JOHN DENIS SARGAN AT THE LONDON SCHOOL OF ECONOMICS

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John Denis Sargan at the London School of Economics

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Abstract

During his period at the LSE from the early 1960s to the mid 1980s, John Denis Sargan rose to international prominence and the LSE emerged as the world's leading centre for econometrics. Within this context, we examine the life of Denis Sargan, describe his major research accomplishments, recount the work of his many doctoral students, and track this remarkable period that constitutes the Sargan era of econometrics at the LSE.

Keywords: John Denis Sargan; London School of Economics; Econometrics; Asymptotic theory; Small-sample distributions; Dynamic models; Autocorrelated errors; Empirical modelling; Doctoral training.

JEL Codes: A14, B23

1 Introduction

John Denis Sargan played a key role in LSE's astonishingly rapid emergence as the world's leading centre for econometrics during the two decades from the early 1960s to the mid 1980s. This period produced an enduring legacy for the profession. Denis's theory contributions spanned much of the econometrics spectrum, and included asymptotic and small-sample distributions of estimators and tests, as well as Edgeworth expansions, identification of parameters in models, the existence of moments of estimators, continuous time analyses, semi-parametric estimation, the properties of instrumental variables and related estimators, Monte Carlo, numerical methods and computing, and dynamic models with autocorrelated errors that produced major insights on methodology. He also undertook important empirical studies, yet devoted a great deal of his time to doctoral training. Indeed, many of his innovative ideas were implemented by his students, and he produced a cadre of brilliant and technically trained doctoral graduates. Within this active milieu of research and mentorship, Denis rose to prominence at the LSE and gained worldwide attention.

Following a brief description of his life, we review Denis's major research accomplishments, the work of the students that he supervised, and the research environment at the LSE that he helped to create.

2 The life of John Denis Sargan

John Denis Sargan was born on August 23, 1924, in Doncaster, Yorkshire, where he spent his childhood. Harry, Denis's father, was the youngest of 8 surviving children, who grew up on a farm and smithy

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in Conisburgh near Doncaster. Harry had gained a place at the local grammar school, but the family could not afford further education for him. As he had an ability to handle horses, Harry joined the Life Guards Cavalry on the outbreak of war in 1914, and when the war ended in 1918, he became a mounted policeman in Doncaster. Denis's mother, Gertrude Porter, was one of 4 children, and being musically gifted, she loved to sing in the parish church at Askern (near Doncaster) where her father was the organist and choirmaster.

Denis and his sister were brought up in a household where money was scarce. He attended the local Church of England primary school, then won a place at Doncaster Grammar School for his exceptional ability in mathematics. He taught himself to play the piano, and enjoyed playing it throughout this life. Aged just 17, Denis won a State Scholarship to St. Johns College, Cambridge, attended at the same time by both Sir David Cox and Jim Durbin, albeit in different years. On completing his degree in mathematics, Denis was drafted into war work on testing new weapons systems. In 1945, he read John Maynard Keynes's *General Theory of Employment, Interest and Money* which made him decide to apply his knowledge of mathematics and statistics to tackle the many economic problems facing the UK in the post-war years. He first returned to Cambridge to read economics, completing his BA degree in a single year, then accepted a lectureship (assistant professorship) in the Economics Department at Leeds University, where he met Mary Millard whom he married in 1953.

Denis was awarded a 2-year Fulbright Scholarship in 1958, so (with all the family) visited the Economics Departments of the Universities of Minnesota and Chicago, which kindled his interests in econometrics and the use of computers therein. He returned to Leeds University as a Reader (Associate Professor) in July 1960, then moved to the London School of Economics as a Reader in Statistics in 1963, (where Jim Durbin was a Professor: see the Chapter by Richard Baillie), and in 1965, joined A.W.H. (Bill) Phillips (see the superb biography by Alan Bollard, 2016, and the Chapter by James Forder) as Professor of Econometrics in the Economics Department where he remained for the rest of his career.

Even though Durbin, Phillips, Roy Allen (see the Chapter by Jim Thomas), Richard Lipsey (see the Chapter by Max Steuer) and Rex Bergstrom all worked at LSE at the time, and an earlier tradition around Arthur Bowley and William Beveridge (see the Chapters by Adrian Darnell and Atsushi Komine) was empirically centered, LSE was a surprising choice of institution for an econometrician in the early 1960s. Certainly, quantitative research was respected by the Methodology, Measurement and Testing (M^2T) group (see the Chapter on Richard Lipsey by Max Steuer). However, the tradition following from LSE's two most famous economists Lionel Robbins (see the Chapter by Sue Howson) and Friedrich Hayek (see the Chapter by Peter Boettke) was far from favourable to econometrics, and notoriously against attributing a substantive role to empirical studies as discussed by Hendry and Morgan (1995, Ch. 4). Nevertheless, in 1965, Denis helped introduce a Masters' level course in Quantitative Economics and Econometrics which set new standards for advanced teaching, creating a generation of econometricians trained to high technical levels. Many of these students went on to undertake doctoral research with him: in total Denis supervised 36 theses (including our own).

The combination of Denis's own outstanding research with the production of a cadre of brilliant and technically trained doctoral students rapidly raised LSE to be the world's leading econometrics centre during the period 1965–1985, with an enduring legacy for the profession. Other key figures in econometrics at LSE during that period included Terence Gorman, Ken Wallis, Grayham Mizon, Jim Thomas, Meghnad Desai (see the Chapter by Raja Junankar), Steve Nickell (see the Chapter by Jan van Ours), Jan Magnus, Cliff Wymer, Mary Morgan, and David Hendry, many of whom were attracted to the School by Denis's presence, as were numerous distinguished visitors, including Frank Fisher, Jean-François Richard, and Ted Anderson. This group was complemented by an evolving and growing phalanx of LSE doctoral students, almost all of whom were supervised by Denis, who contributed in concert a substantial group of econometrically-oriented participants at LSE seminars in economics, transforming discussion and research to evidence-based analysis and econometrically-informed thinking.

From 1982–84, Denis was Tooke Professor of Economic Science and Statistics, and on retirement in 1984, he became Emeritus Professor of Economic Science and Statistics at the University of London. At that time, an international conference was held in his honour at Oxford University (with the proceedings published as Hendry and Wallis, 1984). He died at his home in Theydon Bois, Essex, on Saturday 13 April, 1996. His spouse Mary died on May 1st 2013, and their daughter Barbara died in 2015, but they are survived by their two sons, John and David, and three grandchildren.

We have many fond memories of Denis, as a teacher, a supervisor, a colleague and a giant of the discipline. He ran a workshop for PhD students, at which Hendry remembers starting a presentation on polynomial matrices when a senior professor—who had earlier remarked that econometricians were 'failed mathematicians'—asked what they were, and Denis leapt up to take revenge and used all the rest of the time lecturing to him! Mention a technical problem to Denis at lunch in the LSE, and an innovative solution sketched on a slip of paper came under one's office door later that afternoon. Denis was renowned at the LSE for his unequaled generosity to faculty and to doctoral students. He promptly got all his doctoral students engaged in research, and sometimes ended up essentially proving the most challenging theorems of their theses. One student told us years later that he couldn't prove a key result needed for his thesis, so provided Denis with a 'proof' that worked carefully from the front and from the back but filled in the middle with junk. A week later Denis gave the student some handwritten notes, saying in his typical euphemistic way that he "didn't think the previous proof worked, so here was a new one that did".

3 Denis Sargan's contributions to econometrics

For the first two decades of his career, Denis worked on his own and sole-authored all his publications, half of them in *Econometrica*. From the early 1970s, Denis worked regularly with his students, inspiring them to take on challenging problems in a host of different fields, covering a broad swathe of the discipline of econometrics as it stood at the time. Much of Denis's later research came through his many doctoral students. After a discussion with him about potential projects when none had been well received, Denis would open a desk drawer, take out a draft of a paper and suggest you thought about that, an approach that often led to a successful thesis. In what follows, we have noted the PhDs of his students by using the notational system 'name (date, PhD)'.

