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THE LONG RUN IMPLICATIONS OF AN IS-LM SIMULATION MODEL

Gary Smith
THE LONG RUN IMPLICATIONS OF AN IS-LM SIMULATION MODEL*

by

Gary Smith

The familiar IS-LM analysis is concerned only with the impact
effects of shocks to an economy. However even when population growth,
technological progress and other exogenous changes are ignored, ripples
from the initial shock will in fact persist for some time.

In a continuous framework, there is by direct assumption insuffi-
cient time during each temporary equilibrium for the flow accumulation
of capital and financial assets to alter existing stocks. As time passes
however stock demands will become satisfied as actual stocks do change,
and a sequence of new temporary equilibria will evolve.

In contrast, a discrete time period framework does permit changes
in all stocks within a single period's equilibrium. But if stocks do
change, then the subsequent period will begin with a new stock endowment
and in general not replicate the preceding period. If the system is
stable one might expect that the longer the period and the more fully
stock demands are satisfied within one period, the smaller the accumula-
tion demands will be in the succeeding period. As the demand for capi-
tal is satisfied, for example, the demand for new commodities will fall,
probably reducing employment and/or commodity prices.

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National Science Foundation.
It is sometimes argued that the impact effects of policies may give a distorted and misleading picture of the longer run consequences. For example, from an initially balanced budget an increase in government spending will create a budget deficit leading to a growth in asset supplies as long as the deficit persists. This cumulative growth in assets will affect the economy in ways which may reinforce or reverse the initial impact of the increased federal spending. If the deficit is bond financed there will be a direct continuing upward pressure on the bond rate, which will crowd out more and more investment. If the capital stock falls, the tax base may deteriorate and the deficit grow larger. This gloomy spiral of higher prices, interest rates, and deficits with less capital and output might end only with the collapse of the economy. On the other hand, the changes in real income and wealth may increase the demand for capital and/or may cause the flow demand for bonds to outpace the flow supply.


These implications are unfortunately varied and too often ambiguous. In addition, we are often as interested in the size of a multiplier as its sign and whether it is significantly different under different policy regimes. It is then at least aesthetically dismaying to possess relatively little information about the long run implications of the IS-LM model. In part this is due to the fact that the profession has only
TABLE 1. Long Run Implications of the Tobin-Buitner Model

<table>
<thead>
<tr>
<th>Bond Financing ($\gamma = 0$, $\pi = 0$)</th>
<th>Money Financing ($\gamma = 1$, $\pi = 0$)</th>
<th>Mixed Financing ($\gamma = M/B$, $\pi &gt; 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta G'$  $\Delta M$</td>
<td>$\Delta G'$  $-\Delta B$</td>
<td>$\Delta G'$  $\Delta \gamma$</td>
</tr>
<tr>
<td>$\Delta r$     +   -</td>
<td>-          -</td>
<td>0          -</td>
</tr>
<tr>
<td>$\Delta K$     ?   +</td>
<td>+          +</td>
<td>+          +</td>
</tr>
<tr>
<td>$\Delta Y$     +   0</td>
<td>+          0</td>
<td>+          +</td>
</tr>
<tr>
<td>$\Delta m$     0   1</td>
<td>+          +</td>
<td>+          -</td>
</tr>
<tr>
<td>$\Delta b$     +   -</td>
<td>0          -</td>
<td>+          -</td>
</tr>
<tr>
<td>Stability      stable</td>
<td>stable</td>
<td>stable</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Full Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond Financing ($\gamma = 0$, $P$ endog.)</td>
</tr>
<tr>
<td>$\Delta G'$  $\Delta M$</td>
</tr>
<tr>
<td>$\Delta r$     -   0</td>
</tr>
<tr>
<td>$\Delta K$     +   0</td>
</tr>
<tr>
<td>$\Delta Y$     +   0</td>
</tr>
<tr>
<td>$\Delta m$     +   0</td>
</tr>
<tr>
<td>$\Delta b$     ?   0</td>
</tr>
<tr>
<td>$\Delta P$     -   $P/M$</td>
</tr>
<tr>
<td>$\Delta M$     0   1</td>
</tr>
<tr>
<td>$\Delta B$     -   $B/M$</td>
</tr>
<tr>
<td>Stability      $\beta \to 0$: unstable</td>
</tr>
<tr>
<td>$\beta \to \infty$: ambiguous</td>
</tr>
</tbody>
</table>
recently begun to think seriously about such questions. A more important reason is the failure of nature to perform these artificial experiments which are so easily concocted on paper. The economy is never so serene or uninteresting as to permit a glimpse at these hypothetical steady states. Similarly any unstable spirals that might have begun have always been aborted by institutional changes, including the willingness of the government to pursue different policies.

