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THE DEMAND FOR CASH AND WORKING BALANCES OR CORPORATIONS

Alan W. Heston

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THE DEMAND FOR CASH AND WORKING BALANCES OF CORPORATIONS

By

Alan W. Heston

SECTION I

Introduction

The models of corporate cash and working balances developed in this paper are extensions of those presented by William Baumol [1], and James Tobin [14]. The models are tested on data for individual firms with time series and cross section observations. The models incorporate both a transactions and wealth demand for cash and in contrast to earlier studies for firms the results are consistent with a square root rule for cash, i.e., cash balances change in proportion to the square root of transactions.

In the models of Baumol and Tobin the demand for cash is determined by the interest on non-cash assets and the transactions cost of moving into and out of cash. A firm with surplus cash is induced to move into bills or bonds because they yield a return, but deterred because of the transactions costs of exchanging cash for bills. A firm that borrows its cash holdings similarly finds transactions costs leading it to borrow larger amounts less frequently, while interest charges induce the firm to keep its borrowings frequent and its cash balances low. When transactions are a constant amount per unit time, then profit maximizing behavior is given by Baumol's square root rule for cash.

Empirical evidence for firms has not been consistent with the square root rule. Rather than obtaining elasticities of cash with respect to transactions for firms of near 0.5, the studies of Kesselgoff [8], Meltzer [11], and the present writer [5] have found elasticities closer to 1.0.
In the above studies the principal explanatory variables have been the level of transactions alone, or transactions and the interest rate. In the remainder of this section we argue that the demand for cash and working balances should depend on some additional variables, variables that are also necessary for an adequate test of the square root rule.

In empirical work it cannot be assumed that transactions are certain. If the variance of transactions were unrelated to the level of transactions we would expect the elasticity of cash with respect to transactions to be smaller than if the variance of transactions were positively related to transactions. Since the variance of transactions is frequently positively related to the level of transactions it would be desirable to include the variance of transactions as a variable in the demand relation for cash and working balances. However, the measure of the variance of transactions we probably wish is inter-annual rather than intra-annual. Some work was done with annual data and since the annual variance of sales did not appear to be an important explanatory variable, it has not been used in this study. In a recent study by Keaten [7], the quarterly variance of transactions was used, and these results are summarized in the concluding section.

Two other influences on the demand for cash and working balances are compensating balances and the wealth of a firm. Since observed cash balances satisfy all motives for holding cash, it is important to consider the growing practice of holding compensating balances in banks where firms customarily borrow. If compensating balances are systematically related to the level of transactions of firms, their presence would introduce a bias in the estimation of the coefficient on transactions. Unfortunately, we do not know enough about compensating balances to know if such a bias exists, and one purpose of this study is to see if we can infer from bank loans of firms the extent to which they hold compensating balances.
Corporations hold wealth as well as earn income. One effect of wealth is to give rise to a flow of income, and Meltzer [12] in his study of aggregate money holdings from 1900 to 1949 has measured permanent income as the product of the interest rate and wealth. In addition to giving rise to flow of income for which cash balances must be held, wealth implies a portfolio of assets, and cash balances will be needed to lubricate a portfolio regardless of the level of income. We would expect that a person with $10,000 in income and $1 million in assets would hold more cash than a person with no assets and the same income. This idea is due to Marshall and is illustrated in his discussion of motives for holding money balances where he says,

"To give definiteness to this notion, let us suppose that the inhabitants of a country...keep by them on the average ready purchasing power to the extent of a tenth part of their annual income, together with a fiftieth part of their property." [10, p. 44]

We apply Marshall’s argument to firms and suggest that given transactions, their observed cash holdings will be larger the greater their net worth. The introduction of a wealth demand for cash is important in this study because there is some tendency for the ratio of total assets or net worth to sales to increase with the level of sales. This means that across firms we may observe the ratio of cash to sales as fairly uniform, not for lack of economies of scale in cash holdings, but because the ratio of wealth to sales is increasing. This hypothesis of a wealth demand for cash is tested by including net worth in the demand equations for cash and working balances.
A final variable that we include in our models is the profitability of the firm. Following Tobin and other writers [6] we view the firm as adjusting its portfolio to the rates of return on the various assets. In fact, we deal with only two rates, the rate on securities (the bill rate), and the rate on other assets (gross profits).
SECTION II

Exposition of the Models

In this section we write the equations for our models of cash and working balances. Our estimation is carried out on two sets of data. One set of data is time series-cross section, which we will refer to as the time series, for 42 firms for 12 years, while the other set of data is pure cross section data for 237 firms. While the basic model for the two sets of data is the same, there are a sufficient number of differences to make it more convenient to present the models separately. We will develop the time series model first.

A. The Time Series Model.

The discussion of section 1 leads to our equations for desired cash and working balances

(1.0) \[ C_t^* = \alpha_{10} T_t^{\alpha_{11}} N_t^{\alpha_{12}} R_t^{\alpha_{13}} P_{t-1}^{\alpha_{19}} U_{1t} \]

(2.0) \[ W_t^* = \alpha_{20} T_t^{\alpha_{21}} N_t^{\alpha_{22}} R_t^{\alpha_{23}} P_{t-1}^{\alpha_{29}} U_{2t} \]

where: \( C_t^* \) is desired cash holdings in period \( t \)

\( T_t \) is the level of transactions

\( R_t \) is the rate on Treasury Bills

\( P_{t-1} \) is profits of the previous period

\( N_t \) is Net Worth of the firm
$U_{lt}$, and $U_{2t}$ are error terms assumed independent over $t$, and log-normally distributed.

