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THE OPTIMALITY OF REGULATED PRICING:

A GENERAL EQUILIBRIUM ANALYSIS

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I. Introduction

Consider a world where consumers, with diminishing marginal rates of substitution, maximize utility subject to their budget constraints and firms, with constant or decreasing returns to scale technologies, maximize profits subject to the prevailing prices. Then every competitive allocation is Pareto optimal and every Pareto optimal allocation can, with lump sum redistribution of endowments and share holdings, be supported as a competitive equilibrium. These two propositions, the first and second welfare theorems, form the foundation of neoclassical welfare economics. Unfortunately, in a world where some firms with increasing returns to scale technologies are price-setting profit maximizers, both of these theorems fail to be true. In this case, there is a need for government intervention, which may take the form of regulated pricing of firms with increasing returns to scale technologies.

In this paper, we use the two sector general equilibrium model to investigate the optimality of several pricing schemes that have been proposed for the regulation of a public monopoly. Here a public monopoly is taken to be a multiproduct firm having a nonconvex production set.

A recurring concern in the literature on public utility pricing is the merit of cross-subsidization. That is, if a public enterprise produces

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two or more products, each of which can be priced separately, is it in the public interest to allow this firm to satisfy only a single break even constraint over its total menu of outputs? Under this single constraint, some product lines will be making a profit while others will be sold at a loss. Moreover, the prices need not reflect the marginal cost to society of producing these outputs. Simply put: Should I have to pay for your consumption?

Invoking the benefit principal, some economists have argued against cross-subsidization. Other economists have argued that the government can use commodity taxes and cross-subsidization to improve social welfare. The prices which emerge from this approach are the so-called Ramsey prices. A recent criticism of Ramsey pricing is that it takes the number of firms as fixed and that cross-subsidization may induce socially undesirable entry, i.e., the public enterprise is a natural monopoly (see the illuminating discussion in Faulhaber (7)). If one considers the conditions for total second best optimality which includes the optimal number of firms, then the government should require the regulated prices to be sustainable, i.e., prices at which no other firm with the same technology can profitably enter the markets of the public enterprise. In the best of all possible worlds, one might hope that the Ramsey prices are sustainable. In which case, the government, using commodity taxation, has obtained total second best optimality.

Suppose the public enterprise produces only two products, say electricity and grain, from two inputs, capital and labor, which are inelastically supplied. The production function for electricity and the production function for grain both exhibit decreasing average costs. Hence, the production possibility set for this firm will, in general, be nonconvex. Suppose also that this is the only active firm in the economy, but there are potential entrants. We assume that there are two consumers, each endowed only with capital and labor, who consume only electricity and grain.

Clearly, in this model, Ramsey pricing and sustainable pricing are incompatible, without some special assumption on the technology of the public enterprise. This can be seen as follows.

In order that the consumer prices of electricity and grain are sustainable, they must equal the average costs of producing the equilibrium outputs. Moreover, in this world sustainability also requires that the factor price ratio facing consumers must be the factor price ratio at which the equilibrium outputs are produced at minimum cost. Given the prices of electricity, grain, capital and labor, we lose one degree of freedom by normalization. The remaining degrees of freedom are consumed by the sustainability conditions. Hence, there is no freedom to trade off the elasticities of demand through cross-subsidization to obtain the Ramsey optimum.

If we now assume that both the production function for grain and the production function for electricity are homogenous of degree r, then the average cost of producing electricity equals r times the marginal cost of producing electricity and similarly for grain. In this case, if there is a single houshold or, equivalently, there are no income effects so that we can aggregate the two households, then there exist a Pareto optimal allocation which can be supported by average cost pricing, i.e., where output is sold at average cost and produced at minimum cost.

Under these very restrictive assumptions, the Pareto optimal allocation will be a sustainable Ramsey optimal outcome for any individualistic social welfare function. This is a version of the weak invisible hand theorem in the two sector model (1).

Can we establish a more general result? The answer is yes, if we remember that consumer prices and producer prices can be chosen independently in a world with a government, where there are no profits in equilibrium. This is the funadmental insight in the seminar paper of Diamond and Mirrlees (D-M, (6)). Let q and p be consumer and producer prices, respectively. In D-M, the intended interpretation of p is that it is the vector of shadow prices at the social optimum. Indeed, this has been the prevailing interpretation throughout the optimal taxation literature and the related literature on public sector pricing, as for example, in Baumol-Bradford (2). This interpretation is, of course, necessary for the optimal tax formula derived in D-M.

But if we recall the structure of the D-M argument, we see that it proceeds in two steps. The first is a demonstration that optimal social welfare requires productive efficiency. Next is an observation that profit-maximizing, price-taking firms, with constant or decreasing returns to scale, can be made to produce the socially desired output in the competitive sector by simply announcing the shadow prices associated with the social optimum and requiring the firms to maximize profits at these prices.

