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Some Empirical Estimates of Short Run Price and Output Policies

Edwin S. Mills

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Price and Output Policies *

by

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1. Introduction

The purpose of this paper is to report some attempts to estimate short run reactions of firms to changes in their inventory, sales anticipations, etc. The paper is a summary of four industry studies which are part of a larger project on optimum price and output policies on which the author is engaged. The technique used in the study summarized in this paper is the very simple one of finding linear approximations to two complicated price and output decision rules and then estimating the linear approximations from relevant monthly (or, in one case, quarterly) data.

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2. The theoretical model

The decision model which underlies the regressions estimated in this study is an extension of the model presented in [4], and can only be summarized here. It is a model of short run adjustments of a firm's price and output in response to changes in its sales expectations, inventory level and other variables.

Assume that a firm behaves as if it produces a single, homogeneous, non-perishable commodity. In any period, it may incur the following four costs. Production cost is a function only of the output produced in the period in question. Inventory cost in any period is a function of the inventory carried between that period and the next. There is a cost associated with changes in the rate of production. In any period, this cost is a function of the absolute difference between the production in that and the preceding periods. Although not included in the formal model, a similar cost may attach to changes in price. Allowance is made for the possibility of such a cost in the regression estimates presented below. Finally, there is a cost of shortage which the firm incurs in any period in which the amount demanded exceeds the sum of its production in that period and the inventory remaining from the preceding period. This cost arises because the firm is assumed to have a multi-period planning horizon, but is assumed to approximate its multi-stage programming problem by a single-stage problem in which it attaches a value to inventory and a cost to shortage whenever either of these results from a period's activity. The value attached to a unit of inventory is an es-
estimate of the increase in the present value of future profits which will result from the availability of this unit. The cost of a unit of shortage is an estimate of the decrease in the present value of future profits which will result (owing to loss of goodwill, etc.) from an inability to supply a unit of current demand. If the firm knew these two figures it would have solved its multi-stage programming problem. The approximation results when it is assumed that the firm behaves as if these two figures were known constants. The cost of shortage is only relevant in an imperfectly competitive market and such a market is implied by the assumptions in the next paragraph. It is assumed that all the above cost functions are unchanging over the period of observation.

The demand for the firm's product is assumed to consist of a systematic part and a non-systematic part. The systematic part is specified to be a decreasing function of the price set by the firm and to depend also on other factors -- such as time, national income, etc. --which are beyond the firm's control. The non-systematic part consists of an additive random variable with a distribution which does not depend on the price set by the firm and does not change over the period of observation. The assumption that the random term is independent of the price set cannot be true at all prices, since demand cannot be negative, but may be a good approximation over the range of observed prices. These assumptions imply that the demand equation can be written

\[ x_n = x_n^e + u_n \]  
(1)
where \( x_n \) is the observed demand, \( x^e_n \) is the systematic part and \( u_n \) is the random term. It is assumed that the firm knows \( x^e_n \) (that is, it knows the effect of its price and of the systematic exogenous factors on its demand) and the probability distribution of \( u_n \). Equivalently, it can be assumed that the influences on its demand which the firm does not know are included in \( u_n \), that these unknown influences have stable statistical properties, and that the firm has estimated these statistical properties from past experience. For convenience, it is assumed that \( u_n \) has a zero expected value, although a non-zero expected value would be absorbed in the constant terms in the regressions and would therefore not be perceived by the estimating technique employed in this study. \( x^e_n \) is therefore the expected value of demand in period \( n \) and will sometimes be referred to simply as anticipated demand.\(^1\)

\(^1\) This is perhaps a misnomer. The model assumes that the firm's anticipations consist of a known density function of demand at each price, all the moments of which are relevant to the firm's decisions. It is the assumption that only the first moment of the density function changes from period to period that justifies the terminology.

On the basis of these cost and demand conditions, it is assumed that the firm maximizes the expected value of its profits with respect to its price and output decisions. This implies that price and output are determined by two complicated simultaneous decision rules involving not only these dependent variables but also expected demand and the predetermined
variables lagged production and lagged inventory.

