DEFICIT SPENDING AND CROWDING OUT IN SHORTER AND LONGER RUNS

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September 14, 1977
Deficit Spending and Crowding Out in Shorter and Longer Runs*

Does expansionary fiscal policy fail because of the restrictive effects of the accumulation of non-monetary debt? Are the direct effects of government spending on aggregate demand cancelled or reversed with the passage of time as the public debt grows? Does explicit recognition of the government budget identity overturn standard Keynesian doctrine regarding fiscal policy? These questions have been much discussed in recent years. 1/

In this paper, the main point I wish to make is the following: Suppose it is agreed that the public's demand for money, given their total wealth, is negatively related to interest rates. Then it will also be agreed that the very short run impact effect of increasing the rate of government expenditure is expansionary. Although the money supply is unchanged, its velocity will rise along with interest rates. This impact effect will be cancelled or reversed only if the passage of time turns the negative response of money demand to interest rates into a zero or positive response. This in turn will happen only if wealth effects on demand for money come to dominate the substitution effect. Specifically the condition is (1) that

* The research reported in this paper was in part assisted by a grant from the National Science Foundation. I am grateful to William Brainard, Willem Buiter, and Gary Smith for helpful discussions of the problems and issues under study, but I absolve them of responsibility for what I say.

the public's demand for wealth, and their saving, are positively related to interest rates, and (2) that part of the accumulation induced by higher interest rates on assets other than money is held in money.

I begin in section 1 with some general remarks on the treatment of stock accumulation in macro models. Section 2 gives a heuristic explanation of the main point. In the remainder of the paper, it is substantiated by two formal models, one without and one with an endogenous capital stock.


According to Keynes, his General Theory refers to a short run in which "the existing quality and quantity of available equipment," among other "elements in the economic system," are taken as given. Yet the solution of the model determines a rate of net investment in capital equipment which is in no way constrained to be zero. In general it will not be zero, and with the passage of time, therefore, the stock of capital -- one of the "given" or "independent" variables of the system -- will change. But "the schedule of the marginal efficiency of capital depends partly on the existing quantity of equipment." Therefore the investment function, one of the crucial equations, will shift as capital accumulates or decumulates. This is an obvious reason, though not the only reason, why the Keynesian solution cannot

2/. J. M. Keynes, General Theory of Employment, Interest, and Money, New York: Harcourt Brace, 1936, p. 245. He goes on to say, "This does not mean that we assume these factors to be constant; but merely that, in this place and context, we are not considering or taking into account the effects and consequences of changes in them."

3/. General Theory, p. 246.
be a steady state in time.\(^4\) The solution contains the seeds of its own destruction.

Abba Lerner long ago recognized the confusion of stock and flow in the treatment of capital investment in the General Theory. He proposed a model in which the marginal efficiency of investment depends inversely on both the stock of capital and the flow of investment. If equation of this marginal efficiency to the interest rate requires positive net investment, the capital stock will gradually increase, lowering the rate of investment induced by the same interest rate. Ultimately net investment will dwindle to zero, and the capital stock will be stationary. In this equilibrium the marginal efficiency of investment can be identified with the marginal productivity of capital, and both will be equal to the interest rate. This was one of Lerner's many brilliant clarifications of macro-economic theory.

The investment/capital dynamic is not the only flow-stock relationship which makes the Keynesian "equilibrium" temporary. Saving adds to wealth; wealth affects the propensity to consume and possibly the demand for money. Government deficits add to public debt and thus to the outstanding stocks of non-monetary and monetary government liabilities; changes in the supplies

\(^4\). Writing before Harrod inaugurated modern growth macroeconomics, Keynes embedded his model in a stationary setting: constant labor force, technology, etc. Thus the only steady state would be one with constant capital stock. But it is easy to embed it instead in a setting with a non-zero natural rate of growth. The same problem of interpretation arises when solution of the Keynesian equations give a rate of net investment relative to the capital stock different from the natural growth rate.

\(^5\). A. Lerner, Economics of Control (Macmillan, 1944), 330-345. Lerner had previously presented the ideas in a 1937 paper of which a summary was published in Manchester Statistical Society Report of Group Meetings 1936-37.
of these assets may change interest rates. It is this latter observation which has attracted so much attention in the literature of macro-economic theory in recent years. Some authors claim to have discovered a fatal flaw in the Keynesian macro model as exemplified in the common IS/LM apparatus. The flaw is described as "ignoring the government budget identity."

It is more accurate, however, to regard the failure to track the cumulation of deficits into debt as just one aspect of the model's temporary and short-run character. Investment is not cumulated into capital or saving into wealth. Keynes's excuse was that he was concerned with so short a time period that, whatever the rate of investment, its effects on the capital stock would be negligible. He refers to "factors in which the changes seem to be so slow ... as to have only a small and comparatively negligible short-term influence on our quaesitum. Our present object is to discover what determines at any time the national income of a given economic system." Even though he was not explicit about other assets, the spirit of the approach is that there is not enough time for the flows to alter the stocks significantly. Deficits do add to public debt, but even a $60 billion per year federal deficit adds only $5 billion a month to a $500 billion stock. Curiously, the latter-day discoverers of the government budget equation have confined to government debt their objection to the constant-stock assumption. Indeed they have generally been content to

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acquiesce in Keynes's assumption that the physical capital stock is constant. But if the time span of the model is to be extended enough to allow flows to affect stocks, all stocks should be tracked, not just government debts.