Previous overviews of Denis Sargan's contributions to econometrics include the interview of him by Phillips (1985a), the laudatio by Espasa (1993), Hendry and Pesaran (2001), Ericsson, Maasoumi, and Mizon (2001), Hendry and Phillips (2003), Hendry (2003), Phillips (2003), and Robinson (2003), with several obituaries including Desai, Hendry, and Mizon (1997). His lectures are published as Sargan (1988d), and his collected works are published as Maasoumi (1988). Denis's research work was celebrated on his 60th birthday in Hendry and Wallis (1984). LSE contributions to econometrics, including those by Denis, are described in the Chapter by Jim Thomas.

Recently, Stan Hurn uncovered and reconstituted some video recordings of Denis lecturing at the Australian National University: see http://www.ncer.edu.au/resources/historical-archive.php.

While we have slotted his research into a variety of pigeon holes, it must be remembered that many of Denis's papers spanned several of these categories. Moreover, despite the appearance of disparate contributions, there is the over-riding theme of improving the quality and reliability of empirical modelling. That required Denis to venture into issues of model specification (Section 3.10), modelling methodology (Section 3.11), methods of estimation inference and evaluation (Section 3.5, Section 3.6) in samples of the size then available for macroeconomics (Section 3.3), the theoretical properties of those methods (Section 3.2, Section 3.7, including their asymptotics, Section 3.1) for different types of data (Section 3.4), how to evaluate them in practice (Section 3.8), their operational implementation (Section 3.9), and the resulting findings in a wide range of empirical studies (Section 3.12).

3.1 Asymptotic distributions

The major intellectual influence from past econometric work in Denis's early career came from the research of the Cowles Commission in the 1940s, which was largely embodied in two monographs (Koopmans, 1950, and Hood and Koopmans, 1953). Most of that work focussed on simultaneous equations and simple time series dynamic models. Identification, estimation, computation and, to a more limited extent, testing were the main preoccupations. Given the mathematical complexity of these models in comparison with linear regression equations with fixed regressors, it was natural to develop asymptotic theories, although the short historical time series samples available to researchers at that time meant that finite sample properties were always acknowledged to be of great importance. This intellectual background cast a long shadow over econometric research for the next few decades and empowered much of Denis's research from his early contributions on instrumental variable (IV) estimation (Sargan, 1958a, and Sargan, 1959) in the 1950s through to his work on exact distributions and asymptotic expansions in the 1970s discussed in (Section 3.3).

The two Sargan papers on IV estimation just noted developed limit distribution theory in general cases that opened the way to an inferential framework whose legacy now includes the vast literature on GMM estimation following Hansen (1982), including the Sargan test for overidentification (now more generally known as the J test from Hansen, 1982). Also foreshadowed in Denis's papers on IV is a large body of subsequent work on autocorrelated errors, intensively studied in Hendry (1970, PhD), and issues of near unidentification that Denis later systematically investigated in his 1980 World Congress presidential address (Sargan, 1983a) and that gave birth to work on partial and weak identification (Phillips, 1989, and Staiger and Stock, 1997) and associated asymptotic theory that connects closely with much ongoing research in microeconometrics (overviewed by Tamer, 2010). Even more general work on the asymptotic theory of estimation in the presence of autocorrelated errors was pursued in Toni Espasa (1975, PhD) and published in Espasa and Sargan (1977). That work pursued a semi-parametric approach to the estimation of simultaneous equation models and is one of Denis's two contributions to the field of semi-parametric estimation mentioned below in (Section 3.5).

Denis's early and later work on IV allowed for nonlinear-in-parameter estimation, where matters of computation as well as weak identification are manifest. Computational algorithms and numerical optimization methods are important in practical work on nonlinear modeling; and at the LSE they played a sustained role in Denis's research and teaching (reflected in his lectures on advanced econometrics in Sargan, 1988d), as well as his thesis supervision of Grayham Mizon (1972, PhD), who developed approaches to tackle the non-linearities intrinsic in estimating vintage-capital models, Jerzy Sylwestrowicz (1975, PhD), Bahram Pesaran (1977, PhD) and Yock Chong (1982, PhD). For estimation and inference in nonlinear models, asymptotic theory is now a universal econometric tool for guidance, informing empirical researchers about statistical properties in both correctly specified and misspecified settings. Denis's contributions, and those of his students David Hendry (1970, PhD) and Peter Phillips (1974, PhD), to IV and other estimator limit theory (Sargan, 1971a, and Sargan, 1975a), continuous time econometrics (Section 3.4), reduced form estimation by Esfandiar Maasoumi (1977, PhD), and large models (Sargan, 1975a) all influenced subsequent work within this general avenue of asymptotic research.

A further contribution by Denis to our understanding of simultaneous equations estimation arrived with his 1964 *Econometrica* paper (Sargan, 1964b), which analyzed the two systems estimators three stage least squares (3SLS) and full information maximum likelihood (FIML). This paper established the asymptotic equivalence of 3SLS and FIML, thereby confirming the asymptotic efficiency of 3SLS under the same conditions as FIML and providing empirical researchers a short iterative journey to efficient systems estimation. Related work at the LSE by Jim Durbin (1988) (first presented in 1963) (see the Chapter by Richard Baillie) and, somewhat later and more generally, by Hendry (1976) showed the remarkable property of FIML as an IV estimator with appropriately designed updated instruments. This is an idea that has much more recently been vigorously pursued in work on continuous updating estimators in GMM-based econometric procedures (Hansen, Heaton, and Yaron, 1996).

3.2 Identification

The theory of parametric identification for linear simultaneous equations models was first addressed in generality in the Cowles Commission research of the 1940s, where the textbook rank and order conditions were originally developed. By the early 1970s, the subject was thought to be reasonably mature after further extensions to some nonlinear models by Fisher (1966), inclusion of system-wide restriction information by Wegge (1965), and formulation in terms of information matrix criteria by Rothenberg (1971). Yet somewhat surprisingly by that point, only Sargan (1959) seems to have considered cases of near-unidentification and examined the possible implications for asymptotic theory failure. In that early work on IV estimation asymptotics, Denis briefly mentioned singular cases of non-linear in parameter estimation where the extremum function manifested quartic rather than quadratic behavior in the limit, thereby producing a reduction in identification capability that manifested in a reduction of the usual $n^{1/2}$ convergence rate to $n^{1/4}$ and led to non-normal asymptotic theory. Nor do such failures of normal asymptotics seem to have been considered in statistical work, which favored general treatments of locally asymptotic quadratic likelihoods (LeCam, 1986, and LeCam and Yang, 1990) and generally did not treat these further complications.

Denis's first attempt (Sargan, 1975, Sargan, 1988c) at a general treatment of identification appeared in discussion paper form in 1975 at the LSE. Numbered as A1, this paper initiated the famous red-covered small booklet LSE discussion paper series in econometrics. The paper addressed identification in terms of a general extremum estimation framework involving the finite sample objective function and (upon suitable normalization) its limit function, which was permitted to have a set of 'asymptotic maxima'. This is the first appearance, to our knowledge, of the notion of set identification, on which there is now a large literature in microeconometrics, although this paper by Denis is little known and has sadly never been cited in this subsequent literature. The paper goes on to analyze the limit properties of local and global maxima of the finite sample objective function, sufficient conditions for a unique maximum, and applies the theory to linear models with analytic restrictions, as well as simultaneous equations models.

In other work on this topic, Sargan (1981) and Sargan (1983b) considered problems of identification in dynamic models with autoregressive errors, where there exists potential for common matrix polynomial factors between system and error dynamics, and explored conditions that enabled the separation of these dynamics. This subject relates to statistical and engineering research on model identification (Hannan and Deistler, 1988). In doctoral thesis work at the LSE, some related problems had been earlier considered in the study of solutions to matrix polynomial equations in economic models with vector autoregressive (VAR) errors by David Hendry (1970, PhD) and later Julia Campos Fernandez (1982, PhD) studied IV estimation in similar structural dynamic models with VAR errors.