$W^*_t$ is desired working balances (cash plus security holdings)

$t$ is an index of the year: $t=48, 49, \ldots, 59$.

Equations (1.0) and (2.0) would ideally contain a variable relating to compensating balances of firms. However, for the time series we were not able to obtain data on bank loans for all the years. Consequently for the time series model we are not able to introduce the proxy variable, bank loans, for compensating balances. However, it will be possible to relate estimates of the error terms, $U_{lt}$ and $U_{2t}$, to some data on bank loans, and this analysis is discussed in Section 3.\(^1\)

\[\text{---}\]

\[1. \text{ We have followed Baumol's models in using a multiplicative relation for desired cash and working balances. Use of a multiplicative relation has raised one minor problem, namely that profits can be negative. A common method for handling this situation is to introduce two profits variables, the first equal to one if profits are positive, and the second equal to the value of profits, or if profits are negative the first variable equals the absolute value of profits, and the second variable is set equal to 1. In fact, our sample of firms had only one firm whose profits were negative, during any part of the period and our procedure was to eliminate that firm.}\]

\[\text{---}\]

Equations (1.0) and (2.0) could be estimated directly if we had observations on desired cash and working balances. However, it is our hypothesis that observed annual levels of cash and working balances are not necessarily desired levels. In our earlier work [5] evidence was presented supporting the view that firms respond to any short-run change
in sales by first adjusting their inventory and net receivables (receivables minus payables), and only when these variables are approaching their desired relation to sales do they adjust their cash and working balances. These considerations lead us to our lagged adjustment equations for cash and working balances,

\[(1.1) \quad C_t = C_{t-1} \left( \frac{C_t^*}{C_{t-1}} \right)^{\beta_1} \]

\[(2.1) \quad W_t = W_{t-1} \left( \frac{W_t^*}{W_{t-1}} \right)^{\beta_2} \]

where \(C_t\) and \(W_t\) are observed values of cash and working balances, and \(\beta_1\) and \(\beta_2\) are "reaction" or "speed of adjustment" coefficients. Our hypothesis is that, \(0 < \beta_1, \beta_2 < 1\), which implies that firms only partially adjust their cash and working balances towards their desired levels, within a year. Should these reaction coefficients be greater than \(1\) it would imply that firms overshoot their desired levels within a year, while values less than zero would imply they move in the wrong direction.

Substituting equations (1.0) and (2.0) for \(C_t^*\) and \(W_t^*\) in equations (1.1) and (2.1) respectively,

\[(1.2) \quad C_t = \alpha_{10} T_t^{\beta_1} N_t^{\beta_1 \alpha_{11}} R_t^{\beta_1 \alpha_{12}} P_t^{\beta_1 \alpha_{13}} C_{t-1}^{\beta_1 \alpha_{14}} (1-\beta_1)^{\beta_1} U_{1t} \]

\[(2.2) \quad W_t = \alpha_{20} T_t^{\beta_2} N_t^{\beta_2 \alpha_{21}} R_t^{\beta_2 \alpha_{22}} P_{t-1}^{\beta_2 \alpha_{23}} W_{t-1}^{\beta_2 \alpha_{24}} (1-\beta_2)^{\beta_2} U_{2t} \]
we have the equations for cash and working balances which we will estimate.

B. The Cross Section Model

Our purpose in estimating equations for the demand for cash and working balances from a cross section of firms is twofold. First, we believe that in time series analysis it is important to use a lagged adjustment model to explain observed data. However, lagged adjustment models frequently lead to regression equations involving current and lagged values of the dependent variable, which it would be desirable to avoid if possible. Our cross-section data consists of the annual averages of the variables for the four years, 1953-56, a period that corresponds roughly to a business cycle. By averaging our variables over a business cycle, we hope to obtain observations which may be considered desired, planned, or equilibrium values. For this reason it is unnecessary to employ a lagged adjustment model.

A second advantage of a cross-section model for the present purpose is that the sample is large enough (237 firms) to allow a richer analysis of the behavior of cash and working balances than is afforded by the time series estimates. The cross section also provides a check on the time series results, a check which is largely independent since only 42 of the 237 firms are included in the time series analysis.

Our cross section model consists of the following equations,

\[(3.0) \quad C_t = \alpha_{20} x_1^{31} x_2^{32} x_3^{34} x_4^{35} T^{36} N^{37} L^{38} U_3\]
\[(4.0) \quad W_t = \alpha_{40} x_1^{\alpha_{41}} x_2^{\alpha_{42}} x_3^{\alpha_{43}} x_5^{\alpha_{45}} T^{\alpha_{46}} N^{\alpha_{47}} L^{\alpha_{48}} U_4\]

where \( x_1 = 10 \) if the firm is in the electric power industry, and 1.0 otherwise

\( x_2 = 10 \) if the firm holds no securities, and 1.0 otherwise

\( x_3 = 10 \) if the firm holds no bank loans and 1.0 otherwise

\( x_4 = 10 \) if the firm has neither bank loans nor holds securities and 1.0 otherwise

\( U_3 \) and \( U_4 \) are error terms and are assumed independent over firms (\( U_3 \) and \( U_4 \) may be correlated) and log normally distributed.

and \( L \) is short term and long term bank loans of the firm.