Partly on the basis of the structure of the decision rules and partly on the basis of intuitive plausibility, the following linear equations are suggested as approximations to the two decision rules:

\[ z_n = \beta_{10} + \beta_{11}x_n^e + \beta_{12}z_{n-1} + \beta_{13}I_{n-1} + \epsilon_{1n} \]  
\[ p_n = \beta_{20} + \beta_{21}x_n^e + \beta_{22}z_n + \beta_{23}I_{n-1} + \beta_{24}p_{n-1} + \epsilon_{2n} \]

(2)  
(3)

Here, \( z_n \) is production during period \( n \), \( I_n \) is the inventory level at the end of period \( n \) and \( p_n \) is the price set for period \( n \). \( \epsilon_{1n} \) and \( \epsilon_{2n} \) are random terms which represent misspecifications of the model resulting from the approximate nature of the linear system and, in application, from the influence of variables omitted from the model.

If there is no cost of changing production, \( \beta_{12} = 0 \); if there is no cost of changing price, \( \beta_{24} = 0 \). Intuition and formal analysis of the decision rules support the following restrictions on the \( \beta \)'s.

\( \beta_{11} \) should be positive since an increase in anticipated demand makes profitable an increase in production. \( \beta_{12} \) should be positive since the cost of changing production introduces a tendency for this variable to be positively autocorrelated. \( \beta_{13} \) should be negative since a large inventory remaining from the previous period is a substitute for a high level of production.

\( \beta_{21} \) should be positive if movements in \( x_n^e \) are mostly exogenous.
so that most changes in price are responses to shifts in the demand curve rather than movements along it. $\beta_{21}$ should be negative if changes in price are mostly movements along the demand curve. There can be little doubt that the former is the case in most applications of the kind considered in this study. These applications are to are to monthly and quarterly industry totals from the decade of the 1930's. Presumably, such short run movements in sales result mainly from seasonal, cyclical and secular shifts in demand rather than from period-to-period changes in the price of the commodity in question. $\beta_{22}$ should be negative. If $z_n$ is large because, say, $z_{n-l}$ was large and it is costly to reduce production quickly, then it pays to reduce price in order to stimulate demand and reduce the probability of accumulating a costly inventory. $\beta_{23}$ should be negative because a price reduction is one way of reducing excessive inventory. $\beta_{24}$ should be positive for the same reason that $\beta_{12}$ should be positive.

(2) and (3) were estimated from several sets of experimental observations which had been generated by a Monte Carlo study of the decision rules. This study, which cannot be reported in detail here, confirmed the above sign restrictions on the $\beta$'s. Although variation in the cost and demand parameters had considerable effect on the estimates of the $\beta$'s, in no case were the sign restrictions violated. The same study also suggested that observations generated by the decision rules can be very closely approximated by (2) and (3).

Unfortunately, economists do not normally have observations of $x^e_n$. 
which appears in (2) and (3), among their sample data. Hence, the procedure followed in the empirical studies is to eliminate $x_n^e$ by (1). This gives

$$z_n = \beta_{10} + \beta_{11}x_n + \beta_{12}z_{n-1} + \beta_{13}I_{n-1} + \epsilon_{1n}$$

$$p_n = \beta_{20} + \beta_{21}x_n + \beta_{22}z_n + \beta_{23}I_{n-1} + \beta_{24}p_{n-1} + \epsilon_{2n}$$

(4) and (5) now contain only observable variables. The use of $x_n$ as a proxy for $x_n^e$ introduces errors of observation in one of the independent variables in the model. This means that (4) and (5) include disturbances of both the error-in-variable and the error-in-equation types. Depending on the correlations between $u_n$ and the $\epsilon_{in}$, on their relative variances and on the identifiability of the system, one estimating procedure or another might be appropriate. In fact, in this investigation only single-equation, least squares estimates have been made. At best, this will result in estimates of $\beta_{11}$ and $\beta_{21}$ which have a small sample bias toward zero.

