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AT YALE UNIVERSITY

Box 2125, Yale Station
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THINKING ABOUT CARBON DIOXIDE: THEORETICAL AND EMPIRICAL ASPECTS OF OPTIMAL CONTROL STRATEGIES

William D. Nordhaus

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Summary

The scientific and policy discussions about the CO₂ problem can be divided into three different parts: (a) understanding the carbon cycle and climatic effects of CO₂ elevation; (b) understanding the economic and social impacts of CO₂ and estimating abatement strategies; and (c) weighing the costs and benefits in (a) and (b) to set appropriate policies. Up to now, almost all scientific work has gone into (a); a handful of papers discusses (b); but there have been no attempts to pursue (c), that is to weigh scientific and economic evidence so as to give guidance on national or global policies. The present study attempts to use the basic principles of growth economics to start a discussion of policy responses to the CO₂ problem.

The first part of an analysis of CO₂ is to outline a methodological and empirical framework:

---The first step is a model and description of the carbon cycle and climatic effects of CO₂ elevation. There appears to be reasonable agreement on the basic causes and climatic effects of the CO₂ buildup.

---A second step is to provide a model and estimates of the costs of reducing or abating CO₂ emissions. The paper relies on a crude set of figures on abatement prepared in a linear programming model of the long-run energy system.

---The third and most difficult step is the estimate of the economic impacts of the CO₂ buildup. As current research provides only a few hints at these costs, the paper provides a very crude attempt to quantify a range of estimates. Optimistic and pessimistic sets of costs are derived--the outcomes chiefly reflecting uncertainties about the effect of CO₂ on agriculture or the level of the oceans. The highly tentative conclusions about economic impacts are:

* We cannot now judge whether a CO₂ buildup will be favorable or unfavorable to the global economy.

* The extreme outcomes indicate that the unfavorable outcomes represent a larger loss than the favorable outcomes gain.

---The final element in the discussion is a theory of intertemporal choice between consumption paths. A discussion of discounting starts with a value judgment about the value of real incomes of different generations ("time discounting"). It also asks how much redistribution between poorer and richer generations is desirable (hence "growth discounting"). The overall discount rate is the sum of time and growth discounting. In a world where economic progress occurs, we should discount future consumption if we wish to redistribute from richer to poorer generations.

---Using this line of reasoning, when we make a critical judgment about how to weigh CO₂ control strategies:
\( \text{CO}_2 \) control strategies should be judged by their effects on
the paths of consumption that are generated by the control
strategies.

\( \text{CO}_2 \) control strategies should be chosen to maximize the dis-
counted value of consumption streams, where the discount
rate is the time plus growth discount rate discussed above.

---One of the key variables for both analysis and implementation of con-
tral strategies is the concept of the "shadow price on \( \text{CO}_2 \)." The shadow price
represents (in an efficient control program) the incremental cost of reducing
\( \text{CO}_2 \) emissions by one more ton, and the incremental damage from increasing at-
mospheric concentrations by one ton. In an efficient policy, it is useful to
use the shadow price (perhaps through taxation) as a way of providing market
signals to economic agents as to the value of abatement activities.

The second part of an analysis of \( \text{CO}_2 \), contained in Part II, describes a
set of extremely crude and tentative illustrative calculations which show how
the methodology can be applied.

---A simple first example considers a "no-growth" world economy. The paper
examines (in Table 6 below), the steady state values of important variables for
3 different sets of assumptions about the carbon cycle-economic impacts and 3
different discount rates. In the optimistic case, of course, the optimal strat-
egy is to encourage \( \text{CO}_2 \) buildup as it increases consumption. In the pessimis-
tic case, the optimal steady state \( \text{CO}_2 \) controls range from light to complete
(more precisely, to reduce emissions by one-fifth to 100 percent). The steady
state \( \text{CO}_2 \) shadow price ranges from $9 to $200 per ton carbon. Because the lags
in adjustment of \( \text{CO}_2 \) concentrations to emissions are so long, the other impor-
tant variable for steady state analysis is the time discount rate. For a zero
discount rate, heavy controls are in place in the steady state for either aver-
age or pessimistic parameters. For a high annual discount rate (4 percent)
steady state control levels are light to negligible.

---The examples without growth are clearly unrealistic as it is highly
likely that the global economy and global \( \text{CO}_2 \) emissions will continue to in-
crease for some time to come. We therefore consider control strategies with a
growing economy. In the model used here, economic growth has an unambiguous
effect on the steady state solution. As the global economy grows, uncontrolled
emissions rise and the climatic damage increases (except, of course, in the op-
timistic case). Therefore the ultimate optimal steady-state \( \text{CO}_2 \) control rate
increases and the steady-state \( \text{CO}_2 \) shadow price is higher.

---From a policy perspective, the key finding concerns the optimal current
policy--a policy for a growing world economy well before a steady state has
been reached. Generally, (as shown in Figure 7) the control rate and shadow
price will be lower at the beginning of the transition than in the steady state.

Using the model presented here--and assuming that the parameters are
known with certainty—we might ask what a "best guess" strategy would involve today. The paper suggests (in Table 9) that the current CO\textsubscript{2} emission rate should be about 80 percent of uncontrolled emissions, implying a shadow price of about ten dollars per ton carbon. Along the optimal path, this price and control rate would grow modestly over the next few decades, reaching a price of $20 per ton carbon and a control rate of 30 percent in the year 2000.

The economic model developed below is highly speculative and crude. The analytical approach used in this study should be taken as an attempt to develop a methodology for thinking about carbon dioxide control strategies. One of the central results is that, given present knowledge, we are highly uncertain about the direction and stringency of CO\textsubscript{2} controls. This suggests that, in the near term, further detailed scientific research and modeling efforts are the best investment strategy for coping with the CO\textsubscript{2} problem. The three key uncertainties in the economic analysis are: (i) the economic and social impact of elevation in CO\textsubscript{2} concentrations; (ii) the economic costs of controlling CO\textsubscript{2} emissions; and (iii) the relevant value judgments that we should apply in weighing alternative paths. Work refining the details of the carbon cycle or its first order global climatic impacts may be less critical than those three mentioned above.
A. Introduction

Over the last decade, there has been an outpouring of scientific inquiry into the "carbon dioxide" problem. The preponderance of this research has concerned the carbon cycle and the global climatic effects of elevated atmospheric concentrations, with virtually no serious research on the social and economic impacts of climatic change.

Accompanying the scientific research has been a growing number of scientists who have expressed alarm over the long term impacts of CO₂ and called for changes in current energy policy. The most prominent of these was probably a report to the U.S. Council on Environmental Quality by four leading scientists (hereafter called "the CEQ report"). In this report they conclude:

The CO₂ problem is one of the most important contemporary environmental problems, is a direct product of industrialization, threatens the stability of climates worldwide and therefore the stability of all nations, and can be controlled. Steps toward control are necessary now and should be part of the national policy in management of sources of energy.

To my knowledge, the call for immediate action by the report to the CEQ was without empirical foundation: they neither undertook nor referred to any serious analysis comparing the costs of CO₂ control with its benefits. Indeed, neither the need for such analysis nor the lack of it was mentioned in the CEQ report. The report may be correct or alarmist, but there is no support for the conclusions.

The present paper is written in response to the clear need for an economic analysis of the CO₂ problem. It must be emphasized that this study is

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highly preliminary, and that the empirical analysis is only illustrative given the paucity of relevant data. Nevertheless, given the potential importance of the CO₂ problem, it is necessary to begin such analysis. What follows is, therefore, basically a highly simplified methodological analysis. Its purposes are: (i) to introduce the economist and social scientist to the nature of the CO₂ problem; (ii) to introduce to the natural scientist the analytical tools of the economist, particularly those relating to optimal growth theory; and (iii) to give an order of magnitude description of an optimal control strategy for CO₂.

An outline of the paper is as follows. We have divided the paper into two parts, a theoretical section in Part I and a crude empirical application to follow in Part II. The present part contains two sections. In Section B we outline a simplified model of the economy and the carbon cycle, followed by a discussion of how we may choose between alternative CO₂ strategies. Section C analyzes the optimal CO₂ control path under a number of alternative assumptions.

The second part of this study can be described briefly. The first section reviews briefly alternative empirical models of the carbon cycle, climatic impact, and economic effects. A second section uses these findings to describe alternative optimal paths. Finally, the last section discusses realistic considerations and the implications for current research and policy.

A final important word to natural scientists who may be suspicious of economic reasoning. The physical and economic models prescribed here are drastically oversimplified. This is intentional. The purpose of this paper is not realism—rather it is to simplify the system so that it can be readily and intuitively understood, and so that we can see which variables should be focused on in research and policy. It is easy to construct more complex and realistic models once the goal is clear.

B. Climate in Models of Economic Growth

The purpose of this section is to describe a stylized model of the relation between economic growth and climatic modification. It must be emphasized that the purpose of the model described here is not complete realism, but to discover the elements which are critical to intelligent policy choice.

The model has three components: (i) a simplified view of the carbon cycle; (ii) an economic model which introduces carbon into productive activity; and (iii) a view of how society chooses between alternative consumption paths.

(i) The carbon cycle

Figure 1 gives a pictoral guide to the CO₂ problem. It indicates schematically how fossil fuel combustion influences climate and, eventually, the economy. In what follows we will attempt to simplify a large and complicated system so that it is easily understandable.
FIGURE 1. A pictorial guide to the CO₂ problem. (a) Fossil fuel combustion leads to CO₂ emissions, (b) increasing atmospheric concentration of CO₂, (c) warming and redistributing climate, and (d) affecting over time agriculture, energy consumption, level of oceans, and as-yet-unforeseen variables.
The literature on the CO\textsubscript{2} problem has paid particular attention to the sources and sinks of carbon—yet very great uncertainties remain in "the case of the missing CO\textsubscript{2}."

For our purposes, it is adequate to analyze a case where there are only two reservoirs for carbon—those that are linked to emissions with a short lag (atmosphere, biosphere, mixed layer of ocean) and those where deposition is very slow (deep ocean). We will thus take a "two well-mixed box" model.

In this view, we have the following simple equations:

\[ \dot{M}_1 = E - \delta_{12} M_1 + \delta_{21} M_2 \]
\[ \dot{M}_2 = \delta_{12} M_1 - \delta_{21} M_2 \]

where M's generally indicate the deviation of total mass from a pre-industrial level. More precisely:

- \( M_1 \) = elevation in mass of CO\textsubscript{2} in the atmosphere and quick response reservoirs from preindustrial times;
- \( M_2 \) = elevation in mass of CO\textsubscript{2} in deep ocean;
- \( \delta_{ij} \) = transfer coefficient for CO\textsubscript{2} from reservoir i to reservoir j (fraction of mass per year);
- \( E \) = emissions of CO\textsubscript{2} into atmosphere per unit time.

Dots over variables indicate time derivatives. One further simplification is extremely helpful. We will assume that the "deep ocean" is an infinite sink, so that at all times \( \delta_{12} M_1 \) is much greater than \( \delta_{21} M_2 \). Given the very slow transfer rates in most studies (\( \delta_{12} \approx .002 \)) such an assumption is probably a good first approximation.\textsuperscript{2}

Let us consider then only the carbon in the atmosphere (M for notational simplicity). If \( \beta \) is the fraction of the increase of CO\textsubscript{2} in short-lived reservoirs that resides in the atmosphere, we have:

\[ (1) \quad \dot{M} = \beta E - \delta M \]

where

- \( M \) = increase of mass of atmospheric CO\textsubscript{2} from pre-industrial levels;
- \( E \) = rate of emissions of CO\textsubscript{2} into the atmosphere;
- \( \beta, \delta \) = parameters of process.

We have illustrated the simplified model of the carbon cycle in Figure 2.

(ii) The economic model

The economic model used here is also highly simplified, but the basics are as follows. The economic impacts of CO\textsubscript{2} are similar to those of other pollutants. Economic activity will be affected by the level of CO\textsubscript{2} emission
Figure 2. Simplified Model of Carbon Cycle
because abatement is costly; and in addition there will be economic costs or benefits associated with different levels of atmospheric concentration of CO₂. Given that the structure of the two costs is very different, I will assume that the costs of emission control and of atmospheric concentration are independent.

First starting with emissions. I assume that we are examining a market economy in which capital, labor, land, and resources are combined in productive activities. The outputs of these activities are consumption goods, new capital goods, and CO₂ emissions.

One of the (unfortunate) facts of life in a market economy is that some important outputs are not properly "priced"—these being externalities like pollution, CO₂, noise, or new ideas. As a result, consumers of firms who are primarily concerned about their individual well-being or profits will treat the unpriced good as a free good. In such circumstances, firms will limit the good—CO₂ in this case—up to the point where it has zero incremental value in production. That is, they will spend nothing on CO₂ abatement.

Figure 3 indicates that outcome for CO₂ emissions in a market economy. The central point to note here is that there is no presumption that the current level of emissions or atmospheric concentration is in any way optimal for the economy.