In the present paper, I have conducted the experiments which nature resists by constructing a simple IS-LM simulation model. I have assigned values to the various parameters in the Tobin-Buiter model and then calculated the precise implications of this specific model. To keep the results simple and comparable I have only analyzed G' policies in which the government fixes the level of government purchases plus net debt interest.

My intention was to obtain a rough idea of the size of the multipliers and the likely signs for some of the theoretically ambiguous results. It turns out that none of my results are implausible. Not surprisingly, I am more confident that government expenditures will be expansionary in situations of unemployment than full employment. Money financed expenditures seem more likely to be both expansionary and stable than bond financed expenditures. More generally, the composition of the financing of government budget imbalances is important in the long run. A mixture of monetary and fiscal policies is surely preferable to the rigid dogma that is convenient in mathematical exercises.

The simplicity of my analysis should not trick us into forgetting that in practice the government can design quite specific subsidies, tax incentives or direct purchases to pursue particular objectives, such as the maintenance or enlargement of the nation's productive capacity. I
should also warn that when these models are unstable, private as well as governmental shocks to the system will trigger explosive departures from a steady state equilibrium. My focus on the effects of policy changes will hopefully not give the misleading impression that all would be well if only the government would leave things alone. The maintenance of a steady state equilibrium will instead often require the government to abandon passive policies and promptly offset private shocks to the system.

The Model

The model's variables are defined below:

\[ Y = \text{real net national product} \]

\[ C = \text{real private consumption} \]

\[ G' = \text{real government purchase plus net debt interest,} \]

\[ G + (1-t)(r + \pi^e)b \]

\[ K = \text{capital stock} \]

\[ P = \text{price level} \]

\[ \pi = \text{rate of inflation,} \quad \dot{P}/P \]

\[ \pi^e = \text{anticipated rate of inflation} \]

\[ B = \text{nominal government interest bearing debt} \]

\[ M = \text{nominal government monetary debt} \]

\[ b = \text{real government interest bearing debt,} \quad B/P \]

\[ m = \text{real government monetary debt,} \quad M/P \]

\[ r = \text{anticipated real rate of return on debt} \]

\[ \dot{x} = \text{time derivative of} \quad x \]

The dynamic IS-LM simulation equations are
(1) \[ \dot{K} + G' - .25Y = .05(3.6Y - W) + \pi^e(m + b) \] (I = S)

(2) \[ \left(-.005 + \frac{.000695}{r + \pi^e} + .064\frac{Y^e}{W}\right)W = m \] (L = M)

(3) \[ W = m + b + .30Y/(r + .05) \] (private wealth)

(4) \[ \dot{K} = .05\left(\frac{.30Y}{r + .05} - K\right) \] (investment)

(5) \[ \dot{m} = \gamma(G' - .25Y) - \pi m \] (growth of \( m \))

(6) \[ \dot{b} = (1-\gamma)(G' - .25Y) - \pi b \] (growth of \( b \))

In the version with full employment and endogenous prices these equations are supplemented by

(7) \[ Y = 1000K^{.30}/3000^{.30} \] (production function)

