THE SHORT-RUN MACROECONOMICS OF FLOATING EXCHANGE RATES:

AN EXPOSITION

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Egon Sohmen was ahead of his time. During the heyday of Bretton Woods, when floating exchange rates were a thing of the past and international capital movements were still restricted, he began to examine the macroeconomics of open economies with floating rates and capital mobility. His 1958 M.I.T. dissertation, expanded into his Flexible Exchange Rates published in 1961 while he was a colleague of one of us at Yale, undertook among other things to compare fixed and floating rate regimes with respect to an economy's vulnerability to external shocks and with respect to the workings of fiscal and monetary policies. Sohmen anticipated qualitative results that later became standard. His strong support of floating rates was, so far as it depended on macroeconomic grounds, based on his views that floating rates provided greater insulation from external shocks and that monetary policy would be relatively more effective than under fixed parities.¹

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Formal macroeconomic analysis of open economies with capital mobility began with the work of Fleming and Mundell. They extended standard "IS/LM" analysis to open economies and examined the effects of domestic demand management policies and other shocks, comparing fixed and floating exchange rate regimes. For the floating rate regime, their analysis implied three strong propositions: (1) A market-determined floating rate would enable a "small" open economy to use domestic monetary policy to control its own macroeconomic outcomes, national output or price or some domestically feasible combination of them. (2) With money stock given, the exchange rate would wholly absorb changes in foreign demand for exports or other shocks to the current external account. In this sense the domestic macro-economy would be "insulated" from external disturbances. (3) Fiscal policy would be impotent as a tool of macroeconomic policy. Indeed any shifts in aggregate domestic demand for goods and services—IS shifts—would be, via exchange rate adjustment, completely offset by changes in the external balance on current accounts.

Twenty years of theoretical and empirical research, and six years of experience with floating rates, have raised doubts about these propositions. But they are still widely held. Our purpose in this paper

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is to review the application of macroeconomic theory to the questions addressed by Fleming and Mundell, and many successors. Our perspective is the same as theirs, the extension of simple short-run macroeconomic models to open economies. We do not consider long-run equilibria or the dynamics of adjustment to them, nor do we treat the formation of expectations of exchange rates and other variables. We do try to provide a more careful, more appropriate, and more general specification of the "IS/LM" model for open economies, and we show that this model does not support the three strong propositions.

The crux of the matter is the modeling of asset markets. There are several related issues:

(1) Omission of the exchange rate from the asset demand functions is necessary for the three strong propositions. It means that all the mutual adjustments of aggregate demand and exchange rate must occur within the IS equation. That is, exchange rate movements do not feed through financial markets and interest rates into domestic investment and consumption demand. This assumption, not as sometimes thought perfect substitutability between foreign and domestic assets, is the crucial one. If the exchange rate does not belong in the asset demand equations, then the strong propositions apply in a Fleming-Mundell type model even if the interest differential between domestic and foreign assets varies endogenously.

(2) Following traditional practice, Fleming and Mundell took interest rates and asset prices to be determined in markets that equate stock demands to existing stocks. Yet the solution of the model generally implies that stocks are changing, and conclusions about effects of policies and other shocks may be misleading if the changes in stocks are not tracked. This caution applies to domestic assets, capital and government debt, and thus applies to closed economy models. It could be even more important for the net external position of an open economy, where the flow, the current account surplus or deficit, may be large relative to the stock.

(3) Twenty years ago, the current account was regarded both by practical men and by economists as the locus of exchange rate determination. Or, if the rate was pegged, the current account was expected to absorb the shocks that would otherwise move the exchange rate. In a world of controls restricting inter-currency capital movements, this was not surprising. Though emphasizing capital mobility, Fleming and Mundell were still in this tradition, thanks to the features of their model just discussed. More recently, analysts of the international monetary scene have discovered that the exchange rate is an asset price, determined by portfolio preferences in markets for asset stocks. This useful insight is overdone if it leads to neglect of the relation between exchange rate and current account, as occurs for example in models where domestic and foreign goods are perfect substitutes.  

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1 An analogous insight is that the commodity price level is an asset price, in the sense that its reciprocal is the real value of money. It is likewise overdone when the next step is to ignore the flow markets for commodities themselves, in which the price level is determined.
These points are elaborated in Sections 1 and 2 of the paper. We show how exchange rates enter asset market equations, and what difference their inclusion makes. In the course of the analysis, we also emphasize the macroeconomic importance of export and import elasticities, and of the net creditor or debtor position of the country vis-a-vis the rest of the world. To handle the stock-flow issues mentioned in points (2) and (3) above, we introduce and analyze a discrete time model with four asset markets. Finally, in Section 3, we depart from the small country assumption and show how to extend our modeling procedure to a two-country world. Unfortunately, the results of such an analysis depend on more restrictions of the behavior equations than are required in the single country analysis.

1. A Standard IS-LM Model of a Small Open Economy

Suppose there are just three distinct assets available to savers and wealth-owners in a small open economy. Their values in domestic currency and their descriptions are as follows:

- $H$: Government-issued base (high-powered) money with zero nominal interest.
- $q_V$: Domestic interest-bearing assets with a market-determined nominal yield $r_V$. Their aggregate value, in domestic currency, is $q_V(r_V)$. Where the asset valuation $q_V$ is an inverse function of the yield. $V$ includes both the par value of government bonds, all of which we take for convenience to be consols paying the same annual coupon, and equities in the domestic capital stock valued at current commodity prices. Following the usual assumption of the Hicksian IS-LM framework, these are perfect substitutes and must yield the same real rates of return (or one rate of return must be an invariant function of the other).
Foreign assets, denominated in foreign currency, and bearing a foreign currency yield of \( r_F^* \) exogenous to our small country. The domestic currency price of foreign currency is \( e \). The domestic currency yield \( r_F \) is \( r_F^* \) plus the expected change in the exchange rate, \( (e/e)_{\text{exp}} \).

Domestic private economic agents must have, individually and in aggregate, non-negative holdings of \( H \) and \( V \). They may have either positive or negative holdings of \( F \). The government does not hold any foreign assets or have a foreign debt; it does not intervene in the exchange market. Foreigners do not hold any part of \( H \) or \( V \). We will not consider here changes in expected inflation rates. Thus the nominal rates of interest specified above also stand for real rates.