Next, turn to the ultimate effects of the emissions on economic activity. Note that the level of emissions is not harmful; rather it is the effect of levels of atmospheric concentrations (or perhaps change in concentration) which lead to economic harm or good. The details of the economic effects are discussed in Part II of this study; simply note that they include effects on agriculture, energy consumption, and ocean levels. Thus the costs of the CO₂ build-up are a function of the atmospheric concentration, M in Figure 4. We have shown a hypothetical relation between real output and atmospheric concentrations.

There are four important points to mention concerning the cost function in Figure 4. Unlike the shape of f(E) in Figure 3, there are no presumptions about the effects of changes in M on economic activity. The reason is that M is not a decision variable of individual economic agents—it is a background parameter. This parametric nature of the atmosphere is, indeed, the reason why an externality exists.

Secondly, there is no clear evidence that increased CO₂ levels will be harmful. The evidence is mixed, as I will point out in Part II of this study. Third, in what follows I will assume that h(M) and f(E) are independent. This simply means that the kinds of activities that abate CO₂ emissions (switching from coal to nuclear power or solar energy) are different from those that are hurt by or respond to CO₂ concentrations (agriculture, the tourist industry, or building dikes). While the independence assumption is not unassailable, it seems a good first approximation.
Figure 3. Real output is shown as a function of CO₂ emissions. Because CO₂ is unpriced, a market economy will tend toward that level of emission \( \hat{E} \) which maximizes real output, \( f(\hat{E}) \).
Figure 4. Hypothetical relation $h(M)$ between real income or consumption and changes in CO$_2$ atmospheric concentration, $M$. The wavy shape of the $h(M)$ function represents the view that we are highly uncertain about the relation between CO$_2$ and real income.
Finally, it is assumed that there are no delays in the reaction of the economy to climate. The assumption is unrealistic, and of some importance for policy, but complicates the analysis unnecessarily. Again, when more realistic quantitative models are constructed, lags in response are easily incorporated.

The macroeconomic environment will also be treated in a highly stylized way in the present analysis. We will start by assuming that the economy has no technological progress or population growth and that capital accumulation has ceased. Thus real income equals real output equals consumption. In this case real consumption per capita is constant over time unless effects of the carbon cycle intervene. (An extension to include technological progress will be included later.) Under these assumptions we can represent our stylized economy very simply as:

\[ c = f(E) - h(M) \]

where

- \( c \) = per capita consumption;
- \( E \) = emissions of \( \text{CO}_2 \);
- \( M \) = concentration of \( \text{CO}_2 \) in atmosphere.

At present \( E \) and \( M \) are very small, so \( c(1980) = f(\hat{E}) - h(0) \), where \( \hat{E} \) is the uncontrolled level of \( E \) shown in Figure 2. Note that if the functions \( f(E) \) and \( h(M) \) are as drawn in Figures 3 and 4, consumption declines over time as either \( E \) is curbed or \( M \) increases.

One further oversimplification in the present model is that we assume that both sinks and sources of carbon are inexhaustible. Thus the deep ocean is assumed to receive carbon at a very slow rate, but is assumed to be able to absorb an indefinite amount of carbon. Similarly, there is assumed to be an inexhaustible source of carbon, perhaps high cost shales. Again, omission of the realistic features of depletion of fossil carbon or of the absorptive capacity of the deep oceans is necessary to simplify the analysis. As has been shown elsewhere, it is possible to incorporate exhaustion of both types in a growth model, but the complications take the analysis beyond the intuitive comprehension we are seeking here. The effect of depletion has counteracting effects: it acts to reduce fossil-fuel consumption (as extraction costs rise) and to discourage carbon emissions (as the carbon increasingly remains in the atmosphere). However, if the \( \text{CO}_2 \) programs are relatively stringent, and the discount rate above 1 percent or so, the finiteness of source and sink becomes relatively unimportant for today's decisions; in this case the flows for many decades are determined by abatement costs and \( \text{CO}_2 \) damage rather than by resource limitations.

(iii) **Choice between consumption paths**

Up to now, the problem has been to describe the technological possibilities faced by the global economy. The technology described above will generate alternative consumption paths depending on how \( \text{CO}_2 \) emissions unfold over time.
(that is, depending on our control strategies). We have invoked no value judgments up to this point. To complete our analysis, we need a description of how society chooses between alternative CO$_2$ paths or control strategies. This topic is one which must bring to bear value judgments on the desirability of different possible time paths.

In what follows, I propose that CO$_2$ control strategies be weighed by examining their implications for the consumption (or real income) possibilities of different generations. I will briefly describe the issues involved in such an analysis: (a) what is meant by consumption; (b) how do countries weigh present against future generations; (c) how do countries weigh different levels of consumption; (d) is the actual political process of making choices (b) and (c) consistent or defective. We discuss these in turn.

(a) In the analysis that follows, we assume that the goal of economic activity is consumption. Consumption consists of a "bundle" of goods and services and should be interpreted in a very broad way—including not only conventional items such as food, clothing, and shelter, but also services and intangibles such as culture, recreation, leisure, and enjoying the environment. Clearly, an economy can choose between such alternative consumption bundles and we leave that choice to the individuals or generations. We make the key judgment that higher levels of per capita consumption are desirable; more or better food or shelter, cleaner air are preferable to less or poorer quality. In this view, the purpose of economic policy—and CO$_2$ control—is to enhance to the greatest extent the total consumption bundle.  

Unfortunately, even if we agree that consumption is the central goal of economic activity, we have not gotten very far. More difficult is to know how we should weigh or trade off the consumption of different groups—different generations in the same nation, different groups in today's world. In our oversimplified economic model, we see that the CO$_2$ externality will force a reduction in consumption at some point—either because of damages from increased concentrations or because of lower output due to CO$_2$ abatement. How should the consumption reductions be spread over time and space?

A reasonable way to start would be to begin by assuming that the economic costs of the CO$_2$ externality should be equally spread over space and time. This would be a plausible outcome if all generations and countries were equally well off to begin with and had equal voting power or representation. In any analysis of how to spread consumption over time, however, we must recognize the possibility that the costs will be differentially spread because of (b) time, (c) differences in consumption levels, or (d) unequal representation in political choice. To get ahead of our story, we may feel comfortable about (b) or (c) but uncomfortable about (d).

(b) The first reason why we might want to spread the costs of CO$_2$ control unequally over time might be that we have different levels of concern about consumption of different generations. We call this phenomenon time discounting. For example, if, as between equally well-off generations, society is indifferent
between an increment of real income today and \((1+p)^{100}\) increments in 100 years, we say that the "social rate of time preference" is \(p\) per year. Generally, except for the possibility that later generations may not be here to enjoy consumption, it is hard to defend a social rate of time preference above zero.

(c) A second and much more compelling reason why we might want to spread consumption or \(CO_2\) control costs unequally comes from the fact that different generations have different levels of consumption. Industrial countries have witnessed more or less continual growth in living standards for more than a century; thus the per capita real consumption in the U.S. has grown by a factor of 2.4 over the last 50 years. Society might well feel less urgency about the \(CO_2\) control costs being levied upon a richer than a poorer generation. We might then discount future costs if average living standards were improving--a phenomenon we call growth discounting.

A more precise analytical representation would be as follows: it is generally thought that society feels a greater urgency of delivering real consumption to poorer over richer generations. Thus if \(c\) is per capita consumption, \(u(c)\) is the social valuation (or "utility") of consumption;\(^5\) it is generally felt that \(u''(c) < 0\). For example, society might make the judgment that it is indifferent between delivering 1 unit of consumption today and delivering \((1 + k/100)\) units for a generation which is 1 percent poorer. In this case, the redistributive parameter \(\alpha \ [ = -u''(c) \ c/u'(c), \ or \ the \ elasticity \ of \ the \ marginal \ utility \ of \ consumption \ with \ respect \ to \ consumption\] \ has \ value \ \(\alpha = k\).

In considering growth it is important to distinguish whether growth is a result of population growth or growth in per capita income. The growth discount is probably different in these two cases. The analysis presented here is strictly applicable to the case where population is constant and growth occurs through an increase in per capita consumption. At the other extreme consider a case with constant per capita consumption but growing population. In that case, as all generations are equally well off, it seems inappropriate to apply a growth discount. Put differently, in matters such as energy and \(CO_2\), growth in per capita consumption is good news but growth in population is bad news.\(^6\)

To return to our technical discussion, in the case of varying populations a natural social valuation function for consumption at different points of time would be \(Nu(c)\) -- \(N\) being population growing at rate \(g\) and \(c\) being constant per capita consumption. Assuming \(u(c)\) is positive (i.e., that life is on balance worthwhile) the discount rate on goods in this case is simply \(p\). The redistributive parameter drops out because there are no differences in average consumption levels.

If real income is growing over time, society will discount future consumption if it values incremental consumption of poorer groups more than that of richer groups. If real consumption is growing at \(g\) percent annually, then the growth discount will be \(\alpha g\), where \(\alpha\) was the distributional parameter defined above.
One must take into account both time and growth discounting to understand the phenomenon of goods discounting. Goods discounting is the familiar concept of the relative value of a unit of GNP of consumption at different points of time; if the goods discount rate is 10 percent per annum, this implies that we will forego a unit of consumption, or make an investment, today if the process will yield at least 1.1 units of consumption next year. Ignoring taxes and risk, a goods discount rate can be observed as a real (inflation-corrected) market interest rate. When we observe a real return on capital, \( r \) (say around 7 percent per annum after taxes in the U.S. today), this refers to how society can transform consumption today into consumption tomorrow. In an optimal growth framework, with no population growth and in steady state, the discount rate on goods is related to the other concepts as follows:

\[
(*) \quad r = \rho + \alpha g
\]

where

- \( r \) = real return and discount rate on goods;
- \( \rho \) = pure social rate of time preference;
- \( \alpha \) = redistributive parameter (elasticity of marginal utility of income);
- \( g \) = growth rate of real income.

In the starred equation \( \rho \) is the time discount rate while \( \alpha g \) is the growth discount rate. If, for example, \( \rho \) is .01, \( g = .02 \), \( \alpha = 3 \) --parameters which might reflect today's conditions--the real interest rate on goods would be 7 percent. However, if economic growth slows and the growth discount therefore evaporates, the discount rate on goods should fall toward 1 percent. Note that a high discount rate can be generated either by a high rate of time preference or (in a growing economy) by high aversion to inequality.

(d) The final question is whether the allocation of consumption over time (or setting the discount rate on goods) is determined in the rational and consistent fashion that is described above, or whether it is defective. Many people feel that there is a systematic bias in decisionmaking in favor of the current generation, and there can be little doubt that national decision processes place much higher weight on the nation's citizens than on the rest of the world. The ultimate question is then whether we should accept the current discount rate on goods as the appropriate one to use in making central allocational decisions across generations.

This question is theoretically difficult, perhaps unanswerable. Kenneth Arrow and other political theorists have shown that there is no general democratic process which will translate individual preferences into overall social decisions without inconsistencies (sometimes known as voting paradoxes). In addition, the decision processes that operate do so with no representation from future generations other than today's altruists. Moreover, the key institutions for determining interest rates--central banks--appear more concerned with
TABLE 1

Alternative Discount Concepts in Steady State, with Illustrative Values in Parentheses

<table>
<thead>
<tr>
<th>Item</th>
<th>Without Growth</th>
<th>With Growth in per capita Consumption</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>on Utility,</td>
<td>ρ</td>
<td>ρ</td>
<td>To compare overall wellbeing or &quot;utility&quot; of different generations</td>
</tr>
<tr>
<td>Human Life</td>
<td>(1%)</td>
<td>(1%)</td>
<td></td>
</tr>
<tr>
<td>Discount Rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>on Goods</td>
<td>ρ</td>
<td>ρ+αg</td>
<td>To compare alternative consumption paths; to make investment decisions</td>
</tr>
<tr>
<td></td>
<td>(1%)</td>
<td>(7%)</td>
<td></td>
</tr>
<tr>
<td>Discount Ratea/</td>
<td>ρ+δ</td>
<td>ρ+αg+δ</td>
<td>To compare the value of CO₂ postponement or &quot;pickling&quot; arrangements</td>
</tr>
<tr>
<td>on CO₂</td>
<td>(1.2%)</td>
<td>(7.2%)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The definition of the parameters and illustrative values are as follows:

ρ = pure rate of social time preference - .01 per annum;
α = redistributive parameter = 3;
g = rate of growth of per capita consumption = .02 per annum;
δ = disappearance rate of CO₂ from the atmosphere = .002 per annum.

a/ The CO₂ discount rate is generally defined below.
inflation and trade balances than with ethical judgments about the consumption tradeoffs between current and future generations. Both from a theoretical and institutional viewpoint, then, it is difficult to argue convincingly that the process by which social investment decisions are made would necessarily lead to the appropriate balance between the present and future and to the appropriate discount rate. Ultimately, we must (and will later) fall back on home-baked value judgments about whether the current market discount rates appear reasonable.