The major advantage in the use of (4) and (5) is that they permit estimation of the structural parameters without prior estimation of demand expectations and without imposing on these expectations the frequently-made assumption that expectations are generated by some weighted average of past observed demand levels.
3. The data used

The empirical study reported in this paper consists of the estimation of (4), (5) and an alternative form specified below from time series observations of price, output, sales and inventory for four U. S. industries during the 1930's. Firm or establishment data would be preferable to industry data, but are generally unavailable. The four industries are southern pine lumber, cement, pneumatic tires and department store shoes. All the data are either published or analyzed extensively in easily accessible sources. Therefore, only brief comments and references to original sources will be included here.

The southern pine lumber data consist of 96 monthly observations of each variable from January 1933 to December 1940. The data were originally collected by the Southern Pine Association and the physical volume series are published in [7]. The price series was obtained directly from the Association. The physical volume series are analyzed in Abramovitz's well known study, [1], to which reference is made for further detail.

The cement series come from two distinct sources. The physical volume series consist of 58 quarterly observations of each variable from the third quarter of 1927 to the end of 1941. These series are published in [5]. They have been analyzed by Abramovitz in [1] and by Modigliani and Sauerlender in [6]. Also available for this industry are published demand expectations in the form of the well known shippers' forecasts. This series has been analyzed extensively, especially by Ferber in [2], from
which the series was obtained for this study. The shippers' forecasts were converted to a base which is comparable with that of the other series by a procedure similar to that followed in [6]. The adjusted shippers' forecasts provide a second proxy for demand anticipations and a second set of calculations was made for cement using this proxy instead of observed sales in the regressions. The price series used in the calculations is the index of wholesale cement prices compiled by the Bureau of Labor Statistics and published in [8]. This index is available on a monthly basis only since 1933. Therefore the price equations for cement are computed from only 36 observations for each variable, from the first quarter of 1933 to the end of 1941. Since the shippers' forecasts are available on only a quarterly basis, the other cement variables were converted to this base for all the calculations.

The pneumatic tire data consist of series from two sources as well. Each physical volume series contains 144 monthly observations from January 1929 to December 1940. The price series covers the same period except that the index is not available for the first six months in 1939 and the first six months in 1940. Therefore, the price regressions are based on only 132 observations. The physical volume series were collected by the Rubber Manufacturers' Association and published in [7]. These data are also analyzed in [1]. The price series is the pneumatic tires component of the wholesale price index published by the Bureau of Labor Statistics in [8]. In these data there is a major discrepancy in coverage between the price and physical
volume series since the former represent only tires sold to dealers for replacement purposes whereas the latter represent, in addition, tires sold to automobile manufacturers for original equipment. It is well known that prices of original equipment tires are generally lower than those of replacement equipment tires, but it has not been possible to make an adjustment in the index.

The department store shoe data consist of 108 monthly observations of each variable from January 1932 to December 1940. These series are analyzed by Ruth Mack in [3], and were kindly furnished by her. The price index is series 8, p. 264, in [3]. It was originally compiled by the National Industrial Conference Board. The sales and inventory data are respectively series 29, p. 268, and series 50, p. 272, in [3]. They are based on sample observations of seasonally adjusted dollar value figures collected by the Federal Reserve Board. The index was constructed from the Federal Reserve data by the National Bureau of Economic Research and deflated by the National Bureau with the price series discussed above. The production series used in the computations was calculated from the sales and inventory series by using the identity \( z_n = x_n + \Delta I_n \), Mrs. Mack's inventory index having first been adjusted to its correct base relative to sales.