If there is only one firm with a nonconvex technology in the economy, then obviously profit maximization at the shadow prices of the social optimum need not lead to the socially optimal outcome. Suppose we posit a contestable market rather than a competitive market: a market where entry and exit are costless; assume also that if there is entry, the public utility is required to maintain its pre-entry price; and that every firm has access to the technology of the public utility. If each product line is produced with decreasing average costs and the public utility faces a common carrier constraint to meet all residual demand in the event of entry, then entry will result in a loss of profits for the public enterprise, since we have assumed that the public utility must meet the residual demand at the pre-entry prices. Hence, the public enterprise, in order to maximize profits, will attempt to set prices at which entry is unprofitable, i.e., it will attempt to set sustainable prices.

This suggests that in the case of a natural monopoly, the p in the D-M analysis should be interpreted as sustainable prices rather than shadow prices. Of course, no such p need exist. If it does exist (for which a sufficient condition in this model is that each product line is produced with decreasing average costs) then the excise tax $t \equiv q - p$ is to be interpreted as the optimal excise tax imposed by the government to prevent socially undesirable entry. That is, q are the prices prevailing in the market but the prices facing the public utility and potential entrants is q net of t or p. It is easy to see that the net tax revenue is zero. Hence, there is no surplus to dispose of. If each product line is produced with decreasing average costs, then p is simply average cost pricing, i.e., each product is sold at average cost and produced at minimum cost.

If the conditions of the weak invisible hand theorem are met, then $q = p$ and $t = 0$.

It may seem strange at first reading that the government should use a per-unit tax to prevent entry rather than simply forbidding entry or requiring a licensing fee, once it has determined that the public utility is a natural monopoly. But without the threat of potential entry, average cost pricing is not incentive compatible. Even if the public enterprise is breaking even, it need not be producing at minimum cost. Also, the government wishes to encourage innovation. If some entrepreneur discovers a more efficient technology for producing the outputs of the natural monopolist, we want her to enter the market. Finally, a license fee acts as a fixed charge for entry, hence innovations whose profitability does not exceed this fee will be lost to society.

A rigorous argument that if both products are produced with decreasing average costs and if there is potential entry, then average cost pricing is the profit maximizing strategy for the public enterprise requires a game-theoretic analysis.

Consider the following pseudo game between households, the government, the public enterprise (incumbent firm), and the potential entrants. First, the government announces factor prices and the consumer prices of goods; households then maximize utiltiy subject to their budget constraint, generating an excess demand vector. Next, the government solicits from the public utility the prices at which it will meet the excess demand for products. (All players are price takers in the factor markets which are assumed to be competitive.) Then each potential entrant submits a bid to the government to supply part or all of the excess demand at specified prices. Finally, the government accepts those bids which are less than the bid of the public enterprise. If these bids do not exhaust the excess demand, then the government purchases the residual demand from the public enterprise at its pre-entry price.

It is easy to see that a Nash equilibrium of this pseudo game is where the government announces to households the prices which maximize social welfare; the public enterprise produces all the excess demand for products at minimum cost and sells it to the government at average cost, and no firms enter the markets of the public enterprise.

The genesis of this game originates with Demsetz (5), who argued that if the government auctions off the right to produce products which have an increasing returns to scale technology to the firm with the lowest per-unit cost, then the public enterprise must price at average cost.

We suggest that in addition to auctioning production rights, the government use its taxing power to maximize social welfare.

This proposal answers Telser's criticism of Demsetz that average cost pricing violates the first order conditions for Pareto optimality if there are decreasing average costs; hence, the Demsetz proposal, which is referred to in the literature as franchise bidding, although it prevents monopoly pricing, need not lead to efficient pricing, with price equal to marginal cost (10). However, by using commodity taxation together with a franchise bidding scheme, the government can ensure total second best optimality.

Both the theory of contestable markets (3) and franchise bidding schemes (9) have been proposed as alternatives to the regulation of markets where products are produced with decreasing average costs. But both theories implicitly employ the state (regulatory commission) at a crucial point in their analysis. In the case of contestable market theory, it must be the state, through regulation, that prohibits a price response from the public enterprise post entry. In the franchise bidding scheme, the state is the auctioneer.

The debate is thus about the degree of state intervention rather than about the absence or presence of regulation. The additional burden of computing the optimal excise tax does not seem excessive given the optimal tax formulae which we establish below. The first order conditions necessary for consumer prices to maximize social welfare and for average cost pricing to support the social optimum require that the marginal social utility of an increase in the public utility's revenue, for each product, equals the marginal social utiltiy of a decrease in the public utility's cost, for each factor, at the social optimum.

In summary, the main contribution of this paper is to suggest that the government can use commodity taxes to improve social welfare if firms are pricing at average cost. Moreover, we expect to see average cost pricing if firms are quantity-taking profit maximizers who compete to supply the market demand, e.g., franchise bidding* or contestable markets with decreasing average costs. If there is average cost pricing and products are being produced with decreasing average costs, then entry will occur only if the entrant possesses an innovative technology.