At any point in time, the net wealth of domestic residents is given by their past savings and past capital gains or losses. Subject to this constraint, they are in portfolio equilibrium, holding the stock of each asset that they desire at prevailing interest rates, prices, and incomes. Likewise, government's total debt in money and consols is determined by the past history of its budget, though the market value of its consols depends endogenously on the current interest rate. By open market operations the government can change instantaneously the form of its liabilities, buying or selling a $1 consol for $q_V$ high-powered money. The real capital stock is predetermined, but its nominal market value is endogenous and need not be the same as its replacement value. The quantity of foreign assets, positive or negative, is predetermined by the history of current account surpluses and deficits. The domestic value of this stock depends on the exchange rate, which is endogenous. These predeterminations of stocks do not mean that they are not changing. A government deficit may be increasing \( H \) or \( V \) or both, capital investment
may be occurring, and the country may be earning or losing foreign assets in trade. But at a point in continuous time, these rates of flow do not affect stocks.

The analysis can be carried out either for the extreme Keynesian case, with price level fixed at least for the moment and with output endogenous, or for the classical case, with price level flexible and output supply-determined. Indeed it can be carried out with any intermediate rule relating price level and national product. Like Fleming and Mundell, we present the analysis for the polar Keynesian case and assume the price level predetermined arbitrarily at 1.

Let domestic private purchases of goods and services, including both consumption and investment, be \( E_r V, r_F, Y \) where \( Y \) is real (and nominal) national product. The signs over the arguments, here and elsewhere in the paper, refer to the respective partial derivatives.

Let \( G \) be government purchases, the quantity to be varied exogenously by fiscal policy. Taxes are, for the purposes of this model, behind the scenes; the \( E(\cdot) \) function allows for their influence on private demands. Let \( X(e, Y) + x \) be the export surplus, positive or negative. The shift parameter \( x \) is a favorable shock to the export surplus, e.g. an improvement of export demand. If the Bickerdike-Robinson-Metzler condition for successful devaluation is satisfied, \( X_e > 0 \). The current account surplus also includes \( er_F \), the earnings from foreign investment.

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Our rendition of a Fleming-Mundell model is:

$$E(r_V, r_F, Y) + C + X(e, Y) + x = Y \quad \text{(IS equation)}$$

$$A^H(r_V, r_F, Y) = H \quad \text{(High-powered money)}$$

$$A^V(r_V, r_F, Y) = q_Y(r_V)V \quad \text{(Domestic interest-bearing assets)}$$

$$A^F(r_V, r_F, Y) = eF \quad \text{(Foreign assets)}$$

$$H + q_Y(r_V)V + eF = H_0 + q_Y(r_V)V_0 + eF_0 \quad \text{(Wealth constraint)}$$

$$X(e, Y) + x + eF^* = eF \quad \text{(Balance of payments)}$$

The $A^S$ ($S = H, V, F$) are demand functions for asset stocks as valued in domestic currency. They are functions of income and interest rates; the indicated signs correspond to the assumption of gross substitutability. These stocks, as equation (5) says, must add up to the current value of home residents' wealth. This depends on the composition, as well as the amount, of their previous accumulations ($H_0, V_0, F_0$). Discrete instantaneous transactions are allowed between the public and the government in domestic assets at the initiative of the government. Otherwise the public cannot change its holdings without the passage of time. Thus $F = F_0$; by assumption foreigners are not interested in acquiring $H$ or $V$.

By Walras's law, the sum of the three $A^S$ functions is equal to the sum of the right-hand sides of equations (2), (3) and (4) for any
arguments in the functions. One of these equations may be derived from the other two and (5). The system will determine five endogenous variables. For the flexible exchange rate regime here analyzed, these can be \((r_V, Y, e, V, \hat{P})\) given the exogenous variables \((H, H_0, V_0, F_0 = F, G, X, r_F^*\) . Here \(r_F\) is tied to \(r_F^*\) if exchange rate expectations are not endogenous.

If \(V\) and \(F\) were, for domestic wealth-owners, perfect substitutes, then equations (3) and (4) added together become a single equation and \(r_V\) and \(r_F\) collapse into a single exogenous interest rate. This leaves two asset equations, of which one is redundant. Dropping the combined equation (3) & (4), we see that (2) will determine \(Y\) . Given \(Y\) so determined, equation (1) determines \(e\) . The three strong propositions cited at the beginning follow immediately. Clearly \(H\) determines \(Y\), and neither \(G\) nor \(X\) can affect \(Y\) . All demand shocks can do is to change the exchange rate. Under our assumptions about the \(X\) function, an increase in \(G\) or \(X\) will lower \(e\), i.e., appreciate the exchange rate. These are the Fleming-Mundell conclusions for perfect substitutability of foreign and domestic assets.

But perfect substitutability is not necessary to obtain these conclusions from system (1)-(6). They still follow if the domestic interest rate \(r_V\) can diverge from \(r_F\) . Equations (2) and (3)--letting (4) be the redundant asset equation--together determine \(Y\) and \(r_V\) . Then equation (1) again determines only \(e\), and the conclusions of the preceding paragraph still apply. With \(E_Y < 1\), \(X+X\) will be larger the higher is \(Y\), and so \(e\) will be higher (the currency depreciated) with higher \(Y\) .

Monetary policy, all-powerful under floating rates, may be analyzed
from the subsystem of equations (2) and (3). An open market operation that increases \( H \) by a dollar diminishes \( q_{V}V \) by a dollar. Thus differentiation of (2) and (3) with respect to \( H \) gives:

\[
\begin{bmatrix}
-H \\
A_{r_{V}}^{H}
\end{bmatrix} + \begin{bmatrix}
H \\
A_{Y}^{H}
\end{bmatrix} \begin{bmatrix}
\frac{3r_{V}V}{\partial H} \\
\frac{Y}{H}
\end{bmatrix} = \begin{bmatrix}
1 \\
-1
\end{bmatrix}
\]

(7)

We know from Walras's law that \( A_{r_{V}}^{H} + A_{r_{V}}^{V} + A_{r_{V}}^{F} = 0 \), and the gross substitutability assumption says that the cross-effects \( A_{r_{V}}^{H} \) and \( A_{r_{V}}^{F} \) are both non-positive. Likewise \( A_{Y}^{H} + A_{Y}^{V} + A_{Y}^{F} = 0 \). Customarily we set \( A_{Y}^{H} \) to be positive, to reflect transactions demands for money. Suppose also \( A_{Y}^{V} \geq -A_{Y}^{H} \), implying \( A_{Y}^{F} \leq 0 \). In words, the public does not shift into foreign assets at the same time that increased transactions demands compel them to add to their holdings of domestic money. These assumptions suffice to make negative the determinant of the Jacobian of (7), \([-A_{r_{V}}^{V}A_{Y}^{H} + A_{r_{V}}^{H}A_{Y}^{V}] + A_{Y}^{H}q_{V} = \Delta \) and:

\[
\frac{\partial Y}{\partial H} = \frac{A_{r_{V}}^{F} + q_{V}V}{\Delta} > 0
\]