The only precise way to justify the Keynesian procedure is to regard the IS/LM model as determining the values of variables at a point in time. Then this model must be regarded as a slice, in time of measure zero, of a continuous-time dynamic model. Asset stocks are among the state variables of the model at that time; they are constant, i.e., independent of the solution of the model, insofar as they are completely inherited from the past. They change as time passes -- moving the instantaneous IS and LM curves, if you like. The "short run" model has a new solution each micro-second; whether and where it settles down requires dynamic analysis.

It is still possible, of course, to answer certain questions by comparative static analysis of the temporary solution: How will the solution at that moment be different if, for given values of stocks and other state variables, government expenditure is different? How will the solution be different if the government, by open-market transactions with the public which take zero time, instantaneously alter the supplies of its several liabilities? How would the solution be different if past history had been different and had beenqueathed to the present different stocks of assets? But the relevance of these exercises is limited because the same changes in exogenous variables will affect subsequent temporary solutions as well.
Certainly, it must be admitted, the Keynesian model, particularly the IS/LM version, has not usually been acknowledged to be so evanescent. The equilibrium language of Keynes suggests that he had a more durable solution in mind. Moreover, since the General Theory and even before, the dynamics and stability of Keynesian equilibrium have been discussed on the tacit presumption that it is the steady state of a dynamic system rather than itself a momentary stage of a dynamic process. Examples of such discussion include the theory of the multiplier as an infinite series, Samuelson's application of his correspondence principle to the Keynesian model, and the analysis, pioneered by Hicks himself, of the stability of IS/LM "equilibrium." Presumably, such dynamic analysis would be unnecessary and irrelevant if the model itself determined the values of its variables at every moment of time. If it did so, its structural equations would allow for the adjustment lags which are considered in the stability analyses cited. To make the point another way, it is somewhat inconsistent to assume that the Keynesian "equilibrium" is the asymptotic result of a long process of behavioral adjustment while ignoring the changes in stocks which are bound to occur during the period of convergence.

Interpreting the Keynesian equilibrium as the momentary solution of a dynamic continuous-time process is subject to another class of objections. The model is, after all, a set of simultaneous equations. To be sure, looking at the economic interdependence in this way is one of the most useful and

insightful abstractions of economic science. But it is perhaps an un-
usual strain on credulity to imagine that a new set of simultaneous
equations, finding the prices and quantities that clear several markets
at once, is solved every instant. The weight on that much burdened deus ex
machina, the Walrasian auctioneer, would be extremely heavy.

An alternative to the continuous-time abstraction is a discrete-time
model. Time is broken into periods of finite duration, during which each of
the simultaneously determined endogenous variables assumes one and only one
value. Flows add finite amounts to stocks: saving during the period makes
wealth larger at the end of the period, investment adds to the capital stock,
government deficits add to public debt, etc. In deciding their consumption,
investment, and asset demands, economic agents are determining their end-of-
period stocks; their behavior takes this into account. Thus the government
budget identity, for example, is explicitly respected. Bonds issued to finance
a government deficit must be absorbed into savers' portfolios. The same is
true of bonds or equities issued to finance private capital formation.

In this way, the solution of a discrete-time Keynesian IS/LM model
accounts for some phenomena which the instantaneous model does not. The dis-
crete-time IS and LM curves include effects which, in the continuous-time
approach, are displayed by shifting the curves as stocks change and tracking
the moving solutions. Of course the discrete-time solution also is only a
temporary equilibrium; the new stocks will generally lead to different solutions
in the next period.

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9/ Some authors consider a beginning-of-period stock equilibrium, in
which existing asset supplies are priced, followed by a within-period
"flow equilibrium" in which asset accumulations, among other variables,
are determined. This tortured construction makes no sense to me.
Taken literally, both approaches are implausible. If it strains credibility to imagine simultaneous market clearings repeated every instant, it is certainly arbitrary to require the famous Auctioneer to clear every market on the same periodic schedule. Both treatments of time are imperfect and unrealistic representations of simultaneous and intertemporal interdependence. Theorists had better avoid dogmatism in favor of either method.

2. Deficit financing and "crowding out."

The previous section is prelude to the main subject of the paper, reconsidering the issue of financial crowding out, the effectiveness of expansionary fiscal policy unaccompanied by monetary expansion. Financial crowding out refers to an underemployment situation, in which displacement is not necessary to release resources for the use of the government or its transferees.