While we retain the same notation for variables, all real variables are the average of 1953-1956 values for each firm. Before dealing with the classificatory or dummy variables, let us first note the differences between equations (1.0) and (2.0), and equations (3.0) and (4.0).

The cross-section equations do not contain the bill rate as a variable since the bill rate does not vary among firms. Also the profits variable is excluded from the cross section equations. We are not primarily interested in differences in profits across firms, but rather in the effect of changes in profits on the portfolio of an individual firm over time. Another reason for excluding profits from the cross section equations is that across firms profits are closely related to net worth.
For the cross section observations are available on bank loans of firms. Bank loans are included in equations (3.0) and (4.0) as a proxy for compensating balances and it is expected that the sign on bank loans for cash will be positive. The first dummy variable is introduced into the cross section equation because some firms hold no bank loans. The variable $L$ is the value of bank loans for firms with bank loans, and otherwise is 1.0. The variable $x_3$ is equal to 1.0 for firms with bank loans, and 10.0 for firms with no bank loans. (The values 1.0 and 10.0 for the dummy variables correspond to 0 and 1 when equations (3.0) and (4.0) are expressed in logarithms).

Firms have also been classified according to their security holdings in order to find out if firms that hold no securities exhibit any peculiarities in their demand for cash and working balances. If firms hold no securities because it is economical, we would expect their cash holdings to be larger than other firms, and the coefficient on $x_2$ for cash to be positive. Regardless of whether the coefficient on $x_2$ is positive for cash, we would expect the coefficient on $x_2$ for working balances to be negative. That is we would expect firms with no securities to hold less working balances than firms with securities since, whatever the liquidity needs of the firm, the "liquidity" provided by holding a given amount of working balances in cash is greater than holding the same amount in both cash and working balances.

The final classificatory variable that we have used is for the electric power industry. While our sample of firms includes firms from
such diverse industries as petroleum and retail trade, preliminary classification of firms by industry did not yield very consistent differences, with the exception of electric power. Electric power displays steady growth in sales and inventories are relatively unimportant. Both these factors should lead to less liquidity requirements than in the other industries represented -- food, tobacco, petroleum, chemicals, iron and non-ferrous metals, machinery, transportation, and retail trade -- so we expect the coefficient on $x_\perp$ to be negative.

Before presenting estimates of our equations, we should like to briefly discuss our choice of variables, and the data. Cash holdings of firms are currency, demand deposits, and a small amount of time deposits. Certificates of deposit, which offer firms perhaps a better substitute for cash than bills, were not important during the period of this study. However, their development of certificates of deposit may mean that several of our results will have less applicability in the future. Securities include all non-equity holdings, and are predominately bills [13, pp. 111-114].

Sales are used to measure transactions, and are a close proxy [5, p. 130]. The bill rate is the rate for the fourth quarter of the year of the balance sheet. Thus cash balances of December 31, 1956 would be related to the bill rate of October-December, 1956. Net worth is paid in capital, surplus and reserves.

Other measures than net worth could have been used, and perhaps the most appealing alternative is total assets. However, total assets fluctuate over time very closely to sales, while net worth shows less
cyclical variation. Since we are primarily interested in the effect of wealth on the demand for cash and working balances across firms, rather than over time, there is no advantage in using a measure of wealth, like total assets, which fluctuates markedly over time.

Our measure of profits may be least satisfactory. As mentioned we are not primarily interested in differences in profits across firms, but rather in the effect of changes in profits on the portfolio of a firm over time. In our time-series study we therefore decided to use a profits measure that was an index for each firm. Our measure of \( P \) for the time-series model is gross profits in period \( t-1 \), divided by 1951 gross profits for each firm. The reasons for lagging our profits measure are two. First, for fixed plant and equipment, and perhaps to some extent inventories, there is necessarily some lag between increased profitability and acquisitions of these assets (and therefore some lag until cash and working balances are decreased). Second, current profits are not only the return on assets, but usually also a cash inflow which in the short-run will be reflected in increases in cash and working balances.

The reason for using gross profits, versus profits after taxes, is that the latter measure is unsatisfactory due to excess profits adjustment of the Korean war. Finally, our profits measure for the time series analysis is an index of profits for each firm, rather than an index of profits as a ratio to some assets as inventories, or fixed capital. It was felt that the profits expressed as a rate of return on some asset would be unduly influenced by fluctuations of the divisor (inventories or net fixed capital). Clearly, over a very long period, the measure of profits
we have used would be so dominated by trend as to be unworkable, but for present purposes the measure should be satisfactory.

Our empirical work is on data for corporations, with assets over $10,000,000. The data were collected for the period 1939-1956 by the Board of Governors of the Federal Reserve System, [3], and the data for 1947-1956 for 237 of the firms were available for this study. In previous work a cross section-time series regression equation were estimated for 204 of the firms for a different model in which all variables were deflated by total assets. Since then an attempt was made to extend the data to a later date, but this effort was somewhat unsatisfactory. ¹

¹. The data collection could not be made comparable to the original Federal Reserve data with respect to investment outlays, and certain borrowings of the firms, without greater effort than seemed justified. In addition, mergers and acquisitions in recent years made data collection more difficult than originally anticipated. I would like to thank David Segal and Seong Park of Yale University for their assistance in this work.