(8)

\[
\frac{\partial r_{V}}{\partial H} = \frac{-A_{Y}^{F}}{\Delta} \leq 0
\]

The effect of monetary policy on the exchange rate is not the same, possibly not even the same in sign, as if perfect capital mobility tied the domestic interest rate to the exogenous foreign rate. It hinges on whether the rise in \( Y \) engineered by monetary expansion creates more
room in the economy for the export surplus $X$ or less room. If it creates more room, the exchange rate must depreciate (e rise) to induce the net export demand to fill it, the more so because the direct effect of $Y$ is to diminish $X$ via the marginal propensity to import. If it creates less room, while the $Y$ effect in contracting net export demand is weak, then the exchange rate will have to appreciate (e fall). We can't be sure whether it creates more room or less. By itself expansion of $Y$ creates more, by our assumption that $E_Y < 1$. But in addition there is the increase in $E$, most likely investment, induced by the concurrent interest rate reduction. The ambiguity is due to this effect, which is absent when the domestic interest rate is tied to the foreign rate by perfect substitutability.

The situation is picture in Figure 1. In traditional $(Y,r)$ space the asset equations (2) and (3) jointly determine a locus $HV$ as open market operations change the quantities of the two assets. Monetary expansion moves the economy down and to the right. The slope of the locus, from (8) is $-A_Y^F / (A_Y^F + q_Y^V)$. Great (negative) sensitivity of foreign asset demand to income makes it steep; high substitutability of foreign for domestic assets makes it flat. The economy must be on this locus. The IS curves come from equation (1), and each is drawn for a given value of exchange rate $e$. In Figure 1a, the IS curves are steeper than $HV$. If monetary policy shifts the economy from point 1 to point 2, the IS curve must be shifted out to go through point 2, and this requires exchange depreciation, $e_2 > e_1$. In Figure 1b, the IS curves are flatter than $HV$, and exchange appreciation $(e_2 - e_1)$ brings the requisite leftward shift. In case domestic and foreign assets are perfect substitutes, $HV$ is horizontal. This is
just an extreme case of Figure 1a.

A related graphical version is Figure 2, in \((Y,e)\) space. The LM locus (from equation (2)) is vertical, shifted right by monetary expansion. For a given domestic interest rate the IS curve is upward sloping: to keep equation (1) satisfied, an (export-increasing) increase in \(e\) must be offset by an (import-increasing) increase in \(Y\). Figure 2 also shows, of course, how IS shifts resulting from increases in \(G\) and \(x\), with no LM shift, will be completely absorbed in exchange appreciation. But a decline in the interest rate moves the IS locus to the right. Thus an expansionary monetary policy shifts both curves, and the result may be either Figure 2a or Figure 2b.

Formally, using (1) and (8) gives:

\[
\frac{\partial e}{\partial H} = \frac{(1 - E_Y - X_e) \frac{\partial Y}{\partial H} - E_r \frac{\partial r_v}{\partial H} + \left( A_r \frac{F_r}{r_v} + q_t^v Y (1 - E_Y - X_e) + A_t F_t r_v \right)}{X - \frac{\partial}{\partial e} + \Delta X_e}
\]  

(9)

The second term in the numerator is what introduces the ambiguity regarding the sign. This expression has the sign of the difference between the slope of the IS locus and the slope of the HV locus. If \((\frac{dr}{dy})_{IS} > (\frac{dr}{dy})_{HV}\) as in Figure 1a, then \((\frac{\partial e}{\partial H}) > 0\) as in Figure 2a.

Equation (6) reminds us that foreign asset holdings will generally be changing as a byproduct of the solution. In Figures 1 and 2 we also show a locus for \(F = 0\), but nothing compels the solution to lie on this locus. In \((Y,r)\) space this is essentially the IS curve with \(X\) deleted, and it is flatter than the true IS curve because the import
FIGURE 2

FIGURE 2a

FIGURE 2b
leakage is omitted. For similar reason, it is flatter than the true IS in \((Y,e)\) space, Figure 2. In each case points to the right have \(\dot{P} < 0\), to the left \(\dot{P} > 0\). These current account balance curves will move right in Figure 1 by increases in \(G\), and move down in Figure 2 by increases in \(x\). As for monetary expansion, in the normal case, Figures 1a and 2a, it is not possible to say whether \(\dot{P}\) will be increased or lowered. But in the abnormal case, Figures 1b and 2b, when \(Y\) increases and the currency appreciates, clearly \(\dot{P}\) will decline.

Unless monetary or fiscal policy forces a solution with \(\dot{P} = 0\), the solution will not be stationary but change as \(F_0\) changes with the passage of time. This impermanence has rightly worried theorists. But it would nonetheless be a mistake to impose the condition \(\dot{P} = 0\) on this model. Its solution is transient anyway, because other stocks—domestic financial assets, capital, total wealth—are not stationary either. Point-of-time models should be used with caution, especially for policy implications.\(^1\)

An obvious objection to the above analysis is the omission of the wealth constraint from the portfolio equations. If wealth-owners have a non-zero position in foreign assets, their domestic value depends on the exchange rate \(e\). So, of course, does the total value of their wealth, \(W_0\), given in (6) as \(H_0 + q_VV_0 + eF_0\). Omitting this total or its separate constituents, from the asset demand functions \(A^S\) amounts to assuming that, for example, increases in value of wealth due to exchange depreciation are held entirely in the foreign assets whose domestic value has risen. If there were spillovers into domestic money and other domestic assets, then an increase \(e\) would

\(^1\)On the transition from temporary to steady-state equilibrium in the standard macroeconomic model, see Tobin and Buiter, op.cit.
shift upward the LM curve in \((Y,r)\) space (not drawn in Figure 1) and make the LM curve in \((Y,e)\) space (Figure 2) downward sloping instead of vertical. An IS shift due to fiscal policy or a favorable foreign demand shock lowers \(e\). Therefore it increases \(Y\), as it would in a closed economy or with fixed exchange rates. The strong propositions of the Fleming-Mundell model no longer hold. They do not hold even if, because of perfect substitutability, the domestic interest rate is controlled by the foreign rate.

What if the country is a net debtor on foreign account? Then depreciation, an increase in \(e\), augments the domestic currency value of the debt, lowers wealth, and induces some depletion of domestic financial assets. Hence the LM curve in \((Y,e)\) space is upward sloping. A positive IS shift which raises \(Y\) also raises \(e\), depreciating the currency. But LM might be steeper than IS in \((Y,e)\) space. Then the IS/LM comparative statics gives perverse results: increases in \(G\) or \(x\) lower both \(e\) and \(Y\).