We have collected our different discount concepts together in Table 1. This shows how different discount rates relate to different technological parameters or value judgments. In addition, we have indicated some possible numerical values for a set of assumptions. To reiterate the fundamental point about discounting, we see that a high goods discount rate can arise in a growing economy when there is aversion to inequality. Put differently, if the future is much better off than we are, we will leave a lower capital stock and resource endowment for them, thereby raising the productivity of capital and the goods discount rate.

Coming to our carbon problem, I would propose the following criterion for choosing between alternative carbon strategies:

Society should choose CO₂ controls so as to maximize the discounted expected utility of consumption.

More precisely, we should maximize

\[
(3) \quad \text{max} \int_{0}^{\infty} e^{-\rho t} u[c(t)] dt
\]

where

\[ W = \text{welfare functional to be maximized}; \]
\[ \rho = \text{pure rate of social time preference (per annum)}; \]
\[ t = \text{time}; \]
\[ c(t) = \text{real consumption}; \]
\[ u[c(t)] = \text{utility of real consumption (utils per annum)}. \]

The criterion function in (3) is a way of expressing our notions about how we should weigh consumption paths which naturally grows out of the considerations we have discussed above.

C. Optimal Controls on CO₂

We next turn to solution paths for our simple optimal growth problems in this section. We will discuss two alternative models—one without technological change at some length and then a brief extension to include technological progress.
1. Strategies without growth

Recapitulating our model without growth, we have

\[
\text{(1)} \quad \max W = \int_0^\infty e^{-\rho t} u[c(t)] \, dt \\
\text{(2)} \quad c(t) = f[E(t)] - h[M(t)] \\
\text{(3)} \quad \dot{M}(t) = \beta E(t) - \delta M(t).
\]

Solving such a system explicitly is straightforward but tedious using the Pontryagin Maximum Principle. A more intuitive approach will be used here. We first assume that the functions take the "right" shape—\(u(c)\) and \(f(E)\) are concave and \(h(M)\) is convex. These assumptions are probably warranted except for that about \(h(M)\); we have little precise information about the \(h(M)\) function but is likely to be nonlinear, perhaps even convex.

Next we separate the future into two parts—the transition period (phase I), and a steady state (phase II). The system will clearly tend toward a steady state, so it is useful to begin with an analysis of that.

Phase II (steady state). In the steady state, the atmospheric concentration of CO\(_2\) has stabilized, so \(M(t)\) is constant at \(M^*\). This implies

\[\beta E^* = \delta M^*\]

and

\[c^* = f(E^*) - h(M^*) .\]

The easiest way to find the optimal steady state is by a simple variational technique, illustrated in Figure 5. In the optimum, a small increment of \(E^*\) will leave welfare, \(W\), unchanged. An increase in \(E\) by \(\Delta\) at time 0 raises consumption by \(f'(E^*)\Delta\) where \(f'(E^*) = df(E)/dE\) is the incremental consumption gained by a unit increase in emissions at \(E^*\). In addition, however, it raises CO\(_2\) concentrations by \(\beta e^{-\delta \Delta} \Delta\) and lowers consumption by \(h'(M^*)\beta e^{-\delta \Delta} \Delta\).

Thus \(W\) is changed by

\[\Delta W = u'(c^*)f'(E^*) \Delta - \int_0^\infty \beta e^{-\rho \nu} u'(c^*) h'(M^*) e^{-\delta \nu} \Delta d
\]

which will equal zero when

\[
f'(E^*) = h'(M^*) \left[ \frac{\beta}{\rho + \delta} \right]
\]
FIGURE 5. Figure shows the fundamental variational derivation of optimum. In panel (a) an increase of $\Delta$ in emissions is allowed in period $t = 0$. This results in an increase in atmospheric concentrations as shown in panel (b). In panel (c) the effect on consumption is illustrated. An optimum path is one in which the discounted value of changes in consumption path is zero.
The left hand side is the increment to current consumption, while the right hand side is the present value of the decrement to future consumption, each with respect to a unit increase in emissions of CO₂.

We have illustrated the optimal control strategy for CO₂ in Figure 6. We have first drawn in the marginal cost of reducing emissions--f'(E). The upward sloping lines show the marginal cost of increased atmospheric concentration of CO₂. At the bottom is h'(M) = h'(E/δ)--the first year marginal cost of increasing atmospheric concentrations. The curve above that--h'(E/δ)(δ/(δ+δ))--is the discounted marginal cost of increased emissions.

The optimal control strategy arises where the two marginal cost curves intersect at point A. Note also that at point A the marginal abatement cost f'(E*) equals the present value of incremental emissions, δh'(M*)/(δ+δ). This value is the **efficiency or shadow price of CO₂**--q*.

The importance of the new concept--the CO₂ shadow price--will be briefly discussed. We will take consumption goods to represent the unit of account. In this case, we note that the discount rate on goods in the steady state is simply ρ. This indicates that a unit of foregone consumption today returns ρ units incremental consumption per year forever. Note that this corresponds to our formula in Table 1 above.

We are particularly interested in the "price of CO₂" in our optimal solution. Let us define q as the **shadow price of CO₂ emissions**. That is, q(t) represents the incremental cost in terms of consumption c(t) of increasing CO₂ emissions, E(t).

Along our optimal path we can calculate our shadow price q in two different ways because the marginal emission benefits equal the marginal concentration costs. Thus in a steady state

\[
q^* = f'(E^*) = h'(M^*) \left[ \frac{\delta}{\rho+\delta} \right]
\]

(5)

where q* is steady state shadow price of CO₂. Equation (5) states that the shadow price is equal to both the marginal cost of reducing emissions as well as the discounted cost of the atmospheric concentration from one unit of emission. This correspondence is shown in Figure 6.

The shadow price is an absolutely essential part of any rational calculation of efficient control of CO₂. It is even more important a part of an efficient decentralized control strategy for it allows individual decisions to be made as to the optimal CO₂ emissions. This point will be further discussed in the conclusive section.

A further word on discounting. We have noted that there are different discount rates for different things. We can introduce the concept of a CO₂ discount rate. This states the tradeoff we should make between CO₂ today and in
FIGURE 6. Diagram shows marginal abatement cost and discounted cost of abatement. Optimal strategy described in text.
the future. In our simple model, the CO₂ discount rate is equal to \( \rho + \delta \)--the discount rate on goods and utility plus the disappearance rate on CO₂. Table 1 above shows the relation between the CO₂ discount rate and alternative concepts.

**Phase I (transition).** As we will mention below, the transition phase is much more relevant for current policy toward CO₂ than is a steady state. For this reason we must consider in detail the features of a transition path. Unfortunately phase I is considerably more complicated. Using the same variational calculation as before, we find that an optimal path is the solution to the following equation for all values of \( E(t) \).

\[
0 = e^{-\rho t} u'[c(t)] f'[E(t)] - \int_0^\infty e^{-(\rho + \delta) v} u'[c(v)] h'[M(v)] dv.
\]

We can also provide a condition for our shadow price during the transition. Here again the shadow price, \( q(t) \), must equate costs and benefits of future CO₂ abatement in terms of current period consumption.

\[
q(t) = f'[E(t)] = \int_0^\infty e^{-(\rho + \delta) v} u'[c(v)] h'[M(v)] dv.
\]

This equation is identical to the previous one except that it has been divided by \( u'[c(t)] e^{-\rho t} \), which is the number of "utils" per unit of \( c(t) \) on the margin. It states that the shadow price on CO₂ is (i) the consumption cost of reducing CO₂ by one unit--\( f'[E(t)] \)--and (ii) the consumption cost of increasing atmospheric concentration one unit discounted as that passes through the carbon cycle.

Two interesting special cases for the transition path can be described. In both we assume that the marginal utility of consumption is constant, so that without loss of generality we can take \( u(c) = c \).

**Case 1.** Threshold level of \( M \). Assume that there is a threshold level of CO₂ concentration, \( M^* \), such that for CO₂ concentrations up to \( M^* \) there is no damage from increased CO₂, but that when \( M \) passes \( M^* \) the costs become catastrophic. (It should be emphasized that this is an extremely unlikely scenario, but it is very useful analytically.) Further assume that \( f'(E) = \lambda \), i.e., that the marginal cost of CO₂ is constant at a fixed level.

Clearly the steady state equilibrium is for \( \beta E^* = \delta M^* \), and the long run shadow price is \( q^* = f'(E^*) = f'(\delta M^*/\beta) = \lambda \). We can also determine the shadow price in the transition. Denote time \( T \) at the year in which concentrations reach the fateful level, \( M^* \). Because costs are constant, it will pay to do no abatement until time \( T \). In this case, the shadow price of CO₂ along the transition path is from (7) simply:

\[
q(t) = q^* e^{-(\rho + \delta) (T-t)} = \lambda e^{-(\rho + \delta) (T-t)}
\]
FIGURE 7. Illustration of approximation to shadow price for Phase I, or transition period. Assumption is that there is no cost of increased concentration until time $T$. The time path for concentrations after an increase of $\Delta$ in emissions is shown in panel (a). Calculation of shadow price is given in panel (b). Cost in transition period is simply discounted value of steady state value.
In words, the current shadow price of CO$_2$ in this special case is equal to the steady state value discounted at the CO$_2$ discount rate for the period of time until the steady state is reached. This case is illustrated in Figure 7.

**Case 2.** Constant costs. A second tractable case for investigating the transition is where the damage of increased CO$_2$ is linear with respect to increasing concentrations.

In the case of a fixed marginal damage with respect to CO$_2$ concentration [$h'(m) = h'^*$] we can integrate out the right hand side of equation (7) directly obtained [again where $u'(c)$ is constant]

$$q(t) = \int_{0}^{\infty} e^{-(p+\delta)t} v h'^* dt = \frac{sh'^*}{p+\delta} = q^*.$$  \hspace{1cm} (9)

In this extreme case, the shadow price is equal to the steady state shadow price even in the transition period.

In this case, the optimal control strategy will be invariant over time—that is, the strategy will be to go immediately to the steady state control level, $E^*$.

2. **Strategies with Growth**

The model described in Section 1 above is both unrealistic and pessimistic in that it assumes there are no improvements in technology over time. While it is impossible to predict the future, it is instructive to note that over the past several decades, improvements in technology have allowed outputs to grow 1 to 4 percent faster than inputs depending on region and time period.

Such a possibility changes the outcome in two fundamental ways. First, whereas the no-growth model above projects a gradual decline in consumption over the future, a growth model builds an upward trajectory into consumption. Second, in the earlier model, no growth meant that steady state emissions would be stable and no progressive buildup in the atmosphere would occur; with growth, on the other hand, the only limitation to CO$_2$ buildup is fossil fuel supplies.

Growth models include endless varieties of technological change. It seems reasonable for our purposes here to assume that growth does not affect either the CO$_2$ intensiveness or climate intensiveness of consumption. Such an assumption would be met if there is no correlation between the income-elasticity of goods and their CO$_2$-intensity or their climate-intensity. This implies that as the productivity and output of the population double, for given prices, CO$_2$ emissions double. Further, if a doubling of CO$_2$ reduces consumption by 10 percent with today’s technology, I assume it will do the same with an improved technology. We can thus write our production (and consumption) function as $c(t) = \exp(gt) [f(e(t)e^g) - h(M)]$. 
The treatment of economic growth used here will simplify the analysis by making an approximation to the growth path. Designate "potential consumption," \( x(t) \), to be the level of consumption in year \( t \) if no emissions controls were necessary and with the preindustrial levels of atmospheric CO\(_2\). Thus \( x(0) = f(\bar{E}) - h(0) \) in terms of earlier notation. We assume that potential consumption per capita grows at a fixed rate, \( g \), for \( T \) years and then ceases growing. Thus at time \( T \), \( x(t) \) has grown from its initial level, \( x(0) = 1 \), to a steady state level of \( x^* = \exp(gT) \) and \( c(t) = \exp(gt) \) \([f(E(t)e^{-gT}) - h(M)]\). We will want to investigate how our control strategy varies with different ultimate levels of \( x^* \).

The other important question concerns the decomposition of overall economic growth into population growth and growth in per capita income. This question is of no importance for the steady state analysis that follows; it does assume considerable importance for the transition path.