The standard IS/LM analysis says that complete crowding out will not occur unless (a) investment is perfectly interest-elastic, IS "curve" horizontal, or (b) demand for money is perfectly interest-inelastic, LM "curve" vertical. Condition (a) may be dismissed on the ground that adjustment costs and lags prevent investment from responding instantaneously or quickly to small deviations of the interest rate from the marginal efficiency of capital. Condition (b) may be dismissed, at least in principle, on the theoretical logic and empirical evidence of substitution between money and interest-bearing assets. Consequently, the standard conclusion is that the
impact of fiscal expansion will be expansionary and crowding out will be only partial.

However, as suggested briefly at the outset, the force of the dismissal of condition (b) is weakened as longer time periods are considered. At the instant when government outlays are increased, wealth and its component asset stocks are given. The public can be reconciled to the existing money stock, even if its income-related transactions needs are greater, by higher interest rates. But with the passage of time wealth increases; the supply of government bonds is greater than it would have been under less expansionary fiscal policy, while the money supply is unchanged. This is an additional source of upward pressure on interest rates. In continuous-time models, it would be represented by upward and leftward shifts of the LM curve. In a discrete time model, part of this wealth effect would be included in the LM curve, which would for that reason be steeper than in the momentary model.

Do these wealth effects overcome the initial impact of expansionary fiscal policy? The monetarist answer is affirmative. This is, I believe, the rationale for (a) Milton Friedman's assertion that no important proposition of monetary theory requires zero interest elasticity of the demand for money -- with respect to crowding out, I interpret him to mean that the instantaneous LM curve may be positively sloped but shifts backward as deficits enlarge the stock of bonds relative to money; (b) monetarist claims that Keynesian analysis of fiscal effects does not survive explicit recognition of the government budget identity; (c) empirical findings, as in the St. Louis monetarist econometric model, that positive fiscal effects last only a few quarters.
Figures 1 and 2 illustrate this monetarist scenario, carried to the extreme of more than 100% crowding out. In Figure 1 the process is one of moving momentary equilibrium. In Figure 2 it is telescoped into a single solution for a discrete time period, in which the increment of wealth is enough to make the LM locus negatively sloped. In Figure 1, \( E_1 \) is the initial equilibrium at \( t_1 \), and, let us assume, one that would persist until \( t_2 \) in the absence of policy change. \( E_2 \) is the equilibrium at \( t_2 \) as a result of the new fiscal policy begun at \( t_1 \). In Figure 2, \( E_1 \) and \( E_2 \) are alternative solutions for the period \( t_2-t_1 \), \( E_2 \) with the more expansionary fiscal policy.

Within either of these frameworks, I contend, there are two necessary (but not sufficient) conditions for this scenario, or more precisely for output \( Y_2 \) at \( E_2 \) to be less than or equal to the output \( Y_1 \) at \( E_1 \). One is that the demand for money is positively related to wealth. The second is that the demand for wealth is positively related to the interest rate.

The argument is simple enough. At \( E_2 \) the demand for money must be the same as at \( E_1 \). But the interest rate is higher, and income is no higher. So what keeps the demand for money up? It can only be that wealth is higher. Why do people want more wealth when their income is no higher? It can only be because the interest rate is higher.

Let the LM locus of Figure 2 be defined by \( M = L(r,Y,W) \) and suppose that \( W = W(r,Y) \). The slope of the locus \( \frac{\partial r}{\partial Y} \) is

\[
- \frac{L_Y + L_W W_Y}{L_r + L_W W_r} .
\]

Given that \( L_Y, W_Y > 0 \) and \( L_r < 0 \), this slope will be negative only if \( \frac{L_W W_r}{W} > - L_r > 0 \).
Is it plausible to expect this condition to be met? "Money," in this context, is an asset on which the nominal interest rate is institutionally or legally fixed, at zero or some other ceiling. As a constituent of public wealth, it is "outside" money, printed by the government to finance government deficits, an alternative to the issue for the same purpose of non-monetary liabilities bearing market-determined interest rates. The real world counterpart in the U.S. is the stock of high-powered money, defined as currency outside banks plus unborrowed bank reserves. The demand for this stock is derivative, via banks' reserve requirements and reserve behavior, from the public's demand for fixed-rate deposits.

In the scenario sketched above, interest rates on non-monetary assets rise while those on monetary assets are unchanged; under this inducement, the public saves more and their wealth increases. Certainly they will hold more of the assets whose yields have risen. Will they also hold more money? On portfolio-theoretic grounds this seems unlikely; after all, the motivation for the additional saving is the higher yield, which is lost to the extent that saving is diverted to money. (In the denominator of $\frac{dM}{d\alpha}$ above $(L_wW_x + L_x)$, the relevant $L_w$ is not the same as would apply to an exogenous increment of wealth, or to one induced by an increase of income $Y$.)