Qualitatively similar modifications arise from endogenous expectations of changes of exchange rates.\(^1\) The rate of return on foreign assets, expressed in domestic currency, is \(\frac{r_s^*}{r} + (\hat{e}/e)_{\text{exp}}\). If the exchange expectation term is lower the higher is current \(e\), then a

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\(^1\)The importance of modeling exchange rate expectations was noted by Sohmen, The Theory of Forward Exchange, Princeton Studies in International Finance, No. 17, 1966, p. 34, n. 29, where he criticizes Mundell's analysis for assuming that "spot and forward exchange rates as well as expected future spot rates are identical, even though exchange rates may be perfectly free to find their momentary equilibrium levels at all times." The formal introduction of exchange rate expectations in the Fleming-Mundell apparatus is due to Victor Argy and Michael Porter, "The Forward Exchange Market and the Effects of Domestic and External Disturbances under Alternative Exchange Rate Systems," IMF Staff Papers, 1972, pp. 503-528.
high current e means bigger demands for money and other domestic assets. Substitution effects with respect to the foreign interest rate so adjusted work in the same direction as wealth effects on positive foreign holdings. They apply even if those holdings are initially zero, and even if \( r_V \) must be equal to the foreign interest rate corrected for exchange expectation.

Since these amendments to the Fleming-Mundell model and their implications are familiar, we will not show them formally. Instead in the next section we will incorporate them in a somewhat different model.

2. A Discrete-Time Model with Four Assets

The model we propose uses discrete rather than continuous time.
The motivation is to include some of the effects of finite stock accumulations, which as we observed above are not captured at all when asset markets are modeled as stock equilibrium at a point in time. The same technique has been applied in closed-economy analyses of government fiscal policies and of capital accumulation.

Here we split \( V \) into its constituents, capital \( K \) and government bonds \( B \), no longer assuming them to be perfect substitutes. The market valuation of an equity claim to a unit of physical capital is \( q_K \), inversely related to yield \( r_K \) from equity ownership for a period and positively related to the net rate of return \( R \) earned by the capital in use. The marginal efficiency \( R \) may depend on income \( Y \) of the period, as well as on future \( Y \)'s. Bonds are consols paying $1 a period, valued at \( q_B(r_B) \), where \( r_B \) is the yield from bond ownership for a period. Clearly in both cases, the interest rate depends on the \( q \) expected to prevail next period. In this regard all we need for present
purposes is to assume that the expected \( q \) moves if at all less than proportionately to current period \( q \). In other respects, we follow the first model and its notation.

\[
\begin{align*}
A^K(\cdot) - q_K(r_K)K_{-1} &= I(r_K, R) \\
A^B(\cdot) - q_B(r_B)B_{-1} &= \gamma_B(G + B_{-1} - T(Y)) + Z_B \\
A^F(\cdot) - e_{F_{-1}} &= X(e, Y) + x + \epsilon^{*}_{F_{-1}} \\
A^H(\cdot) - H_{-1} &= \gamma_H(G + B_{-1} - T(Y)) + Z_H \\
S_P &= G + B_{-1} - T + I + X + x + \epsilon^{*}_{F_{-1}}.
\end{align*}
\]

(Capital)

(Government bonds)

(Foreign assets)

(High-powered money)

(Private saving)

Here the \( A^S \) (\( S = K, B, F, H \)) are the domestic currency values of asset stocks desired at the end of the period. The arguments of the functions (\( \cdot \)) include variables endogenous within the period \((r_K, r_B, r_F(e), e, Y)\), predetermined variables \((H_{-1}, B_{-1}, K_{-1}, F_{-1})\), and in the background parameters such as those of the tax function \( T(Y) \), which we will not vary in the present analysis. All interest rates are one-period yields. The foreign interest rate in domestic currency \( r_F \) allows for exchange rate expectations; as argued before, these may be a function of the current exchange rate; \( r_F(e) \), with \( r_F^e \) negative. Of course, \( r_F \) also depends on the exogenous foreign interest rate \( r_F^* \). The exchange rate enters additionally as a carrier of wealth effects when \( F_{-1} \) is non-zero. Capital appreciation will be distributed among the several assets. We assume therefore that each partial derivative \( A^S_e \) has the same sign as \( F_{-1} \) but is smaller than \( F_{-1} \) in absolute value.
The parameters $\gamma_H$ and $\gamma_B$, which are non-negative and sum to one, are the shares of this period's government deficit financed by base money and bonds respectively. Open market purchases of bonds with money occur in amount $Z_B$, equal to $-Z_B$.\footnote{Here in this clean floating regime the government is assumed to stay out of the foreign asset markets; otherwise a $\gamma_F$ and $Z_F$ would be added for foreign assets. This is further elaborated in Kouri and de Macedo, "Exchange Rates and the International Adjustment Process," \textit{Brookings Papers on Economic Activity}, No. 1, 1978.} \(\ell\) is the amount of capital investment during the period valued at $q_K$, thus $I = q_K \Delta K$.

The strategy of the model is simple. Each asset equation has the period's incremental demand on the left, and the new supply on the right. The incremental demand is the difference between the stock desired at end of period and the value of the pre-existing stocks at this period's asset prices, $q_{K-1}$, $q_{B-1}$, $e_{F-1}$, $H_{-1}$. On the right, new investment adds to the capital stock, government deficits and open market operations supply money and bonds, and the current account balance changes the stock of foreign assets. The sum of the first four equations gives the IS equation (14), private saving $S_p$ on the left equal to government dissaving plus domestic investment plus foreign investment on the right. One of the five equations is redundant. In what follows we find it convenient to drop the IS relation.

Walras's law and gross substitutability among assets, used in the analysis of the first model, take somewhat different form in this one. The partial derivatives of the $A^S$ with respect to any yield or to income do not sum to zero. Their sum is the partial derivative of total desired end-of-period wealth with respect to the same variable. We assume
this sum to be non-negative for every interest rate and positive for $Y$. That is, an increase in any interest rate increases saving, or at least does not reduce it. We assume, however, that such increase occurs wholly in the asset whose rate is increased, and that to this is added any pure portfolio substitutions against other assets. Thus cross-partial of the $A^S$ with respect to interest rates are non-positive. A standard assumption will also be that all $A^S_Y$ are positive.