Denis's World Congress Presidential address Sargan (1983a) to the Econometric Society also focused on identification, exploring linear models that are nonlinear in parameters where there is first order lack of identification, resulting in non-normal limit distribution theory in place of conventional asymptotics. This was a major paper that pursued the idea and implications of near-unidentification that Denis initiated in his 1959 article. The new paper dealt with the special case of a one dimensional rank deficiency in the first order condition for identification, so that the immediate implications of the rank failure could be isolated in a single new parametric coordinate after transformation, with spillover effects impacting the remaining parametric coordinates. With his customary algebraic flare, Denis works out the asymptotic theory, shows some intriguing limit theory involving Cauchy-like distributional behavior and the possibility of double minima that leads in turn to a combined estimator with 'improved' convergence from the coupling. The paper ends with an exploration of one of Denis's favorite nonlinear IV models and a simulation study that shows clear evidence of the relevance of the new asymptotics. This Presidential Address is a tour de force with echoes that have persisted through to the latest work on weak identification in econometrics.

3.3 Small sample properties and Edgeworth expansions

As emphasized earlier, Denis's work was motivated by a thematic vision of where econometric research might usefully progress in the future toward the general goal of aiding our understanding of economic phenomena. His research largely followed a path inspired by this vision, an important part of which arose from a desire to improve inferential methods in a manner that respected the relatively small samples of data being used in econometric work. Most empirical models at the time involved structural and dynamic elements, so endogenous and lagged endogenous variables featured prominently in the regressor set. These complications produce formidable difficulties in the development of exact distribution theory for econometric estimators and tests. In the early 1960s, only work by Basmann (1961) and Bergstrom (1962), the latter then at LSE, had penetrated the field, examining single equation estimates of small static simultaneous equations models with two endogenous variables. That research revealed interesting small sample properties and distributions quite distinct from the asymptotic normal, notably in miscentering and the heavy tails associated with potential non-existence of moments. General theory in this field of research was to come more than a decade later.

In the meantime, the complications of exact theory led Denis to develop a substantial research agenda that started in the mid 1960s at the LSE concerned with asymptotic approximants obtained by Edgeworth expansions. This approach, while still of great complexity, at least enabled the consideration of estimators and test statistics in structural and dynamic models whose realism matched that of models typically used in empirical practice. The feasibility of this approach also opened the field to Denis's doctoral students, beginning with the thesis work of William Mikhail (1969, PhD) on Edgeworth expansions for IV estimators, which partly appeared in published form as Denis's first joint research paper with one of his students (Sargan and Mikhail, 1971). By the mid 1970s, significantly general results on the validity and algorithmic construction of these approximants arrived in two major papers by Denis in *Econometrica* (Sargan, 1975b, and Sargan, 1976a) dealing with expansions of asymptotically normally distributed criteria, well-suited to econometric estimators and t-tests. This work was followed by a further major contribution dealing with approximants for asymptotically chi-squared criteria (Sargan, 1980c). Articles by Phillips (1977a,1977b) opened the door to valid Edgeworth expansions in dynamic models and Denis's doctoral students worked on this topic with theses by Steve Satchell (1981, PhD) and Yiu Kuen Tse (1981, PhD) on expansions in various dynamic models, some of which appeared in a joint paper with Denis in *Econometrica* (Sargan and Satchell, 1986) and in published volumes (Sargan and Tse, 1981, Sargan and Tse, 1988a, and Sargan and Tse, 1988b). A later thesis by Ignacio Mauleón Torres (1983, PhD) dealt with extensions and applications of the expansions for asymptotic chi-squared criteria.

One significant contribution by Denis to this field of asymptotic expansions that remained unpublished for nearly two decades was his 1970 World Congress paper on Edgeworth expansions for full information maximum likelihood estimators in simultaneous equations models, which eventually appeared in his collected works volume (Sargan, 1988b). Another late contribution (Sargan (1993b)) considered alternative approaches to approximation. A further important contribution, this time to exact distribution theory, was given by Denis in an Appendix of his Walras Bowley Lecture (Sargan, 1976a) that appeared in *Econometrica*. This Appendix provided the first published attempt to find the long-sought exact distribution of the instrumental variable estimator of a single equation in a simultaneous equations model, allowing for an arbitrary number of endogenous variables and succeeding in obtaining the analytic form for the just identified case. The completely general result for IV estimation that allowed for an arbitrary degree of overidentification as well as any number of endogenous variables was obtained by Phillips (1980) and for LIML in Phillips (1985b), completing this line of research. Later work in the 1990s proved the broad relevance of this research for practical work, by showing that the finite sample theory delivered the correct asymptotic theory for applications in which the instruments used for estimation are weak or even irrelevant.

3.4 Continuous time

While Bill Phillips (see the Chapter by James Forder) and Rex Bergstrom were at the LSE, interest emerged in the development of methods for estimating econometric models formulated in continuous time as systems of stochastic differential equations. Early work on this subject was done at the school by Phillips (1959) and Phillips and Quenouille (1960). Bergstrom (1966) later opened up the investigation of the asymptotic effects on estimation and asymptotics of the misspecification induced by non-recursive discrete approximations to the differential equation system. Subsequently, in a long paper Sargan (1974a) that was characteristically circulated in an even longer form in 1971 (ultimately appearing in part as Sargan, 1976b), Denis provided a full asymptotic development of the effects of such misspecification. An important element in Sargan's approach that has subsequently been heavily utilized in statistical research on infill asymptotics is that the misspecification asymptotics were studied by allowing the sampling interval (h) between observations to tend to zero. This artifice conveniently enabled a full set of asymptotic results to be obtained for a suite of single and multiple equation econometric estimators of the discrete approximate system.

During this period at the LSE two of Sargan's doctoral students produced theses on continuous systems. First, Cliff Wymer (1970, PhD) examined higher order systems of differential equations, applying the results to models of financial markets where data were, even at this time, observed far more frequently than macroeconomic time series. Wymer's contribution to continuous time estimation via discrete approximations also involved the substantial development of nonlinear estimation algorithms and software, which had a substantial influence on the success of the empirical program of research on continuous time econometric models of national economies such as the model of the UK by Bergstrom and Wymer (1976). Importantly, that software included one of the first developments of computerized algebra and calculus, which facilitated use of the software for users who only needed to enter code for the specific nonlinear functions appearing in the system to be estimated. Second, Peter Phillips (1974, PhD) developed methods and associated asymptotic theory for estimating the exact discrete model corresponding to a continuous time system, covering models with identities, exogenous variables, and both stock and flow data. This approach was first explored in Phillips (1972) and has now become widespread in practical work, especially with financial data, modern forms using nonparametric methods (Bandi and Phillips, 2003). Phillips's thesis also showed that the aliasing (identification) problem of fitting continuous systems with discretely observed data may be resolved, including close diophantine approximations to the true system, by simple exclusion and other restrictions on the coefficients of the original continuous system, some of this work appearing in Phillips (1973).

Research on continuous time modeling at the LSE during this time was a morning star of the major role that continuous stochastic process econometrics later came to play in the analytic study of nonstationary time series and the emergent field of financial econometrics. The unit root and cointegration revolution (Phillips, 1987, and Engle and Granger, 1987) forever changed the face of time series econometrics through use of functional limit theory to stochastic processes and long run balance among integrated processes. With the advent of ultra high frequency data, financial econometric methods now make extensive use of infill ($h \to 0$) as well as large span asymptotics in studying the limiting form and properties of the trajectories of financial asset prices. This vast body of subsequent research owes, at least in part, a debt to early work on continuous time econometrics originating at the LSE.

3.5 Semi-parametric estimation

The first statistical work on semi-parametric estimation was done by Whittle (1951), who developed the so-called Whittle likelihood (see Hannan and Deistler, 1988), and by Hannan (1963), who introduced the method of spectral regression. Both developments have been influential in econometrics, but spectral regression was immediately relevant in econometrics because it offered a very general way of estimating a regression model with stationary errors by treating the error process nonparametrically, so that it was not necessary to specify (or approximate) the data generating process of the errors. Denis saw the potential of this approach and, working with Toni Espasa (1975, PhD) succeeded in extending the approach to handle simultaneous equations models in which the structural coefficient matrices were parametrically specified and the errors were treated nonparametrically as stationary processes. This work was an early econometric contribution to semi-parametric methods.