One possible rationalization is the transactions requirement for cash associated with portfolio management. Let $\alpha$ be the fraction of additional yield-induced wealth that goes into high-powered money to meet this transactions requirement. Let $m$ be the ratio of high-powered money to wealth. Then the
condition that \( \frac{\alpha}{\delta r} \) be positive is \( \frac{\Delta r/W}{-Lr/M} > \frac{m}{\alpha} \). Now \( m \) is very much larger than \( \alpha \), given the many purposes for which money is held other than portfolio transactions. So the elasticity of wealth with respect to the interest rate would have to be much larger than the substitution elasticity of demand for money.
3. **Crowding Out in a Short-run Discrete Time Model.**

I propose now to explore the issues raised in section 2 in two simple formal models. They are discrete time models, and the government budget identity is explicitly respected. They are short-run models, not in the sense that stocks are fixed but in the sense that stocks and other state variables are not stationary. I shall consider first an economy without capital, and show that a positive total interest-elasticity of money demand, derived as explained in section 2 from a positive interest-elasticity of saving, is necessary (but not sufficient) for the monetarist result. I shall turn then to an economy with an endogenous capital stock, and show the same proposition. Comparison of the two models, moreover, will indicate that the condition is both less likely to be fulfilled and less likely to be sufficient when capital accumulation is recognized.

**A model without capital.** The national product identities are:

\[ Y = C + G = C + S + T - B_{-1} \]

Here \( Y \) is national product, \( C \) private expenditure on goods and services, \( G \) government purchases, \( S \) saving, \( T \) tax payments, \( B_{-1} \) government interest payments (equal to bonds outstanding at beginning of period.) All variables refer to a time period of finite duration. Within the period the price level is taken as fixed, determined by events in previous periods. It may be different next period
because of events in this one, but within the one-period model it is not endogenous. The same is true of price expectations. Thus for present purposes the variables can be considered both real and nominal.

Taxes, net of transfers other than interest, will be taken throughout as a function of income \( T(Y) \), with \( T_Y > 0 \). The points about fiscal policy can be made by regarding \( G \) as the policy variable. Accordingly \( S \) may be taken as a function of two endogenous variables, \( Y \) and the interest rate \( r \), and of lagged variables like \( B_{-1} \). The implications of the sign of \( S_r \) is a matter of central interest; the previous discussion suggests that \( S_r > 0 \) is necessary, though not sufficient, for complete or more than complete crowding out. Of course \( C \) can be derived from \( S \). Thus \( C_Y = 1 - T_Y - S_Y \) and \( C_r = - S_r \). Although \( C \) connotes consumption, it could be interpreted to include investment expenditure in hybrid models of this genre which allow investment but hold the capital stock constant. Indeed one way in which the important assumption \( S_r > 0 \) slips into such models is the natural assumption that investment is inversely related to the interest rate, \( C_r < 0 \).

The government deficit for the period is:

\[ D = G + B_{-1} - T \]
The deficit is financed in proportion $\gamma_B$ by selling bonds
and in proportion $\gamma_M = 1 - \gamma_B$ by printing high-powered money. Un-
less otherwise noted, I consider only non-negative values of both
$\gamma_B$ and $\gamma_M$. The fraction $\gamma_B$ is a policy parameter; I shall
be particularly interested in the case $\gamma_B = 1$. Bonds are assumed
to be consols paying $1$ net of tax per period. Their quantity $B$
is measured by the coupon liability. During the period the in-
crease in quantity of bonds is $\Delta B = B - B_{-1}$. The price of bonds is
$q_B$.

Similarly, the increase in the stock of money is $\Delta M = M - M_{-1}$.
Thus:

$$\begin{align*}
\gamma_B \Delta B &= q_B^D \\
\Delta M &= q_M^D
\end{align*}$$

(3)

In this model all private saving is absorbed by the government deficit,
as implied in (1):

$$S = D = q_B \Delta B + \Delta M$$

(4)

The public's wealth changes not only by saving but by capital gain
or loss on their initial bond holdings $B_{-1}$. The capital gain, positive
or negative, is $(q_B - q_{B_{-1}}) B_{-1} = \Delta q_B B_{-1}$. The relevant interest rate
is the one period rate on bonds $r_B$, which depends on the expected price
of bonds next period $q_B^e$. A bond costing $q_B$ held until next period will yield
the holder \( \frac{1 + q^e_B}{q_B} - 1 = r_B \). Thus \( q_B = \frac{1 + q^e_B}{1 + r_B} \). Now \( q^e_B \) may depend on \( q_B \), but I assume some regressivity in this dependence, so that \( \frac{\partial q^e_B}{\partial q_B} < 1 + r_B \). This assures that \( \frac{\partial q_B}{\partial r_B} \), which is equal to \( \frac{-q_B}{1 + r_B} - \frac{\partial q^e_B}{\partial q_B} \) is negative. This derivative also describes the relation of \( \Delta q_B \) to \( r_B \). I shall represent it as \( q^*_B \) below.