Differentiating totally the system (10)–(13) gives (15). Note that for symmetry the differential of the exchange rate $de$ is entered with a minus sign.

\[
\begin{bmatrix}
A^K_{r^K} - I_{r^K} - q^K_{r^K-1} & A^K_{r^B} & -A^K_{r^F e} & -A^K_{r^Y} & -I_{R^Y} \\
A^B_{r^K} & A^B_{r^B} - q^B_{r^B-1} & -A^B_{r^F e} & -A^B_{r^Y} & \gamma^B_{r^Y} \\
A^F_{r^K} & A^F_{r^B} & -A^F_{r^F e} & -A^F_{r^Y} & -X^F_{r^Y} \\
A^H_{r^K} & A^H_{r^B} & -A^H_{r^F e} & -A^H_{r^Y} & \gamma^H_{r^Y}
\end{bmatrix}
\begin{bmatrix}
dr^K \\
dr^B \\
dr^F \\
dr^H
\end{bmatrix}
\]

\[
\begin{bmatrix}
dr^K \\
dr^B \\
dr^F \\
dr^H
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
\gamma^B & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
\gamma^H & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
dG \\
dx \\
dZ^H
\end{bmatrix}
\]

Beyond the assumptions already described, two more suffice to establish the sign pattern of the Jacobian of (15). One refers to the entry in the last column first row. It is that $A^K_Y > I_{r^K_Y}$. This means that an
increase in $Y$ generates investment. It is analogous to the assumption in standard IS/LM analysis that the marginal propensity to save exceeds the marginal propensity to invest, the assumption that keeps the IS curve from sloping upward. The other is that $F_{-1}$ $\geq$ 0. (We have already alluded to the reversals that occur via the wealth effect of changes in exchange rates when the country is a debtor on external accounts, and we shall return to this question.) With these assumptions—ignoring for the moment the possibility of zeros, and recording at the bottom the signs of column sums—the sign pattern of the Jacobian is

$$
\begin{bmatrix}
+ & - & - & + \\
- & + & - & + \\
- & - & + & + \\
- & - & - & +
\end{bmatrix}

$$

The determinant $\Delta$ of such a matrix is positive. The pattern $\frac{\partial^3 Y}{\partial \xi \partial \eta \partial \zeta}$ is

$$
\begin{bmatrix}
+ & - & - & 0 \\
- & + & - & \gamma_B \\
- & - & + & 0 \\
- & - & - & \gamma_H
\end{bmatrix},

$$

also positive. That of $\frac{\partial^3 Y}{\partial \xi \partial \eta \partial \zeta}$ is
\[
\begin{bmatrix}
+ & - & - & 0 \\
- & + & - & 0 \\
- & - & + & 1 \\
- & - & - & 0 \\
+ & + & + & 1
\end{bmatrix},
\]

likewise positive. That of \( \frac{\partial^2 Y}{\partial Z^2_H} \) is

\[
\begin{bmatrix}
+ & - & - & 0 \\
- & + & - & -1 \\
- & - & + & 0 \\
- & - & - & 1 \\
+ & + & + & 0
\end{bmatrix},
\]

positive too, as can be seen by adding the fourth row to the second, changing it to \([- + - 0]\) without altering the value of the determinant.

These results contradict the strong propositions cited at the beginning. More important, those propositions do not all stand even under the following assumptions: (i) \( F_{-1} = 0 \), erasing the wealth effect of exchange rate variation, (ii) \( \gamma_H = 0 \), eliminating any monetary accumulation of fiscal policy and unbalanced budgets, (iii) \( r_{Fe} = 0 \), eliminating any change in expectation of exchange rate movement accompanying variation in the current level of \( e \). With these restrictions, the pattern of the Jacobian is
\[
\begin{pmatrix}
+ & - & 0 & + \\
- & + & 0 & + \\
- & - & + & + \\
- & - & 0 & + \\
+ & + & + & + \\
\end{pmatrix}
\]

and \( \Delta \) is still positive. \( \Delta_{\delta \xi}^{2Y} \) is

\[
\begin{pmatrix}
+ & - & 0 & 0 \\
- & + & 0 & 1 \\
- & - & + & 0 \\
- & - & 0 & 0 \\
+ & + & + & 1 \\
\end{pmatrix}
\]

also positive. The essential reason for this may be seen by going back to the first model and observing that a hypothetical increase in \( c_Y V \) in (3)—not offset by a reduction of \( H \)—would raise \( Y \). (If \([0,1]\) replaces the second column of the Jacobian in (7), the determinant remains negative.) In the second model, with discrete time, this is precisely what happens when there is an increase in \( G \) wholly financed by issuing bonds.

However, the insulation proposition, \( \frac{\partial Y}{\partial X} = 0 \), holds in the special case under consideration:

\[
\begin{pmatrix}
+ & - & 0 & 0 \\
- & + & 0 & 0 \\
- & - & + & 1 \\
- & - & 0 & 0 \\
+ & + & + & 1 \\
\end{pmatrix}
\]

\( \Delta_{\delta \xi}^{2Y} \) is

\[
\begin{pmatrix}
+ & - & 0 & 0 \\
- & + & 0 & 0 \\
- & - & + & 1 \\
- & - & 0 & 0 \\
+ & + & + & 1 \\
\end{pmatrix}
\]

= 0 .
An increase in export demand, at a given $Y$, lowers $e$. But now this appreciation alters neither the amounts the public wants to save in domestic assets at that $Y$ nor the incremental supplies of those assets. So if the exchange rate appreciates enough to keep the trade surplus unchanged, the same old $Y$ will still be a solution.

The conclusions just reached could have been obtained by dynamic analysis of a continuous-time model tracking the moving equilibrium of stock demands and supplies. The discrete time model here analyzed reaches the conclusions in a simpler way. The showing that fiscal policy works even when $e$ is not in asset demand functions arises naturally from the balance of payments equation (12), where $e$ is a price equilibrating capital and current account transactions.