However, annual data for the period 1947-1959 for 42 firms in the chemical, steel and machinery industries were available.
SECTION III

Estimates of the Models

Part A. Time Series

In equations (1.3) and (2.3) we assume the error terms are log normally distributed. Taking the logarithms of these equations and suppressing the coefficient on the error term we may estimate the coefficients by least squares. These estimates of the coefficients, and the standard errors of the coefficients are presented in Table 1. Also, $R^2$ and the F values of the equations are presented.

The high $R^2$'s are to be expected since cash and working balances are being correlated with their lagged values. While a better index of the amount of variance explained by the regression equation could be had if we could have used the changes in cash and working balances and the dependent variables, this could not be easily done as long as a multiplicative relationship was used.

For the cash equation all coefficients are of the expected sign, and for working balances the same holds true except for the coefficient on the bill rate. We would expect the coefficient on the bill rate for working balances to be positive, and even if it were to be negative, we would not expect it to be algebraically smaller than the coefficient on the bill rate for cash. In short, our estimates of the bill rate coefficient for working balances is unsatisfactory. An explanation of this is that when the bill rate rises inventories and other assets are also rising, which leads firms to run down their securities. We would
TABLE 1
Estimates of Regression Equations for Cash and Working Balances
TIME SERIES MODEL
(Standard errors in Parentheses)

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dependent</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cash (C)</td>
<td>Cash Plus Securities (W)</td>
</tr>
<tr>
<td>Common Log of Intercept</td>
<td>-.3407</td>
<td>-.4322</td>
</tr>
<tr>
<td>$T_t$ (Sales)</td>
<td>.0894</td>
<td>.0726</td>
</tr>
<tr>
<td></td>
<td>(.0364)*</td>
<td>(.0504)**</td>
</tr>
<tr>
<td>$N_t$ (Net Worth)</td>
<td>.1042</td>
<td>.1659</td>
</tr>
<tr>
<td></td>
<td>(.0432)*</td>
<td>(.0648)**</td>
</tr>
<tr>
<td>$P_t$ (Bill Rate)</td>
<td>-.0937</td>
<td>-.1267</td>
</tr>
<tr>
<td></td>
<td>(.0309)**</td>
<td>(.0422)**</td>
</tr>
<tr>
<td>$P_{t-1}$ Gross Profits $t-1$</td>
<td>-.2820</td>
<td>-.0762</td>
</tr>
<tr>
<td></td>
<td>(.0234)</td>
<td>(.0328)**</td>
</tr>
<tr>
<td>$C_{t-1}$ (Lagged Cash)</td>
<td>.7935</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0274)**</td>
<td></td>
</tr>
<tr>
<td>$W_{t-1}$ (Lagged Working Balances)</td>
<td></td>
<td>.7789**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.0311)**</td>
</tr>
</tbody>
</table>

N 504 - 42 firms, 12 observations per firm.

$R^2$ (corrected for degrees of freedom) .939** .916**

F 1300 915

* Coefficient to which standard error refers is different from zero at the 5% level of significance.

** Coefficient or correlation coefficient is different from zero at the 1% level of significance.
expect this effect to show up in our profits variable, and to be sure
profits have a negative sign, but does not allow us to make a satisfactory
estimate of the bill rate, on working balance.

The coefficients presented in Table 1 are what we might call short
run coefficients, since they tell the immediate reaction of a firm to a
change in each of the variables. Writing the estimates of equation (1.3),
where ' is an estimate.

\[(1.4) \quad \log C_t = -0.3407 + 0.0839 \log T_t + 0.1042 \log N_t - 0.0937 \log R_t
- 0.0280 \log P_{t-1} + 0.7935 \log C_{t-1} \quad .\]

In the long-run, \( C^* = C_t = C_{t-1} \), and a firm has adjusted to an equilibrium.
Setting \( C_{t-1} = C_t = C^* \) in (1.4) and rearranging terms we derive estimates
of the long-run coefficients on our variables, which are,

\[(1.5) \quad \log C^*_t = -1.6495 + 0.439 \log T_t + 0.512 \log N_t - 0.460 \log R_t
- 0.138 \log P_{t-1} \quad .\]

Following the same procedure for working balances we have,

\[(2.4) \quad \log W^*_t = -1.9354 + 0.328 \log T_t + 0.750 \log N_t - 0.573 \log R_t
- 0.344 \log P_{t-1} \quad .\]

We may first note that as expected, the profits variable is more
negative for working balances than for cash balances. Thus increased
profitability of the firm takes more away from securities than cash.
We may also note from the above derivation of the long-run coefficients that the reaction coefficients $\beta_1$ and $\beta_2$ are similar in magnitude, namely .207 and .221 for cash and working balances respectively. These coefficients are between zero and one, and suggest that firms add just cash and working balances to their desired levels quite slowly.

For cash both the short-run and long-run negative elasticities on the bill rate are quite substantial. A 10% change in the bill rate, say from 3% to 3.3% would produce an estimated short-run decrease in cash holdings of firms of .94% and a long-run decrease of over 4%. These are not modest decreases in cash, since non-financial corporations as a whole hold about $25 billion in money.

The coefficient on net worth for both cash and working balance is positive and significant as hypothesized. The larger coefficient on net worth for working balances than cash is consistent with the fact that larger firms hold most of the securities held by businesses.