Wealth owners decide the values of the two assets they will hold at the end of the period, or equivalently the increments in these values during the period. \( F^B(\ ) \) and \( F^M(\ ) \) represent these increments for bonds and money respectively. The targets \( F^B \) and \( F^M \) are achieved by saving and, in the case of bonds, by capital gain. They are both functions of within-period endogenous variables, \( r_B \) and \( Y \), of predetermined initial stocks \( q_B, Y_{-1}^B, Y_{-1}^M \), and of other lagged or exogenous variables. Their relation to saving is:

\[
(5) \quad S(\ ) = F^B(\ ) - \Delta q_B Y_{-1}^B + F^M(\ )
\]

This identity may be regarded as a definition of saving in terms of asset accumulations, and I will dispense with the saving function in the formal model below. The marginal propensity to save \( S_Y \) is the sum \( F^B_Y + F^M_Y \), which will be assumed, as is traditional, to be positive but less than \( 1 - T_Y \).
Note that the capital gains term in (5) may make \( S_{r_B} > 0 \) even if 
\[
F^B_{r_B} + F^M_{r_B} \leq 0,
\]
becaues the public will recoup by saving capital losses due to an increase in the bond interest rate. In order to concentrate on the monetarist issue raised in section 2, I shall assume that 
\( S_{r_B} > 0 \).

Here is the model, relating three within-period endogenous variables \( (r_B, Y, D) \) to the policy parameters \( (G, Y_B) \).

\[
\begin{align*}
F^B( & ) - \Delta q^B_{B-1} - \gamma_y D = 0 \\
F^M( & ) - \gamma_m D = 0 \\
\tau(Y) + D &= G + B_{-1}
\end{align*}
\]

Our principal interest is in the effects of variation of \( G \) on the solution of (6):

\[
\begin{bmatrix}
F^B_{r_B} - \frac{\partial q^B}{\partial G} \frac{B_{-1}}{G} & F^B_Y - \gamma_y & \frac{\partial q^B}{\partial G} \\
F^M_{r_B} & F^M_Y - \gamma_m & \frac{\partial Y}{\partial G} \\
0 & \tau_y & 1 \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial A_B}{\partial G} \\
\frac{\partial Y}{\partial G} \\
\frac{\partial D}{\partial G}
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]
Let $\Delta$ be the determinant of the Jacobian, and $\Delta_{ij}$ its minor with respect to the element in row $i$, column $j$. Then:

$$\Delta = \Delta_{33} - T_Y \Delta_{32}.$$  

(8) \[ \frac{\partial r_a}{\partial G} = \frac{\Delta_{31}}{\Delta}, \quad \frac{\partial Y}{\partial G} = -\frac{\Delta_{31}}{\Delta}, \quad \frac{\partial D}{\partial G} = \frac{\Delta_{33}}{\Delta}. \]

The two principal cases to consider are

I) $F^M_B < 0$ and

II) $F^H_B \geq 0$. In the first case, the substitution effect is dominant, as in conventional short-run Keynesian analysis. In the second case, the wealth effect dominates the substitution effect; an increase in interest rate induces additional saving, and some of it goes into money.

Case I: The sign pattern of the determinant is:

$$\begin{bmatrix}
+ & ? & -\delta_d \\
- & + & -\delta_m \\
0 & +T_Y & +1
\end{bmatrix}$$

(9) $\Delta_{33}$ is positive, even if $F^B_Y$, the $?$ in (9), is negative. (If the second row of $\Delta_{33}$ is added to the first, $\Delta_{33}$ becomes $\begin{bmatrix}+ & +\end{bmatrix}$

on our assumption that $S_Y = F^B_Y + F^M_Y > 0$.)
\( \Delta_{32} \) is negative. Hence \( \Delta, \frac{\partial Y}{\partial G}, \text{ and } \frac{\partial D}{\partial G} \) are all positive. 
\( \Delta_{31} \) is \( F_Y^B(\gamma_B - 1) + F_Y^M \gamma_B \). If \( F_Y^B \) is negative, \( \frac{\partial n_B}{\partial G} \) is certainly positive, however the deficit is financed. (It would of course be possible to reduce the interest rate while increasing \( G \) if not only the deficit but part of the pre-existing debt were monetized, i.e., if \( \gamma_B \) were sufficiently negative. This is true because \( F_Y^B + F_Y^M \) is positive.) If \( F_Y^B \) is positive, \( \frac{\partial n_B}{\partial G} \) will be positive for high values of \( \gamma_B \), notably 1, but negative for low values, notably 0.

These are standard Keynesian results, and here serve only to show that they are altered neither by explicit respect for the government budget equation nor by the assumption that saving responds positively to interest rate.

**Case II:** The sign pattern of the Jacobian is:

\[
\begin{bmatrix}
+ & ? & -\gamma_B \\
+ & + & -\delta_M \\
0 & + & T_Y \\
\end{bmatrix}
\]

(10)

The sign of \( F_Y^M \) in the second row, first column, is positive instead of negative. Here it is convenient to consider first \( \gamma_B = 1 \), and \( F_Y^B < 0 \). Then \( \Delta_{33} \) is positive, but \( \Delta_{32} \) is also positive. So \( \Delta \) may have either sign. It will be positive if and only if

\[
\frac{F_Y^B}{F_Y^M} \frac{r_B}{F_Y} > \frac{F_Y^B + T_Y}{F_Y}.
\]
Roughly and allegorically speaking, the condition is that a rise in interest rate has a comparative advantage in absorbing bonds, a rise in income in absorbing money. (Taxes generated by income increases are a way of absorbing bonds when the deficit is bond financed -- hence the term \( T_Y \) on the right-hand side of the inequality.)