We shall assume that the reader is familiar with the two-sector model as a general equilibrium model of increasing returns to scale, say, as in the paper of Brown-Heal (4). In fact, the present paper should be viewed as a continuation of the investigations initiated there.

Before proving our major propositions concerning the use of commodity taxes to improve social welfare, we first discuss first-best situations where lump sum taxation is feasible.

In the final section of the paper, we discuss the equity and efficiency of the recent FCC decision to impose a flat monthly "interstate access charge" on all subscribers to local telephone service.

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*By franchise bidding, we mean that firms are bidding to produce given quantities of outputs. Our notion is similar, in spirit, to the recurrent short term contracts of Posner. See Williamson (11) for a discussion and critique of various forms of franchise bidding, including the scheme of Posner.

II. The Model

We consider an economy with two households, and a single firm producing two products from two factors which are inelastically supplied by the households. The factors are capital (K) and labor (L). The products are grain (G) and electricity (E). Households have utility functions denoted U_x and U_y, respectively. Endowments and shareholdings in the firm are given by (K_x, L_x), (K_y, L_y); θ_x and θ_y. The production function of the firm is separable, where the production functions for the products are F_G and F_E or equivalently, cost functions C_G and C_E. Let $K = K_x + K_y$ and $L = L_x + L_y$.

We make the standard assumptions regarding households and production functions, with the following exceptions:

(1) although we assume that factor markets are competitive, i.e., that the production functions for both products exhibit diminishing marginal rates of technical substitution, we do not assume constant or decreasing returns to scale;

(2) household's indifference curves do not cut the coordinate axes;

(3) both marginal and average costs are well behaved at zero output.

Under these assumptions, we construct the Edgeworth-Bowley box for production and the associated production possibility set for the firm. In general, the production possibility set for the firm (public monopoly) is nonconvex.

Let P_G and P_E denote the prices of grain and electricity, and

w and r denote the prices of labor and capital.

A point (G, E) is said to be production efficient if it lies on the frontier of the firm's production possibility set. Each point on the frontier uniquely determines the factor price ratio at which these outputs are produced at minimum cost. Moreover, the marginal rate of transformation (MRT) at this point is equal to the ratio of the marginal costs.

The income distribution is said to be fixed if $(K_x, L_x) = \theta_x(K, L)$ and $(K_y, L_y) = \theta_y(K, L)$.

A marginal cost pricing (MCP) equilibrium is a family of consumption plans, production plans, prices, and lump sum taxes such that households are maximizing utility subject to after-tax income; the public monopoly is producing at minimum cost and selling at marginal cost, where losses are covered by the lump sum taxes; and all markets clear.

An average cost pricing (ACP) equilibrium is a family of consumption plans, production plans and prices such that households are maximizing utility subject to their budget constraint; the public monopoly is producing at minimum cost and breaking even; and all markets are clear.

An average cost pricing equilibrium without cross-subsidization is a family of consumption plans, production plans and prices such that households are maximizing utility subject to their budget constraint; the public monopoly is producing at minimum cost and selling each product at average cost; and all markets clear.

In general, neither MCP nor ACP equilibria will be Pareto optimal. ACP equilibrium violate the first order conditions necessary for Pareto optimality and MCP equilibria need not satisfy the sufficient conditions for Pareto optimality, if the production possibility set is nonconvex.

III. Welfare Theorems

Proposition (1): If the income distribution is fixed, then every Pareto optimal allocation, where both households consume a positive amount of each good, can be supported as a MCP equilibrium with lump sum transfers to households.

Proof: If $(\bar{E}_x, \bar{G}_x)$, $(\bar{E}_y, \bar{G}_t)$ is the Pareto optimal allocation, let $\bar{E} = \bar{E}_x + \bar{E}_y$, $\bar{G} = \bar{G}_x + \bar{G}_y$ and $A = \{(E, G) | E = E_x + E_y, G = G_x + G_y$ where $U_x(E_x, G_x) \geq U_x(\bar{E}_x, \bar{G}_x)$ and $U_y(E_y, G_y) \geq U_y(\bar{E}_y, \bar{G}_y)\}$. Then $(\bar{E}, \bar{G})$ is on the production possibility frontier and A is a convex closed set which is tangent to the production possibility set at $(\bar{E}, \bar{G})$. Let w/r , P_E/r , P_G/r be the factor price ratio at which $\bar{E}$ and $\bar{G}$ are produced at minimum cost, the marginal cost of producting $\bar{E}$, and the marginal cost of producing $\bar{G}$. These prices define the budget constraint for households as:

$$\frac{P_E}{r} E_j + \frac{P_G}{r} G_j = \frac{w}{r} L_j + K_j + \theta_j \Pi + T_j \text{ , where } j = x \text{ or } y \text{ ,}$$

where Π is the firm's profit and T_j is the lump sum transfer to household j . The firm is required to produce $(\bar{E}, \bar{G})$ at minimum cost and sell at marginal cost. Since the income distribution is fixed, the right hand side of the budget constraint reduces to:

$$\theta_j (\frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}) + T_j \text{ ,}$$

Let
$$T_j = \frac{P_E}{r} \bar{E}_j + \frac{P_G}{r} \bar{G}_j - \theta_j (\frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}) \text{ .}$$

Recalling that the MRT at $(\bar{E}, \bar{G})$ is the ratio of the marginal costs, i.e., P_E/P_G, we see that the set A is supported by the price line with slope P_E/P_G. Hence, $(\bar{E}_x, \bar{G}_x)$ and $\bar{E}_y, \bar{G}_y)$ maximize U_x and U_y, subject to the budget constraints as defined above.

