8	8	8	4	5	4
69	8	8	6	6	5	3
70	8	8	8	5	5	3
77	8	8	8	5	5	2
78	8	8	8	4	5	2
82	8	8	8	8	5	3
83	8	8	8	8	5	3
85	8	8	8	5	5	3
92	8	8	8	6	5	2
93	8	8	8	5	5	2
102	8	8	8	8	5	2
106	8	8	8	5	5	3
107	8	8	8	8	5	3
111	8	8	8	4	5	4
113	8	8	8	8	5	3
122	8	8	5	5	5	2
124	8	8	5	4	5	2
131	8	8	8	8	5	4
132	8	8	8	8	5	3
140	8	8	5	5	5	3
	288	288	264	230	180	144
%	1.000	1.000	0.917	0.799	0.625	0.500
Worst	1.000	1.000	0.833	0.597	0.250	0.000

Appendix 1

60 MIXED MTIVE GAMES: N=7

	JM	NCEH	NCEL	MAXM	ENT	WORST
2	7	6	6	6	5	3
5	7	7	7	4	5	4
9	7	5	5	6	5	2
10	7	7	5	4	5	2
13	7	7	7	5	5	3
16	7	7	7	7	5	3
17	7	7	7	7	5	3
18	7	7	7	7	5	3
23	7	7	7	7	5	4
27	7	7	7	7	5	4
30	7	7	7	7	5	3
33	7	7	7	7	5	2
34	7	7	7	7	5	2
36	7	7	7	5	5	2
37	7	4	4	4	5	4
44	7	5	5	7	5	4
46	7	7	5	6	5	3
47	7	7	7	7	5	3
50	7	5	5	5	5	3
51	7	7	7	7	5	2
52	7	6	6	7	5	2
56	7	5	5	5	5	3
57	7	5	5	4	5	4
63	7	7	7	7	5	3
66	7	7	7	7	5	4
67	7	7	5	7	5	3
68	7	7	7	6	5	3
71	7	7	7	7	5	3
73	7	5	5	5	5	2
74	7	7	5	4	5	2
76	7	7	5	5	5	2
79	7	7	7	7	5	2
80	7	7	7	6	5	2
81	7	7	7	7	5	2
86	7	7	7	7	5	3
87	7	7	7	7	5	4
90	7	7	7	7	5	4
94	7	7	7	7	5	3
95	7	7	7	7	5	3
96	7	7	7	6	5	4
98	7	7	7	7	5	3
101	7	6	6	7	5	2
104	7	7	7	7	5	2
105	7	5	5	6	5	2
109	7	7	7	7	5	3
110	7	5	5	5	5	3
115	7	6	6	6	5	3
116	7	4	4	4	5	4
117	7	7	7	4	5	4
119	7	7	7	7	5	3
125	7	7	7	5	5	2
126	7	5	5	7	5	3
128	7	7	7	5	5	3
129	7	7	5	7	5	3
133	7	7	5	6	5	3
135	7	7	7	7	5	4
137	7	7	7	7	5	3
139	7	7	5	5	5	2
141	7	5	5	7	5	4
143	7	5	5	5	5	3
	420	388	372	368	300	176
%	1.000	0.924	0.886	0.876	0.714	0.419
Worst	1.000	0.869	0.803	0.787	0.508	0.000

42 MIXED MTIVE GAMES: N=6

	JM	NCEH	NCEL	MAXM	ENT	WORST
1	6	4	4	4	5	4
4	6	6	6	6	5	3
6	6	5	5	6	5	4
12	6	6	6	6	5	4
14	6	6	6	6	5	4
21	6	6	6	6	5	2
22	6	6	6	6	5	4
24	6	6	6	6	5	3
25	6	5	5	5	5	4
31	6	6	6	6	5	3
35	6	6	6	6	5	2
38	6	5	5	5	5	4
39	6	5	5	4	5	4
43	6	5	5	4	5	4
45	6	6	6	5	5	4
49	6	6	6	6	5	3
53	6	6	6	6	5	2
58	6	5	5	6	5	3
60	6	6	6	6	5	4
64	6	5	5	6	5	4
65	6	5	5	5	5	4
72	6	5	5	6	5	3
75	6	6	6	6	5	2
84	6	6	6	6	5	3
88	6	5	5	5	5	4
89	6	6	6	6	5	3
91	6	6	6	6	5	4
97	6	6	6	6	5	4
99	6	6	6	6	5	4
103	6	6	6	6	5	2
108	6	6	6	6	5	3
112	6	6	6	6	5	4
114	6	6	6	6	5	4
118	6	5	5	5	5	4
121	6	6	6	6	5	3
123	6	6	6	6	5	2
127	6	6	6	6	5	3
130	6	5	5	6	5	4
134	6	7	7	5	5	4
138	6	5	5	4	5	4
142	6	5	5	6	5	3
144	6	5	5	5	5	4
	252	236	236	236	210	144
%	1.000	0.937	0.937	0.937	0.833	0.571
Worst	1.000	0.852	0.852	0.852	0.611	0.000