Three important factors distinguish the department store shoe study from the other three industry studies. First, the shoe series, but none of the others, are seasonally adjusted. In principle, the decision model should apply to firms whose demand follows a seasonal pattern as well as to firms whose demand follows other cyclical patterns. Hence, the use of
seasonally adjusted data is not generally desirable since it may interfere with structural estimation. Nevertheless, it is of interest to compare the estimates from adjusted data with those from unadjusted data. Second, the physical volume series in the shoe data are obtained by deflating corresponding dollar value series with a price index which also appears as a variable in the system, whereas the physical volume data in the other three studies are direct physical counts. The difference involves the choice of weights in the aggregation procedure and it is not possible to say that one procedure is in principle better than the other. Each involves the possibility of introducing spurious correlation if it is the wrong procedure. Third, the shoe data represent the only retail trade included among the four studies. In this case, "production" represents deliveries of goods from wholesalers or manufacturers rather than the physical or chemical transformation of materials. It is a plausible conjecture that the decision model underlying (4) and (5) is less relevant to retailers' order-delivery decisions than to manufacturers' production decisions. For example, the cost of changing the rate of production has no obvious interpretation in the case of retailing.

4. Empirical results

For purposes of comparison, a second price equation, which excludes the term in lagged price, has been estimated from the data for each industry. This equation is
\[ p_n + \beta_{30} x_n + \beta_{31} x_{n-1} + \beta_{32} z_n + \beta_{33} z_{n-1} + \epsilon_{3n} \]  

(6)

Estimates of the coefficients in (4), (5) and (6) for each of the four industries are presented in Table 1. For the cement industry the table also shows estimates of the coefficients in the analogs to (4), (5) and (6) obtained by using the shippers' forecasts as the expectations proxy instead of current sales. Standard errors are shown in brackets below the coefficients to which they refer. Also shown are the squared multiple correlation coefficients and, for (4) and (5), the Durbin and Watson \( d \)-statistics. Values of the latter statistic which indicate positive autocorrelation among regression residuals at the five percent significance level are followed by *. Significance at the one percent level is indicated by **. Indeterminacy at the five percent level is indicated by +.

There is a much greater tendency toward autocorrelated residuals among the estimates from monthly than from quarterly data. This, and the results of experimental calculations using first differences, suggest that autocorrelated residuals are much more of a problem in monthly than in quarterly data.

Consider first the four production regressions which employ current sales as an independent variable. In these regressions, only two coefficients, both of lagged inventory, are less than three times their standard errors. Each coefficient except one has the sign predicted in Section 2 on the basis of theoretical considerations. The one coefficient with the wrong sign, that of lagged inventory for the southern pine lumber data, is not significantly
<table>
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<tr>
<th>Equation and Dependent Variable</th>
<th>Coefficient</th>
<th>Southern Pine Lumber</th>
<th>Cement Using Current Sales</th>
<th>Cement Using Shippers Forecasts</th>
<th>Pneumatic Tires</th>
<th>Shoes</th>
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<td>(4)</td>
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<td>R^2</td>
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<td>0.964</td>
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<td>0.808</td>
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<tr>
<td></td>
<td>d</td>
<td>1.33**</td>
<td>1.98</td>
<td>1.82</td>
<td>1.39**</td>
<td>2.27</td>
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<td>0.572</td>
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<td>1.57 *</td>
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<td>2.22</td>
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<td>0.48 **</td>
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<td>-0.017</td>
<td>0.002</td>
<td>0.003</td>
<td>0.172</td>
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<td>0.124</td>
<td>0.105</td>
<td>0.353</td>
<td>0.523</td>
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different from zero at the 95 percent level. The only surprise among these estimates is the generally small values of the lagged inventory terms. This is presumably partly explained by the fact that three of the commodities are very durable and therefore reducing excessive inventories quickly by lowering production is of small importance relative to other considerations. In the case of the southern pine data, it may be partly explained by the fact that the production decision (the decision to cut timber) was normally made by the owner of the trees, whereas the inventories were held by mills which were mostly under separate control. Thus, the effect of lagged inventory on production is interfirm and must operate through a market, possibly making the relationship less direct over short intervals. In the case of the shoe data, a special explanation may be that deliveries of shoes are intended to cover sales during the coming season, whereas inventories are at least partly of shoes which are going out of season. In this case inventories would have little effect on deliveries.