The sales coefficient for cash is of particular interest, since it is certainly well below 1.0. The long-run coefficient of .433 ± .187 is not different from 0.5 at the 1% or 5% levels of significance. This test is performed using an approximation for the standard error of the long-run coefficient [5, p. 150]. The approximation is the same as used in estimating the variance of the coefficient of variation, [15]. Thus the results of our time series model are consistent with a square root rule for cash.
Analysis of the Residuals:

The residuals from the equations of Table 1 have been estimated, and some tests performed. The sums of the estimated residuals by year are presented in Table 2 for both cash and working balances. An F statistic was used to test whether there was a significant difference in the average residual by year. For cash the F value was 1.77, with 11 degrees of freedom in the numerator and 406 degrees of freedom in the denominator, while the corresponding value for working balances was 5.08. The critical value of F is 1.81 (2.29) at the 5% (1%) level of significance, so we would accept the null hypothesis that the means of residuals of cash were equal and reject the hypothesis for working balances.

These results suggest that for working balances use of additional explanatory variables that are related to firm activity over time might improve the regression relation. This is also suggested by the perverse relation that was found between the bill rate and working balances. Several indices have been plotted against these residuals without any close correspondence emerging. In his study Meltzer [11] estimated coefficients by year, and the pattern of his average coefficients (averaged over industries) in the six years in which the studies overlap, did not seem to have any relation to the residuals of Table 2. We conclude that our time series model for working balances does not present an adequate picture of year to year changes.

F statistics have also been used to test whether there was a significant difference among the mean residuals of the 42 firms of the
TABLE 2
Sums of Residuals by Year
Time Series Models

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimate of Average Residual For Cash</th>
<th>Estimate of Average Residual For Working Balances</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948</td>
<td>- .00098</td>
<td>- .00115</td>
</tr>
<tr>
<td>1949</td>
<td>.00003</td>
<td>.00025</td>
</tr>
<tr>
<td>1950</td>
<td>.00099</td>
<td>.00168</td>
</tr>
<tr>
<td>1951</td>
<td>.00080</td>
<td>.00161</td>
</tr>
<tr>
<td>1952</td>
<td>- .00040</td>
<td>- .00135</td>
</tr>
<tr>
<td>1953</td>
<td>- .00023</td>
<td>- .00014</td>
</tr>
<tr>
<td>1954</td>
<td>- .00022</td>
<td>- .00075</td>
</tr>
<tr>
<td>1955</td>
<td>- .00053</td>
<td>.00170</td>
</tr>
<tr>
<td>1956</td>
<td>.00052</td>
<td>- .00055</td>
</tr>
<tr>
<td>1957</td>
<td>- .00068</td>
<td>- .00230</td>
</tr>
<tr>
<td>1958</td>
<td>- .00020</td>
<td>- .00025</td>
</tr>
<tr>
<td>1959</td>
<td>- .00016</td>
<td>.00125</td>
</tr>
</tbody>
</table>
sample. The critical values of \( F \) were 1.42 (1.64) at the 5\% (1\%) levels of significance with 41 and 456 degrees of freedom. The \( F \) value for cash was 1.70 and for working balances, 1.36, and we accept the hypothesis that the means of residuals of firms are equal for working balances, and for cash we reject the hypothesis. To further check the result for cash balances we examined differences in means of residuals by industry for cash. There was no significant difference in the industry means. In fact one steel firm causes all the trouble, and it contributes about one third of the variance among the mean residuals of cash of the 42 firms. Exclusion of this firm would mean that there would be no significant difference in the mean residual of the 41 remaining firms.

Our final work with the residuals relates to compensating balances. As was mentioned earlier bank loan information was available for the time series data for only a part of the period. The maximum value of short-term bank loans and the maximum of the sum of short-term and long-term bank loans was collected for each firm during the years the data were available (1948-1956). If compensating balances were related to the size of bank loans of the firm, then we would expect a positive relation between bank loans and the estimated value of the mean residual for the firm. No relation was found, but only 26 of the 42 firms held any bank loans, so the number of observations was small. We will present more evidence on compensating balances in our discussion of the cross section model to which we now turn.
Part B. Estimates of the Cross-section Model

The estimates of equations (3.0) and (4.0) are presented in Table 3. At the bottom of Table 3 we have presented the intercepts for the various classes of firms in an alternate form that may be more intelligible. We will first discuss these coefficients on the classificatory variables.

The five intercepts (the four dummy variables plus the common intercept) were used in a regression equation (or analysis of variance) without the other variables. These classificatory variables by themselves gave an $R^2$ of .078 for cash with an $F$ ratio of 3.93 which is significant at the 5% and 1% levels. For working balances the corresponding $R^2$ was .159 and the $F$ ratio was 8.75, also significant. Although a few of the classificatory variables do not have significant coefficients in some equations we have retained all of them in both equations.

Analysis of the separate intercepts indicates that electric power holds less cash and working balances than firms in other industries. This is as one might expect, since electric power is apt to have a smooth growth in sales and little need for liquidity to meet inventory fluctuations, since the latter are likely to be negligible compared to other industries.