In a second contribution to this topic, Denis wrote an article on large econometric model estimation (Sargan (1975a)) as a contribution to a symposium on large macroeconometric models of national economies and emerging work on global economic models. At the time his paper was written in the early 1970s macroeconomic models had grown to huge systems of many hundreds of equations that were continually growing in size to satisfy the demands of policy analysis and forecasting. Estimation of such systems was consequently conducted in a framework of a very high dimensional system, with vast numbers of endogenous and (presumed) exogenous variables. Denis's first contribution was to formulate the system in operator form in terms of infinite matrices, giving an infinite dimensional structural and reduced form system, but requiring only a finite number of variables (and hence parameters) in each equation. This framework may therefore be viewed as semi-parametric, as suggested in a fine exposition of the paper by Robinson (2003). In spite of the finite number of parameters in each equation, use of estimation methods such as IV involves projections on instruments and these projections have properties that depend on the infinite dimensional reduced form system. Sargan succeeded in showing that, under certain regularity conditions, use of iterative types of IV procedures designed for large systems produced estimators which were asymptotically equivalent to the (infeasible) versions that utilized the full reduced form system. Robinson confirmed the same finding for conventional two-stage least squares estimates and gave a central limit theory justifying inference. In addition to studying IV procedures in this paper, Denis verified a standing conjecture by Lawrence Klein concerning the data requirements for full information maximum likelihood being that the total number of variables in the system be less than the sample size.

This line of research on infinite dimensional systems broke entirely new ground at the time and anticipated later modeling and estimation concerns by several decades. Since the turn of the century, much attention has been given to high dimensional modeling methods and the application of machine learning in statistical models. Denis's work on high dimensional structural systems foreshadows some of the big data concerns that have recently occupied the statistics and econometric professions. Moreover, working on large dimensional models constituted a major departure from the mainstream of Denis's own research and illustrates his remarkable versatility, revealing a capability to fashion whatever mathematical tools might be required to address provocative new lines of research, while remaining alert to the needs of practitioners.

3.6 Instrumental variables and estimation methods

Two of Denis's early contributions to econometrics, Sargan (1958a) and Sargan (1959), developed instrumental variables estimation for single equations with possibly autocorrelated errors. Several special cases were already known, such as indirect least squares (see Tinbergen, 1930), and precursors (see Reiersøl, 1941) or related methods like limited information maximum likelihood (see Anderson and Rubin, 1949), but no general treatment existed before Denis's research. His formulation included 'method of moments' estimators (see Hansen, 1982), although that aspect was not emphasised. Denis then began to focus more on the small-sample distributions, as in Sargan (1964a), Sargan and Mikhail (1971) and Section 3.3, and on full-information estimators (Sargan, 1964b). However, others of his papers also indirectly contributed to a more complete understanding of the properties and limitations of instrumental variables methods, especially Sargan (1964c).

Other areas Denis fostered included recovering missing data in a likelihood framework by Emmanuel Drettakis (1971, PhD), comparing estimators of seemingly unrelated regressions by Tony Hall (1976, PhD), and likelihood estimation of models with unobservable variables by Kirti Mehta (1979, PhD).

3.7 Existence of moments

Because the formulae for many econometric estimators, including instrumental variables, require inverting matrices that are not guaranteed to be non-singular for all possible values of the data, the issue arises as to whether or not they will have finite moments (means, variances, etc.) in small samples. In Sargan (1974b), Denis considers when expansions of moments as used by Nagar (1959) will be appropriate. He offers a new interpretation, which transpires to be highly relevant in Monte Carlo distribution sampling, as discussed in the next section. Although separated in time, Sargan (1978) and Sargan (1988a) are closely related, the former establishing the absence of moments in small samples of reduced-form estimators derived from 3 stage least squares (3SLS), and the latter showing those existed for full-information maximum likelihood (FIML), whereas the converse held for 3SLS and FIML estimators of the parameters of the simultaneous equations representation. In his thesis, Esfandiar Maasoumi (1977, PhD) showed how to improve estimates of the 3SLS reduced-form coefficients, using data-based combined estimation, a subject that has recently achieved much renewed attention.

3.8 Monte Carlo methods

Hendry and Trivedi (1972) in fact drew on inspiration from both Denis and Bill Phillips to use more sophisticated simulation methods than just distribution sampling, including antithetic variates. These had the potential to lead to more general findings that did not depend on the specific parameter values used in the simulations, and indeed immediately led to such a result for forecasting. Control variables used in Hendry and Harrison (1974) were based on an unpublished memo by Denis, and these together with the insights in Sargan (1982) on settings where finite-sample moments might not exist for the methods being simulated led to the survey in Hendry (1984).

3.9 Numerical methods and computing

The complicated calculations for Sargan (1964c) required Denis to hardwire the then available computer, and on arrival at LSE he programmed in Atlas Autocode for that machine. The small word length, little memory storage and limited speeds forced him to think carefully about the appropriate numerical methods for the estimation and tests he required, an interest that persisted. Two notable studies are those by his doctoral students Jerzy Sylwestrowicz (1975, PhD) and Yock Chong (1982, PhD), where the latter developed analytic differentiation software, but his influence was far broader, extending to a large number of studies by his students responding to his encouragement to develop appropriate software, including for example Hendry and Srba (1980) and Sargan and Tse (1988b).

3.10 Dynamic models and autocorrelated errors

Directly, and through his doctoral students, Denis advanced the analysis of dynamic models with autocorrelated errors in many key ways. His first incursion in Sargan (1953) was describing non-random time series, but his early research included economic-theory analyses in Sargan (1955), Sargan (1958b) and Sargan (1961a). However, beyond the papers noted in Section 3.6 and the following subsections, perhaps the key paper is Sargan (1961b), which tackles the estimation of systems of dynamic equations with autoregressive errors, leading to David Hendry (1970, PhD). In addition, Sargan and Drettakis (1974) and Emmanuel Drettakis (1971, PhD) addressed using dynamics to improve estimates of missing data, whereas Bahram Pesaran (1977, PhD), tackled the problems created by measurement errors in dynamic models. Denis also considered modelling rational expectations in Sargan (1993a) and John Hunter (1989, PhD), as well as specification tests for dynamic models in Sargan and Mehta (1983) and Neil Ericsson (1982, PhD). An overview is provided by Hendry, Pagan, and Sargan (1984).

Although he had considered a special case of distinguishing between observable-variable dynamics and autoregressive errors in Sargan (1964c), the general case of 'common factor dynamics' (Comfac) was not published until Sargan (1980d). In its simplest form, consider the following data generation process (DGP):

$$y_t = \beta_0 + \boldsymbol{\beta}_1' \mathbf{x}_t + \beta_2 y_{t-1} + \nu_t \tag{1}$$

where:

$$\nu_t = \rho \nu_{t-1} + \epsilon_t \quad \text{with} \quad \epsilon_t \sim \mathsf{IN}\left[0, \sigma_\epsilon^2\right] \tag{2}$$

where $|\beta_2| < 1$, $|\rho| < 1$, $\mathbf{x}_t$ is a vector of n strongly exogenous variables and $\mathsf{IN}\left[0, \sigma_\epsilon^2\right]$ denotes an independent identically distributed normal random variable with mean zero and variance σ_ϵ^2. Then substituting (1) into (2) and rearranging:

$$y_t = (1-\rho)\,\beta_0 + \boldsymbol{\beta}_1' \mathbf{x}_t - \rho \boldsymbol{\beta}_1' \mathbf{x}_{t-1} + (\beta_2 + \rho)\, y_{t-1} - \rho \beta_2 y_{t-2} + \epsilon_t \tag{3}$$

Comparing (3) to an unrestricted dynamic relation between the same variables:

$$y_t = \gamma_0 + \boldsymbol{\gamma}_1' \mathbf{x}_t + \boldsymbol{\gamma}_2' \mathbf{x}_{t-1} + \gamma_3 y_{t-1} + \gamma_4 y_{t-2} + e_t \tag{4}$$

reveals that there are $2n+3$ parameters in (4) but only $n+3$ in (3) as (2) entails $\boldsymbol{\gamma}_2 = -\rho \boldsymbol{\gamma}_1$. Thus, although at first sight (2) appears to generalise (1), autoregressive errors are a restriction on a dynamic relation; and that restriction is testable. Despite the date on Sargan (1980d), this idea was developed by Denis during 1977 and explained in advance by Hendry and Mizon (1978) and Mizon and Hendry (1980), but its fundamental implications still needed reiterationby Mizon (1995).