A comparison between the two production regressions for the cement data indicates that the one which contains current sales is much more satisfactory than the one which contains the shippers' forecasts. Not only is $R^2$ much smaller when the shippers' forecasts are used, but also the lagged inventory term has the wrong sign. This suggests strongly that the shippers' forecasts omit important information concerning future sales which is available and used when production decisions are made.

Consider now the price regressions, (5) and (6). Here, the results are considerably less satisfactory. Except for the cement data, all the $R^2$'s
for (5) are large. However, these regressions are heavily dominated by the terms in lagged price, as can be seen from the fact that the corresponding $R^2$'s for (6) are much smaller. This is not in principle inconsistent with the theory, since it allows for the possibility of a cost of changing price which will introduce lagged price into the price decision rule. However, the quantitative importance of lagged price in the estimated regressions suggests strongly that lagged price is serving as a proxy for other variables which are omitted from these regressions. Furthermore, in only some of the estimates do the other coefficients have the right signs. For southern pine lumber, all coefficients in (5) have the correct sign and are highly significant. In (6), $z_n$ takes on part of the job done by $p_{n-1}$ in (5) and its coefficient becomes positive. For cement, all the coefficients in both versions of (5) have the correct signs. However the coefficients of the expectations proxy and of production are not significant at the 95 percent level in either case. Indeed the two versions of (5) estimated from the cement data are very similar in every respect. This is because both are dominated by the term in lagged price. In (6), only the coefficient of lagged inventory in the version using current sales is significant at the 95 percent level. For pneumatic tires, only the term in lagged price in (5), and only the term in lagged inventory in (6), are significant at the 95 percent level. In (6), the significant coefficient has the wrong sign. (5) suggests that, in the tire industry, price can almost be represented by a random walk during the sample period. For department
store shoes, the only coefficients that are significant at the 95 percent level are those in production and lagged price in (5), and that in lagged inventory in (6). Of these, only the term in lagged price has the right sign.

It is interesting to note that only in the shoe data is $b_{21}$ negative. It was asserted in Section 2 that this should be true when most changes in demand represent movements along the demand curve rather than shifts in the demand curve. Since the shoe data, being seasonally adjusted, have had one of the largest causes of shifts in demand removed, it would not be surprising if the remaining movements were primarily endogenous.

In summary, although the estimates of (5) have large $R^2$'s on the whole, these estimates are unsatisfactory in that they are excessively dependent on the lagged price terms. Estimates of the non-price terms in (5) are satisfactory in two of the industries (southern pine lumber and cement), but are nonsignificant and/or have the wrong signs in the other two industries (pneumatic tires and department store shoes). Finally, it makes little difference whether current sales or the shippers' forecasts are used as a proxy for demand anticipations in the price equations.

Some further insights can be gained by an examination of graphs of the regressions. Most of these cannot be presented here, but Figure 1, representing southern pine lumber, is included. The graphs in this figure illustrate many of the following remarks, which refer to all the industries studied. Part (a) refers to the production equation, (4), and part (b) to
the price equation, (5). The top panel in each part shows the observed (solid line) and predicted (broken line) values of the dependent variable. The panels below show the contributions of each independent variable to the prediction. The bottom panel shows the residual, i.e. the difference between the two lines in the top panel. Within each part, all panels use the same units on the vertical scale.

Parts (a) of these graphs support the contention that the production regressions are, on the whole, quite satisfactory. Except in the case of pneumatic tires, they are remarkably successful in predicting turning points in production, whether of a seasonal or other nature. In cement, for example, in which seasonal movements in demand are particularly strong, (4) predicts each of the turning points in production in the correct quarter. In the case of department store shoes, turning points are predicted somewhat less accurately since the value of $R^2$ is smaller than in the other estimates of (4), but there appears to be no systematic tendency for the regression to lag behind observed turning points in production.

A comparison of the graph of the production equations for cement using current sales with that using the shippers' forecasts supports the contention made above that the former are a much better proxy for demand anticipations in explaining production decisions than are the latter. The regression containing the shippers' forecasts shows considerable tendency to predict turning points in production one period too late, or occasionally,
to fail to predict them altogether.