If the country is a debtor to foreigners ($F_{-1} < 0$), wealth effects are reversed. This does not necessarily change the qualitative comparative static results, because portfolio substitution and trade substitution effects may still dominate. An interesting special case occurs if the signs of the $-de$ column are all reversed, as could happen with negative $F_{-1}$ and wealth effects— or a fortiori if $r_e$ is zero or positive and the trade elasticities are perverse, $X_e < 0$. The analysis then stands with the exception that the exchange rate moves in the opposite direction. For example, monetary expansion makes it appreciate, and a jump in export demand makes it depreciate. Wonders about the dynamic mechanics and the stability of this case are beyond the scope of this paper. More complex troubles arise from other combinations of the entries of the third column of the Jacobian. The results so far are collected in Table 1.
TABLE 1

<table>
<thead>
<tr>
<th>Fiscal Policy</th>
<th>Foreign Demand</th>
<th>Monetary Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effects of G on:</td>
<td>Effects of x on:</td>
<td>Effects of Z_H on:</td>
</tr>
<tr>
<td>Y (1) G_Y = 1</td>
<td>Y (1) x_Y</td>
<td>Y (1) Z_H</td>
</tr>
<tr>
<td>Y (2) G_Y = 1</td>
<td>e (2) x_e</td>
<td>e (2) Z_e</td>
</tr>
<tr>
<td>Y (3) G_Y = 1</td>
<td>e (3) x_e</td>
<td>e (3) Z_e</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard case</th>
<th>+</th>
<th>+</th>
<th>+</th>
</tr>
</thead>
<tbody>
<tr>
<td>No wealth or asset substitution effects of exchange rates</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Negative foreign assets, zero or positive substitution effects of exchange rates, perverse relation of trade balance to exchange rate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

(1) General case
(2) All bond finance of government deficits
(3) All money finance of government deficits
We can improve the model by allowing the large rest of the world to hold the small home country's assets. Unfortunately notation becomes complex. An asset stock $S_{-1}$ ($S = K, B, H$) is held partly by domestics, $A_{S_{-1}}$ and partly by foreigners $J_{S_{-1}}$. At this point we continue to represent foreign investments available to domestic savers simply by a single asset $F$ with a fixed interest rate in foreign currency $r_F^*$. The demand for a stock by foreigners will be denoted by $J^S(\cdot)$. Those demands are for stocks at market value in the foreigners' currency, let's call it marks. They must be converted into domestic currency, dollars, in the asset equations. The endogenous variables on which they depend are the home country yields $r_S = (\Delta e/e)_{\text{exp}}$, interpreting $r_H$ to be zero. In keeping with the designation of rest-of-world as "large," we ignore at this stage any wealth effects of $e$ on foreign portfolios. The current level of $e$ may, however, affect foreign demands via expected changes in $e$. For domestic savers, the endogenous variables in $A^S(\cdot)$ are as before the two local $r_S$, the foreign interest rate $r_F^* + (\Delta e/e)_{\text{exp}}$, domestic income $Y$, and $e$ as a carrier of wealth effects. The model now looks like this:

<table>
<thead>
<tr>
<th></th>
<th>Domestic Demands</th>
<th>Foreign Demands</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>(16) $A^K(\cdot) - q_K^{A_{K_{-1}}} + eJ^K(\cdot) - q_K^{J_{K_{-1}}} = I(r_K, R)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(17) $A^B(\cdot) - q_B^{A_{B_{-1}}} + eJ^B(\cdot) - q_B^{J_{B_{-1}}} = Y_B(G + B_{-1} - T) + Z_B$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(18) $A^F(\cdot) - e^{A_{F_{-1}}} - \sum_S (e J^S(\cdot) - q_S^{J_{S_{-1}}}) = X(e, Y) + x + e r_F^* - R^{J_{K_{-1}}} - J_{B_{-1}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(19) $A^H(\cdot) - q_H^{A_{H_{-1}}} + eJ^H(\cdot) - J^H_{-1} = Y_H(G + B_{-1} - T) + Z_H$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the balance of payments equation (18), $S$ takes on $(K,B,H)$
and \( q_H \) is identically 1. On the right-hand side interest and dividend payments to foreigners must now appear. The arguments of the \( A^S \) and \( J^S \) functions have been described above.

With some plausible additional assumptions, the amendment of the model leaves the conclusions unaltered. Consider, as an example that applies to all the asset equations, the differentiation of the capital equation (16) with respect to \((r_K, r_B, -e, Y)\):

\[
\begin{align*}
A^K_K & - q^*_K - \lambda^K_B \\
A^K_B & + e^K_B \\
- A^K_F & - A^K_Y \\
A^K & - I^K_Y
\end{align*}
\]

\(20\)

\[
\begin{align*}
- I^K & + e^K_F \\
- J^K & + e^K_F \sum_S \frac{J^K_S}{r^K_S}
\end{align*}
\]

This row, we assert, has the same sign pattern as in (15), namely \([+ - - +]\). Consider the four entries in turn:

To the first, for \(dr_K\), is added foreigners' positive response to an increase in equity yield.

To the second, for \(dr_B\), is added foreigners' negative response to an increase in bond yield.

The third entry, for \(-de\), now includes two effects of a decline in the exchange rate, an appreciation of the dollar, on foreigners' demand for equity in dollars. The first term \((-J^K\) says the same demand in marks amounts to less in dollars. The second term says that a decline in the exchange rate brings expectation of a subsequent rise, which makes dollar investments, whether in equity or bonds or money, less attractive. Our new assumption, which seems innocuous, is that \(\sum_S J^K_S / r^K_S\) is positive.

That is, if the yields of all three assets decrease by the same amount—
the yield differences among them remain unchanged but their differential over foreign assets declines—then foreigners' demand for equity will be smaller. Thus the third entry remains unambiguously negative.

The fourth entry is unamended, since $Y$ is not in foreigners' asset demand functions.

The same argument maintains the sign patterns for the bond and money rows, from equations (17) and (19).

The balance of payments equation (18) now gives the following partial derivatives:

$$
\begin{align*}
\frac{r_K}{A^F_K - e \sum J^S_K,} & \quad \frac{r_B}{A^F_B - \sum J^S_B,} & \quad -e & \quad Y \\
+ q^J_K & \quad -q^J_B & + A^F_e & - A^F_e + F_F (1 + r^F_F) + X_e & + A^F_Y - X_R
\end{align*}
$$

(21)

In (15) this row had the signs $[- - +]$ for $F_F > 0$, $e_e < 0$, $X_e > 0$. Recall that a negative sign means that an increase in the variable tends to worsen the home country's balance of payments. We must look at the additions to the entries.

To the first two entries, for $r_K$ and $r_B$, the additions reinforce the negative sign. We assume $\sum J^S_K$ and $\sum J^S_B$ are both positive, meaning that an increase in any one domestic yield attracts capital from abroad, not just substitutions by foreigners among domestic assets.

Naturally, foreigners' holdings $J^S_{-1}$ are all nonnegative.

The third entry, for $e$, looks the most complicated, but the new entries reinforce its positive sign. We already justified, using the capital equation as an example, the assumption that $J^S_K + J^S_B + J^S_H$
is positive.

The only addition to the fourth entry, for \( Y \), is positive, reflecting a possible income-associated increase in the earnings of foreign owners of domestic equity.

Thus the amendments to (15) leave intact its sign pattern. They also leave intact the dominant-diagonal quality of the matrix, indicated above by the positive signs for column sums. The sum of the four equations (16)-(19) is still the home country's IS equation, (14).