(8) \[ \pi^e = \beta(\pi - \pi^e) \] (price expectations)

Among the specific assumptions embedded in this model are that capital's share of factor income is .30; the tax rate is .25; the equilibrium ratio of wealth to total disposable factor income is 5.33; and there is a risk premium of .05 between the anticipated real return \( r \) on government bonds and the anticipated return on capital. One minor departure from Tobin-Buit is that I allow the valuation of capital to differ from its replacement cost. This does not affect any of their qualitative results as long as I maintain their assumption of static expectations regarding the valuation of capital.

The consumption function from (1) is implicitly
\[ C = 0.76(1 - 0.25)Y + 0.25(r+m)b + 0.05W - \pi^e(m+b). \]

In a steady state with \( W = 3.6Y \), this becomes

\[ C = (1 - 0.25)Y + (1 - 0.25)(r+m)b - \pi^e(m+b). \]

At the long run equilibrium, the demand for money from (2) has a wealth elasticity of 0.36, an income elasticity of 0.64, and an interest elasticity of -0.50.

With \( M = 100 \) and \( G' = 250 \) the full employment stationary state is at \( P = 1 \), \( Y = 1000 \), \( K = 3000 \), \( r = 0.05 \), and \( B = 500 \). For comparability \( P \) is set equal to one in the unemployment version of the model. For comparability during inflationary steady states, the demand for money function is then changed to

\[ L = \left( -0.0039 + \frac{0.00695}{\pi + \pi^e - 0.03} + 0.064Y \right)W. \]

With \( G' = 268 \) and \( \gamma = 1/6 \), the full employment long run steady state is at \( \pi^e = 0.03 \), \( M/P = 100 \), \( B/P = 500 \), and the same values for the remaining endogenous variables as in the stationary state model. In the steady state unemployment model, these same values are obtained by omitting the production function and setting \( \pi^e \) at 0.03. With these adjustments all of the elasticities, short run multipliers, and steady state solution values are in agreement.

The price expectations parameter \( \beta \) was not assigned a specific value since there is little information about it and yet it is critically important to the stability of the full employment model. I will instead report the values of this parameter for which the model is stable or unstable.
The Unemployment Version

Equations (1), (2), and (3) with prices exogenous give the familiar short run IS-LM unemployment model. The impact effects from the initial steady state of an increase in \( G' \) and an expansionary open market operation are given in Table 2 and are depicted in Figure 1.

### TABLE 2. Impact Multipliers with Unemployment

<table>
<thead>
<tr>
<th>( \Delta G' )</th>
<th>( \Delta M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta x )</td>
<td>.00021</td>
</tr>
<tr>
<td>( \Delta Y )</td>
<td>2.88</td>
</tr>
</tbody>
</table>

![Diagram](image)

FIGURE 1. Impact Multipliers with Unemployment

The long run effects of these monetary and fiscal policies depend critically upon how subsequent budget imbalances are financed. The three financing policies considered here are bonds only \((\gamma = 0)\), money only \((\gamma = 1)\), and mixed \((\gamma = M/B = 1/6)\). Since one of the asset stocks is held constant under the first two policies, the steady states are stationary states in which the price level and the nominal stock of the other asset are also constant. For the reasons given earlier the price level
is set at 1.0. With the mixed financing policy, the price level need not be constant but must grow at the same rate as the nominal asset stocks. In this regime, the rate of inflation was exogenously set at .03.