Phase II (steady state). Using this approximation to the growth process, we can then simply modify our earlier technique to indicate the optimal path. Note that—because after \( T \) years the growth process has stopped—we can draw upon our earlier steady state analysis. Our steady state level of consumption is given by

\[
(10) \quad c^* = x^*f(E^*/x^*) - h(M^*)
\]

where

\[
(11) \quad M^* = \bar{E}E^*/\delta.
\]

Using the same analysis as for Phase II above, the optimal level of emissions, concentrations, and CO\(_2\) shadow price is:

\[
(12) \quad q^* = f'(E^*/x^*) = h'(M^*) \frac{\delta}{\rho + \delta}.
\]

A graphical exposition is shown in Figure 8. In this diagram we have shown how the optimal steady state CO\(_2\) control strategy depends upon how much the economy has grown over the transition, \( x^* \). In this case, note we have put on the horizontal axis the level of emissions as a fraction of the uncontrolled level. By assumption the marginal cost per ton CO\(_2\) at a given fractional control rate is constant, so the marginal cost of abatement curve, \( AB \), does not shift with differing growth paths.

The cost of atmospheric concentrations is represented by the upward sloping curve, \( EC \)—this represents the discounted marginal cost of CO\(_2\) concentrations as a function of the control rate. Higher levels of growth (i.e., higher \( x^* \)), mean that, at a given control rate, emissions \( (E^*) \) increase proportionally. Therefore, growth shifts the EC curve to the left by the factor \( x^* \), to the ED curve. Thus for \( x^* = 2.5 \), the curve shifts left so that at each level of cost on the vertical axis the emissions as a fraction of uncontrolled emissions,
Marginal cost of abatement, concentrations

\[
h'(M^*) \frac{\beta}{\rho + \delta} = h' \left[ \frac{E^*}{E_0 x^*} \frac{\beta \tilde{E}_0 x^*}{\delta} \frac{\beta}{\rho + \delta} \right]
\]

\[f'(E^*/E_0 x^*)\]

\[x^* = 2.5\]

\[x^* = 1\]

Emissions as fraction of uncontrolled level

\[(E^*/E_0 x^* = \lambda)\]

FIGURE 8. Illustration of effect of economic growth on steady state controls. Horizontal axis shows emissions \((E^*)\) as fraction of uncontrolled emission, \((E_0 x^*)\). By assumption marginal cost of emissions—AB or \(f'(E^*/E_0 x^*)\)—at given fractional control rate is unaffected by growth. Marginal environmental cost—\(h'(M^*)\)—shifts to the left by factor \(x^*\) in steady state as emissions rise by factor \(x^*\) with given control rate.
FIGURE 9. Normal effect of growth on CO₂ control.
FIGURE 10. Extreme case of constant marginal costs of CO$_2$ concentration.
FIGURE 11. Extreme case of threshold level of $M$. 
is 40 percent of its original level.

Alternative solutions are shown in Figures 9 through 11. Figure 9 shows the "normal" case of increasing marginal costs of atmospheric concentrations; the marginal cost function shown is linear. In this case, emissions, the control rate, and the CO₂ shadow price are higher in the steady state if growth occurs. Figure 10 shows the extreme case of a constant marginal cost of increased concentrations. In this case, the marginal cost at a given control rate grows proportionately with the economy, so the shadow price and the control rate rise with the size of the economy. The final example is Figure 11, which illustrates the outcome with a threshold or catastrophic level of concentration. In this case, emissions are fixed—as they are given by the level of the threshold. If marginal costs of abatement are as pictured, the shadow price on emissions rises more than proportionately with \( x^* \).

In summary, we can form a rough idea of how the economy is likely to look if there is economic growth for a period of time. In the normal case or rising marginal abatement costs and rising marginal damage costs (shown in Figures 8 and 9), the optimal steady state policy will show higher levels of controls per unit output, for the damage per unit emissions is higher. Total emissions and atmospheric concentrations are likely to be higher, however, for an optimal plan will allow some compromise between the higher costs from abatement and the higher damages from increased atmospheric concentrations. The increased economic and environmental stress will be reflected in a higher steady-state CO₂ shadow price.

If all this sounds grim, we might ask is growth worth the costs? It is not logically impossible that higher levels of growth will lead to lower ultimate levels of consumption, but the conditions are extremely unlikely. If we take the worst case of a catastrophic level of concentrations, then an increase in output will force an equal reduction in the emissions per unit output. It can be shown that in this case if the shadow price on CO₂ is greater than consumption per unit output economic growth will lower total consumption. This condition is equivalent to saying that the tax collections from a carbon tax would equal national income—not impossible, but quite unlikely if we believe the results of the second part of this study.  

Phase I (transition path). The principles which lie behind the transition path are virtually identical to those for the no growth path. The only difference is that the discount rate may differ because of the presence of the phenomena of "growth discounting" discussed above.

Recall that, in the presence of economic growth, the discount rate on goods will be higher if there is a desire to redistribute consumption from richer to poorer generations. Following the analysis above, we decompose the annual growth rate \( g \) into a fraction \((1-\mu)\) which is population growth and \( \mu \) which is growth in per capita consumption. Assume that the effect of CO₂ on the level of consumption can be neglected. Denote \( r(t) \) as the discount rate on goods. Then, under these circumstances, the discount rate on goods during the
transition period will be \( r(t) = r_1 = \rho + \mu g \), while after the transition period \( r(t) = r_\infty = \rho \). We can write our transition equation as

\[
q(t) = f'[E(t)] = e^{-\int_{t}^{T} \beta e^{-(\delta + \delta r_1) v} h'[M(v)] dv}
\]

\[
- e^{\int_{t}^{T} \beta e^{-(\delta + \delta r_\infty) v} h'[M(v)] dv}.
\]

We can easily solve (7') for our polar extremes. In case 1—the threshold—let us assume the threshold date \( T > T \). Then we have

\[
q(t) = q^* e^{-(\delta + \delta r_1)(T-t)} e^{-\int_{t}^{T} \beta e^{-(\delta + \delta r_\infty) v} h'[M(v)] dv}.
\]

Further note that at time 0, since \( x^* = e^{T} \), we have

\[
q(0) = q^* e^{-(\delta + \delta r_1) x^* - \rho}.
\]

Equation (8'') shows that the shadow price is calculated in the same way as in (8) except that the growth term \(-\rho\) is tagged onto the end.

The second case of transition, constant marginal damage from emissions, is one we will examine in the second part of this study and we therefore examine that more carefully. Again it is easiest to see the results through a variational argument. Let us assume 1 unit of \( \text{CO}_2 \) is emitted in period 0, so that \( \beta \exp(-\delta t) \) survives in the atmosphere at future time \( t \). In today's economy, this yields a fixed marginal damage, \( h'^* \). But the damage is proportional to the size of the economy and is therefore growing at rate \( g \) for \( T \) years. Thus the damage per year is given by

\[
\text{damage} = \begin{cases} 
\beta e^{-\delta t h'^* e^{T}}, & t < T \\
\beta e^{-\delta t h'^* e^{gT}}, & t > T.
\end{cases}
\]

Similarly, future damage is discounted at rate \( r = \rho + \mu g \) until \( T \), then at \( \rho \) after time \( T \). This yields discounted future damage, \( D \), as

\[
D = \int_{0}^{T} e^{-(\rho + \mu g) v} \beta e^{-(\delta + \delta r_1) v} h'^* e^{g v} dv
\]

\[
+ h'^* g e^{-\delta T} e^{-\rho \int_{T}^{\infty} \beta e^{-(\delta + \delta r_\infty) v} h'[M(v)] dv}.
\]

Since the current shadow price is equal to future incremental damage, we have
\[ q(0) = \beta h^* \frac{1 - e^{-[\delta + \rho + (\alpha - 1)g]T}}{\delta + \rho + (\alpha - 1)g} e^{-\frac{[\delta + \rho + (\alpha - 1)g]}{\rho + \delta}}. \]

The interesting question is how—in a world with growth—equation (9') compares with the steady state value of \( q^* \) in (12). It can be shown that the current shadow price is lower than the steady state if the transition period discount rate is above the steady state discount rate by at least the growth rate. This will occur if \( \alpha \mu \) is greater than 1. This condition corresponds to either a low desire for redistribution, or a high ratio of population growth to per capita income growth. Implicit in a very high goods discount rate is a social decision that either \( \rho \) is high or \( (\alpha - 1) \) is positive.

If we have a high transition period discount rate, the main point to note is that in both growth cases, the shadow price of CO\(_2\) will be rising along the transition path while economic growth continues. This implies that the level of CO\(_2\) control at the beginning is lower than at the end of the period. The logic behind this result is that the current generation (including poor in developed and underdeveloped countries) should sacrifice less current consumption to improve the climate if future generations (i) can do something themselves and (ii) will be richer.

This concludes our theoretical analysis of optimal strategies for the control of carbon dioxide. We have shown that it is possible to put the "carbon dioxide problem" into a more general framework involving questions of the pace of economic growth and the value judgments about the allocation of costs of CO\(_2\) between current and future generations. The advantage of such a framework is that it allows a comparison of programs for the abatement of CO\(_2\) with investment decisions and rates of return on investment, rather than treating CO\(_2\) in isolation of other economic problems and constraints.

The framework used for the analysis is highly simplified to allow easy comprehension of the effects of major variables. Yet even in this highly simplified world, there are no simple rules for deciding the timing and stringency of CO\(_2\) control strategies. One key question is the ultimate level of CO\(_2\) control. As is shown, the level will depend upon the appropriate discount rate, the costs of CO\(_2\) control and damage, the rate of economic growth, and the parameters of the carbon cycle. Another key question is the shape of the optimal carbon control strategy over time. Again, the control rate will vary from flat to tilted depending on the shape of the damage function and the discount rate.

Clearly much work needs to be done before we will have a clear idea about the stringency and timing of good policies to control carbon dioxide. Part II of this study attempts to use existing data on the carbon cycle and its impacts, together with the theoretical model developed here, to see if a rough idea can be formed as to the shape of good control strategies.
THINKING ABOUT CARBON DIOXIDE
PART II. EMPIRICAL AND POLICY ASPECTS OF A MODEL
OF OPTIMAL CONTROL STRATEGIES

A. Introduction

In the first part of the present study, I presented and analyzed a
highly stylized model of the CO₂-economy interaction; that work will not be
reviewed here. Suffice it to reiterate that—even in that highly simplified
model—no clear and simple formula for a control strategy can be derived.
The optimum steady state depends on such questions as whether the economy is
growing; on the rate of time preference; on the costs of damage from and control
of CO₂; and on the disappearance rate of CO₂ from the atmosphere. A
transition path adds further complications and depends—in addition to the
parameters listed above—on the shape of cost functions, the growth of con-
sumption along the path, and attitudes toward income distribution across
generations.

The purpose of this part of the study is to apply the analytical reason-
ing of the first part of the paper to the real policy problem faced today. The
outline is as follows. In Section B, the data on the relations will be briefly
examined. Section C uses the empirical estimates to calculate the optimal
steady state and transition control strategies. In the final section, we dis-
cuss implications for policy and research.

B. Review of Empirical Results

We begin with a brief review of the scant knowledge about the empirical
relations and describe what an optimal path might look like. It must be empha-
sized that—given the paucity of knowledge about the economic effects of CO₂—
the results presented here are only illustrative.

In what follows, I will describe in turn the carbon cycle, the costs of
CO₂ emission abatement, and the climatic consequences of increased CO₂
concentrations.

1. The Carbon Cycle

The carbon cycle can be briefly described as follows: Man's activities,
particularly the combustion of fossil fuels, lead to significant releases of
CO₂ into the atmosphere. Through a process of diffusion these emissions are
distributed through the oceans and biosphere, but the rate of distribution is
so slow that a large fraction of industrial CO₂ remains in the atmosphere for a
very long time.

In modeling the carbon cycle, reservoirs are divided into those which mix
very slowly (the deep oceans) and those that mix relatively quickly (all others).
(A more complete model would include the interaction between the different
reservoirs and possible non-linearities.) In addition, I assume that the deep ocean eventually absorbs all of the CO$_2$ rather than the 80 to 95 percent usually assumed. This assumption leads to the following model for atmosphere concentrations ($\dot{M}$):

$$\dot{M} = \beta E - \delta M$$

where

$\beta$ = effective fraction of mass of CO$_2$ in atmosphere as proportion of all mass in quick mixing reservoirs;
$M$ = change in mass of CO$_2$ in atmosphere since preindustrial times;
$E$ = CO$_2$ emissions;
$\delta$ = fraction of CO$_2$ in quick mixing reservoirs which transfers annually into deep oceans.

There are a large variety of models for calibrating equation (1). Table 1 presents one of the recent surveys plus results from an earlier work by the present author.

2. Costs of Reducing CO$_2$ Emissions

The second important relation concerns the costs of reducing CO$_2$ emissions or other control strategies. In reviewing control strategies, there are three broad groups as shown in Table 2—do nothing, prevent atmospheric concentrations from rising, and offset climatic impacts.

Clearly, we should attempt to follow that strategy or set of strategies which impose least cost on the global economy. Unfortunately we do not know at this time what a least-cost strategy would entail. In addition, virtually every strategy has side-effects so that the remedy may be worse than the disease:

---Is it desirable to build up nuclear power very quickly so as to prevent CO$_2$ buildup?

---If we avoid the nuclear route, is it worth the very high cost of substituting solar energy?