<u>Proposition (2)</u>: If the income distribution is fixed and the utility functions are concave and homogeneous of degree one, then there exists a Pareto optimal MCP equilibrium.

<u>Proof</u>: Under the assumptions on preferences and endowments, it follows from Eisenberg's Theorem (see Theorem 3 in (8)) that the market demand function is generated by a utility function $U(E, G)$, where

$$U(E, G) = \max[U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y}$$

such that $E_x + E_y = E$ and $G_x + G_y = G$.

Suppose U achieves its maximum over the production possibility set at $(\bar{E}, \bar{G})$. Then, assuming monotonicity of U_x and U_y, $(\bar{E}, \bar{G})$ is on the production possibility frontier. Let $(\bar{E}_x, \bar{G}_x)$ and $(\bar{E}_y, \bar{G}_y)$ be the Pareto optimal allocation, corresponding to $U(\bar{E}, \bar{G})$. That is,

$$U(\bar{E}, \bar{G}) = [U_x(\bar{E}_x, \bar{G}_x)]^{\theta_x}[U_y(\bar{E}_y, \bar{G}_y)]^{\theta_y}$$

If w/r, P_R/r, and P_G/r is the factor price ratio at which $\bar{E}$ and $\bar{G}$ are produced at minimum cost, the marginal cost of producing $\bar{E}$ and the marginal cost of producing $\bar{G}$, then the optimal solution of max $U(E, G)$

such that $\frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}$

is $(\bar{E}, \bar{G})$, since U is a concave function of (E, G).

$$U(\bar{E}, \bar{G}) = \max\, [U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y}$$

such that $E_x + E_y = \bar{E}$ and $G_x + G_y = \bar{G}$

$$\leq \max\, [U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y}$$

such that $\frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}$, where $E_x + E_y = E$ and $G_x + G_y = G$

$$\leq U(E, G)$$

such that $\frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}$

$$\leq \max\, U(E, G)$$

such that $\frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}$

$$= U(\bar{E}, \bar{G}) .$$

Hence, $(\bar{E}_x, \bar{G}_x)$ and $(\bar{E}_y, \bar{G}_y)$ is the optimal solution of

$$\max[U_x(E_x, G_x)]^{\theta_x}[U_y(E_y, G_y)]^{\theta_y}$$

such that $\frac{P_E}{r}(E_x + E_y) + \frac{P_G}{r}(G_x + G_y) = \frac{P_E}{r}\bar{E} + \frac{P_G}{r}\bar{G}$.

Therefore, by Eisenberg's Theorem the optimal solutions to

$$\max U_j(E_j, G_j)$$

such that $\frac{P_E}{r} E_j + \frac{P_G}{r} G_j = \theta_j(\frac{P_E}{r}\bar{E} + \frac{P_G}{r}\bar{G})$, $j = x$ or y

are $(\bar{E}_x, \bar{G}_x)$ and $(\bar{E}_y, \bar{G}_y)$. Recalling the fixed income distribution, we see that the right hand side of each household's budget constraint can be expressed as $\frac{w}{r} L_x + K_x + \theta_x \Pi$ and $\frac{w}{r} L_y + K_y + \theta_y \Pi$, where Π is the profit of the natural monopoly that is producing $(\bar{E}, \bar{G})$ at minimum cost and selling at marginal cost.

Proposition (3): Every Pareto optimal allocation, where both households consume a positive amount of each good, can be supported as an ACP equilibrium with lump sum transfers to households.

Proof: If $(\bar{E}_x, \bar{G}_x)$, $(\bar{E}_y, \bar{G}_y)$ is the Pareto optimal allocation, let $\bar{E} = \bar{E}_x + \bar{E}_y$ and $\bar{G} = \bar{G}_x + \bar{G}_y$. Let w/r be the factor price ratio at which $(\bar{E}, \bar{G})$ is produced at minimum cost. Cross-subsidization gives us one degree of freedom in determining the relative prices for products, i.e., we only have the single break-even constraint $\frac{P_E}{r}\bar{E} + \frac{P_G}{r}\bar{G} = \frac{w}{r}L + K$.

Hence, we can require that the product price ratio equals the MRT at $(\bar{E}, \bar{G})$, i.e., $\frac{P_E}{r} = \text{MRT}\,\frac{P_G}{r}$. These two equations uniquely determine the values of $\frac{P_E}{r}$ and $\frac{P_G}{r}$, where $\frac{P_G}{r} = \dfrac{\frac{w}{r}L + K}{(\text{MRT})\bar{E} + \bar{G}}$.