Under the all or nothing financing regimes \((\gamma = 0 \text{ or } 1)\), the substitution of the stationary conditions \(\pi = \pi^e = \dot{m} = \dot{b} = \dot{k}\) into the dynamic IS-LM model, \((1)-(6)\) yields the long run relations

\[
\begin{align*}
(9) \quad G' &= .25Y \quad \text{(balanced budget)} \\
(10) \quad \left(0.05 + \frac{0.0025}{r}\right)Y &= M \quad \text{(long run LM)} \\
(11) \quad .3Y &= (r + .05)K \quad \text{(capital marginal productivity)} \\
(12) \quad 3.6Y &= M + B + K. \quad \text{(desired wealth)}
\end{align*}
\]

With pure bond financing, \(m\) is exogenous and these relations \((9)-(12)\) recursively determine \(Y, r, K\) and \(B\). With pure money financing the long run LM curve becomes

\[
(10') \quad \left(0.05 + \frac{0.0025}{r}\right)Y = 3.6Y - B - .3Y/(r + .05).
\]

Equations \((9), (10'), (11)\) and \((12)\) then recursively determine \(Y, r, K,\) and \(M\).

Comparing \((10')\) with \((10)\), the long run relation between \(Y\) and \(r\) with money financing is negative rather than positive. The long run wealth-income ratio is constant and at any interest rate the demands for money and for bonds plus capital rise proportionately with income. On the supply side, the marginal productivity condition implies that at any interest rate the capital stock increases proportionately with income.
If \( m \) is held constant then \( m/Y \) falls as \( Y \) increases and \( r \) must rise in order to persuade people to hold a smaller proportion of their wealth as money. When \( b \) is instead held constant, \( r \) must fall as \( Y \) rises to persuade people to hold a smaller proportion of their wealth as bonds plus capital.

Using the IS (1), LM (2), CT (9), and LLM (10 or 10') curves, the short and long run effects of an increase in \( G' \) are depicted in Figures 2a and 2b. The long run multipliers for the specific simulation model are collected in Table 3.

\[ \text{FIGURE 2. An Increase in } G' \text{ with Unemployment} \]
TABLE 3. Long Run Multipliers with Unemployment

<table>
<thead>
<tr>
<th></th>
<th>Bond Financing</th>
<th>Money Financing (γ = 1, π = 0)</th>
<th>Mixed Financing (γ = 1/6, π = 0.03)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ΔG' ΔM</td>
<td>ΔG' -ΔB</td>
<td>ΔG' Δγ</td>
</tr>
<tr>
<td>Δr</td>
<td>0.00040 -.00100</td>
<td>-.00008 -.00003</td>
<td>0 -.04</td>
</tr>
<tr>
<td>ΔK</td>
<td>.01 30</td>
<td>14.32 .97</td>
<td>11.19 1602</td>
</tr>
<tr>
<td>Δy</td>
<td>4 0</td>
<td>4 0</td>
<td>3.73 134.3</td>
</tr>
<tr>
<td>Δm</td>
<td>0 1</td>
<td>.08 .03</td>
<td>.373 -186.5</td>
</tr>
<tr>
<td>Δb</td>
<td>14.39 -30</td>
<td>0 -1</td>
<td>1.865 -932.5</td>
</tr>
</tbody>
</table>

Stability stable stable stable

The impact increase in Y raises the desired capital stock while the increase in r lowers it. On balance,

\[ \frac{d(.30Y/(r+.05))}{dG'} = 3(2.88) - 30,000(.00021) = 2.44, \]

so that the desired capital stock increases by 2.44 units. Investment is temporarily stimulated, and there will be leftward pressure on the IS curve as the demand for capital is satisfied and investment returns to zero. At the same time the government deficit created by the increase in G' will increase asset supplies as long as the deficit persists; i.e., as long as income is to the left of the new GT curve. This growth in asset supplies will exert a continuing rightward pressure on the IS curve. The LM curve will shift leftward if the deficit is bond financed and rightward with money financing. In either case the increase in national income is larger in the long run than in the short run. With bond financing the interest rate rises sufficiently to keep the capital stock essentially unchanged. With money financing, the interest rate falls in
the long run and the capital stock increases significantly. Since the increase in desired wealth is the same in both cases, this demand is satisfied under bond financing by a relatively large expansion of government debt.