---Will CO$_2$ scrubbers introduce enormous environmental effects of another kind?

---Where will we store all the pickled trees?

In what follows, I will examine a particular subset of control strategies whose side effects are easier to describe—these are an efficient combination of energy conservation and fuel switching. This estimate is drawn from my earlier paper, although the focus in that work was slightly different.
### TABLE 1a

Some Estimates of Parameters of CO₂ Distribution Model

<table>
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<tr>
<th>Author</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$M_0$</th>
<th>$E_0$</th>
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<td>Siegenthaler and Oeschger (1978)</td>
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</tbody>
</table>

**Note:** The parameters were derived by fitting an equation to two points on a curve which showed logarithm of atmospheric CO₂ against time to points 100 years and 300 or 400 years after a pulse input.

**Symbols:**

- $\beta$ = fraction of increase mass in quickly-mixing reservoirs residing in atmosphere;
- $\delta$ = annual transfer rate from quickly-mixing reservoirs to deep oceans;
- $M_0$ = initial mass of CO₂ in atmosphere (10⁹ tons);
- $E_0$ = emissions of CO₂ per annum (10⁹ tons at 1979 world output levels).

## TABLE 2

Strategies for Controlling Effects of CO₂ Buildup

1. **Laissez Faire**—Do nothing

2. **Reduce atmospheric concentrations**
   - A. Reduce energy consumption
   - B. Reduce CO₂ emissions
     - Shift to low CO₂ fuels
     - Mix CO₂ into deep oceans
     - Introduce CO₂ scrubbers
   - C. Remove CO₂ from atmosphere
     - Grow and pickle trees
     - Recycle CO₂ in methanol

3. **Offset climatic impacts**
   - A. Climatic engineering
     - Paint roofs and roads white, put latex into oceans
   - B. Adapt to warmer climate and higher oceans
     - Move corn belt
     - Build dikes
The purpose of the calculation is to estimate \( f(E) \)—the relation between reduced emissions and output discussed in Part I. Calculation of abatement cost is complicated because the underlying resource base is shifting over time as certain fossil fuels become depleted. At the beginning, it might be relatively cheap to shift from high to low carbon fuels because of the availability of natural gas. After a century of growth, it might again be cheap to switch as fossil fuels are depleted and energy prices are already very high. In addition, switching takes time, as the economy's infrastructure, capital stock, and moves adjust to different technologies.

Figure 12 gives an estimate of the cost of reducing CO\(_2\) emissions as a function of the percent reduction. These are calculated by examining the shadow price on CO\(_2\) emissions in each period and for different control programs from my earlier paper.\(^{11}\) The shape of the cost is theoretically plausible in that it indicates that a greater reduction has a higher marginal cost. According to these figures, the cost per ton removed starts at zero for the first ton and rises to $200 per ton carbon as virtually all CO\(_2\) is removed. Figure 13 illustrates what kinds of activities would be performed to abatement CO\(_2\) at different levels of marginal abatement costs.

We can parameterize the relation shown in Figure 12 as follows:

\[
f'(E) = 200(1 - E/E_0)^2
\]

where

- \( f'(E) \) = marginal cost of reducing CO\(_2\) emissions;
- \( E_0 \) = uncontrolled level of CO\(_2\) emissions;
- \( 1-E/E_0 \) = rate of CO\(_2\) control.

It may be useful to introduce the notion of a "backstop technology" for CO\(_2\) abatement. For example, if CO\(_2\) scrubbers or tree picking can remove unlimited amounts of CO\(_2\) at a fixed cost, this would constitute such a backstop technology. The only careful study of the costs of CO\(_2\) abatement technologies that has been uncovered is by Albanese and Steinberg.\(^{12}\) They estimate that for electricity generation plants 50 percent removal of CO\(_2\) would cost approximately $170 per ton carbon while 90 percent removal would cost about $300 per ton carbon. It appears that use of non-fossil fuels is likely to be considerably cheaper.

A less speculative example would be growing trees and burying them, thereby locking up carbon for extended periods. Depending on location and type, trees sell for $50 to $100 per ton, and land fill might add $10 to $20 per ton. This would apparently be competitive with other CO\(_2\) abatement strategies once the control rate reached 50 to 80 percent.

Figure 14 illustrates the notion of a backstop technology for CO\(_2\) abatement. The \( f'(E) \) curve is the marginal abatement cost illustrated in Figures 1 and 2. If there is an inexhaustible source of CO\(_2\) removal costs MC\(t\), this is
Marginal cost of controlling CO$_2$ emissions ($/\text{per ton}$)

\textbf{fitted curve is}

\[ f' \left( \frac{E}{E_0} \right) = 200 \left( 1 - \frac{E}{E_0} \right)^2 \]

Fraction of emissions uncontrolled (percent)

Figure 12. Relation between fraction of CO$_2$ emissions abated and marginal cost of abatement.

Source: Nordhaus, "Strategies."
Marginal Cost of Abatement

- Run all transportation systems on electricity or hydrogen
- Switch small uses from carbon-based fuels to electricity
- Use "carbon-efficient" synfuels
- Produce heat with nuclear power instead of coal
- Generate electricity with nuclear power instead of coal
- Switch to low carbon fuels

Figure 13. Illustrative example of progression in CO₂ abatement strategies.
FIGURE 14. If an "inexhaustible" source of abatement appears at cost \( MC^* \), then abatement curve shifts from CBD to CBA.
represented by the heavy line AB. An efficient emissions reduction strategy would be to follow \( f'(E) \) to point B, then to use the backstop for further reductions.

3. Economic Impacts of CO\(_2\)-Induced Climatic Changes

The final and most difficult part of the estimation is to measure economic impacts of the climatic changes induced by CO\(_2\) elevation. To my knowledge, no thorough study of such a question exists, although it is indispensable for rational decisions on control strategies. We must emphasize that what follows is only a crude guess at the order-of-magnitude costs or benefits of such climatic changes.

Rather than give the spurious precision of a single figure, I will provide both optimistic and pessimistic estimates. The "optimistic" calculation can be translated roughly as what might happen if unforeseen events are minimal or offset and if adaptiveness of human society and technology are high. "Pessimistic" would be the reverse. For concreteness, we might say that there is a one-in-ten chance that events might be worse than the pessimistic or better than the optimistic outcome. In addition, we will average the two extremes for a middle-of-the-road approach.

a. Basic assumptions

The basic assumption about the impact of CO\(_2\) is as follows: as a result of CO\(_2\) elevation, there will be a significant modification in the global climate. It is generally agreed that the global climate will become warmer, and that this warming will be greater near the poles. Most climatologists expect a significant change in the distribution of other important climatic factors, particularly rainfall and wind patterns, and, sooner or later, an effect on the level of the oceans.

The optimistic scenario assumes a doubling of CO\(_2\) concentrations leads to a 2°C rise in global temperature and a neutral pattern of precipitation changes. The pessimistic scenario has a 3°C rise in global temperature and an unfavorable change in temperature and precipitation distribution.

b. Agriculture

World agricultural production in 1975 is taken to be $1 trillion, of a total $6 trillion Net World Product.

(i) CO\(_2\) effect. In the pessimistic scenario, other factors are limiting and there is no effect of CO\(_2\) on production. In the optimistic scenario, the increase in agricultural production is one-fifth of the increase in CO\(_2\) concentration. This is two-thirds of the figure in Ravelle and Shapero (who estimate a factor of 0.3) because such a high number simply seems implausibly high.

(ii) Temperature effect. There are a large number of contradictory
<table>
<thead>
<tr>
<th>Sectors Studied</th>
<th>No Change in Precipitation</th>
<th>12.5% Increase in Precipitation</th>
<th>12.5% Decrease in Precipitation</th>
<th>6.25% Increase in Precipitation</th>
<th>6.25% Decrease in Precipitation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Natural Resources</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corn production</td>
<td>-420</td>
<td>-400</td>
<td>-420</td>
<td>280</td>
<td>260</td>
</tr>
<tr>
<td>cotton production</td>
<td>220</td>
<td>200</td>
<td>180</td>
<td>-60</td>
<td>-60</td>
</tr>
<tr>
<td>wheat production</td>
<td>1,840</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>rice production</td>
<td>19,120</td>
<td>21,660</td>
<td>18,360</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>forest production (U.S. only)</td>
<td>13,220</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Douglas fir production</td>
<td>9,500</td>
<td>7,300</td>
<td>11,760</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>marine resources</td>
<td>28,620</td>
<td>35,720</td>
<td>48,680</td>
<td>-12,360</td>
<td>-14,100</td>
</tr>
<tr>
<td>water resources</td>
<td>-60</td>
<td>200</td>
<td>2,100</td>
<td>100</td>
<td>1,080</td>
</tr>
<tr>
<td><strong>Urban Resources (U.S. only)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>health impacts (excluding skin cancer, U.S. only)</td>
<td>47,720</td>
<td>129,700</td>
<td>37,920</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>wages</td>
<td>73,340</td>
<td>37,220</td>
<td>103,900</td>
<td>-31,090</td>
<td>-31,000</td>
</tr>
<tr>
<td>residential, commercial</td>
<td>3,520</td>
<td>3,520</td>
<td>3,520</td>
<td>-1,760</td>
<td>-1,760</td>
</tr>
<tr>
<td>and industrial fossil fuel demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>residential electricity demand</td>
<td>-14,960</td>
<td>?</td>
<td>?</td>
<td>7,100</td>
<td>?</td>
</tr>
<tr>
<td>commercial electricity demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>housing, clothing expenditures</td>
<td>10,140</td>
<td>?</td>
<td>?</td>
<td>-5,060</td>
<td>?</td>
</tr>
<tr>
<td>public expenditures</td>
<td>480</td>
<td>720</td>
<td>300</td>
<td>-220</td>
<td>-340</td>
</tr>
</tbody>
</table>

*Negative sign denotes benefit.

studies of the effect of changed temperatures on agriculture. One of the most comprehensive is d'Arge (1977) shown in Table 3, who gives an estimate of about $5 billion increase in annual agricultural production per degree C. temperature increase. These will represent the "optimistic" estimates.

(iii) Precipitation. Early studies estimated that there will be some changes in precipitation patterns, with increases of 0 to 10 percent in most latitudes and decreases of 0 to 10 percent in the 30° to 40° latitude. A more detailed recent study by the same Manuabe and Wetherald shows a quite different pattern, with precipitation patterns showing erratic patterns--increasing in most areas except near the equator and in the 40° to 50° latitude. According to d'Arge, as reported in Table 3 above, a 12.5 percent increase in precipitation would lead to less than 1 billion in increased annual output (this amounts to about 0.1 percent of output). Given the uncertainties, it seems likely that a -2% and +2% increase in agricultural production per degree C. would be reasonable pessimistic and optimistic estimates.

(iv) Redistribution. The most difficult effect to measure is what I will call the redistribution. It appears that climate patterns are much more sensitive than are global or continental averages to changes in external conditions like CO₂. That is, the world's climate will be redistributed. The fundamental "aggregation theorem" is that if the response functions to temperature and precipitation are linear over the relevant range, and if the response rates are uncorrelated with the local climatic changes, then the redistribution effect will be zero. Thus the economic impact of climatic change through redistribution will depend on the nonlinearity of response functions, the degree of adaptation to local climates, the extent of local climate variation, and the time scale on which redistribution occurs.

A different way of analyzing the redistribution effect is by comparing the "island" and the "strip" views of the world. In the island view, each person, firm, or nation is an island--endowed with its own immobile land, labor, and capital and its own unadaptable technology. As the climate is redistributed around the islands, some fertile and heavily populated ones will become barren, while other barren ones will bloom without a population to reap the harvest. In such a case, depending on the exact nature of the production functions, there could be a very serious loss in overall output even though the average climate does not change at all.

At the other extreme is the "strip" view of the world. In the view, the globe is seen as a continuous strip running from north to south, with populations and technologies highly mobile and adaptable to changing conditions. In such a world, as the global climate changes—with all functional zones moving 500 miles toward the pole—the population and the technology move with the climate at little cost. In the "strip" view, then, there can be major changes and redistributions of climate without major social or economic losses.

One other important factor in the adaptation of human societies is the time factor. Much of the productive capacity of an economy is fixed or
Figure 15: Estimated Effect of a Doubling of Atmospheric CO$_2$ on Precipitation (mm) at Selected Longitudes of Northern Hemisphere (1979). The Figures are Change from Mass Balance Areas (from Muhle and Wetherald [1979]).
quasi-fixed. Buildings generally are completely immobile, established populations are moved with cost and sometimes personal hardship, agricultural and touristic technologies are often fine-tuned to specific local conditions. If a climatic change occurs extremely rapidly, as in the dramatic case of coastal flooding, then much of this productive capacity may be lost. If the climatic change occurs slowly, and the natural mobility of new investment and young people along with the changing technology are given time to adjust, the economic and social losses may be much smaller. Unfortunately, aside from the natural time constants of labor and capital, it is extremely difficult to guess what time paths might be too slow or too fast for society to adapt to.