These prices define the following budget constraint for households as:

$$\frac{P_E}{r}E_j + \frac{P_G}{r}G_j = \frac{w}{r}L_j + K_j + T_j ,$$

where T_j is the lump sum transfer to household j and $j = x$ or y . Since the firm breaks even, the profit $\Pi = 0$. Let

$$T_j = \frac{P_E}{r}\bar{E}_j + \frac{P_G}{r}\bar{G}_j - \frac{w}{r}L_j - K_j .$$

Then $(\bar{E}_j, \bar{G}_j)$ maximizes $U_j(E_j, G_j)$ subject to the budget constraint. Proposition (3) was suggested to us by T. N. Srinivasan.

<u>Proposition (4)</u>: If the income distribution is fixed and the utility functions are concave and homogeneous of degree one, then there exists a Pareto optimal ACP equilibrium .

Proof: As in the proof of Proposition (2), we invoke the Eisenberg aggregation theorem to obtain $U(E, G)$. We then maximize U over the production possibility set, obtaining a maximum at $(\bar{E}, \bar{G})$ on the production possibility frontier. Let w/r be the factor price ratio at which $(\bar{E}, \bar{G})$ is produced at minimum cost given the MRT at $(\bar{E}, \bar{G})$, we set

$$\frac{P_E}{r} = \text{MRT}\,\frac{P_G}{r} \quad \text{and} \quad \frac{P_G}{r} = \frac{\frac{w}{r}L + K}{(\text{MRT})\bar{E} + \bar{G}} .$$

As in the proof of Proposition (3), these relative product prices are chosen to make the firm break even and the product price ratio equal the MRT. The rest of the argument is the same as the proof of Proposition (2), noting that in the family of maximization problems the budget constraints depend only on the MRT at $(\bar{E}, \bar{G})$.

Proposition (5): If F_E and F_G are both homogeneous of degree $s \geq 1$, then every Pareto optimal allocation, where both households consume a positive amount of each good, can be supported as an ACP equilibrium without cross-subsidization and lump sum transfers to households.

Proof: If $(\bar{E}_x, \bar{G}_x)$, $(\bar{E}_y, \bar{G}_y)$ is the Pareto optimal allocation, let

$$\bar{E} = \bar{E}_x + \bar{E}_y \quad \text{and} \quad \bar{G} = \bar{G}_x + \bar{G}_y .$$

Let w/r be the factor price ratio at which $(\bar{E}, \bar{G})$ is produced at minimum cost and P_E/r, P_G/r be the average cost of producing $\bar{E}$ and $\bar{G}$, respectively. Since F_E and F_G are both homogeneous of degree s, the ratio of the average costs of producing $\bar{E}$ and $\bar{G}$, i.e., P_E/P_G, equals the ratio of the marginal costs of producing $\bar{E}$ and $\bar{G}$, i.e., the MRT. Hence, P_E/P_G = MRT. These prices define the following budget constraints:

$$\frac{P_E}{r} E_j + \frac{P_G}{r} G_j = \frac{w}{r} L_j + K_j + T_j \quad \text{for } j = x \text{ or } y$$

where the lump sum transfers

$$T_j = \frac{P_E}{r} \bar{E}_j + \frac{P_G}{r} \bar{G}_j - \frac{w}{r} L_j - K_j \ .$$

Then $(\bar{E}_x, \bar{G}_x)$ and $(\bar{E}_y, \bar{G}_y)$ maximize U_x and U_y subject to the given budget constraints.

Proposition (6): If the income distribution is fixed, the utility functions are concave and homogeneous of degree one, and both F_E and F_G are homogeneous of degree $s \geq 1$, then there exists a Pareto optimal ACP equilibrium without cross-subsidization.

Proof: The argument is the same as the proof of Proposition (4), except that P_E/r and P_G/r are the average costs of production. At these prices the firm breaks even and the product price ratio is the MRT.

We now present the major two theorems in this paper.

Theorem (1): If the social welfare function is individualistic, then every socially optimal allocation can be supported as an ACP equilibrium without cross-subsidization and with commodity taxation. Moreover, if C_E and C_G exhibit decreasing average costs, then the producer prices for products are sustainable against partial entry. That is, no potential entrant, using the technology of the public monopolist, can enter the market for electricity or grain and make a profit by producing any part of the given total demand at the prevailing factor price ratio and selling it at prices no greater than the prevailing producer prices for products.