Figures 3a and 3b depict the effects of an expansionary open market operation. The multipliers in Table 3 strikingly illustrate the moral

![Diagram](image)

a. Bond financing \((γ = 0)\)  
b. Money financing \((γ = 1)\)  
c. Mixed Financing \((γ = M/B)\)

FIGURE 3. Expansionary Monetary Policy with Unemployment

that the ultimate effects of policies do depend upon how subsequent budget imbalances are financed. The impact effects are an increase in \(Y\) and a decline in \(r\). The desired capital stock jumps upward by 28.65 units. As this demand for capital is satisfied the IS curve moves leftward. There is also a budget surplus as long as income is to the right of the \(GT\) curve, and the decline in financial assets puts further leftward pressure on the IS curve. With bond financing, the supply of money
is maintained as the demand for money declines, shifting the LM curve rightward. The long run equilibrium is with a significantly lower interest rate and larger capital stock. Since income and hence wealth are unchanged, the capital will have displaced an equal amount of government debt. With money financing, on the other hand, the budget surplus reduces the money supply, shifting the LM curve leftward. The interest rate rises almost back to its original level and the change in the capital stock is slight. Since the initial unit decline in the bond supply is maintained and the money supply increases, the ultimate increase in capital stock must be less than one unit.

An inflationary steady state with mixed financing is described by the equations derived from the substitution of \( \hat{K} = \hat{m} = \hat{b} = \pi = \pi^e \) into the dynamic IS-LM model (1)-(6):

\[
(13) \quad \left( \frac{.05 + .0025}{r} \right) \frac{m}{Y} = \gamma \frac{m+b}{Y} = \gamma \frac{W-K}{Y} = \gamma \left( 3.6 - \frac{.3}{r + .05} \right)
\]

\[
(14) \quad G' - .25Y = .03(m+b) = .03 \left( 3.6Y - \frac{3Y}{r + .05} \right)
\]

\[
(15) \quad .3Y = rK
\]

\[
(16) \quad 3.6Y = m + b + K
\]

Equations (13), (14), (15) and (16) recursively determine \( r, Y, K, \) and \( m+b \).

The LLM curve (13) is now horizontal. With both the demand for and supply of money proportional to income, the interest rate is determined solely by asset market equilibrium.

The short and long run effects of an increase in \( G' \) are depicted
in Figure 2c. Since the wealth elasticity of the demand for money is less than one, the mixed financing policy shifts the LM curve rightward as with a purely money financed deficit. The long run increase in $Y$ is reduced somewhat since the government's capital gains on the increased stock of debt reduce the need for additional tax revenue. The increase in the stock of debt is modest because the constant interest rate permits a significant expansion of the capital stock to satisfy most of the increased demand for private wealth.

The effects of an increase in the financing parameter $\gamma$ are depicted in Figure 3c. $M$ grows faster (and $B$ more slowly than $P$), continuously shifting the LM curve to the right. As income rises the deficit shrinks, $M+B$ grows more slowly than $P$, and the IS curve shifts leftward. In the new long run equilibrium the increased in tax revenue has been fully offset by the reduced capital gains on the smaller stock of real debt. Equation (13) shows that money velocity must fall, necessitating a fall in the interest rate. With income high the capital stock must expand for the marginal product of capital to fall with $r$. Table 3 indicates how strongly a change in $\gamma$ affects the economy in the long run. When $\gamma$ increases by .05 to .217, income increases by 6.7 units; the capital stock increases by 80 units; the interest rate falls by two tenths of a percentage point; and real government debt falls by 56 units.

A one shot open market operation (not depicted) has no lasting effect on the economy since the steady state composition of government debt is fixed by the government's deficit financing parameter $\gamma$. The impact effects of an expansionary open market operation are to raise $Y$ and lower $r$. Investment demand is temporarily increased along the IS curve, but as the capital stock grows the IS curve will move leftward,
The larger tax revenue reduces the deficit, leading \( M \) and \( B \) to grow more slowly than \( P \). Both the IS and LM curves consequently shift leftward until the original deficit is reestablished with both \( M \) and \( B \) growing at the same rate as \( P \).