If this condition is met, then the monetarist configuration of Figure 2 results:

\[
\frac{\partial \Pi_B}{\partial G} > 0, \quad \frac{\partial Y}{\partial G} < 0, \quad \frac{\partial D}{\partial G} > 0.
\]

Letting \( F_Y^B \), the ? in (10) become positive does not change this outcome as long as the condition for positive \( \Delta \) is fulfilled. However, a rise in \( F_Y^B \) relative to \( F_Y^M \) makes the condition less likely. A priori portfolio substitution effects justify the assumption that \( F_Y^B \) exceeds \( F_Y^M \) and transactions considerations the assumption that \( F_Y^M \) exceeds \( F_Y^B \). This is enough to make \( \Delta_{33} \) positive but guarantees positive \( \Delta \) only if \( T_Y \) is zero.

If the condition is not met and \( \Delta \) is negative, then the counter-intuitive conclusions are:

\[
\frac{\partial nH}{\partial G} < 0, \quad \frac{\partial Y}{\partial G} > 0, \quad \frac{\partial D}{\partial G} < 0.
\]

Figure 3 shows this outcome in IS/LM terms. Both IS and LM are negatively sloped, but IS is steeper. It is tempting to dismiss this solution as "unstable." But if system (6) really describes the equations which
the economy somehow solves simultaneously within the period, we have no right to do so. There is no shorter run, no dynamic process in real time of which this solution is an equilibriu. To say any solution is unstable is just to impugn gratuitously the iterative computer program of the Walrasian auctioneer who simultaneously clears the markets.

In any event, there is always a positive value of \( \theta_B \), less than or equal to 1, below which \( \frac{\partial W}{\partial \theta} \) is positive. \( \frac{\partial x_B}{\partial \theta} \) will also be positive if \( F_Y^B \) is negative. But if \( F_Y^B \) is positive, \( \frac{\partial x_B}{\partial \theta} \) will be negative. Still excluding the possibility that \( \Delta_{33} \) is negative, \( \frac{\partial D}{\partial \theta} \) will always be positive.

A model with capital. In this model part of each period's production is added to the capital stock available for use in subsequent periods. A unit of capital stock in use earns an after-tax return of \( R \) per period. The public owns equity titles to the whole stock, and the market value of equity per unit of capital is \( q_K \). The yield to the equity holder for the period is \( r_K \). This depends, of course, on \( q_K^e \). Analogously to the relation between the valuation and one-period yield of bonds,

\[
q_K = \frac{R + q_K^e}{1 + r_K}
\]

I assume, as in the bond case, sufficient regressivity of expectations so that \( q_K \) is negatively related to \( r_K \). However, equity valuation is also a function of another within-period endogenous variable, \( Y \), for two reasons. One is that the contemporary earnings rate \( R \) varies directly with \( Y \). The other is that expected future earnings \( R \) and interest rates \( r_K \), summarized in \( q_K^e \), are affected by current business
activity. This calculation may go either way. As daily stock market reports remind us, we cannot generalize about the effect of current economic news on equity values, the relation between \( q_K \) and \( Y \).

Taking \( q_K = q(r_K, Y) \), I will denote the partial derivatives as \( q_{r_K}, q_Y \).

As previously assumed for other assets, savers have a demand for accretion of the value of their holdings of equity during the period \( F^K(\cdot) \). This demand is met in two ways. One is the capital gain \( \Delta q_{K-1} \) on the stock of capital at the beginning of the period. This capital gain depends indirectly on \( r_K \) and \( Y \) via the valuation function \( q(r_K, Y) \). The other source of supply is by new net investment \( q_K \Delta K \). This is taken to be \( I = \varphi(q_K)K_{-1} \) where \( \varphi \) is an increasing function of \( q_K \) and thus related to \( r_K \) (negatively) and to \( Y \). The quantity \( I \) serves the same role for capital as \( \gamma^D_B \) and \( \gamma^D_M \) serve for bonds and money.