Since Bill Phillips (1966) had researched moving-average errors, Hendry and Trivedi (1972) investigated how well each form (autoregressive and moving average) approximated the other, and concluded that having the correct order (say 2nd) was more important than knowing the correct form.

By the 1980s, attention in time series analysis had started to focus on non-stationarity, and in Sargan and Bhargava (1983b) and Alok Bhargava (1983, PhD), Denis reconsidered the issue of unit roots and the age-old contentious problem of model specification in levels or differences, the latter being (for example) the special case of (3) when $\rho = 1$. Antecedents in that debate included Hooker (1901), Smith (1926), and Granger and Newbold (1974), with a key clarification of the last of these in Phillips (1986). As ever, the multifaceted Sargan (1964c) plays a key role by introducing empirical models of what are now called equilibrium correction mechanisms (EqCMs) where past real wages deviating from productivity fed into determining changes in wages in the next period. This class of models was a precursor to formal cointegration analysis, and redirected the debate to one of the significance or not of the relevant EqCM in a model otherwise in differences: for a recent re-appraisal, see Castle and Hendry (2017). Returning to (4), reparametrize it as:

$$\Delta y_t = \boldsymbol{\gamma}_1' \Delta \mathbf{x}_t - \gamma_4 \Delta y_{t-1} + \lambda \left(y_{t-1} - \kappa_0 - \boldsymbol{\kappa}_1' \mathbf{x}_{t-1}\right) + e_t \tag{5}$$

where $(y_{t-1} - \kappa_0 - \boldsymbol{\kappa}_1' \mathbf{x}_{t-1})$ is the EqCM, with $\lambda = (\gamma_3 + \gamma_4 - 1) \neq 0$, $\kappa_0 = -\gamma_0/\lambda$, and $\boldsymbol{\kappa}_1 = (\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2)/\lambda$ where the κ_i represent the 'long-run' effects, and would correspond to a cointegrating relation when the process determining $\{\mathbf{x}_t\}$ was integrated of first order, $\mathsf{I}(1)$. Conversely, when $\lambda = 0$ in (5), the equation reduces to one in first differences of the variables. However, tests of $\lambda = 0$ are non-standard, although appropriate critical values have been tabulated by Ericsson and MacKinnon (2002), and are often coded in econometric modelling software.

The possibility of a root on the unit circle in a moving average was investigated by Sargan and Bhargava (1983a). Another source of non-stationarity from time-varying parameters had also been investigated by his students in Michael Fitzpatrick (1976, PhD) and Louisa Franzini-Bhargava (1983, PhD). Panel data dynamic models were also considered, as in Manuel Arellano Gonzalez (1985, PhD), relating back to differencing equation specifications.

3.11 Econometric methodology

Denis made major contributions to the methodology of econometrics with many lasting insights. The first was in Sargan (1957) which discussed three methodological issues that concerned him about Fisher (1956): (i) that economic theory was highly abstract despite economic data being complicated; (ii) all too often this led to over-simplified regression models which probably excluded substantively important variables; and (iii) the interpretation of tests was doubtful when large numbers of hypotheses had been investigated. Both Sargan (2001a), which describes Denis's insights into model selection, and Sargan (2001b), which contributes to understanding and avoiding what is often called 'data mining', namely finding adventitiously significant coefficients due to testing many effects at inappropriate significance levels, followed up his worries in 1957 about that last issue, although these two papers were only published long after being written.

As Hendry (2003) explains, however, Sargan (1964c) changed existing approaches substantively in a valuable direction. As noted above, in that paper Denis introduced early forms of both Comfac and EqCMs, and based the formulation of the latter on an economic analysis of long-run behaviour, as well as developing mis-specification tests applicable to dynamic equations and providing a way of comparing linear with log-linear formulations. He formulated an iterative algorithm to estimate the parameters of the resulting specifications—using a non-linear-in-parameters instrumental variables approach designed to counteract the adverse effects of data measurement errors—and proved that it would converge with near certainty, all embodied in a computer program he also developed. His test for instrument validity has been widely used. These developments fostered other innovations at the LSE (discussed in the Chapter by Jim Thomas), but here we note the emergence of an emphasis on general-to-specific modelling (see Mizon, 1977, and Davidson, Hendry, Srba, and Yeo, 1978), since extensively developed (see Hendry and Doornik, 2014) with rigorous diagnostic testing (Meghnad Desai described AUTOREG—see Hendry and Srba, 1980—as destroying pet theories), and the associated requirement of encompassing (see Mizon, 1984, Mizon and Richard, 1986, and Bontemps and Mizon, 2008).

3.12 Empirical studies

Denis's eclectic interests led to a wide range of empirical analyses including several prior to his arrival at LSE in M.M. El Imam (1957, PhD), J. Hortala-Arau (1966, PhD), and Madan Handa (1968, PhD). Once he was settled at the School, a veritable flood of work appeared, including three studies of demand systems in Ray Byron (1968, PhD), Julia Hebden (1974, PhD), and Ranjan Ray (1977, PhD) all with distinct theoretical orientations, as well as Ross Williams (1969, PhD) modelling the demand for consumer durables. There were also analyses of inventory and investment behaviour by Pravin Trivedi (1969, PhD) and Robin Rowley (1969, PhD) respectively; and of the demand for imports by Michael Feiner (1970, PhD). There were three studies estimating production functions by Eleftherios Charatsis (1970, PhD), Sargan (1971b), which is the origin of the trans-log production function, and Mizon (1974). There was even a study of the rubber industry by K.C. Cheong (1972, PhD).

A major empirical interest of Denis's that lasted throughout much of his career was econometric modelling of wages and prices in the UK. Again, Sargan (1964c) represents an important development, where his empirical wage-price model broke new ground and provided new insights into their joint determination. That research was followed by studies in Keith Vernon (1970, PhD), Sargan (1971c), Toni Espasa (1975, PhD), and the companion papers Sargan (1980a) and Sargan (1980b). His approach provided a marked contrast to the nominal wage model in Phillips (1958), and Hendry remembers vivid discussions about the appropriate specification between these two giants in the Quantitative Economics module of the LSE MSc in 1966–67.

4 Conclusion

Denis Sargan was remarkable for what he achieved, but even more remarkable for what he was capable of doing, even if he did not get around to doing it. No problem seemed too difficult for him to conceptualize and, once conceptualized, to solve or to create approximants that opened up new understandings. His track record of research is replete with examples. One that is still unappreciated is that, while many of the world's leading statisticians were working on constructing general theories of locally asymptotically normal large-sample theory, Denis was tackling the finite sample properties of full information maximum likelihood. He did this not just with asymptotic approximations but with refreshingly original insights into exact small-sample properties. In short, undaunted by technical difficulty his creative mind saw nothing as impossible.

Denis was almost unique in the profession at the time for his extraordinary intellectual power, his technical insight, his originality, and his prescience regarding productive research directions. Standing on the frontier of the discipline in the 1950s he saw the massive untapped potential of instrumental variables. In the early 1960s, he carved a new path forward in empirical research with a boldly innovative approach to dynamic specification in an econometric methodology that gave birth to the 'LSE approach' to modelling. In the late 1960s and 1970s, his work on the finite-sample properties of econometric estimators took the subject to previously unimagined heights of generality. The mid 1970s and 1980s brought forth studies of identification that explored entirely new regions of nearly unidentified models that have subsequently opened up an industry of research.

Resonating through all his research and the advanced mathematics that he used with such detached confidence in its deployment, was Denis's deep concern to develop econometric methods for the betterment of empirical research. Denis Sargan's primary gift to the profession is as a consummate toolmaker. He was peerless in his time at creating econometric methods. His lasting legacy is the benefit that these methods have since bestowed on the econometric community. That achievement is buttressed by the huge effort he devoted to bringing forward a generation of technically trained doctoral students who further advanced his approach, often starting from one of Denis's original insights.

Writing this essay has given the authors a further opportunity to reflect on Denis's genius, his generosity to his students (not least ourselves), his colleagues and his remarkable contribution to creating the unique habitat for econometrics that existed under his aegis at the LSE over two decades from the mid 1960s. In conjunction with Denis's talents, it was this habitat that propelled the LSE to extraordinary heights by 1980 at the forefront of the global expansion of econometrics.