Consequently all the qualitative conclusions of the simpler model stand. Note, however, that even stronger conditions are now necessary for insulation. It is not enough that \( r^F_e \) and \( F_{-1} \) be zero, or even that the net foreign assets of the country be zero. It is necessary that \( F^{-1} \) and each \( J^S \) be zero. Otherwise there will be non-zero off-diagonal entries in the third (−de) column of the matrix. The reason the \( J^S \) are involved is that they are stock demands in marks. The exchange rate is involved in converting them into dollars, even though it appears nowhere inside the \( J^S \) functions directly or indirectly.

3. A Two-Country World

In this section, we will discuss briefly the problems of modeling a two-country world, with each country large enough to affect the asset markets of the other. Think of North America vis-a-vis the Common Market, dollars vis-a-vis marks. Each country will be described in the way the home country was modeled in Section 2. However, for simplicity of exposition, we will return to one of the assumptions of Section 1, that the capital and bonds of a country are perfect substitutes for each other, though not for the capital and bonds of its partner. No issue of principle
is involved in this condensation. In this world there are four distinct assets, two for each country, money and capital-cum-bonds. The corresponding four equations, plus the balance of payments equation, determine five endogenous variables: two interest rates, two incomes, one exchange rate.

To write down the model, we need one bit of additional notation beyond that of Sections 1 and 2. The variables for the second region will be distinguished by asterisks. Here then is the model:

\[
\begin{align*}
\text{American Demands} & \quad (22) \quad (\text{American capital \\ & \quad \text{& bonds)} \\
& \quad A^V(\cdot) - q^A_{V-1} + eJ^V(\cdot) - q^J_{V-1} = I(r^V, R) \\
& \quad \quad + \gamma_B(G - B_{-1} - T) \\
& \quad \quad + Z_B \\
\text{European Demands} & \quad (23) \quad (\text{European capital \\ & \quad \text{& bonds)} \\
& \quad A^V(\cdot) - e q^A_{V-1} + eJ^V(\cdot) - e q^J_{V-1} = eI^*(r^V_{*}, R^*) \\
& \quad \quad + e y_B^*(G^* - B^*_{-1} - T^*) \\
& \quad \quad + Z_B^* \\
\text{Supply} & \quad (24) \quad (\text{American money)} \\
& \quad A^H(\cdot) - A_{H-1} + eJ^H(\cdot) - J_{H-1} = \gamma_H(G - B_{-1} - T) + Z_H \\
\text{European money} & \quad (25) \quad (\text{European money)} \\
& \quad A^H(\cdot) - e A_{H-1} + eJ^H(\cdot) - eJ_{H-1} = e y_H^*(G^* - B^*_{-1} - T^*) \\
& \quad \quad + Z_H^* \\
\text{Balance of payments} & \quad (26) \quad (\text{Balance of payments)} \\
& \quad \frac{1}{S^*} \sum_{S^*} (A^S(\cdot) - q^A_{S-1}) - \frac{1}{S} \sum_{S} (eJ^S(\cdot) - e J_{S-1}) = X(e, Y, Y^*) + x \\
& \quad \quad + e y^A_{K-1} + x^A_{B-1} \\
& \quad \quad - R^K_{-1} - J_{B-1} \\
\end{align*}
\]

Note that the sum of the four equations (22) through (25) is the world IS equation, the sum of the two American asset equations and the balance
of payments equation \((22)+(24)+(26)\) is the American IS equation, the sum of the two European asset equations less the balance of payments equation \((22)+(24)-(26)\) is the European IS equation in dollars.

The endogenous arguments of the \(A\) functions are \((r_\nu, r_{v*}, Y, e)\). Those of the \(J\) functions are \((r_\nu, r_{v*}, Y^*, e)\). American income does not affect European residents' asset demands, or vice versa. The trade balance function \(X\) now includes both \(Y\) and \(Y^*\), with partial derivatives of opposite signs. The assumptions and reasoning of the last part of Section 2, now applied to both countries, gives a set of simultaneous equations in differentials as follows:

\[
\begin{bmatrix}
+ & - & + & + & - \\
- & + & + & + & \cdot \\
- & - & + & + & + \\
- & + & + & - & + \\
+ & + & + & + & ?
\end{bmatrix}
\begin{bmatrix}
\text{dr}_\nu \\
\text{dr}_{v*} \\
\text{dY} \\
\text{dY}^* \\
\text{-de}
\end{bmatrix}
= 
\begin{bmatrix}
\gamma_B & 0 & -1 & 0 & 0 \\
0 & \gamma_B^* & 0 & -1 & 0 \\
\gamma_H & 0 & 1 & 0 & 0 \\
0 & \gamma_H^* & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\text{dG} \\
\text{dG}^* \\
\text{dZ}_H \\
\text{dZ}_H^* \\
\text{dx}
\end{bmatrix}
\]

The sign pattern for \(-\text{de}\) in the final column requires explanation. In connection with equation \((20)\), we concluded that the effect of an increase in the mark price of dollars on European demand for an American asset would be negative. To hold the same value in marks, European investors would sell some of their holdings. Against this is the wealth effect, which we did not previously allow. Having gained wealth, European investors might wish to increase the mark value of any one of their dollar holdings. But the wealth elasticity would have to exceed one—indeed equal the reciprocal of the asset's share in the portfolio—to induce
them not to sell any of the dollar asset. For example, if the holding was 1/4 of the portfolio, then a 1% increase in the exchange rate would increase wealth by 1/4 of 1%, and it would take a wealth elasticity of 4 to make the exchange rate elasticity unity. Formally in equation (20), third entry, we add a term $-\varepsilon e^K > 0$. But it can be expected to be smaller in absolute value than $-J^X$.

For the same reasons that depreciation of the mark decreases dollar demand for American assets, it increases dollar demand for European assets. That is why the signs in the -de column alternate. The last entry says that depreciation of the mark deteriorates the American balance of payments, which does not require any new assumptions.

The two income columns are shown in (27) as non-negative, as before. But the ++ for the diagonals denote a new condition, namely that the maximum non-diagonal entry in the column is smaller than the average of the first four entries. This ensures that the first four rows and columns are a dominant diagonal matrix. The economic meaning is the same that motivated our assumption in the continuous time model of Section 1, that transactions demands for cash are met by substitutions against all other assets. In that model, money was the only asset with a positive income elasticity. Here, however, wealth is not predetermined and saving offsets the portfolio substitutions. Transactions demand gives money that dominant positive income effect.