It was argued earlier that firms who held no securities ought to hold less working balances, given the values of other variables, than firms holding both cash and securities. This would be true because a given amount of cash provides more liquidity for the firm than the same amount of cash and securities. Looking at the pattern of intercepts we find this is the case. The intercept for working balances for firms with securities


<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>Cross Section</th>
<th>Long-run Time Series$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cash</td>
<td>Working Balances</td>
<td>Cash</td>
</tr>
<tr>
<td>Common log of</td>
<td>-.8029</td>
<td>-.8355</td>
<td>.433</td>
</tr>
<tr>
<td>intercepts</td>
<td>(.0942)**</td>
<td>(.0956)**</td>
<td>(.187)*</td>
</tr>
<tr>
<td>Sales(S)</td>
<td>.4063</td>
<td>.2771</td>
<td>.512</td>
</tr>
<tr>
<td></td>
<td>(.0579)**</td>
<td>(.0588)</td>
<td>(.253)*</td>
</tr>
<tr>
<td>Net Worth(N)</td>
<td>.4787</td>
<td>.7447</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0579)**</td>
<td>(.0589)**</td>
<td></td>
</tr>
<tr>
<td>Bank Loans(L)</td>
<td>.0643</td>
<td>-.0165</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0298)*</td>
<td>(.0303)</td>
<td></td>
</tr>
<tr>
<td>X ,No Bank Loans</td>
<td>.1285</td>
<td>.1548</td>
<td></td>
</tr>
<tr>
<td>(34 firms)</td>
<td>(.0373)*</td>
<td>(.0581)**</td>
<td></td>
</tr>
<tr>
<td>X ,No Securities</td>
<td>-.1257</td>
<td>-.2624</td>
<td></td>
</tr>
<tr>
<td>(39 firms)</td>
<td>(.0415)**</td>
<td>(.0422)**</td>
<td></td>
</tr>
<tr>
<td>X ,No Bank Loans nor</td>
<td>.3224</td>
<td>.1314</td>
<td></td>
</tr>
<tr>
<td>Securities (4 firms)</td>
<td>(.1093)**</td>
<td>(.1110)</td>
<td></td>
</tr>
<tr>
<td>X ,Electric Power</td>
<td>-.1964</td>
<td>-.3397</td>
<td></td>
</tr>
<tr>
<td>(28 firms)</td>
<td>(.0530)**</td>
<td>(.0539)**</td>
<td></td>
</tr>
</tbody>
</table>

237 observations for each equation.

$R^2$ (corrected for degrees of freedom) .842** .875**

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The coefficients on the Classificatory variables are here presented in an alternative form:

<table>
<thead>
<tr>
<th>Working Balances</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Intercept if firm has bank loans and securities</td>
<td>.8355</td>
</tr>
<tr>
<td>2. Intercept if firm has no securities</td>
<td>-1.0971</td>
</tr>
<tr>
<td>3. Intercept if firm has no bank loans</td>
<td>-.6807</td>
</tr>
<tr>
<td>4. Intercept if firm has no bank loans nor securities</td>
<td>-.8117</td>
</tr>
<tr>
<td>5. If firm is in electric power industry, .3397 should be subtracted from the above intercepts for working balances, and .1964 should be subtracted for cash.</td>
<td></td>
</tr>
</tbody>
</table>

1. Derivation of coefficients is discussed on p. 16 above, and approximation of standard errors of coefficients is discussed on p. 17 above.
* Coefficient different from zero at 5% level of significance.
** Coefficient different from zero at 1% level of significance.
is -.855 and for firms with no securities, - 1.097. The group of firms with no bank loans, those with securities have an intercept for working balances of - .6807 and those without securities, - .8117. So within each bank loan category firms with no securities hold less working balances than firms with securities.

However, the above result for firms with bank loans, because contrary to hypothesis, firms with no securities hold less cash than firms with securities. An interpretation of this result is that firms without securities are not simply in a position where it is unprofitable to substitute bills for cash. Were this the case, we would expect these firms to hold more cash than firms with securities. Rather our dummy variables are correlated with some other attributes of the firm, as of liquidity needs or managerial preferences. Evidently the holding of no securities is a proxy for small liquidity needs, or a desire for small cash holdings. The pattern of intercepts suggest that

2. As a rough check on our results we have classified firms by their coefficient of variation of sales, and found that there is no particular concentration of firms with low variance of sales in the group of firms with no securities. The coefficient of variation of sales was computed from annual figures, so this is not a very satisfactory test. Firms with no securities are 39 of 237 firms or 16.4%. The industry with the largest percent of firms with no securities is retail trade where 10 of 34 firms or almost 30% hold no securities. As was mentioned earlier, retail trade is an industry that might have low cash needs because inventories are quite liquid. The fact that over 1/4 of the firms with no security holdings are in an industry that a priori might have low liquidity requirements is consistent with the proposition that firms with no security holdings is a proxy for other factors.
firms holding no bank loans hold more cash and working balances than firms with bank loans. This result might be interpreted to mean that firms with no bank loans do not or cannot maintain ready access to borrowing and must perforce, keep a more liquid position. However, once a firm borrows from banks its cash balances, which we take to be compensating balances, do increase with the amount borrowed. The coefficient on bank loans for cash is .0643, and significant at the 5% level. This elasticity is low in the sense that most lines of credit require 20% of a loan in balances. We interpret these results to suggest that compensating balances may be of moderate importance in analyzing the demand for cash by firms but bear little relation to working balances.