References

Anderson, T. W. and H. Rubin (1949). Estimation of the parameters of a single equation in a complete system of stochastic equations. *Annals of Mathematical Statistics 20*, 46–63.

Arellano Gonzalez, M. (1985). Estimation and testing of dynamic econometric models from panel data. Phd thesis, Economics Department, London School of Economics.

Bandi, F. and P. C. B. Phillips (2003). Fully nonparametric estimation of scalar diffusion models. *Econometrica 71*, 241–283.

Basmann, R. L. (1961). A note on the exact finite sample frequency functions of generalized classical linear estimators in two leading over-identified cases. *Journal of the American Statistical Association 56*, 619–636.

Bergstrom, A. R. (1962). The exact sampling distributions of least squares and maximum likelihood estimators of the marginal propensity to consume. *Econometrica 30*, 480–490.

Bergstrom, A. R. (1966). Nonrecursive models as discrete approximations to systems of stochastic differential equations. *Econometrica 34*, 173–182.

Bergstrom, A. R. (Ed.) (1976). *Statistical Inference in Continuous Time Econometric Models*. Amsterdam: North-Holland.

Bergstrom, A. R. and C. R. Wymer (1976). A model of disequilibrium neoclassical growth and its application to the United Kingdom. See Bergstrom (1976), pp. 267–327.

Bhargava, A. (1983). The theory of the Durbin-Watson statistic with special reference to the specification of models in levels as against in differences. Phd thesis, Economics Department, London School of Economics.

Bollard, A. (2016). *A Few Hares to Chase*. Oxford: Oxford University Press.

Bontemps, C. and G. E. Mizon (2008). Encompassing: Concepts and implementation. *Oxford Bulletin of Economics and Statistics 70*, 721–750.

Byron, R. P. (1968). The estimation of systems of demand equations using prior information. Phd thesis, Economics Department, London School of Economics.

Campos Fernandez, J. (1982). Instrumental variables estimation of dynamic economic systems with autocorrelated errors. Phd thesis, Economics Department, London School of Economics.

Castle, J. L. and D. F. Hendry (2017). Clive W.J. Granger and cointegration. *European Journal of Pure and Applied Mathematics 10*, 57–80.

Charatsis, E. G. (1970). Statistical methods for estimation of production functions: A cross-section and time-series analysis of the Greek manufacturing industry. Phd thesis, Economics Department, London School of Economics.

Cheong, K. C. (1972). An econometric study of the world natural and synthetic rubber industry. Phd thesis, Economics Department, London School of Economics.

Chong, Y. Y. (1982). Comparative methods of computing maximum likelihood estimates for non-linear econometric systems. Phd thesis, Economics Department, London School of Economics.

Davidson, J. E. H., D. F. Hendry, F. Srba, and J. S. Yeo (1978). Econometric modelling of the aggregate time-series relationship between consumers' expenditure and income in the United Kingdom. *Economic Journal 88*, 661–692.

Desai, M. J., D. F. Hendry, and G. E. Mizon (1997). John Denis Sargan. *Economic Journal 107*, 1121–1125.

Drettakis, E. G. (1971). Missing data in econometric estimation. Phd thesis, Economics Department, London School of Economics.

Durbin, J. (1988). Maximum likelihood estimation of the parameters of a system of simultaneous regression equations. *Econometric Theory 4*, 159–170. Paper presented to the Copenhagen Meeting of the Econometric Society, 1963.

El Imam, M. M. (1957). The consumption function of the U.K., 1929–1938. Phd thesis, Economics Department, London School of Economics.

Engle, R. F. and C. W. J. Granger (1987). Cointegration and error correction: Representation, estimation and testing. *Econometrica 55*, 251–276.

Ericsson, N. R. (1982). Testing non-nested hypotheses in systems of linear dynamic economic relationships. Phd thesis, Economics Department, London School of Economics.

Ericsson, N. R., E. Maasoumi, and G. E. Mizon (2001). A personal retrospective on J. Denis Sargan. *Econometric Reviews 20*, 133–157.

Ericsson, N. R. and J. G. MacKinnon (2002). Distributions of error correction tests for cointegration. *Econometrics Journal 5*, 285–318.

Espasa, A. (1975). A wages-earnings-prices inflation model for United Kingdom 1950–1970: Its specification and estimation by classical and spectral methods. Phd thesis, Economics Department, London School of Economics.

Espasa, A. (1993). Laudatio on the occasion of the investiture of Professor John Denis Sargan with the Degree of Doctor Honoris Causa. Mimeo, Universidad Carlos III, Madrid.

Espasa, A. and J. D. Sargan (1977). The spectral estimation of simultaneous equation systems with lagged endogenous variables. *International Economic Review 18*, 583–605.

Feiner, M. P. (1970). An econometric study of disaggregated UK imports, 1958–66. Phd thesis, Economics Department, London School of Economics.

Fisher, F. M. (1966). *The Identification Problem in Econometrics*. New York: McGraw Hill.

Fisher, M. R. (1956). Exploration in saving behaviour. *Oxford Bulletin of Economics and Statistics* (formerly the *Bulletin of the Oxford University Institute of Statistics*) *18*, 201–278.

Fitzpatrick, M. D. (1976). Varying parameter estimation and trends in the parameters which determine the modal choice associated with long distance international travel. Phd thesis, Economics Department, London School of Economics.

Franzini-Bhargava, L. (1983). Estimating and testing the parameters of time varying econometric models. Phd thesis, Economics Department, London School of Economics.

Granger, C. W. J. and P. Newbold (1974). Spurious regressions in econometrics. *Journal of Econometrics 2*, 111–120.

Griliches, Z. and M. D. Intriligator (Eds.) (1984). *Handbook of Econometrics*, Volume 2. Amsterdam: North-Holland.

Hall, A. D. (1976). The relative efficiency of estimators of seemingly unrelated regressions. Phd thesis, Economics Department, London School of Economics.

Handa, M. L. (1968). Econometrics of import planning in India 1947–65,: A case study of selected commodities. Phd thesis, Economics Department, London School of Economics.

Hannan, E. J. (1963). Regression for time series. In M. Rosenblatt (Ed.), *Proceedings of a Symposium on Time Series Analysis*, New York. John Wiley.

Hannan, E. J. and M. Deistler (1988). *The Statistical Theory of Linear Systems*. John Wiley & Sons: New York.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica 50*, 1027–1054.

Hansen, L. P., J. Heaton, and A. Yaron (1996). Finite-sample properties of some alternative GMM estimators. *Journal of Business and Economic Statistics 14*, 262–280.

Hebden, J. J. (1974). A complete set of dynamic consumer demand equations. Phd thesis, Economics Department, London School of Economics.

Hendry, D. F. (1970). The estimation of economic models with auto-regressive errors. Phd thesis, Economics Department, London School of Economics.

Hendry, D. F. (1976). The structure of simultaneous equations estimators. *Journal of Econometrics 4*, 51–88.

Hendry, D. F. (1984). Monte Carlo experimentation in econometrics. See Griliches and Intriligator (1984), Chapter 16, pp. 937–976.

Hendry, D. F. (2003). J. Denis Sargan and the origins of LSE econometric methodology. *Econometric Theory 19*, 457–480.

Hendry, D. F. and J. A. Doornik (2014). *Empirical Model Discovery and Theory Evaluation*. Cambridge, Mass.: MIT Press.

Hendry, D. F. and R. W. Harrison (1974). Monte Carlo methodology and the small sample behaviour of ordinary and two-stage least squares. *Journal of Econometrics 2*, 151–174.

Hendry, D. F. and G. E. Mizon (1978). Serial correlation as a convenient simplification, not a nuisance: A comment on a study of the demand for money by the Bank of England. *Economic Journal 88*, 549–563.

Hendry, D. F. and M. S. Morgan (Eds.) (1995). *The Foundations of Econometric Analysis*. Cambridge: Cambridge University Press.

Hendry, D. F., A. R. Pagan, and J. D. Sargan (1984). Dynamic specification. See Griliches and Intriligator (1984), pp. 1023–1100.

Hendry, D. F. and M. H. Pesaran (2001). Introduction: A special issue in memory of John Denis Sargan: Studies in empirical macroeconometrics. *Journal of Applied Econometrics 16*, 197–202.