6 GAMES OF PURE OPPOSITION

	JM	NCEH	NCEL	MAXM	ENT	WORST
11	5	5	5	5	5	5
40	5	5	5	5	5	5
59	5	5	5	5	5	5
100	5	5	5	5	5	5
120	5	5	5	5	5	5
136	5	5	5	5	5	5
	30	30	30	30	30	30
	1.000	1.000	1.000	1.000	1.000	1.000

Mixed mptives

	JM	NCEH	NCEL	MAXM	ENT	WORST
7	1	0.868852	0.803279	0.786885	0.508197	0
6	1	0.851852	0.851852	0.851852	0.611111	0
	1	0.861852	0.823279	0.813636	0.550573	0

7 Appendix 2: 144 games

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff		Dom.	Pareto Optima	Transpose
						Row	Col.			
1	(1,4) (3,3) (2,2) (4,1)	22	6	1	Sym	2	2	2	3	NA
2	(1,2) (3,1) (2,4) (4,3)	11	7	1		2	4	2	2	115
3	(1,1) (3,2) (2,3) (4,4)	3	8	1	Sym	4	4	2	1	NA
4	(1,4) (3,3) (4,2) (2,1)	20	6	1		4	2	1	3	108
5	(1,3) (3,4) (4,1) (2,2)	16	7	1		3	4	1	2	117
6	(1,3) (3,2) (4,1) (2,4)	18	6	0		2.5	2.5	0	3	134
7	(1,2) (3,3) (4,4) (2,1)	7	8	2	Sym	4 3	4 3	0	1	NA
8	(1,2) (3,1) (4,4) (2,3)	9	8	1		4	4	1	1	113
9	(1,1) (3,2) (4,3) (2,4)	5	7	1		3	2	1	2	105
10	(1,1) (3,4) (4,3) (2,2)	2	7	2	Sym	3 4	4 3	0	2	NA
11	(1,4) (2,3) (3,2) (4,1)	24	5	1		3	2	2	4	120
12	(1,4) (2,2) (3,3) (4,1)	22	6	1	Sym	3	3	2	3	NA
13	(1,4) (2,1) (3,2) (4,3)	19	7	1		4	3	1	2	128
14	(1,3) (2,4) (3,1) (4,2)	17	6	1		4	2	2	2	112
15	(1,3) (2,2) (3,1) (4,4)	15	8	1		4	4	1	1	131
16	(1,2) (2,3) (3,4) (4,1)	10	7	1		3	4	1	2	137
17	(1,2) (2,4) (3,1) (4,3)	11	7	1		4	3	2	2	86
18	(1,2) (2,1) (3,4) (4,3)	8	7	1		3	4	2	2	109
19	(1,1) (2,3) (3,2) (4,4)	3	8	1	Sym	4	4	2	1	NA
20	(1,1) (2,2) (3,3) (4,4)	1	8	1		4	4	2	1	102
21	(1,1) (2,4) (3,3) (4,2)	6	6	1		3	3	1	3	123
22	(1,4) (2,3) (4,2) (3,1)	23	6	1		4	2	2	3	114