Table 4 presents the long-run estimates for sales and net worth from the time series equations for comparison with the cross section coefficients. For both cash and working balances, the sales and net worth coefficients for the time series are all within one standard deviation of the cross section coefficients, using the standard errors of the latter coefficients which are the smaller. The agreement is thus quite strong. The distribution by size of sales of the 237 firms in the cross-section is given in Table 5. Though the 42 firms in the time series were somewhat larger than the other 195 firms comprising the cross-section, the two sets of firms are fairly evenly distributed by size of sales.
TABLE 4

Distribution of Firms by 1953-56 Average Sales

<table>
<thead>
<tr>
<th>Size Class</th>
<th>42 Time-Series Firms</th>
<th>Other Cross Section Firms</th>
<th>Total Cross Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below $50 million</td>
<td>0</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>$50 to $100 million</td>
<td>8</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>$100 to $150 million</td>
<td>8</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td>$150 to $300 million</td>
<td>9</td>
<td>39</td>
<td>48</td>
</tr>
<tr>
<td>$300 to $500 million</td>
<td>3</td>
<td>29</td>
<td>32</td>
</tr>
<tr>
<td>$500 to $1 billion</td>
<td>8</td>
<td>27</td>
<td>35</td>
</tr>
<tr>
<td>Over $1 billion</td>
<td>6</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>195</td>
<td>237</td>
<td></td>
</tr>
</tbody>
</table>
SECTION IV

Comparisons with Previous Estimates and Conclusions

In Table 5 estimates of the elasticity of cash to sales of business are presented. Table 5 contains point estimates of the coefficients (which are in some cases averages of coefficients) and their standard errors (or standard deviations of coefficients where several elasticities were estimated). There are a number of differences among the studies in both the sample used and in the specification of the regression equation. Some of these sources of difference among the estimates are described in Table 5.

The sample of firms used in this study were large firms -- larger on the average than those included in any of the other studies. In addition, the present estimates (including 4.a of Table 5) are the only ones based on observations of individual firms. The industries in the time series part of this study were chemicals, steel and machinery, and these industries are represented in the samples of Kesselgoff and Meltzer, and partly in the sample used by Keaten. The only industry specific to this study is electric power which has been used in estimating the cross section model of this paper. Industries included in the other studies and not included in this study were textiles, lumber, paper, coal, and stone, clay and glass.
### TABLE 5

Comparative Estimates of Elasticity of Cash to Sales for Businesses

<table>
<thead>
<tr>
<th>Study</th>
<th>Elasticity of Cash to Sales</th>
<th>Standard Error of Coefficient</th>
<th>Nature of Observations</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Kisselgoff(^1)</td>
<td>1.14</td>
<td>.53</td>
<td>Time Series</td>
<td>19 annual observations on the averages of data for a sample of firms for the period, 1921-39. Firms were large</td>
</tr>
<tr>
<td>2. Meltzer(^2)</td>
<td>a. 1.042 b. 1.028</td>
<td>a. .042 b. .032</td>
<td>Cross-Section</td>
<td>126 regression equations, one for each of 14 industries in each of 9 years, 1938, 1944, 1946, 1951, and 1953-57. Each regression equation was estimated from 8 to 12 observations, on size classes in each industry. Data from Statistics of Income.</td>
</tr>
</tbody>
</table>
TABLE 5 (Continued)

<table>
<thead>
<tr>
<th>Study</th>
<th>Dependent Variable(^5)</th>
<th>Form of Regression Equation(s) and Independent variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kisselgoff</td>
<td>Annual Average of Cash Balances of a sample of firms.</td>
<td>Linear regression equation with 2--5 year government bond rate, profits, and an estimate of transactions (as opposed to simply sales) as the three independent variables.</td>
</tr>
<tr>
<td>Meltzer</td>
<td>Mean cash balances of each asset size group.</td>
<td>Linear in logarithms of variables. Sales is the independent variable. (Regression equations employing sales and the square root of sales were also estimated. See footnote 2 below.)</td>
</tr>
<tr>
<td>Keaten</td>
<td>Quarterly cash balances by industry.</td>
<td>Linear in logarithms of variables. Sales, bill rate, and a measure of the variance of sales are the three independent variables in each of the five regression equations.</td>
</tr>
<tr>
<td>Heston</td>
<td></td>
<td>Linear regression equation with lagged cash balances, bill rate, sales, inventories, net receivables, and the change in taxes due as the 6 independent variables. All variables were deflated by mean total assets of the firm, 1943-55. A dummy variable was used (a separate intercept was estimated) for each firm.</td>
</tr>
</tbody>
</table>

- **a.** Annual Change in cash balances by firm and year. |
- **b.** Annual cash balances by firm and year. |
- **c.** Average of 1953-56 cash balances for each firm. |

---

1. Elasticity calculated from linear regression equation at mean values of variables, and standard error calculated assuming variance of means was zero. See [5, p. 152].
TABLE 5 (Continued)

Footnotes continued.

2. Coefficient a. is the mean of the 126 coefficients, and standard error a.1 is the unbiased estimate of the standard deviation of the means of the coefficients by year, and standard error a.2 is the standard deviations of the means of the coefficients by industry. Coefficient b. is the mean of 112 coefficients, the excluded coefficients being those for the only prewar year, 1938. Standard errors b.1, and b.2 are the standard deviations of the means of the coefficients by year and industry respectively, with 1938 coefficients being excluded. The individual standard errors of the coefficients were all less than 0.05, the t values of the coefficients falling between 251.5 and 20.2. [11, p. 10]. The calculations above were done by the author from the information supplied in Tables 1 and 2 of Meltzer's paper [11, pp. 9-10]. Meltzer also estimated equations using a quadratic form and these latter are not discussed in this paper because they are less comparable, and because Meltzer does not show any strong preference for the quadratic form.