The significance of the dominant diagonal structure of the 4 x 4 matrix can be seen if we abstract from all effects of exchange rate on asset demands, via wealth or expected exchange rate appreciation or depreciation, making all the entries except the bottom one zero in the last column. We then find the following: (i) A trade shock $dx$ affects
only the exchange rate; the insulation property of the floating exchange rate holds as before. (ii) Expansionary monetary action—open market purchases or increases in government spending financed wholly or predominantly by monetary issue—in one country raises income in that country and lowers it in the other. (iii) Expansionary fiscal action—increases in government spending financed wholly or predominantly by bond issue—raises income in both countries. These conclusions arise from manipulations of determinants familiar by now to a studious reader, and they are omitted here to spare space and tedium.

The second and third results may seem surprising, especially the second, but the explanations are straightforward. Expansionary monetary policy is essentially a method of exchange depreciation. The resulting trade surplus—always assuming well-behaved elasticities—raises the country's income and lowers its partner's income.\(^1\) Bond-financed fiscal stimuli raises interest rates and induces capital inflows that appreciate the exchange rate. The resulting trade deficit moderates, but does not cancel, the fiscal stimulus in the home country and raises income in the other country.

\(^1\)The practical value of domestic monetary policy in a floating rate system is diminished by the fact that it is a "begger-my-neighbor" policy. In 1972, during the transition from fixed to floating rates, one of us wrote, "Since monetary policy is the more responsive instrument of domestic stabilization, perhaps we should welcome an exchange rate regime that increases its potency relative to that of fiscal policy. However, when the export-import balance becomes the strategic component of aggregate demand, one country's expansionary stimulus is another country's deflationary shock. We can hardly imagine that the Common Market will passively allow the U.S. to manipulate the dollar exchange rate in the interests of U.S. domestic stabilization. Nor can we imagine the reverse. International coordination of interest-rate policies will be essential in a regime of floating exchange rate, no less than in a fixed-parity regime." Tobin, The New Economics One Decade Older, Princeton University Press, 1974, pp. 91-92.
With respect to (ii), remember that if the period of our discrete-time model were very short, the portfolio substitution effects of \( Y \) and \( Y^* \) would dominate the accumulation effects. The two income columns would then have non-positive entries everywhere but the diagonals. Then expansionary monetary policy in either country would raise incomes in both countries. In a short period, the contagion of lower interest rates is stimulative in the second country as well as the first. It is the build-up of desired saving, strong enough to make the income columns non-negative, that reverses the effect on the second country over longer time periods. The reason is that the second country's trade deficit reduces the net wealth available to its savers, and a fall in income is the only way to reconcile them to that fact. This effect is ignored or deferred in a continuous-time snapshot in which exchange rate determination is detached from the asset markets. Imagine the second country to be the small open economy of the model of Section 1, faced with a decline in the outside interest rate \( r_F^* \) combined with a negative shock \( x \) to its trade balance. The decline in \( r_F^* \) will be stimulative, but the model says that in the assumed circumstances the economy is insulated from the trade shock by movement of the exchange rate. Only later would accumulation of foreign debt or decumulation of foreign assets have repercussions on domestic saving and portfolio choices that cause income to decline.

The general case (27), with wealth and expectation effects of exchange rate movements, is very messy. We will not attempt a taxonomy here.

We can, nevertheless, point out the relevance of the simplification used so far by partitioning the Jacobian in (27) as
\[ J = \begin{bmatrix}
J_1 & B \\
(4 \times 4) & (4 \times 1) \\
\Theta & \theta_e \\
(1 \times 4) & (1 \times 1)
\end{bmatrix} \]

Since \(J_1\) is a dominant diagonal matrix, \(\det(J_1) > 0\). Also \(\theta_e > 0\).

Furthermore,

\[ \det(J) = \theta_e \det(J_1 - B\Theta / \theta_e). \]

Denoting the total effects of the exchange rate on first four rows by \(V, V^*, H\) and \(H^*\) and the first four columns of the last row by \(\Theta\) the structure and sign pattern of the \(-B\Theta\) matrix is:

\[
\begin{bmatrix}
V_e \\
V^*_e \\
H_e \\
H^*_e
\end{bmatrix}
\begin{bmatrix}
\theta \\
\theta \\
\theta_Y \\
\theta_{Y*}
\end{bmatrix} =
\begin{bmatrix}
- & + & + & - \\
+ & - & - & + \\
- & + & + & - \\
+ & - & - & +
\end{bmatrix}
\]

The sign of the determinant of the difference \(J_1 - B\Theta / \theta_e\) is thus ambiguous.

The stronger the effect of the exchange rate on the balance of payments \(\theta_e\), the smaller are the elements of the matrix \(B\Theta / \theta_e\) and therefore the less likely it is that the sign of the determinant of the Jacobian \(J\) will be different from the sign of the determinant of the \(J_1\) matrix.

For large \(\theta_e\), in this sense, the effect of an increase in the demand for American exports will appreciate the dollar as before, even though \(B\) is not a zero vector.
4. **Concluding Remarks**

The analysis in the preceding sections has referred to a Keynesian economy, in which real output is flexible but price is fixed for the point or period of time to which the models refer. This follows the Fleming-Mundell analysis with which we began, and it is of intrinsic interest. But the applicability of the method is by no means restricted to this Keynesian case. Indeed, with few qualifications, the analysis applies to the polar opposite classical case, with price level endogenous and output exogenously supply-determined. Anyone is free to re-read the article, substituting $P$ for $Y$ throughout.

We have tried to provide a framework for analysis of the short run effects of macroeconomic policies and other events on economies linked by trade, capital markets, and floating exchange rates. Some conclusions of previous analysis, it turns out, do not survive in our models. The points we emphasize are: (i) Assets, ranging from capital to base money, are imperfect substitutes both within and across countries. (ii) Changes in stocks, including particularly those resulting from imbalances in external current accounts, have important effects not captured in conventional point-in-time specification of asset stock equilibrium. (iii) Floating rates do not insulate economies from shocks in their external trade accounts when exchange rate movements induce portfolio shifts either by influencing expectations of appreciation or depreciation or by altering the wealth of portfolio owners. (iv) Although we are able to obtain some definite qualitative results in standard cases, they depend on a series of conditions that might not be met. It is easy to imagine plausible configurations in which the comparative statics would yield results counter to conventional intuition and wisdom, e.g. when a country
is in debt to the rest of the world, when the trade balance elasticities are perversely low, when exchange rate expectations are extrapolative instead of regressive, when the income effects on asset demands are irregular. (v) Our framework can be applied not only to a small economy in a large world but to two, and by extension more, large economies or currency areas connected by commodity and financial markets.