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
23	(1,4) (2,2) (4,3) (3,1)	21	7	1		4	3	2	2	87
24	(1,4) (2,1) (4,2) (3,3)	20	6	1		3	3	1	3	127
25	(1,3) (2,4) (4,1) (3,2)	18	6	1		3	2	2	3	118
26	(1,3) (2,2) (4,4) (3,1)	15	8	1	Sym	4	4	2	1	NA
27	(1,3) (2,2) (4,1) (3,4)	16	7	1		3	4	1	2	135
28	(1,3) (2,1) (4,4) (3,2)	13	8	1		4	4	2	1	83
29	(1,2) (2,3) (4,4) (3,1)	9	8	1		4	4	1	1	132
30	(1,2) (2,3) (4,1) (3,4)	10	7	1		3	4	2	2	98
31	(1,2) (2,4) (4,1) (3,3)	12	6	1		3	3	2	3	89
32	(1,2) (2,1) (4,4) (3,3)	7	8	1		4	4	2	1	107
33	(1,1) (2,3) (4,2) (3,4)	3	7	1		3	4	2	2	81
34	(1,1) (2,2) (4,3) (3,4)	2	7	1		3	4	2	2	104
35	(1,1) (2,4) (4,2) (3,3)	6	6	1	Sym	3	3	2	3	NA
36	(1,1) (2,4) (4,3) (3,2)	5	7	1		4	3	1	2	125
37	(1,4) (4,3) (2,2) (3,1)	21	7	1		2	2	1	2	116
38	(1,4) (4,2) (2,3) (3,1)	23	6	1		2	3	1	3	88
39	(1,4) (4,1) (2,2) (3,3)	22	6	0		2.5	2.5	0	3	138
40	(1,4) (4,1) (2,3) (3,2)	24	5	1		2	3	1	4	100
41	(1,3) (4,4) (2,2) (3,1)	15	8	2	Sym	4 2	4 2	0	1	NA
42	(1,3) (4,4) (2,1) (3,2)	13	8	1		4	4	1	1	106
43	(1,3) (4,2) (2,4) (3,1)	17	6	1		2	4	1	2	97
44	(1,3) (4,2) (2,1) (3,4)	14	7	0		2.5	2.5	0	2	126

Appendix 2

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
45	(1,3) (4,1) (2,4) (3,2)	18	6	1		2	4	1	3	91
46	(1,2) (4,3) (2,4) (3,1)	11	7	2		4 2	3 4	0	2	133
47	(1,2) (4,3) (2,1) (3,4)	8	7	1		4	3	1	2	94
48	(1,2) (4,4) (2,1) (3,3)	7	8	1		4	4	1	1	82
49	(1,2) (4,1) (2,4) (3,3)	12	6	1		2	4	1	3	121
50	(1,2) (4,1) (2,3) (3,4)	10	7	0		2.5	2.5	0	2	143
51	(1,1) (4,3) (2,2) (3,4)	2	7	1		4	3	1	2	79
52	(1,1) (4,2) (2,3) (3,4)	4	7	1		4	2	1	2	101
53	(1,1) (4,2) (2,4) (3,3)	6	6	2	Sym	4 2	2 4	0	3	NA
54	(1,1) (4,4) (2,2) (3,3)	1	8	1		4	4	1	1	92
55	(1,1) (4,4) (2,3) (3,2)	3	8	2		4 2	4 3	0	1	122
56	(1,4) (4,3) (3,2) (2,1)	19	7	1		3	2	1	2	110
57	(1,4) (4,3) (3,1) (2,2)	21	7	0		2.5	2.5	0	2	141
58	(1,4) (4,2) (3,3) (2,1)	20	6	1		3	3	1	3	84
59	(1,4) (4,1) (3,2) (2,3)	24	5	0		2.5	2.5	0	4	136
60	(1,4) (4,1) (3,3) (2,2)	22	6	1		3	3	1	3	99
61	(1,3) (4,4) (3,2) (2,1)	13	8	2		4 3	4 2	0	1	140
62	(1,3) (4,4) (3,1) (2,2)	15	8	1		4	4	1	1	111
63	(1,3) (4,2) (3,4) (2,1)	14	7	1		3	4	1	2	95
64	(1,3) (4,2) (3,1) (2,4)	17	6	0		2.5	2.5	0	2	130
65	(1,3) (4,1) (3,2) (2,4)	18	6	0		2.5	2.5	0	3	144
66	(1,3) (4,1) (3,4) (2,2)	16	7	1		3	4	1	2	90