Evidence on the degree of variability of climatic response is shown by the Manabe-Wetherald predictions of the effect of CO\textsubscript{2} doubling on precipitation less evaporation, shown in Figure 15. A simple inspection of the changes in the land half, over the 20° to 60° latitude, indicates that the average absolute change (about 0.03 cm per day) is about three times larger than the mean change (about 0.01 cm per day).

My hunch is that it will prove impossible to predict, with any conviction, the magnitude of the distribution effect before it occurs. We would need much more understanding than we now have of regional climatic effects of CO\textsubscript{2} changes as well as of the response of agriculture to climatic changes. It may be that some evidence from year-to-year variation in weather may give a rough indication of the order of magnitude of the distribution effect, for annual variations are not dissimilar to the predicted variability just described.

We can obtain optimistic or pessimistic estimates using the above reasoning. In the optimistic case, we assume that the fundamental aggregation theorem holds, so there is no redistribution effect. In the pessimistic case, we assume that any change is harmful, and use the estimate above that the average absolute change is three times the means. This leads to a redistribution effect three times the pessimistic level. It should be emphasized that these are only illustrative figures.

c. Energy

The major effect of climate changes on energy demand will be on the requirements for space heating and cooling. These are unlikely to be major effects in the aggregate as compared to the agricultural and sea-level effects. We will include as estimates the extreme estimates from d'Arge's survey shown in Table 3.

d. Health, amenities, etc.

In work investigating the costs and benefits of the climatic impacts of ozone depletion, d'Arge and co-workers investigated a number of effects other than those on agriculture and energy. These included health impacts, wages, and expenditure patterns. The results, discussed carefully in the d'Arge reference above, indicate that a 1°C decline in mean temperature is associated with a
one-third percent higher level of wages. This is interpreted as indicating that a family must be compensated by higher income in order to live in a more inclement climate. d'Arge also indicates this is a plausible figure by showing that identifiable costs (higher costs for fuel, clothing, snow removal) are the same order of magnitude. I will include this estimate as the optimistic estimate. The methodology is quite controversial and some of the results are quite implausible. Therefore it seems best to ignore it for the pessimistic estimate until further vetting of the numbers takes place.

e. Changes in sea level

The effect of CO₂ changes on economic activity through changes in the sea level is one of the most uncertain and difficult areas to analyze yet it may be of extreme importance. We must ask (i) the effects of temperature changes on the sea level and (ii) the effects of sea level changes on the economy.

(i) The effects of temperature changes on the sea levels. Little is known about the effects of climatic changes on the ocean levels. In my earlier article, I performed a crude statistical analysis of temperature changes on ocean levels from data on past temperatures and sea levels. This investigation assumed a simple linear model and no catastrophic elements such as those just alluded to by Schneider. The best guess was an eventual rise of 5 (±5) meters in the ocean levels per degree C increase. The time scale of this rise appeared very slow, with the mean lag of 15 to 28 centuries, so that the short-run effect was 2.4 (±1.0) millimeters per year per degree C. I will take this to be an "optimistic" estimate of the effect of temperature on sea-level changes. Figure 16 shows the calculated path of sea-level changes from different control strategies for the optimistic case.

A pessimistic approach would allow for a larger sea level rise and, perhaps, for a surge of the West Antarctic glacier. We assume, in this case, that a doubling of CO₂ would lead to a 5 meter rise in ocean levels in a century, with an additional 5 meters eventually occurring.

(ii) Economic impacts of sea-level rise. The economic effect of sea level changes on the economy depends to a major degree on how fast such changes take place. At one extreme, complete adaptation through gradual retreat of cities, or building of dikes could take place. The most that would be lost is the land that was not worth saving and the cost of dikes. Such a view might be reasonable if the rise in the oceans were at the slow rate anticipated in the optimistic case--around one-half meter per century for the case of CO₂ doubling. At the other extreme, with a sudden surge and rise in ocean levels, all the land and buildings in low-lying regions would be lost--this list could include an impressive list of the world's great cities (New York, London, Tokyo, Shanghai to begin with). The only study which attempts to address this is that of Schneider and Chen for the U.S.¹⁴ They found that approximately 1 percent of U.S. population and immobile property is "lost" for every meter the coastline rises (up to 8 meters). For the pessimistic case, we assume this relation holds for the world--which implies there is absolutely no adaptation--and that the elasticity
Figure 16. Estimates of the effect of temperature on the level of oceans for alternative paths of carbon dioxide concentrations. All calculations assume no rise in coastlines with no change in carbon dioxide concentrations. The percentage increase figures indicate control strategies which limit atmospheric concentrations to the given figure at a maximum.

Source: Nordhaus, "Strategies."
of output with respect to tangibles is one-half.

In the optimistic case, the U.S. would lose only the land that was not worth protecting with levees, and cities would retreat to high ground as buildings depreciate. If the sea-level rise was only 0.25 to 0.50 meters per century (as opposed to the 0.1 to 0.2 meters per century at present), the main casualty would be the land.

The optimistic methodology would value the lost land at the price for undeveloped land. Using this technique, I looked at the loss for the U.S. from a rise of 30 meters in the sea level. Such a rise would absorb approximately 4 percent of all farm land, 5 percent of all land area in the lower 48 states, and 8 percent of the value of all farms. On an economy-wide basis for the U.S., this represents a capital loss of between 1/2 and 1 percent of national tangible wealth per 30 meters of sea level rise. Extrapolating on a world wide basis, taking into account the different intensities of agriculture in the U.S. and world, this represents a loss of from 0.4 to 1.3 percent of global capital. For our purposes we will assume that the elasticity of output with respect to tangibles is one third. This implies that a loss of 30 meters represents a loss in real output of about 1/3 percent for the optimistic case.

f. Summary

The sum total of these effects is shown in Table 4. We can summarize our "empirical findings": Overall, we do not know whether a CO₂ buildup will be beneficial or harmful. The uncertainties about the effects of CO₂, as well as the effects of the climate on agriculture, are not sufficiently clear today to judge the overall effect. More important, we are unsure of the ultimate impact of the rise in sea level, as alternative methodologies give drastically different estimates. These are perhaps clues as to where research on economic impacts should look most carefully.

For what it is worth, we see that in the optimistic case, the effect on the global economy of a doubling of CO₂ is an increase of about 5 percent in total output. In the pessimistic case, the loss is between 7 and 12 percent of world output. These are, clearly, very significant effects for what might appear an insignificant pollutant.

It might also be noted that the pessimistic case looks worse than the optimistic case looks good—there appears an asymmetry in possible outcomes. This has implications for decisionmaking under uncertainty.

For the purposes of our application in the next section, we will use the following equation:

\[ h(M) = f(\hat{E})kM/M_0 \]
TABLE 4

Economic Effects of CO₂ Doubling for "Optimistic" and "Pessimistic" Scenario
[Billions of dollars, 1975 prices and level of output]

<table>
<thead>
<tr>
<th></th>
<th>Optimistic</th>
<th>Pessimistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO₂</td>
<td>$200</td>
<td>0</td>
</tr>
<tr>
<td>Temperature</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Precipitation</td>
<td>40</td>
<td>-60</td>
</tr>
<tr>
<td>Distribution</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>Energy</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>Health, Amenities, Etc.</td>
<td>48</td>
<td>0</td>
</tr>
<tr>
<td>Sea Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100-year</td>
<td>-1</td>
<td>-150</td>
</tr>
<tr>
<td>Asymptotic</td>
<td>-7</td>
<td>-450</td>
</tr>
<tr>
<td>TOTAL*</td>
<td>301 to 295</td>
<td>-391 to -691</td>
</tr>
</tbody>
</table>

Total as percent of 1975 Net World Output 5% -7 to -12%

*The first figure is the 100 year value, while the second is the asymptotic value. Note that this table is somewhat misleading because it attempts to compress into a snapshot processes which have very different time lags.
where
\[ \dot{E} = \text{emissions with no abatement;} \]
\[ h(M) = \text{loss in consumption due to CO}_2 \text{ buildup;} \]
\[ f(E) = \text{consumption with no CO}_2 \text{ abatement;} \]
\[ k = \text{loss parameter, indicating the fractional loss in consumption} \]
\[ \text{per doubling in CO}_2 \text{ concentration;} \]
\[ M = \text{increase in atmospheric concentration of CO}_2; \]
\[ M_0 = \text{pre-industrial concentration of CO}_2. \]

The key parameter in the equation is \( k \), which gives the fractional reduction in output when CO\textsubscript{2} doubles. From Table 4 above, we find that--since consumption is taken to be $6\times10^{12}$ in 1975--\( k \) is -.05 in the optimistic case and from .07 to .12 in the pessimistic case. We take .10 to be the value for the pessimistic case, and the average value is taken to be .03.\textsuperscript{15}

As a final note, it should be emphasized that we are assuming that the cost of the CO\textsubscript{2} buildup is linear in the concentration. This is convenient but clearly oversimplified. One clear non-linearity is in the temperature response; climatic simulations indicate that the change in temperature is a function of the logarithm of CO\textsubscript{2} concentration. Similarly, the response of land-based ice--such as a surge of the West Antarctic mass--may have a threshold effect; clearly a further doubling of CO\textsubscript{2} cannot cause a second surge of the same glacier. This over-simplification can in part be justified as saying that, although \textit{ex post} the response function may be nonlinear, with today's knowledge we cannot identify the nonlinearities. From a decisionmaker's perspective, it can be treated as linear in the expected value. With further research, of course, the shape of the impact function may be determined.

C. Model Results

1. A World without Growth

We now turn to integrating the optimal growth model introduced in Part I of this study with the empirical model outlined above. To restate the model, we want to solve the following problem:

\[
\text{maximise } \int_0^\infty e^{-\rho t} u(c(t)) dt
\]

subject to \( \dot{M}(t) = \beta E(t) - \delta M(t) \)

\[
f[E(t)] = f(\dot{E}(t)) - \frac{200}{3} \dot{E}(t) \left[ 1 - \frac{E(t)}{\dot{E}(t)} \right]
\]

\[
h[M(t)] = f(\dot{E}(t)) k M(t)/M_0
\]

\[
c(t) = f[E(t)] - h[M(t)]
\]
<table>
<thead>
<tr>
<th></th>
<th>k</th>
<th>β</th>
<th>δ</th>
<th>E₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>-.05</td>
<td>.62</td>
<td>.004</td>
<td>6 x 10⁹</td>
</tr>
<tr>
<td>Middle</td>
<td>.03</td>
<td>.50</td>
<td>.002</td>
<td>6 x 10⁹</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>.10</td>
<td>.37</td>
<td>.0015</td>
<td>6 x 10⁹</td>
</tr>
</tbody>
</table>

where

- \( k \) = percentage reduction in consumption per percent increase in \( \text{CO}_{2} \) concentration;
- \( \beta \) = fraction of "quick-mixed" \( \text{CO}_{2} \) in atmosphere;
- \( \delta \) = annual mixing rate from "quick-mixed" to deep oceans;
- \( E_0 \) = initial year emissions (tons carbon)
where

\[ M_0 = \text{preindustrial level of atmospheric concentration of CO}_2; \]
\[ M(t) = \text{increase in atmospheric CO}_2 \text{ from preindustrial level}; \]
\[ \dot{M}(t) = \text{rate of increase of } M(t); \]
\[ E(t) = \text{emissions of CO}_2; \]
\[ E(t) = \text{emissions of CO}_2 \text{ if no abatement takes place}; \]
\[ c(c) = \text{level of consumption (or net world product)}; \]
\[ u[c(t)] = \text{utility or value attached to level of consumption}; \]
\[ \rho = \text{rate of time preference}; \]
\[ \beta, \delta, k = \text{parameters of CO}_2 \text{ impact}; \]
\[ h(\cdot), f(\cdot) = \text{functions relating emissions and concentrations to consumption}. \]

In examining cases, we will examine the optimistic and pessimistic cases described above. In addition, we will discuss an "average" case which simply averages the values for the two extremes.