Proof: We shall find it convenient to represent the production possibility set for the firm by a smooth transformation function $H(E, G)$. The production possibility set is the set of (E, G) such that $H(E, G) \leq 0$ and the set's frontier is characterized by $H(E, G) = 0$. Let Δ be the price simplex in R^4_+ and $q = (q_E, q_G, q_L, q_K)$ be the normalized household prices for electricity, grain, labor and capital. Since households are only endowed with factors which they supply inelastically, let $X(q)$ be the market demand for electricity and grain. Then $D = \{q \in \Delta | H(X(q)) \leq 0\}$. If $V(q)$ is an individualistic (indirect) social welfare function, then the government's problem is to $\max_{s.t.\ q \in D} V(q)$. D is clearly compact, hence we only need to show that $D \neq \phi$ in order for the government's problem to have a solution. Moreover, it follows from Lemma (1) in D-M (6), that the social optimum if it exists will be on the frontier of the production possibility set, e.g., households are endowed with labor but do not consume leisure. Unlike D-M, we cannot show that $D \neq \phi$ by invoking the existence theorem for exchange economies since households are not endowed with electricity or grain. Instead, we invoke the existence theorem for an ACP equilibrium without cross-subsidization, which is Theorem (3) in (4).* Hence, $D \neq \phi$. Let $\hat{q}$ be an optimal solution of $\max_{q \in D} V(q)$, then $X(\hat{q})$ is on the frontier of the production possibility set. Denote $X(\hat{q})$ as $(\hat{E}, \hat{G})$ and let $\hat{P}_L/\hat{P}_K$ be the factor price ratio at which $(\hat{E}, \hat{G})$ is produced at minimum cost and $\hat{P}_E/\hat{P}_K$, $\hat{P}_G/\hat{P}_K$ be the average cost of producing $\hat{E}$ and $\hat{G}$, respectively. Renormalizing $\hat{q}$ so that capital is the numeraire good, we have $\hat{q}_L/\hat{q}_K$ as the factor price ratio facing households and $\hat{q}_E/\hat{q}_K$, $\hat{q}_G/\hat{q}_K$ as the prices which households face in the product markets. The optimal excise taxes are $\hat{t}_E = \hat{q}_E/\hat{q}_K - \hat{P}_E/\hat{P}_K$, $\hat{t}_G = \hat{q}_G/\hat{q}_K - \hat{P}_G/\hat{P}_K$, and $\hat{t}_L = \hat{q}_L/\hat{q}_K - \hat{P}_L/\hat{P}_L$; capital is untaxed. Applying Walras' law and the break-even condition, we see that the net tax revenue is zero.

It is clear that with decreasing average costs for both electricity and grain that no part of $(\hat{E}, \hat{G})$ can be produced for profit at product prices no greater than $\hat{q}_E/\hat{q}_K$ and $\hat{q}_G/\hat{q}_K$, with a factor price ratio of $\hat{q}_L/\hat{q}_K$. This completes the proof.

A production efficient point (E, G) is said to be <u>supported by AC pricing without cross-subsidization</u> if for relative prices $p = (P_E, P_G, P_L)$, where capital is the numeraire good, P_L is the factor price ratio at which (E, G) is produced at minimum cost and P_E, P_G are the average costs of producing E and G .

<u>Theorem (2)</u>: If the economy has a single household and V is an individualistic social welfare function, then a necessary condition for relative household prices $q = (q_E, q_G, q_L)$, to be socially optimal and the demand (E(q), G(q)) to be supported by AC pricing without cross-subsidization

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*If the conditions for this theorem do not hold, e.g., F_E or F_G are Cobb-Douglas production functions with increasing returns, it is still true that $D \neq \emptyset$. Simply choose prices of products so high that aggregate demand is almost zero, hence feasible.

at relative producer prices $p = (P_E, P_G, P_L)$ is that:

$$\frac{E}{\frac{\partial}{\partial q_E}[p_E E + p_G G]} = \frac{L}{\frac{\partial}{\partial q_L}[p_L L + K]} = \frac{G}{\frac{\partial}{\partial q_G}[p_E E + p_G G]}$$

Proof: As in Theorem (1), let $H(E, G)$ denote the transformation function for the production possibility set. The government's optimization problem is $\max V(E(q), G(q))$

s.t. $H(E(q), G(q)) \leq 0$. The first order conditions are

$$(1) \qquad \frac{\partial V}{\partial E}\frac{\partial E}{\partial q_j} + \frac{\partial V}{\partial G}\frac{\partial G}{\partial q_j} = \lambda\left\{\frac{\partial H}{\partial E}\frac{\partial E}{\partial q_j} + \frac{\partial H}{\partial G}\frac{\partial G}{\partial q_j}\right\} \text{ where } j = E, G, \text{ and } L.$$

Since there is only one consumer and V is individualistic, we see that

$$(2) \qquad -\alpha j = \frac{\partial V}{\partial E}\frac{\partial E}{\partial q_j} + \frac{\partial V}{\partial G}\frac{\partial G}{\partial q_j} \quad \text{for } j = E, G, \text{ and } L$$

where α is the marginal utility of income.