The accounting identities (1), (4), (5) are extended in obvious ways:

\[
(1') \quad Y = C + I + G = C + S + T - B_{-1} \\
(4') \quad S = D + I = q^B_K AB + \Delta M + q_K \Delta K \\
(5') \quad S(\cdot) = F^B(\cdot) - \Delta q^B_B B_{-1} + F^M(\cdot) + F^K(\cdot) - \Delta q_{K-1} \]

The model, relating four within-period endogenous variables \( (r_K, r_B, Y, D) \) to the policy parameters \( (C, \gamma_B) \) is the following:
\[
\begin{align*}
(6') & \begin{cases}
F^K(Y) - \Delta g_K K_{-1} - \Phi(q_K) K_{-1} = 0 \\
F^B(\cdot) - \Delta g_B B_{-1} - \gamma_B D = 0 \\
F^M(\cdot) - \gamma_M D = 0 \\
T(Y) + D = G + B_{-1}
\end{cases} \\
(7') & \begin{bmatrix}
F_{x_K}^K - \Phi^r g_{x_K} K_{-1} & F_{x_B}^K & F_Y^K - \Phi^r g_Y K_{-1} & 0 \\
- \Phi^r g_{x_K} K_{-1} & F_{x_B}^B & F_Y^B & -\gamma_B \\
F_{x_K}^B & F_{x_B}^B - \Phi^r g_{x_B} B_{-1} & F_Y^B & -\gamma_B \\
F_{x_K}^M & F_{x_B}^M & F_Y^M & -\gamma_M \\
0 & 0 & T_Y & 1
\end{bmatrix}
\begin{bmatrix}
\frac{\partial n_K}{\partial G} \\
\frac{\partial n_B}{\partial G} \\
\frac{\partial Y}{\partial G} \\
\frac{\partial D}{\partial G}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\end{align*}
\]

\[
(8') \begin{cases}
\Delta = \Delta_{44} - T_Y \Delta_{43} \\
\frac{\partial n_K}{\partial G} = -\frac{\Delta_{41}}{\Delta}, \quad \frac{\partial n_B}{\partial G} = \frac{\Delta_{42}}{\Delta}, \quad \frac{\partial Y}{\partial G} = -\frac{\Delta_{43}}{\Delta}, \quad \frac{\partial D}{\partial G} = \frac{\Delta_{44}}{\Delta}
\end{cases}
\]

Like the previous model, there are two principal cases to consider:

I) $F_{n_K}^M, F_{n_B}^M < 0,$ II) $F_{n_K}^M, F_{n_B}^M > 0.$

**Case I.** The sign pattern of the determinant of the Jacobian is:

\[
\begin{bmatrix}
+ & - & ? & 0 \\
- & + & ? & -\gamma_B \\
- & - & + & -\gamma_M \\
0 & 0 & +T_Y & +1
\end{bmatrix}
\]
The standard Keynesian results apply once again. For 100% bond-
finance, $(\gamma_B = 1), Y, D,$ and $r_B$ all rise with $G$. (As in the first
model, these conclusions do not depend on the signs of the $\gamma$ terms,
provided every column in $\Delta_{44}$ has a positive sum.) But $r_K$ might possibly
fall if the top $\gamma$ is positive, e.g. if prosperity tends to lower expected
equity values.

**Case II.** The sign pattern is:

\[
\begin{bmatrix}
+ & - & ? & 0 \\
- & + & ? & -\gamma_B \\
+ & + & + & -\gamma_M \\
0 & 0 & T_Y & 1 \\
\end{bmatrix}
\]

(10')

To avoid a tedious catalog, I consider solely $\gamma_B = 1$ and confine
myself to the plausible assumption that $\Delta_{44}$ is positive. As in the first
model, positive interest responses in the money row make $\Delta_{43}$ positive.
The monetarist configuration then arises when $\Delta = \Delta_{44} - \theta \Delta_{43}$ is also
positive.

However, a high $T_Y$ can make $\Delta$ negative, particularly if the $\gamma$
entries in (10') are strongly positive. This gives rise to the same type of
counter-intuitive results as in the simpler model: $\frac{\partial r_K}{\partial G} < 0, \frac{\partial Y}{\partial G} > 0, \frac{\partial D}{\partial G} < 0$. However, the bond interest rate may go either way.
Finally, consider a case intermediate between I and II, with $F_M^H$ positive, and $F_M^{TB}$ negative. An increase in wealth induced by higher equity yields raises the demand for money. But bonds and money are strong substitutes. With positive entries in the third (Y) column, and with a pattern of signs indicating that bonds are a closer substitute for money than for capital, $\delta Y/\delta G$ will be positive.


The long-run equilibria of models of this type, and the comparative statics of these stationary states, have been examined in the Blinder-Solow and Tobin-Buiter papers cited in footnote 1. Our paper looks at the stability of those states, using a continuous time model. However, we did not consider the implications of interest-responsive demand for wealth.

Here I shall confine myself to a cursory look at the long-run equilibrium of the second model of section 3. The purpose, here as in previous studies, is not realism. It is not realistic to imagine that policy never changes or that output is demand-determined over so long a run. The purpose is rather to pursue the pure logic of the issue. For example, the persistence of the "Keynesian" conclusions of section 3 would be called into question if under the same behavioral assumptions the stationary state value of $Y$ turned out to be inversely related to $G$.