**a. Steady state**

We start with a description of the optimal abatement and pricing in the steady state. In what follows the figures should be interpreted as order-of-magnitude "illustrations" of optimal policies. Because of the simplifications, the linearity of the \( h(\cdot) \) function and the omission of time delays these results may well depart considerably from the results from more complex models.

\[(13) \quad f'(E^*) = g'(M^*) \left[ \frac{\beta}{(\delta + \rho)} \right] \]

where

\[ E^* = \text{steady state emissions}; \]
\[ M^* = \frac{E^* \beta}{\delta} = \text{steady state atmospheric concentration}; \]
\[ \beta = \text{fraction of quick-mixed CO}_2 \text{ in atmosphere}; \]
\[ \delta = \text{rate of diffusion from quick-mixed reservoirs to deep oceans}; \]
\[ \rho = \text{rate of social time preference}. \]

In our simplified example, (1) becomes:

\[(14) \quad 200 \left[ 1 - \frac{E^*}{\dot{E}} \right]^2 = \frac{8k \dot{f}(\dot{E})}{M_0(\rho + \delta)}. \]

The fraction abated is given by

\[(15) \quad \left[ 1 - \frac{E^*}{\dot{E}} \right] = \left[ \frac{8k \dot{f}(\dot{E})}{200M_0(\rho + \delta)} \right]^{1/2}. \]

The shadow price of CO\(_2\) in the steady state, \( q^* \), is given by
Table 6
Steady State Values for Optimal CO₂ Control Strategy without Further Growth

<table>
<thead>
<tr>
<th>Case</th>
<th>Rate of Time Preference</th>
<th>Fractional Reduction in Emissions (1 - E*/E)</th>
<th>Ultimate CO₂ Concentration as Fraction of Preindustrial Level (1 + M*/M₀)</th>
<th>Shadow Price on CO₂ (1979$ per ton C)</th>
<th>Item: coal price per million btu (inclusive of CO₂ tax) a/</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>This case is not applicable as there are net benefits; CO₂ should be encouraged or subsidized.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ρ = 0.00</td>
<td>.61</td>
<td>2.0</td>
<td>$75</td>
<td>$5</td>
</tr>
<tr>
<td>Average</td>
<td>ρ = 0.01</td>
<td>.25</td>
<td>2.9</td>
<td>12</td>
<td>2.48</td>
</tr>
<tr>
<td></td>
<td>ρ = 0.04</td>
<td>.13</td>
<td>3.5</td>
<td>3.6</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>ρ = 0.00</td>
<td>1.00</td>
<td>1.0</td>
<td>$200</td>
<td>10</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>ρ = 0.01</td>
<td>.39</td>
<td>2.1</td>
<td>31</td>
<td>3.24</td>
</tr>
<tr>
<td></td>
<td>ρ = 0.04</td>
<td>.21</td>
<td>2.9</td>
<td>9</td>
<td>2.36</td>
</tr>
</tbody>
</table>

a/ These calculations assume coal sells for $50 per ton and contains 25 million btu per ton. Moreover, the shadow price on CO₂ is assumed to take the form of an excise tax on CO₂ emissions and is added onto the price of coal. If coal converts to gasoline at two-thirds efficiency with non-coal cost $1 per gallon, then the figure in the last column, plus $1, will buy 7 gallons of gasoline.
FIGURE 17. Illustration of optimal control strategy, average parameters, for discount rates of 0, 1, and 4 percent. Point A is for $\rho = 0.00$; B is for $\rho = 0.01$; C is for $\rho = 0.04$. 
\[ q^* = f'(E^*) = \frac{8kF(E)}{(\rho + \delta)M_0} = 200 \left[ 1 - \frac{E^*}{E} \right]^2 \]

Table 6 gives the results for the steady states for different assumptions about parameters and different discount rates. Figure 17 shows the solution geometrically. The important results are as follows:

*First, for the optimistic case, there should be no controls on CO\(_2\). Indeed, it should be encouraged.*

*Second, as between the average and pessimistic cases, there are modest differences in control strategies. For both these cases, and for all discount rates, there is some CO\(_2\) abatement in the optimal strategy. The degree of control varies between 13 and 100 percent in the cases examined.*

*Third, the discount rate is clearly the important parameter for the optimal CO\(_2\) control strategy. With no discounting, the control strategy ranges from 61 to 100 percent; while for the high discount rate, the rate of control ranges from 13 to 21 percent. These differences are even more apparent in the shadow prices.*

b. **Transition Paths without Growth**

Calculation of optimal transition paths is generally extremely difficult without a full-scale optimization model. Under simplified assumptions, however, we can obtain an approximation to the full optimization which is intuitively appealing.

Recall that we have assumed that the economic impact of CO\(_2\) elevation is approximately linear in the atmospheric concentration. As shown in Part I, this implies—in a world without economic growth—that the shadow price on CO\(_2\) is _invariant over time_. We conclude that _the control rate on CO\(_2\) should be invariant over the transition path and should be equal to the steady state value_.

It should be noted that under such an optimal policy, the buildup of CO\(_2\) is very slow. For example, for average value of the parameters and \(\rho = .01\), the ultimate level of CO\(_2\) is about 3 times the preindustrial level. After 100 years, in the optimal regime, the atmospheric concentration has built to less than 20 percent of that figure.

2. **A Growing Economy**

It seems certain that the example just discussed is unrealistic as it overlooks the possibility of further growth in standards of living in both developed and less developed countries. As noted in Part I, growth complicates the analysis considerably because it both changes the production structure and changes the discount rate.
A couple of simple modifications can be analyzed. The easiest is to assume the economic growth at rate $g$ continues for $T$ years, after which no further growth takes place. We also assume that the "carbon intensity" of production is constant—that is that the total uncontrolled emissions per dollar output is unchanged, and that the fractional loss of output per fractional reduction in CO$_2$ emissions is unchanged.

The only new parameter needed is the value of the redistributive parameter, $\alpha$. Recall from Part I that $\alpha$ defines our societal value judgments about the relative urgency of consumption of generations with different standards of living. Technically, $\alpha$ is the elasticity of the marginal utility of income. A value of zero indicates we are indifferent to redistribution between poor and rich; a value of $\alpha = 1$ indicates that we will redistribute $\$1$ of consumption to a generation one-half as rich if it costs the richer generation no more than $\$2$.$^{16}$

We might start with the presumption that economic outcomes reflect social decisions and ask what parameter values are implicit in these decisions. Given that the real rate of return of capital is around 7 percent annually, we can derive the following combination of parameters to attain that discount rate (assuming the growth of per capita incomes is 2 percent annually):

<table>
<thead>
<tr>
<th>Time Preference $\rho$</th>
<th>Growth Discount $(\alpha x 2%)$</th>
<th>Goods Discount $r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>3.54 x 2%</td>
<td>7%</td>
</tr>
<tr>
<td>1%</td>
<td>3 x 2%</td>
<td>7%</td>
</tr>
<tr>
<td>4%</td>
<td>1.75 x 2%</td>
<td>7%</td>
</tr>
</tbody>
</table>

Clearly, we need to have either a very high rate of time preference or a strong taste for redistribution to obtain the existing market discount rate. While it stretches the imagination to think that the political process has any combination of the parameters above, they are not so outrageous to warrant rejection. In what follows I will assume that the rate of time preference is 1% and that the redistributive parameter is 3. Under this assumption, during the growth era, the discount rate on goods is $\rho + 3g$ and the CO$_2$ discount rate is $\rho + 3g + 6$. After growth ceases, $g$ falls to zero and the discount rates fall to the same level as in earlier sections.

Table 7 gives the estimates of the optimal control strategy for different growth rates and different periods over which growth occurs. The significance results are as follows. With a growth economy, the ultimate level of control of carbon dioxide rises with higher growth rates or higher ultimate levels of consumption. The level of control, in this example, rises as the square root of the size of the economy. Similarly the shadow price on CO$_2$ rises in cases of growth.
It is somewhat more complicated to calculate the current value of the shadow price in a growing economy. In Part I, we showed that the current shadow price could be calculated as follows:

\[ q(0) = \ln \left[ \frac{1 - e^{-(\delta + r_I - g)t}}{\delta + r_I - g} + \frac{e^{-(\delta + r_I - g)t}}{\delta + r_{II}} \right] \]

where \( r_I \) and \( r_{II} \) are the goods discount rates during the growth phase and steady state and \( T \) is the length of the growth phase. In our numerical example, with no population growth, if \( \rho = .01 \), \( \alpha = 3 \), and \( g = .01 \), \( r_I \) is 4% and \( r_{II} \) is 14% per annum. If \( g \) rises to .04, \( r_{II} \) rises to 13%.

Table 8 summarizes the results for the current shadow price and control rate under the alternative growth assumptions shown in Table 7. There is not a great deal of difference in the impact on current \( \text{CO}_2 \) control strategies as between different growth trajectories. The current control rates are in the 20 percent range for any but the most robust growth trajectories. The current shadow price on \( \text{CO}_2 \) ranges from around $10 for modest growth paths up to $70 for the largest growth path.

It should be emphasized that the most important result for current policy is today's shadow price along with its trajectory for the next couple of decades. Table 8 indicates that depending on the discount rate and degree of growth pessimism, the current penalty on \( \text{CO}_2 \) should be in the $10 to $30 per ton range, unless one is very optimistic about economic growth.

It is worth noting why in some sense only the current shadow price is important. Today's decisions concern the amount of abatement that should occur today as well as the kinds of R&D that society should begin. These decisions will, in our optimal plan, be completely determined by today's shadow price and its trajectory over the next few years.

Given the great importance for current policy, it is useful to ask how these results change with alternative parameters. Table 9 gives results for our different policy questions for different distributional parameters, for a pure rate of time preference of 1 percent, for average parameters, and where growth progresses at 4 percent for 50 years. This calculation shows the importance of the distributional parameter when growth occurs. If \( \text{CO}_2 \) paths are chosen in a way that is relatively indifferent to differences in income, the discount rate is quite low and the current \( \text{CO}_2 \) shadow price is actually higher than the steady state. Emissions today are doing a lot of damage in a larger economy in the future! If we are very "egalitarian," preferring to distribute consumption to poorer generations, then the goods discount rate is high over the growth phase, and the current shadow price of \( \text{CO}_2 \) is quite low, or even zero.

It is relatively easy to perform sensitivity analysis in the present model. Other examples are the following:
TABLE 7

Alternative Results in Steady State for CO₂ Control Programs; Average Assumptions, with \( \rho = 0.01 \).

<table>
<thead>
<tr>
<th>Length of Growth Period</th>
<th>Growth Rate</th>
<th>Consumption (1979 = 1)</th>
<th>Control Rate (percent of Emissions Controlled)</th>
<th>Shadow Price (q*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Growth</td>
<td></td>
<td>1.0</td>
<td>25%</td>
<td>$12</td>
</tr>
<tr>
<td>25 years</td>
<td>( g = .01 )</td>
<td>1.3</td>
<td>28</td>
<td>16</td>
</tr>
<tr>
<td>50 years</td>
<td>( g = .01 )</td>
<td>1.7</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>200 years</td>
<td>( g = .01 )</td>
<td>7.3</td>
<td>66</td>
<td>88</td>
</tr>
<tr>
<td>25 years</td>
<td>( g = .04 )</td>
<td>2.7</td>
<td>41</td>
<td>33</td>
</tr>
<tr>
<td>50 years</td>
<td>( g = .04 )</td>
<td>7.0</td>
<td>65</td>
<td>85</td>
</tr>
<tr>
<td>100 years</td>
<td>( g = .04 )</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

Note: This table uses the assumption of a linear effect for calculating the transition shadow price. Other parameters are \( k = .03 \), \( \beta = 0.5 \), \( \delta = .002 \), \( \rho = .01 \), and \( \alpha = .03 \).
TABLE 8

Current CO$_2$ Control Strategy and Shadow Price, Alternative Growth Trajectories

Current (1979) Levels of Optimal:

<table>
<thead>
<tr>
<th>Length of Growth Period</th>
<th>Growth Rate</th>
<th>CO$_2$ Control Rate (Percent of Emissions Controlled)</th>
<th>Shadow Price on CO$_2$ [g (1979)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Growth</td>
<td></td>
<td>25%</td>
<td>$12/ton</td>
</tr>
<tr>
<td>26 years</td>
<td>$g = .01$</td>
<td>22%</td>
<td>$10/ton</td>
</tr>
<tr>
<td>50 years</td>
<td>$g = .01$</td>
<td>22%</td>
<td>$10/ton</td>
</tr>
<tr>
<td>200 years</td>
<td>$g = .01$</td>
<td>41%</td>
<td>$33/ton</td>
</tr>
<tr>
<td>25 years</td>
<td>$g = .04$</td>
<td>18%</td>
<td>$6/ton</td>
</tr>
<tr>
<td>50 years</td>
<td>$g = .04$</td>
<td>22%</td>
<td>$10/ton</td>
</tr>
<tr>
<td>100 years</td>
<td>$g = .04$</td>
<td>59%</td>
<td>$71/ton</td>
</tr>
</tbody>
</table>

Note: Assumptions are the same as in Table 7.
(i) If the social rate of time preference is higher, then both steady state and current CO$_2$ controls are less stringent. Roughly speaking, a doubling of the discount rate will lead to the CO$_2$ control rate falling by 30 percent.

(ii) Similarly, if we use pessimistic rather than average assumptions for the impact of CO$_2$, the economic impacts are estimated to be about three times more severe. In this case, our CO$_2$ control rate would rise by almost a factor of 2.