Hendry, D. F. and P. C. B. Phillips (2003). John Denis Sargan: 1924–1996. *Biographical Memoirs of Fellows of the British Academy 120*, 385–409.

Hendry, D. F. and F. Srba (1980). AUTOREG: A computer program library for dynamic econometric models with autoregressive errors. *Journal of Econometrics 12*, 85–102.

Hendry, D. F. and P. K. Trivedi (1972). Maximum likelihood estimation of difference equations with moving-average errors: A simulation study. *Review of Economic Studies 32*, 117–145.

Hendry, D. F. and K. F. Wallis (Eds.) (1984). *Econometrics and Quantitative Economics*. Oxford: Basil Blackwell.

Hood, W. C. and T. C. Koopmans (Eds.) (1953). *Studies in Econometric Method*. Number 14 in Cowles Commission Monograph. New York: John Wiley & Sons.

Hooker, R. H. (1901). Correlation of the marriage rate with trade. *Journal of the Royal Statistical Society 64*, 485–492.

Hortala-Arau, J. (1966). Capital needs in a developing economy: Spain as a case study. Phd thesis, Economics Department, London School of Economics.

Hunter, J. (1989). Dynamic modelling of expectations with particular reference to the UK labour market. Phd thesis, Economics Department, London School of Economics.

Koopmans, T. C. (Ed.) (1950). *Statistical Inference in Dynamic Economic Models*. Number 10 in Cowles Commission Monograph. New York: John Wiley & Sons.

LeCam, L. (1986). *Asymptotic Methods in Statistical Decision Theory*. Springer: New York.

LeCam, L. and G. L. Yang (1990). *Asymptotics in Statistics: Some Basic Concepts*. Springer–Verlag: New York.

Maasoumi, E. (1977). A study of improved methods of estimating reduced form coefficients based upon 3SLS estimators. Phd thesis, Economics Department, London School of Economics.

Maasoumi, E. (Ed.) (1988). *Contributions to Econometrics: John Denis Sargan*. Cambridge, England: Cambridge University Press. 2 volumes.

Mauleón Torres, I. (1983). Approximations to the finite sample distribution of econometric chi-squared criteria. Phd thesis, Economics Department, London School of Economics.

Mehta, K. B. J. (1979). Maximum likelihood estimation of economic relationships involving unobservable indicators. Phd thesis, Economics Department, London School of Economics.

Mikhail, W. M. (1969). A study of the finite-sample properties of some econometric estimators. Phd thesis, Economics Department, London School of Economics.

Mizon, G. E. (1972). The estimation of vintage capital production relations. Phd thesis, Economics Department, London School of Economics.

Mizon, G. E. (1974). The estimation of non-linear econometric equations: An application to the specification and estimation of an aggregate putty-clay relation for the U.K. *Review of Economic Studies 41*, 253–270.

Mizon, G. E. (1977). Model selection procedures. In M. J. Artis and A. R. Nobay (Eds.), *Studies in Modern Economic Analysis*, pp. 97–120. Oxford: Basil Blackwell.

Mizon, G. E. (1984). The encompassing approach in econometrics. See Hendry and Wallis (1984), pp. 135–172.

Mizon, G. E. (1995). A simple message for autocorrelation correctors: Don't. *Journal of Econometrics 69*, 267–288.

Mizon, G. E. and D. F. Hendry (1980). An empirical application and Monte Carlo analysis of tests of dynamic specification. *Review of Economic Studies 49*, 21–45.

Mizon, G. E. and J.-F. Richard (1986). The encompassing principle and its application to non-nested hypothesis tests. *Econometrica 54*, 657–678.

Nagar, A. L. (1959). The bias and moment matrix of the general k-class estimators of the parameters in simultaneous equations. *Econometrica 27*, 575–595.

Pesaran, B. (1977). Estimation of dynamic economic models when variables are subject to measurement errors. Phd thesis, Economics Department, London School of Economics.

Phillips, A. W. H. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957. *Economica 25*, 283–299.

Phillips, A. W. H. (1959). The estimation of parameters in systems of stochastic differential equations. *Biometrika 46*, 67–76.

Phillips, A. W. H. (1966). Estimation of stochastic difference equations with moving average disturbances. Walras-Bowley lecture, Econometric Society, San Francisco. In Leeson, R. (ed.) *A. W. H. Phillips: Collected Works in Contemporary Perspective*, Cambridge: Cambridge University Press, 2000.

Phillips, A. W. H. and M. H. Quenouille (1960). Estimation, regulation and prediction in interdependent dynamic systems. *Bulletin de l'Institute de Statistique 37*, 335–343.

Phillips, P. C. B. (1972). The structural estimation of a stochastic differential equation system. *Econometrica 40*, 1021–1041.

Phillips, P. C. B. (1973). The problem of identification in finite parameter continuous time models. *Journal of Econometrics 1*, 351–362.

Phillips, P. C. B. (1974). Problems in the estimation of continuous time models. Phd thesis, Economics Department, London School of Economics.

Phillips, P. C. B. (1977a). Approximations to some finite sample distributions associated with a first order stochastic difference equation. *Econometrica 45*, 463–85.

Phillips, P. C. B. (1977b). A general theorem in the theory of asymptotic expansions as approximations to finite sample distributions of econometric estimators. *Econometrica 45*, 1517–34.

Phillips, P. C. B. (1980). The exact finite sample density of instrumental variable estimators in an equation with it n+1 endogenous variables. *Econometrica 48*, 861–878.

Phillips, P. C. B. (1985a). The ET interview: Professor J.D. Sargan. *Econometric Theory 2*, 119–139.

Phillips, P. C. B. (1985b). The exact distribution of LIML: II. *International Economic Review 26*, 21–36.

Phillips, P. C. B. (1986). Understanding spurious regressions in econometrics. *Journal of Econometrics 33*, 311–340.

Phillips, P. C. B. (1987). Time series regression with a unit root. *Econometrica 55*, 277–302.

Phillips, P. C. B. (1989). Partially identified econometric models. *Econometric Theory 5*, 181–240.

Phillips, P. C. B. (2003). Vision and influence in econometrics: John Denis Sargan. *Econometric Theory 19*, 495–511.

Ray, R. (1977). Utility maximisation and consumer demand with an application to the United Kingdom, 1900–1970. Phd thesis, Economics Department, London School of Economics.

Reiersøl, O. (1941). Confluence analysis by means of lag moments and other methods of confluence analysis. *Econometrica 9*, 1–24.

Robinson, P. M. (2003). Denis Sargan: Some perspectives. *Econometric Theory 19*, 481–494.

Rothenberg, T. J. (1971). Identification in parametric models. *Econometrica 39*, 577–592.

Rowley, J. C. R. (1969). An econometric study of fixed capital formation in the British economy, 1956–1965. Phd thesis, Economics Department, London School of Economics.

Sargan, J. D. (1953). An approximate treatment of the properties of the correlogram and periodogram. *Journal of the Royal Statistical Society*, *Series B 15*, 140–152.

Sargan, J. D. (1955). The period of production. *Econometrica 23*, 151–165.

Sargan, J. D. (1957). The danger of over-simplification. *Oxford Bulletin of Economics and Statistics* (formerly the *Bulletin of the Oxford University Institute of Statistics*) *19*, 171–178.

Sargan, J. D. (1958a). The estimation of economic relationships using instrumental variables. *Econometrica 26*, 393–415.

Sargan, J. D. (1958b). The instability of the Leontief dynamic model. *Econometrica 26*, 381–392.

Sargan, J. D. (1959). The estimation of relationships with autocorrelated residuals by the use of instrumental variables. *Journal of the Royal Statistical Society*, *Series B 21*, 91–105.

Sargan, J. D. (1961a). Lags and the stability of dynamic systems: A reply. *Econometrica 29*, 670–673.

Sargan, J. D. (1961b). The maximum likelihood estimation of economic relationships with autoregressive residuals. *Econometrica 29*, 414–426.

Sargan, J. D. (1964a). An approximate distribution of the two-stage least squares estimators. *Econometrica 32*, 660. abstract.

Sargan, J. D. (1964b). Three-stage least-squares and full maximum likelihood estimates. *Econometrica 32*, 77–81.