In the long run, there are steady-state stock demands $K^D$, $B^D$, $M^D$, summing to desired wealth. These are functions of $Y$ and of the interest
rate \( r_B \) and \( r_K \). Actual and expected \( q \)'s are equal: \( r_K = R \) and \( q_K = 1; \ q_B = 1/r_B \). The supply of capital is endogenous, \( K(Y, r_K) \), with \( K_Y > 0, \ K_{r_K} < 0 \). This relationship is technological.

For example, a Cobb-Douglas CRS production function implies \( K = \frac{aY}{r_K} \).

Taking the supply of money \( M \) as exogenous, then the outstanding stock of bonds \( B \) is endogenous. (Alternatively one could specify the fractions of money and bonds in the total value of debt and let that total be the endogenous variable.) The budget is balanced in long-run equilibrium. The equations, in the four endogenous variables \( (r_K, r_B, Y, B) \) are:

\[
\begin{align*}
K^D( ) - K(Y, r_K) &= 0 \\
B^D( ) - B/r_B &= 0 \\
M^D( ) &= M \\
T(Y) - B &= G
\end{align*}
\]

\[(11)\]

\[
\begin{bmatrix}
K_{r_K}^D - K_{r_K} \\
B_{r_K}^D \\
M_{r_K}^D \\
0
\end{bmatrix}
- \begin{bmatrix}
K_{Y}^D - K_Y \\
B_{Y}^D - \frac{1}{r_B} \\
M_{Y}^D \\
T_Y - 1
\end{bmatrix}
\begin{bmatrix}
\frac{\partial r_K}{\partial G} \\
\frac{\partial r_B}{\partial G} \\
\frac{\partial Y}{\partial G} \\
\frac{\partial B}{\partial G}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]

\[(12)\]
The Jacobian has the same structure as (7'), except that the bottom diagonal element is -1 instead of +1. Thus \( \Delta = -\Delta_{44} \).

\( T_Y \Delta_{43} \). A Keynesian sign pattern insures \( \Delta_{43} < 0, \Delta_{44} > 0 \). So \( \Delta \) may have either sign. Positive \( \Delta \) implies positive \( \partial Y/\partial G, \partial B/\partial G, \partial B/\partial G \), ambiguous \( \partial X/\partial G \). Note that because of the increase in bond interest \( B \), \( \partial Y/\partial G \) exceeds \( \frac{1}{T_Y} \), the amount necessary to collect taxes enough to cover the increase in \( G \). Figure 4 is illustrative. LLM is the long-run balance of money demand and supply. GT is the balanced budget locus. Both curves carry with each point the value of \( B \) that maintains portfolio balance. The rightward shift in GT represents an increase in \( G \).

It is also possible that \( \Delta \) is negative -- if \( T_Y \) is low, for example. This means that the LLM curve is steeper than GT, and the comparative statics give perverse results. In the Tobin-Beiter paper previously cited, it is shown -- though for a continuous time model of somewhat different structure -- that this equilibrium is unstable. An increase in government purchases starts the economy off on a track of increasing income and interest rates which left to itself never converges to a balanced budget equilibrium.

With the "monetarist" sign pattern, the two possibilities are illustrated by Figure 5 and Figure 6. In formal terms \( \Delta_{43} \) is now positive. If \( \Delta_{44} \) is positive, then \( \Delta \) is negative and \( \partial Y/\partial G = -\frac{\Delta_{43}}{\Delta} \) is positive, while \( \partial X/\partial G \) and \( \partial B/\partial G \) are negative. Figure 5 applies. But in the analogous
short-run case, \( Y \) is moving in the other direction. This suggests that the equilibria displayed in Figure 5 are unstable and irrelevant. A monetarist nightmare comes true. Deficit spending feeds on itself in an ever weaker economy afflicted with high and rising interest rates. Figure 6 represents a \( \Delta_{44} \) so negative that \( \Delta \) becomes positive. The budget balance line \( GT \) slopes the "wrong" way; moving northeast along it, the demand for bonds declines and taxes fall to match. Expansionary fiscal policy is represented by a downward shift of \( GT \). This long-run configuration is possibly a stable version of the monetarist scenario.

5. Concluding remarks.

I have tried to rationalize the monetarist claim that fiscal policy is ineffective or worse in its effects on aggregate demand, even though the instantaneous demand for money is inversely sensitive to interest rates. I found a rationalization in interest-induced saving, provided greater wealth, whatever motivated its accumulation, entails greater transactions balances. I doubt the empirical importance of such behavior, but that is another question.

At the same time, I have shown that the standard Keynesian results can, in theory, apply for long periods of time. They do not violate the government budget constraint.

What difference does it make? After all, no one need advocate or practice the use of fiscal stabilization policies without the active help of monetary policies. Yet it does make a difference whether and how fiscal
policy makes a difference. With two major tools, we can aim not only at domestic stabilization but at some other target too -- growth or external balance. Monetary policy itself should take account of the economic effects of fiscal measures. The moral is embarrassingly obvious, but these days it is frequently denied or ignored.

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Crowding Out More than 100% Continuous-Time Version

Crowding Out More than 100% Discrete-Time Version
Fiscal Expansion Lowers Interest and Raises Income?