Appendix 2

Game #	Payoff Matrix		Shape	Joint Max	PSNEs	Symmetric	Nash Payoff		Dom.	Pareto Optima	Transpose
							Row	Col.			
67	(1,2)	(4,3)	8	7	2		4	3	0	2	129
	(3,4)	(2,1)					3	4			
68	(1,2)	(4,3)	11	7	1		4	3	1	2	96
	(3,1)	(2,4)									
69	(1,2)	(4,4)	7	8	2	Sym	4	4	0	1	NA
	(3,3)	(2,1)					3	3			
70	(1,2)	(4,4)	9	8	1		4	4	1	1	85
	(3,1)	(2,3)									
71	(1,2)	(4,1)	10	7	1		3	4	1	2	119
	(3,4)	(2,3)									
72	(1,2)	(4,1)	12	6	0		2.5	2.5	0	3	142
	(3,3)	(2,4)									
73	(1,1)	(4,3)	5	7	1		4	3	1	2	80
	(3,2)	(2,4)									
74	(1,1)	(4,3)	2	7	2	Sym	4	3	0	2	NA
	(3,4)	(2,2)					3	4			
75	(1,1)	(4,2)	6	6	1		4	2	1	3	103
	(3,3)	(2,4)									
76	(1,1)	(4,2)	4	7	2		3	4	0	2	139
	(3,4)	(2,3)					4	2			
77	(1,1)	(4,4)	3	8	1		4	4	1	1	93
	(3,2)	(2,3)									
78	(1,1)	(4,4)	1	8	2		4	4	0	1	124
	(3,3)	(2,2)					3	3			
79	(1,1)	(2,2)	2	7	1		3	4	1	2	51
	(3,4)	(4,3)									
80	(1,1)	(2,3)	4	7	1		3	4	1	2	73
	(3,4)	(4,2)									
81	(1,1)	(2,4)	5	7	1		4	3	2	2	33
	(3,2)	(4,3)									
82	(1,2)	(2,1)	7	8	1		4	4	1	1	48
	(3,3)	(4,4)									
83	(1,2)	(2,3)	9	8	1		4	4	2	1	28
	(3,1)	(4,4)									
84	(1,2)	(2,4)	12	6	1		3	3	1	3	58
	(3,3)	(4,1)									
85	(1,3)	(2,1)	13	8	1		4	4	1	1	70
	(3,2)	(4,4)									
86	(1,3)	(2,1)	14	7	1		3	4	2	2	17
	(3,4)	(4,2)									
87	(1,3)	(2,2)	16	7	1		3	4	2	2	23
	(3,4)	(4,1)									
88	(1,3)	(2,4)	18	6	1		3	2	1	3	38
	(3,2)	(4,1)									

Game #	Payoff Matrix		Shape	Joint Max	PSNEs	Symmetric	Nash Payoff		Dom.	Pareto Optima	Transpose
							Row	Col.			
89	(1,4)	(2,1)	20	6	1		3	3	2	3	31
	(3,3)	(4,2)									
90	(1,4)	(2,2)	21	7	1		4	3	1	2	66
	(3,1)	(4,3)									
91	(1,4)	(2,3)	23	6	1		4	2	1	3	45
	(3,1)	(4,2)									
92	(1,1)	(2,2)	1	8	1		4	4	1	1	54
	(4,4)	(3,3)									
93	(1,1)	(2,3)	3	8	1		4	4	1	1	77
	(4,4)	(3,2)									
94	(1,2)	(2,1)	6	7	1		3	4	1	2	47
	(4,3)	(3,4)									
95	(1,2)	(2,4)	11	7	1		4	3	1	2	63
	(4,3)	(3,1)									
96	(1,3)	(2,1)	14	7	1		3	4	1	2	68
	(4,2)	(3,4)									
97	(1,3)	(2,4)	17	6	1		4	2	1	2	43
	(4,2)	(3,1)									
98	(1,4)	(2,1)	19	7	1		4	3	2	2	30
	(4,3)	(3,2)									
99	(1,4)	(2,2)	22	6	1		3	3	1	3	60
	(4,1)	(3,3)									
100	(1,4)	(2,3)	24	5	1		3	2	1	4	40
	(4,1)	(3,2)									
101	(1,1)	(3,2)	5	7	1		2	4	1	2	52
	(2,4)	(4,3)									
102	(1,1)	(3,3)	1	8	1		4	4	2	1	20
	(2,2)	(4,4)									
103	(1,1)	(3,3)	6	6	1		2	4	1	3	75
	(2,4)	(4,2)									
104	(1,1)	(3,4)	2	7	1		4	3	2	2	34
	(2,2)	(4,3)									
105	(1,1)	(3,4)	4	7	1		2	3	1	2	9
	(2,3)	(4,2)									
106	(1,2)	(3,1)	9	8	1		4	4	1	1	42
	(2,3)	(4,4)									
107	(1,2)	(3,3)	7	8	1		4	4	2	1	32
	(2,1)	(4,4)									
108	(1,2)	(3,3)	12	6	1		2	4	1	3	4
	(2,4)	(4,1)									
109	(1,2)	(3,4)	8	7	1		4	3	2	2	18
	(2,1)	(4,3)									
110	(1,2)	(3,4)	10	7	1		2	3	1	2	56
	(2,3)	(4,1)									