From equation systems (1) and (2), we derive

$$(3) \qquad \frac{E}{G} = \frac{MRT\frac{\partial E}{\partial q_E} + \frac{\partial G}{\partial q_E}}{MRT\frac{\partial E}{\partial q_G} + \frac{\partial G}{\partial q_G}} \quad \text{and} \quad \frac{E}{L} = \frac{MRT\frac{\partial E}{\partial q_E} + \frac{\partial G}{\partial q_E}}{MRT\frac{\partial E}{\partial q_L} + \frac{\partial G}{\partial q_L}}$$

Recalling that the MRT is the ratio of the marginal costs, i.e., $MRT = MC_E/MC_G$, we get

$$(4) \qquad \frac{E}{G} = \frac{MC_E\frac{\partial E}{\partial q_E} + MC_G\frac{\partial G}{\partial q_E}}{MC_E\frac{\partial E}{\partial q_G} + MC_G\frac{\partial G}{\partial q_G}} \quad \text{and} \quad \frac{E}{L} = \frac{MC_E\frac{\partial E}{\partial q_E} + MC_G\frac{\partial G}{\partial q_E}}{MC_E\frac{\partial E}{\partial q_L} + MC_G\frac{\partial G}{\partial q_L}}$$

But

$$(5) \qquad MC_E \equiv E \frac{\partial AC_E}{\partial E} + AC_E = E \frac{\partial P_E}{\partial E} + P_E$$

$$\text{and} \quad MC_G \equiv G \frac{\partial AC_G}{\partial G} + AC_G = G \frac{\partial P_G}{\partial G} + P_G$$

where AC_E and AC_G are the average costs of producing E and G. From (5), we see that

$$(6) \qquad MC_E \frac{\partial E}{\partial q_E} = \frac{\partial}{\partial q_E} (P_E E) \; ; \; MC_E \frac{\partial E}{\partial q_G} = \frac{\partial}{\partial q_G} (P_E E) \qquad \text{and}$$

$$MC_G \frac{\partial G}{\partial q_E} = \frac{\partial}{\partial q_E} (P_G G) \; ; \; MC_G \frac{\partial G}{\partial q_G} = \frac{\partial}{\partial q_G} (P_G G)$$

Substituting (6) into equation (4), we have

$$\frac{E}{G} = \frac{\frac{\partial}{\partial q_E} [P_E E + P_G G]}{\frac{\partial}{\partial q_G} [P_E E + P_G G]} \quad \text{and} \quad \frac{E}{L} = \frac{\frac{\partial}{\partial q_E} [P_E E + P_G G]}{\frac{\partial}{\partial q_L} [P_E E + P_G G]}$$

Since $P_E E + P_G G \equiv P_L L + K$, $\frac{\partial}{\partial q_L} [P_E E + P_G G] = \frac{\partial}{\partial q_L} [P_L L + K]$.

Therefore, $\frac{E}{L} = \frac{\frac{\partial}{\partial q_E} [P_E E + P_G G]}{\frac{\partial}{\partial q_L} [P_L L + K]}$, completing the proof.

IV. A Policy Application

The FCC has recently imposed a flat monthly "interstate access charge" on all subscribers to local telephone service. The argument for introducing this charge is that opening up the telecommunications market to competition makes it impossible, in the long run, for AT&T long lines to continue the Bell system's monopolistic practice of subsidizing local telephone service by charging above unit cost for long-distance service. Consequently, those who wish to make local calls must bear the "full cost" of this service. As one might expect, most consumers argue that these charges impose an unfair burden on residential subscribers and that there is little proof that the profits of AT&T long lines will be seriously eroded by competition from Sprint, MCI and other new carriers of long-distance telecommunications.*

Prior to divestiture, AT&T produced both local and long-distance telephone service, and was only required to satisfy a single break-even constraint. In terms of our model, the relevant equilibrium concept is an ACP equilibrium.

Subsequent to divestiture, local service will be provided by the operating companies, which will continue to be regulated monopolies; but long-distance service will be provided by AT&T long lines lines and other carriers. If the regulation of the local operating companies is successful, i.e., output is produced at minimum cost and sold at average cost, and if, as hoped, competition in the decreasing average cost market for long-distance calls leads to

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*For an excellent discussion of the issues, see "Sidestepping the Telephone Company," by Andrew Pollack in the *New York Times*, December 1, 1983, page 29.

average cost pricing, then the relevant equilibrium notion is an ACP equilibrium without cross-subsidization.

Unfortunately, first best optimality for this type of equilibrium can be obtained only under restrictive assumptions on tastes, the income distribution or technology. This is the content of Propositions (5) and (6).

We have shown in Theorem (1) that second best optimality can be achieved in the presence of average cost pricing without cross-subsidization, by using commodity taxation. Hence, we propose an excise tax on long-distance calls, where the resulting tax revenue is used to subsidize local service.

If both services are produced with decreasing average costs and carriers compete to produce the quantities demanded at the subsidized rates, then AT&T long lines, together with the local operating companies, will produce the services demanded at minimum cost and only carriers with an innovative technology will enter the unregulated market.* Of course, this policy may push long-distance rates back up to the values they had under the Bell system monopoly; but society will still obtain two of the three potential cost savings which accrue from competition: (1) the efficient production of all service, i.e., production at minimum cost, (2) the long-run cost savings arising out of innovation. Selling output at unit cost is the third potential cost saving deriving from competition, but in our proposal this is the opportunity cost that society must pay for subsidizing local service.