All of these unemployment scenarios are stable. Tobin-Buiter and Smith show that this stability is assured by the parametric sign assumptions and therefore does not depend upon the particular values that I have selected for this simulation model.

The Full Employment Version

The substitution of the production function (7) into the IS (1) and LM (2) equations gives the full employment determination of \( P \) and \( r \).

![Figure 4. Impact Multipliers with Full Employment](image_url)

The simulation values of these multipliers from the initial steady state are given in Table 4.
TABLE 4. Impact Multipliers with Full Employment

<table>
<thead>
<tr>
<th>ΔC'</th>
<th>ΔM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δr</td>
<td>.00032</td>
</tr>
<tr>
<td>ΔP</td>
<td>.0043</td>
</tr>
</tbody>
</table>

Substituting the production function (7) into the capital marginal productivity condition (11) that is derived from (4) and differentiating

\[
\frac{(r + .05)}{K} \frac{dK}{dr} = \frac{1}{.30 - 1} = -1.43
\]

shows that the optimal and, in the long run, actual capital stock can only be influenced by changes in the real interest rate. And since output depends only upon the capital stock, the long run expansiveness of government policies depends only upon their success in reducing the real interest rate. I am specifically ignoring here the government's ability to structure taxes so as to encourage capital formation and the fact that many government expenditures (such as those for highways, education, and research) directly increase the nation's productive capacity.

The substitution of the production function (7) into equations (9)-(12) gives

(17) \[ G' = 250(K/3000)^{.30} \]
(18) \[ 300(K/3000)^{.30} = (r + .05)K \]
(19) \[ \left(50 + \frac{.5}{r}\right)(K/3000)^{.30} = M/P \]
(20) \[ 3600(K/3000)^{.30} = K + (M+B)/P \].
which recursively determine the stationary state values of $K$, $r$, $P$, and $B$ with bond financed deficits. The full employment multipliers are collected in Table 5. For a graphical depiction in $r-P$ space, the mar-

<table>
<thead>
<tr>
<th>TABLE 5. Long Run Multipliers with Full Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond Financing ($\gamma = 0$, $P$ endog.)</td>
</tr>
<tr>
<td>$\Delta G'$</td>
</tr>
<tr>
<td>$\Delta r$</td>
</tr>
<tr>
<td>$\Delta K$</td>
</tr>
<tr>
<td>$\Delta Y$</td>
</tr>
<tr>
<td>$\Delta m$</td>
</tr>
<tr>
<td>$\Delta b$</td>
</tr>
<tr>
<td>$\Delta \pi$</td>
</tr>
</tbody>
</table>

Stability unstable stable iff $\beta < .145$ stable iff $\beta < .134$

ginal productivity condition (18) can be combined with (17) and (19) to give the full employment $GT$ and $LIM$ curves,

\[
(21) \quad GT : \Delta r = \frac{-7}{(0.25)(3000)} \Delta G' = -0.00093 \Delta G'
\]

\[
(22) \quad LIM : -1429 \Delta r = \Delta M - 100 \Delta P
\]

With money financing (19) is replaced by
(19') \[ 50 + \frac{2.5}{r} (K/3000)^{30} = 3600(K/3000)^{30} - K - B/P \]

and the LLM curve becomes

(22') \[ -28,857 \Delta r = -\Delta B + 500 \Delta P \]

The LLM curve describes a positive relationship between \( r \) and \( P \) with bond financing and a negative association with money financing.

Using the IS, LM, GT (21), and LLM (22 or 22') curves, the effects of an increase in \( G' \) are depicted in Figure 5.