3. The elasticity is the average of the elasticity for the five industries studied and the standard error a. is the unbiased estimate of the standard deviation of the 5 elasticity coefficients. Standard error B. is the average of the standard errors of the coefficients. The estimated coefficients, their standard errors, and $R^2$ for the five regression equations were: Tobacco, .327 (.190), $R^2 = .111$; Textiles, .452 (.201) $R^2 = .788$; Chemicals, .251 (.054), $R^2 = .468$; Electrical Machinery, .504 (.053), $R^2 = .826$; and Motor Vehicles, .516 (.070), $R^2 = .684$.

4. Elasticity is calculated at the mean values of the variables in the regression equation and the standard error is estimated assuming the variance of the mean of cash and sales is zero. See Heston [5, pp. 150-152].

5. In all studies cash balances consist of currency, demand deposits, and time deposits.
In Table 4 we have not presented correlation coefficients and some discussion may be warranted. Though correlation coefficients varied somewhat among the studies, there were ample reasons why high correlations were obtained in certain studies and lower correlations in others. The high correlations ($R^2$ of over .9) of Kisselgoff, Meltzer, and the time series of this study (4.b) are in part due respectively to substantial trend in variables, few observations and use of asset size classes, and correlation of current and lagged values of the dependent variable. The correlation for the cross section part of this study (4.c) is over 80% ($R^2$), and has some size effects working in its favor. The correlation for a previous study by the author (4.a) is about 36% ($R^2$), which should be in the same league with the above studies, since the dependent variable was change in cash holdings rather than the stock of cash. The $R^2$'s for Keaten's study ranged from .11 to .83, (and their average was .58, for what it may mean) and are quite respectable when it is considered that the data was quarterly (which is not an advantage for cash correlations) and that the firm composition of each of the industries changed over the period. In summary, we present the above information on correlations, but will make no use of it in this paper.

Going through the elasticities of Table 4 in order, we find the standard error of Kisselgoff's estimate is large relative to the other studies. The 95% confidence interval of Kisselgoff's estimate includes all of the other estimates. Kisselgoff's study is most directly comparable with Keaten's in that both are strictly time series studies. The differences between these studies might be due to differences in behavior of firms
during the two periods. In Table 4 we have noted that the mean coefficient
for Meltzer's study is 1.042, but if 1938 is excluded, the mean is 1.028.
Excluding 1938, the standard deviation of the means of the coefficients
by year of Meltzer's study is .011. Since the 1938 mean value of co-
efficients was 1.157, there is a substantial difference in pre-war and
post-war behavior as evidenced by Meltzer's coefficients. While the
absolute difference between the average of 1938 coefficients (1.157) and
the average of the remaining coefficients (1.028) is nowhere near as large as
the difference between Kesselgoff's (1.14) and Keaten's (.410) estimates, the
standard errors of the latter are also larger. In short, we would attribute
a part of the difference between the estimates of Kesselgoff and Keaten
to lower demand for cash in the post-war period.

The estimates of this study (4.b and 4.c) differ significantly from
the estimates of Meltzer and (4.a). One major difference in specification
between these studies is that net worth of the firm is included in the
former. If we are correct in arguing that cash balances are needed to
lubricate a portfolio, regardless of the level of income, then the demand
for cash should include some measure of the wealth of a firm. In any event
the inclusion of net worth in the equations of this study probably accounts
for the bulk of the differences between the estimates of Meltzer and 4.a,
and the estimates of 4.b and 4.c. Since the ratio of net worth to sales tends
to rise with the level of sales, inclusion of net worth as a variable will give
rise to a lower sales elasticity so long as firms have a wealth demand for
cash. Thus we think the elasticities of the present study (.439 ± .187, and
.406 ± .058) provide a closer description of the behavior of firms because
they are derived from a model which specifies a wealth demand for cash. However, our regression equations are misspecified to the extent that there exists a systematic relation of net worth and sales across firms. The results of this misspecification, is probably to make our estimates of the sales elasticity too low.

We may incorporate comparisons of other coefficients from the above studies in the conclusions of the paper. The interest elasticity of cash balances was negative in all time series regressions in Table 5. Kesselgoff found a strong relation for the inter-war period, and the other studies representing the post-war period exhibit the same pattern for firms which has been repeatedly documented for aggregative data. Keaten's study which includes observations for the 1950-60 period, found a negative relation in the five industries he studied, the mean elasticity being -.071 with a standard deviation of .050. The present study estimated a short-run elasticity of -.0957 (+.301) and a long-run elasticity of -.460 (+.178), over the time period 1948-1959. Keaten's estimates are the response over a quarter in cash balances to a change in the interest rate, and the results may not differ from the estimates of this study as much as they appear to. In particular, if there is some lag in the response of firms in adjusting their cash balances to desired levels, then his estimates might be on the low side. The present study was not able to provide a satisfactory estimate of the elasticity of working balances to the bill rate.
As noted there is strong evidence of a wealth demand for both cash and working balances. In addition it was found that as would be expected a change in profits reduced the demand for cash balances to working balances, the latter showing the largest decrease. The model produced some evidence that firms with bank loans increase their cash holdings in relation to the volume of their bank loans. We interpret this result as evidence that compensating balances play some part in the demand for cash balances, though not for working balances.

If we allow for a wealth demand for cash and working balances, our results show that demand relations for firms are consistent with the models of cash holdings of Baumol and Tobin. Within the limitations of our sample which consisted of large corporations, we have found evidence consistent with a square root rule for cash, and a similar result for a somewhat different model has been found by Keaten. Put another way, the evidence of this study raises serious doubts about the existence of a simple proportional relation between cash and sales for businesses.
REFERENCES