(iii) The uncertainty about the fate of CO$_2$ or the "case of the missing CO$_2"--to which much of the scientific attention is given--is of relatively little consequence for current policy. If we take the extremes--optimistic and pessimistic in Table 5--the optimal control strategies differ by less than 20 percent in our model.

(iv) The shape of the CO$_2$ impact function is important for the general policy. The linear form used here implies that CO$_2$ emissions are equally important at every level of atmospheric concentration. If there were threshold levels, or increasing marginal impacts, the effect would generally be to postpone CO$_2$ abatement efforts--lowering the current shadow price.

We show in Figures 18 and 19 illustrative trajectories for optimal shadow prices and control rates for the alternative assumptions shown above. In summary, it is clear that the fundamental uncertainties that would be important to resolve, if the analysis here is to be useful for policy, is the future growth patterns, the economic impacts of CO$_2$, and how society feels about redistribution of consumption over different generations.

D. Realistic considerations

Up to now I have analyzed the CO$_2$ problem as an exercise in pure optimal economic growth. In this final section realistic considerations of policy and policies must be acknowledged.

1. The externality problem. Economic literature has recognized an important source of inefficiency in the presence of "externalities." An externality is a process by which the actions of economic agents affect others outside (or external to) the market. In these cases, I produce (or consume) something valuable but do not pay (or earn) for my actions. A standard example is pollution; when I pollute the air others incur health or property damage, but I do not pay the cost. If I were to pay for the damage, I would reduce my pollution up to the point where marginal costs of further reduction cost more than the damage--an efficient outcome. Since I do not have to pay for the cost of my emissions in the pure market economy, I pollute more than would be efficient. Such reasoning has led most industrial nations to control pollution.

It would be natural to suppose that CO$_2$ pollution could be controlled in
### Table 9

Illustration of Effect of Distributive Parameter on Optimal Path for CO₂ Control, Average Assumptions with \( p = 0.01 \), \( g = 0.04 \) for 50 Years

<table>
<thead>
<tr>
<th>Distributive Parameter (a)</th>
<th>CO₂ Control Rate (percent of Emissions Controlled)</th>
<th>Steady State</th>
<th>Transition Path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Discount Rate on Goods When in Growth Phase</td>
<td>Current CO₂ Shadow Price</td>
</tr>
<tr>
<td>0</td>
<td>65%</td>
<td>$85/ton 1%</td>
<td>$125/ton</td>
</tr>
<tr>
<td>(indifferent on differences in distribution)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>65%</td>
<td>$85/ton 5%</td>
<td>$85/ton</td>
</tr>
<tr>
<td>3</td>
<td>65%</td>
<td>$85/ton 13%</td>
<td>$10/ton</td>
</tr>
<tr>
<td>6</td>
<td>65%</td>
<td>$85/ton 21%</td>
<td>$4/ton</td>
</tr>
<tr>
<td>—</td>
<td>65%</td>
<td>$85/ton</td>
<td>0</td>
</tr>
<tr>
<td>(extreme egalitarian)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The solution in the box will be taken as the "best guess" policy today.*
Figure 18. Illustration of the Trajectories of CO₂ shadow prices and control rates, different growth rates.
Figure 19. Illustration of the trajectories of CO₂ shadow prices and control rates, alternative assumptions about the redistributive parameter.
a similar way as particulates or other pollutants. This supposition would be much too optimistic. Most externalities are ones that are internal to nations; thus a government can weigh the costs and benefits of a pollution control program for particulates and decide that, on balance, it is in the interests of its citizens. In other words, most pollution problems are external to individuals but internal to a nation. In such a circumstance, the balancing of costs and benefits of pollution control by a nation's government may lead to policies in the vicinity of an efficient policy.

The CO₂ problem is different from conventional pollutants because it is an externality across space and time. Once in the atmosphere, it disperses across the globe and has a half-life of several centuries. Thus, just as I as an individual have little reason to curtail my particulate emissions, we as a nation have little incentive to curb CO₂ emissions. France burns 5 percent of the world coal production annually, and has approximately 5 percent of Gross World Product. By curbing its CO₂ output, it makes little contribution to the solution and receives only one-twentieth of the benefit from its own actions. It has little incentive to act alone.

The problem is exacerbated by the long time period over which CO₂ affects our economies. It may be that "competitive political systems" have an institutional myopia because of the emphasis on short-term rewards to electorates; this would superimpose a temporal externality on the spacial one.

In such a situation, a CO₂ control strategy could only work if major nations successfully negotiated a global policy. While such an outcome is possible, there are few examples where a multinational environmental pact has succeeded—the nuclear test ban treaty being the most prominent. Other clearly recognized problems—whales, acid rain, undersea mining—provide a living for negotiators but little else.

The multilateral bargaining is severely complicated by the likelihood that some major countries will benefit from the CO₂ buildup. It is sometimes thought that the USSR and Canada will benefit from the warmer climate. Given that these two countries (and the former's allies) burn 25 percent of the world coal, it is hard to see how a CO₂ strategy can succeed without them. On the other hand, given the unlikelihood that the U.S. or other western nations will compensate the USSR for participating it is hard to see why the Soviet bloc would participate. Finally, given that a major bloc does not participate, it is hard to envisage the others, including LDC's, making a major sacrifice. Thus the differences in country experience is likely to doom an international agreement.

In conclusion, the fact that CO₂ is an externality across nations and time poses particularly difficult political problems for its control. The fact that national experiences are likely to differ, and that its costs are so uncertain, make it highly unlikely that an effective multilateral agreement can be attained.
2. Alternative control techniques. Little attention has been paid to the techniques that would be useful for controlling CO$_2$. The analysis here may lead to certain conclusions about the aggregate control rate—we suggested a control rate of around 20 percent would be optimal today. In addition, it is necessary to ask how we might implement such a control strategy—that is how we might design a system so that nations and individuals have the appropriate incentives for controlling emissions.

There are basically two kinds of control devices—prices and quantities. In the former, a "carbon tax" would be placed on all net sources of CO$_2$, basically burning of fossil fuels. In the quantity approach, standards are placed on technologies so that emissions are reduced.

It is generally agreed, particularly for cases such as CO$_2$, that prices are a much more efficient control device than quantities. This is so because the benefits of emission reduction are uniform across all technologies, while the costs of CO$_2$ abatement vary widely. It is efficient to reduce CO$_2$ emissions so that the incremental cost of further reductions are uniform across technologies. This will be approximated by a uniform CO$_2$ tax, since each individual will have an incentive to reduce emissions up to the point where further reductions cost more than the carbon tax.

To be sure, use of prices has been quite rare in pollution control; it has been more usual to use quantitative "command and control" regulations. Thus for sulfur emissions, the U.S. imposes a limitation to x pounds of sulfur per million btu of input instead of a sulfur emission tax of $y per pound sulfur. The major problem with the use of quantity restrictions is that they are inherently inefficient because (i) they do not lead to the rule that marginal costs of abatement are equalized across polluters and (ii) they do not set up strong incentives to develop new technologies. These problems would be exacerbated for CO$_2$ because of its global nature and widespread use.

A price-type scheme for abating CO$_2$ might run as follows. Nations would first agree on a strategy for controlling CO$_2$—something like the framework outlined here in much greater detail and regional and temporal disaggregation. The program would include both a target emissions reduction and a shadow price which reflects that reduction. The shadow price that comes out of the optimal program is the appropriate carbon tax. Thus in our best guess case described above, the carbon tax would be approximately ten dollars per ton carbon combusted.

In this scheme, individual nations would be committed to take steps to assure that the carbon tax was reflected in fuel and other prices. In a market economy like the U.S., such a commitment would be naturally fulfilled by pricing a $10 per ton excise tax on coal, lesser amounts on oil and gas. The U.S. could collect the revenue and could use it to lower other taxes. Nations that have nationalized coal or energy industries would place consumption taxes on CO$_2$ sources.

There will be many who argue for the proceeds of the carbon tax to go into a fund for development, solar power, or other worthy causes. The arguments
over the spoils will be so divisive that the scheme will thereby flounder. For this reason, it is best to separate the carbon tax from general redistributive or other purposes by leaving tax revenues in the nations where they originate.

The carbon tax will, of course, grow in real terms over time depending on the discount rate. In addition, as new knowledge becomes available, new calculations would require changes in the carbon tax.

3. Issues involving uncertainty. We have emphasized that the CO₂ problem is loaded with uncertainties of all kinds. One strategy when faced with uncertainty is to use the "best guess" and proceed as if the world were certain. This is the approach which is almost universally followed. Such an approach is not appropriate under two general conditions--when the effects of actions are highly nonlinear or when the uncertainties are likely to be resolved with the passage of time.

On the first condition, there is no convincing evidence of which I am aware that the effects of CO₂ are known to be highly nonlinear. Possible exceptions are the melting of the Arctic sea-ice pack and disintegration of the West Antarctic ice sheet. These would surely have highly nonlinear effects, but (particularly for the second) we do not have clear ex ante knowledge of where (or when) the nonlinearity will arise. Put differently, if our ex ante (or judgmental) probability of such discontinuous events is smooth, then from a decision-maker's perspective the expected consequences are not highly nonlinear and the presence of uncertainty probably does not change the best guess strategy markedly.

The second condition--reduction in uncertainty over time--may be significant for the CO₂ strategy. Given a strong program of physical and social science research, we should be able to narrow the uncertainties about the parameters of an optimal strategy considerably over the next decade. At that point, for example, we should have a good idea whether a CO₂ buildup will be beneficial or not, and the costs of CO₂ abatement can be clarified. At that point, the chance of making a major mistake will be reduced below that in today's state of ignorance.

To put the point somewhat differently, we can invest in reducing the impacts of CO₂ or in improving our knowledge about the impacts of CO₂. It seems highly likely that, for the very near term, our best investment is to improve understanding about the CO₂ problem. Improvement in knowledge is a particularly important strategy when there are important irreversible elements in policies.

Using the tools of "decision theory" it is possible to fold this progressive reduction of uncertainty into today's decisions. Such an analysis would be an important adjunct to any serious best guess or scenario approach to CO₂ control.

One final point: the presence of uncertainty is per se not a reason for either accelerating or slowing programs for the control of CO₂. It is simply
irrational to say we should wait for further information before we act; or that we should act purely because the consequences may be grave. Unless the structure of the costs or the way in which uncertainties are resolved is highly asymmetric, we should act now on the best guess (or certainty equivalent) basis.
FOOTNOTES


2. The approximation will slightly understate the costs of CO₂ emissions because in steady state a fraction (around 10 percent in most studies) of the emissions remains in the atmosphere (almost) forever. Given the rate of return from deep oceans to atmosphere is about one-tenth of its entry rate, this can be ignored except where the discount rate is zero.


4. The great debate between the growth and no growth advocates would appear to make the central assumption here questionable. On closer examination, however, most of the advocates of a no-growth philosophy argue that growth in consumption is not feasible rather than not desirable. Who voluntarily turns down a 20 percent pay raise?

5. "Utility function" is a technical term which here indicates the social valuation of different levels of consumption. Utility cannot be directly measured but may in principle be inferred from society's decisions.

6. One of the uncomfortable aspects of the framework put forth here is that it would seem to imply that we would have the same attitude towards poorer countries as toward poorer generations. Why should we care less about today's Indians than the 21st century's Americans? Yet the clear reading from the political process is that we have very little aversion to inequality in other countries.

7. If concentrations are fixed at M*, then steady state consumption is \( c^* = x^* \left[ f(\delta M*/\delta x^*) - h(M^*) \right] \). Further, \( \frac{dc^*}{dx^*} x^* = c^* - E^* f' < 0 \) when \( f' E^* > c^* \). Since \( f' = q^* = \text{optimal carbon tax} \), this condition means growth hurts consumption when shadow tax collections exceed total national income.

8. The literature on the carbon cycle is voluminous and statements below will not be individually documented. For a recent survey see, George M. Woodwell, "The Carbon Dioxide Question," Scientific
9. Recall from the footnote no. 2 of Part I of this paper that we ignore the return from deep oceans to atmosphere. Our parameter estimates are consistent with that assumption. They will therefore overestimate \( \delta \) after the first two or three centuries.


11. See Nordhaus, "Strategies." In the analysis that follows we assume that emissions cannot be negative—i.e., that no \( \text{CO}_2 \) removal strategies are possible. For this reason the abatement curve in Figure 12 stays in the northeast quadrant.


15. Note that, as the difference between the values for the pessimistic case is due to the time period over which the effect is calculated, the pessimistic value depends on the discount rate. For any non-zero discount rate, the present value of the slower part of the sea-level rise is small relative to the first part.

16. A value of \( \alpha = 0 \) implies that the utility function is linear as in \( u = c \). A value of \( \alpha = \infty \) indicates that preferences are "Ravlian," as in \( u = \min (c_t, c_{t+1}, \ldots) \). If \( \alpha = 1 \), the utility function is Bernoullian as in \( \ln u = \sum_{t=1}^{\infty} \ln c_t \).