---

**FIGURE 5.** An Increase in \( G' \) with Full Employment
The impact effects of an increase in $G'$ are to raise $r$ and $p$. Since this temporarily discourages investment the capital stock, output, and tax revenue begin falling. The shrinking capital stock and growing federal deficit shift the IS curve rightward. The deficit pushes the LM to the left with bond financing and to the right with money financing. With bond financing, the cycle of higher interest rates, less capital, less tax revenue, and a larger supply of bonds continues indefinitely. With money financing, the deficit induced growth in the money supply checks the rise in interest rates and stabilizes the economy if price expectations are sufficiently inelastic ($\beta < .145$). If price expectations are too elastic the system will be destabilized by private attempts to increase saving in order to preserve purchasing power and to economize on money holdings.

With purely money or bond financing deficits, the level of $G'$ fixes stationary state tax revenue and hence output, the capital stock, and the real interest rate. An increase in $G'$ requires a higher income level, a higher capital stock and a lower real interest rate. These long run values and hence* the changes in the level and composition of wealth do not depend upon whether budget deficits are wholly money or bond financed. However the implications for the price level and the nominal stocks of money and bonds are quite different and help explain the differences in the stability of these long run equilibria. When the nominal money supply is fixed, the required 1.3% increase in the real money supply requires a 1.3% fall in the price level. With the price level reduced

* W if fixed by Y. Real government debt is fixed by W and K. W and r fix the real demand for money and hence the composition of real government debt.
by this amount, the required fall in the real bond supply of 5.4% requires a 6.7% fall in the nominal bond supply. When the nominal bond supply is instead fixed, the 5.4% fall in the real supply requires a 5.4% increase in prices and a 1.3% increase in the real money supply then requires a 6.5% increase in the nominal supply. Thus a bond financed increase in \( G' \) has the unlikely long run effects of lowering the price level and the nominal supply of bonds while a money financed expenditure has the more plausible effects of increasing the price level and nominal supply of money.

The effects of an expansionary open market operation are depicted in Figure 6. Since \( G' \) fixes the real variables in the system, an open

\[ a. \text{ Bond Financing} \quad (\gamma = 0) \]

\[ b. \text{ Money Financing} \quad (\gamma = 1) \]

FIGURE 6. Expansionary Monetary Policy with Full Employment

market operation has no real effects on the stationary state. With bond financing, the initial increase in \( M \) is maintained and the new long
run equilibrium has a higher price level and a larger nominal stock of bonds. With money financing, the initial decline in $B$ is maintained and the new equilibrium has a lower price level and a lower nominal money supply. The first scenario is unstable while the second is stable if expectations are not too elastic.

The impact effects are to reduce the interest rate and raise the price level. As investment proceeds, a larger tax base creates a government budget surplus. The growing stocks of capital and financial assets shift the IS curve leftward. With bond financing the LM curve also shifts leftward and an unending expansionary cycle commences. With money financing, the LM shifts leftward and the price level and money supply fall until the new long run equilibrium is reached. However, with sufficiently elastic price expectations ($\beta > .145$) attempts to consume capital gains and to hold more money will destabilize the system.

An inflationary steady state with mixed financing is not depicted since the price level is time dependent. The calculated multipliers are in Table 5. Although one shot asset exchanges remain neutral, an increased monetization of deficits turns out to be strongly expansionary. An increase in either $G'$ or $\gamma$ lowers the real interest rate, increases income and the capital stock, and reduces the real level of government debt. Thus for a while prices must increase faster than the nominal supplies of debt. An increase in $G'$ raises the long run rate of inflation while, interestingly, an increase in $\gamma$ reduces it. It is also noteworthy that $G'$ is somewhat less expansionary than in the unemployment model, but $\gamma$ is more than twice as powerful. It turns out that a mixed financing regime is stable as long as $\beta$ is less than $.134$. 
REFERENCES


Smith, Gary, "The Long Run Consequences of Monetary and Fiscal Policies When the Government's Budget is Not Balanced" (1977).