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*Here the assumption of competition and decreasing average cost in the market for local service is replaced by the regulatory constraint on the local operating companies.

In effect, this proposal continues the historical policy of making a transfer from long-distance users to local users of telephone service, essentially a transfer from business to residential subscribers, which reflects society's choice of a particular kind of welfare function. Our observation is that opening the telecommunications market to competition can be made consistent with the present rate structure or social welfare function, which subsidizes local service.

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CFDP #
684A

AN OPTIMAL TAX RULE FOR AVERAGE COST PRICING*

by

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At Bill Nordhaus's suggestion, we have derived an expression for the optimal tax formula in CFDP 684 in terms of the elasticities of the demand curves and average cost curves. This expression enables us to compare the tax rates in the two product markets.

The interesting special case is where the demands are independent. In this situation, if τ_E and τ_G are the tax rates for E and G , where $\tau_j = t_j/P_j$ for $j = E$ or G , then the formula reduce to:

$$\frac{\eta_E(1+\eta_{AC_E})}{(1+\tau_E)} = \frac{\eta_G(1+\eta_{AC_G})}{(1+\tau_G)}$$

In this expression, if $f(x)$ is the curve in question, then

$$\eta_{f(x)} = \frac{x}{f(x)} \frac{df(x)}{dx} .$$

This rule says that the optimal tax rate is proportional to the elasticity of the average cost curve and inversely proportional to the elasticity of the demand curve. If the average cost curves have equal elasticities,

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*This note is an addendum to "The Optimality of Regulated Pricing: A General Equilibrium Analysis," Cowles Foundation Discussion Paper #684, December 1983.

then our rule reduces to the Ramsey rule. If the demand elasticities are equal, then the higher tax is imposed on the product with the more elastic average cost curve.

If E is produced with increasing returns, i.e., $\eta_{AC_E} < 0$ and G is produced with decreasing returns, i.e., $\eta_{AC_G} > 0$, then G is taxed more heavily than G . This prescription is reminiscent of Marshall's proposal that increasing returns to scale industries should be subsidized and decreasing returns to scale industries should be taxed. If the marginal cost of producing E is 0 , i.e., $\eta_{AC_E} = -1$ and G is produced with constant average cost, i.e., $\eta_{AC_G} = 0$, then E is subsidized and G is taxed. This is the case of Dupuit's bridge.

For the case of AT&T, we see that if we require no cross-subsidization, i.e., both long distance service and local service is priced at average cost, and we wish to optimize social welfare by using commodity taxation, then *ceribus paribus*, we should impose an excise tax on long distance service and subsidize local service if long distance service has the more elastic average cost curve.

A heuristic argument that the optimal tax rate is proportional to the elasticity of the average cost curve is below (see Figure A).

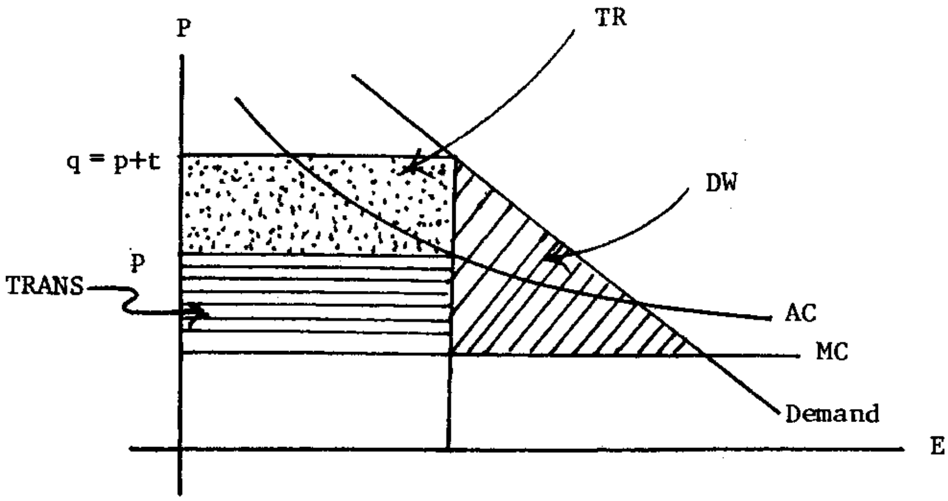

Figure A

DW	is the deadweight loss,
TR	is the tax revenue collected by government,
TRANS	is the consumer surplus transferred to the firm by average cost pricing

If we minimize deadweight loss, then a necessary condition for social optimality is that the deadweight loss per dollar of tax revenue be equal in both markets.

Ignoring the change in q, as we change the elasticity of the average cost curve, we see that DW / TR is a decreasing function of η_{AC} and the tax rate τ is an increasing function of η_{AC}. Hence, the good with the higher elasticity of average cost has the higher tax.

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