Sargan, J. D. (1964c). Wages and prices in the United Kingdom: A study in econometric methodology. In P. E. Hart, G. Mills, and J. K. Whitaker (Eds.), *Econometric Analysis for National Economic Planning*, Volume 16 of *Colston Papers*, pp. 25–54. London: Butterworths. with discussion.

Sargan, J. D. (1971a). Asymptotic expansion for the distribution functions of econometric estimators. *Econometrica 39*, 168. abstract.

Sargan, J. D. (1971b). Production functions. In P. R. G. Layard, J. D. Sargan, M. E. Ager, and D. J. Jones (Eds.), *Qualified Manpower and Economic Performance: An Inter-plant Study in the Electrical Engineering Industry*, pp. 143–204. London: Allen Lane The Penguin Press. Part 5.

Sargan, J. D. (1971c). A study of wages and prices in the U.K. 1949–1968. In H. G. Johnson and A. R. Nobay (Eds.), *The Current Inflation*, London, pp. 52–71. Macmillan Press.

Sargan, J. D. (1974a). Some discrete approximations to continuous time stochastic models. *Journal of the Royal Statistical Society*, *Series B 36*, 74–90.

Sargan, J. D. (1974b). The validity of Nagar's expansion for the moments of econometric estimators. *Econometrica 42*, 169–176.

Sargan, J. D. (1975a). Asymptotic theory and large models. *International Economic Review 16*, 75–91.

Sargan, J. D. (1975b). Gram-Charlier approximations applied to t ratios of k-class estimators. *Econometrica 43*, 327–346.

Sargan, J. D. (1976a). Econometric estimators and the Edgeworth approximation. *Econometrica 44*, 421–448. 1977, "Erratum", *Econometrica*, 45, 1, 272.

Sargan, J. D. (1976b). Some discrete approximations to continuous time stochastic models. See Bergstrom (1976).

Sargan, J. D. (1978). On the existence of the moments of 3SLS estimators. *Econometrica 46*, 1329–1350.

Sargan, J. D. (1980a). The consumer price equation in the post war British economy: An exercise in equation specification testing. *Review of Economic Studies 47*, 113–135.

Sargan, J. D. (1980b). A model of wage-price inflation. *Review of Economic Studies 47*, 97–112.

Sargan, J. D. (1980c). Some approximations to the distribution of econometric criteria which are asymptotically distributed as chi-squared. *Econometrica 48*, 1107–1138.

Sargan, J. D. (1980d). Some tests of dynamic specification for a single equation. *Econometrica 48*, 879–897.

Sargan, J. D. (1981). Identification in models with autoregressive errors. *Journal of Econometrics 16*, 160–161. abstract.

Sargan, J. D. (1982). On Monte Carlo estimates of moments that are infinite. In R. L. Basmann and G. F. Rhodes, Jr. (Eds.), *Advances in Econometrics: A Research Annual*, Volume 1, pp. 267–299. Greenwich, Connecticut: JAI Press.

Sargan, J. D. (1983a). Identification and lack of identification. *Econometrica 51*, 1605–1633.

Sargan, J. D. (1983b). Identification in models with autoregressive errors. In S. Karlin, T. Amemiya, and L. A. Goodman (Eds.), *Studies in Econometrics, Time Series, and Multivariate Statistics: In Honor of Theodore W. Anderson*, pp. 169–205. New York: Academic Press.

Sargan, J. D. (1988a). The existence of the moments of estimated reduced form coefficients. See Maasoumi (1988), pp. 133–157.

Sargan, J. D. (1988b). The finite sample distribution of FIML estimators. See Maasoumi (1988), pp. 57–75.

Sargan, J. D. (1988c). The identification and estimation of sets of simultaneous stochastic equations. See Maasoumi (1988), pp. 236–249.

Sargan, J. D. (1988d). *Lectures on Advanced Econometric Theory*. Oxford: Basil Blackwell. edited and with an introduction by Meghnad Desai.

Sargan, J. D. (1993a). Estimation methods for simple rational expectation models. *Revista Española de Economía* (a.k.a. the *Spanish Economic Review*) *10*, 5–18.

Sargan, J. D. (1993b). Some alternatives to the Edgeworth approximation for econometric statistics. In P. C. B. Phillips (Ed.), *Models, Methods, and Applications of Econometrics: Essays in Honor of A. R. Bergstrom*, Chapter 12, pp. 165–175. Cambridge, Massachusetts: Basil Blackwell.

Sargan, J. D. (2001a). The choice between sets of regressors. *Econometric Reviews 20*, 171–186.

Sargan, J. D. (2001b). Model building and data mining. *Econometric Reviews 20*, 159–170.

Sargan, J. D. and A. Bhargava (1983a). Maximum likelihood estimation of regression models with first order moving average errors when the root lies on the unit circle. *Econometrica 51*, 799–820.

Sargan, J. D. and A. Bhargava (1983b). Testing residuals from least squares regression for being generated by the Gaussian random walk. *Econometrica 51*, 153–174.

Sargan, J. D. and E. G. Drettakis (1974). Missing data in an autoregressive model. *International Economic Review 15*, 39–58.

Sargan, J. D. and F. Mehta (1983). A generalization of the Durbin significance test and its application to dynamic specification. *Econometrica 51*, 1551–1567.

Sargan, J. D. and W. M. Mikhail (1971). A general approximation to the distribution of instrumental variables estimates. *Econometrica 39*, 131–169.

Sargan, J. D. and S. E. Satchell (1986). A theorem of validity for Edgeworth expansions. *Econometrica 54*, 189–213.

Sargan, J. D. and Y. K. Tse (1981). Edgeworth approximations to the distribution of various test statistics. In E. G. Charatsis (Ed.), *Proceedings of the Econometric Society European Meeting 1979: Selected Econometric Papers in Memory of Stefan Valavanis*, Chapter 12, pp. 281–295. Amsterdam: North-Holland.

Sargan, J. D. and Y. K. Tse (1988a). Edgeworth approximations for 2SLS estimates of a dynamic model. See Maasoumi (1988), pp. 172–181.

Sargan, J. D. and Y. K. Tse (1988b). Some experience of numerical computation of Edgeworth approximations. See Maasoumi (1988), pp. 158–171.

Satchell, S. E. (1981). Edgeworth approximations in linear dynamic models. Phd thesis, Economics Department, London School of Economics.

Smith, B. B. (1926). Combining the advantages of first-difference and deviation-from-trend methods of correlating time series. *Journal of the American Statistical Association 21*, 55–59.

Staiger, D. and J. H. Stock (1997). Instrumental variables regression with weak instruments. *Econometrica 65*, 557–586.

Sylwestrowicz, J. D. (1975). Numerical optimization of non-linear functions of several variables using random search techniques. Phd thesis, Economics Department, London School of Economics.

Tamer, E. (2010). Partial identification in econometrics. *Annual Review of Economics 2*, 167–195.

Tinbergen, J. (1930). 'Determination and interpretation of supply curves: An example' [Bestimmung und Deutung von Angebotskurven: ein Beispiel]. *Zeitschrift fur Nationalökonomie 1*, 669–679. Reprinted in English by Hendry and Morgan (1995), *op cit.*

Trivedi, P. K. (1969). An econometric study of inventory behaviour in the UK manufacturing sector, 1956–67. Phd thesis, Economics Department, London School of Economics.

Tse, Y. K. (1981). Edgeworth approximation to the finite sample distribution of econometric estimators and test statistics. Phd thesis, Economics Department, London School of Economics.

Vernon, K. (1970). An econometric study of wage and price inflation in the United Kingdom for the post-war period. Phd thesis, Economics Department, London School of Economics.

Wegge, L. L. (1965). Identification criteria for a system of equations as a whole. *Australian Journal of Statistics 7*, 67–77.

Whittle, P. (1951). *Hypothesis Testing and Time Series Analysis*. Uppsala: Almquist and Wiksell.

Williams, R. A. (1969). An econometric study of post-war demand for consumer durables in the United Kingdom. Phd thesis, Economics Department, London School of Economics.

Wymer, C. R. (1970). Econometric estimation of stochastic differential equation systems with applications to adjustment models of financial markets. Phd thesis, Economics Department, London School of Economics.