Game #	Payoff Matrix		Shape	Joint Max	PSNEs	Symmetric	Nash Payoff		Dom.	Pareto Optima	Transpose
							Row	Col.			
111	(1,3)	(3,1)	15	8	1		4	4	1	1	62
	(2,2)	(4,4)									
112	(1,3)	(3,1)	17	6	1		2	4	2	2	14
	(2,4)	(4,2)									
113	(1,3)	(3,2)	13	8	1		4	4	1	1	8
	(2,1)	(4,4)									
114	(1,3)	(3,2)	18	6	1		2	4	2	3	22
	(2,4)	(4,1)									
115	(1,3)	(3,4)	14	7	1		4	2	2	2	2
	(2,1)	(4,2)									
116	(1,3)	(3,4)	16	7	1		2	2	1	2	37
	(2,2)	(4,1)									
117	(1,4)	(3,1)	21	7	1		4	3	1	2	5
	(2,2)	(4,3)									
118	(1,4)	(3,1)	23	6	1		2	3	2	3	25
	(2,3)	(4,2)									
119	(1,4)	(3,2)	19	7	1		4	3	1	2	71
	(2,1)	(4,3)									
120	(1,4)	(3,2)	24	5	1		2	3	2	4	11
	(2,3)	(4,1)									
121	(1,4)	(3,3)	20	6	1		4	2	1	3	49
	(2,1)	(4,2)									
122	(1,1)	(3,2)	3	8	2		4	4	0	1	55
	(4,4)	(2,3)					3	2			
123	(1,1)	(3,3)	6	6	1		3	3	1	3	21
	(4,2)	(2,4)									
124	(1,1)	(3,3)	1	8	2		4	4	0	1	78
	(4,4)	(2,2)					3	3			
125	(1,1)	(3,4)	4	7	1		3	4	1	2	36
	(4,2)	(2,3)									
126	(1,2)	(3,1)	11	7	0		2.5	2.5	0	2	44
	(4,3)	(2,4)									
127	(1,2)	(3,3)	12	6	1		3	3	1	3	24
	(4,1)	(2,4)									
128	(1,2)	(3,4)	10	7	1		3	4	1	2	13
	(4,1)	(2,3)									
129	(1,2)	(3,4)	8	7	2		4	3	0	2	67
	(4,3)	(2,1)					3	4			
130	(1,3)	(3,1)	17	6	0		2.5	2.5	0	2	64
	(4,2)	(2,4)									
131	(1,3)	(3,1)	15	8	1		4	4	1	1	15
	(4,4)	(2,2)									
132	(1,3)	(3,2)	13	8	1		4	4	1	1	29
	(4,4)	(2,1)									

Game #	Payoff Matrix		Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
133	(1,3)	(3,4)	14	7	2		3	4	0	2	46
	(4,2)	(2,1)					4	2			
134	(1,4)	(3,1)	23	6	0		2.5	2.5	0	3	6
	(4,2)	(2,3)									
135	(1,4)	(3,1)	21	7	1		4	3	1	2	27
	(4,3)	(2,2)									
136	(1,4)	(3,2)	24	5	0		2.5	2.5	0	4	59
	(4,1)	(2,3)									
137	(1,4)	(3,2)	19	7	1		4	3	1	2	16
	(4,3)	(2,1)									
138	(1,4)	(3,3)	22	6	0		2.5	2.5	0	3	39
	(4,1)	(2,2)									
139	(1,1)	(4,3)	5	7	2		4	3	0	2	76
	(2,4)	(3,2)					2	4			
140	(1,2)	(4,4)	9	8	2		4	4	0	1	61
	(2,3)	(3,1)					2	3			
141	(1,3)	(4,1)	16	7	0		2.5	2.5	0	2	57
	(2,2)	(3,4)									
142	(1,4)	(4,2)	20	6	0		2.5	2.5	0	3	72
	(2,1)	(3,3)									
143	(1,4)	(4,3)	19	7	0		2.5	2.5	0	2	50
	(2,1)	(3,2)									
144	(1,4)	(4,2)	23	6	0		2.5	2.5	0	3	65
	(3,1)	(2,3)									

Game # corresponds to the numbering system established in the companion paper
Payoff Matrix gives the normal form of each game with payoffs listed as (row payoff, column payoff)
Shape corresponds to the shape of the payoff set's convex hull as shown in Appendix 1
Joint Max gives the highest possible combined payoff for the two players
Symmetric is marked "Sym" if the game is symmetric, otherwise it is left blank
Nash Payoff lists the payoffs of the noncooperative equilibrium
if there are two equilibria with different payoff sums, the one with the highest sum is listed first
Dom. specifies the number of row and column strategies that are strictly dominated
Pareto Optima gives the number of payoff pairs that are Pareto optimal
Transpose lists the game number corresponding to the transpose of the game shown