Finally, the graphs of the production regressions, including that for southern pine lumber, support the conclusion reached above that the inventory term makes only a small contribution to the explanation of movements in production in most cases.

Parts (b) of the figures strongly support the conclusion that the price regressions are dominated by the terms in lagged price. The principal evidence is that, in all four industries, the price regressions show a strong tendency to lag one period behind major movements in observed price. This suggests the further conclusion that there is very little information in short run movements in sales, production and inventory levels which is useful in predicting short run price movements. For example, in each price series, the largest sustained movement occurs about the middle of 1933. In each case, the regression fails to predict the upward surge in price which took place at that time. The explanation of this pervasive movement is presumably to be found in the attitudes and policies of the New Deal, which was at that time formulating NRA codes, rather than in the state of the markets in which the products were sold.

Beyond this general observation, several remarks can be made concerning the individual series. Much the most volatile of the four price series is that of southern pine lumber, with an average absolute monthly change of 3.98 percent. The least volatile appears to be cement, with an average quarterly change of only 0.71 percent. Shoes showed an average monthly price change of 0.53 percent and tires an average (also monthly) of 1.14 percent. The pattern
of price changes is also very different in the four cases. Southern pine lumber and department store shoe prices tend to change almost every month, whereas cement and pneumatic tire prices tend to change in large, infrequent jumps. In the tire price series, for example, no change is recorded in more than half the months in the sample. These facts presumably reflect the use of industry-wide price administration in the cement and tire industries. If this is so, it is not surprising that prices in these industries are relatively unresponsive to short run changes in sales, production and inventories.

5. Comparison with naive forecasts

It is interesting to compare the forecasting abilities of the price and production regressions with those of some naive forecasts. The naive forecast used most commonly for this comparison is the value of the variable in question lagged one period. The rationale of the use of this naive forecast is that the forecast "no change" is available free and therefore a theory should be judged by its ability to improve on this, i.e. to forecast changes in the variable. For the production regression, there is a second naive forecast with which comparison should be made. This is to forecast that production will equal sales each period. The rationale for this naive forecast is that it is the best forecast that could be made with a static model (such as the textbook theory of monopoly) which does not take into account inventory fluctuations and hence the possibility of divergence between output and sales. Hence this comparison indicates whether better forecasts can be made by taking inventory fluctuations into account.

The criterion by which the accuracy of the forecasts is usually measured is the average absolute percentage forecasting error. Thus, for example, the accuracy of the naive forecast lagged production would be measured
by

\[
\frac{100}{N} \sum_{n=1}^{N} \frac{z_n - z_{n-1}}{z_n}
\]

where \( N \) is the sample size. It should be noted that, even though each regression contains the naive forecasting variables with whose forecasting accuracy it is compared, this does not imply that the regression must have smaller forecasting errors by this criterion. The reason is that the regressions minimize the sum of squared forecasting errors rather than the sum of absolute percentage errors.

The results of this comparison are presented in Table 2. Not surprisingly, even the naive forecasts are quite accurate. This is simply another way of saying that prices, production and inventories do not change much from month to month. In this connection, it is interesting to note that in two of the industries (cement and shoes) the sales variable is a more accurate naive forecast of production than is lagged production, whereas in the other two industries (southern pine lumber and tires) lagged production forecasts production better than sales.

In each case, the production regression, \((4)\), forecasts better than any of the naive forecasts with which it is compared. In southern pine lumber, for example, the production regression forecasts about one-fifth more accurately than the more accurate naive forecast \((z_{n-1})\) and about one-third more accurately than the less accurate naive forecast \((x_n)\). It is also in-
TABLE 2
COMPARISON BETWEEN REGRESSION FORECASTS
AND NAIVE FORECASTS

<table>
<thead>
<tr>
<th>Variable to be Forecast</th>
<th>Forecasting Type</th>
<th>Southern Pine Lumber</th>
<th>Cement</th>
<th>Pneumatic Tires</th>
<th>Department Store Shoes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecasting Variable</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_{n-1}$</td>
<td></td>
<td>6.78</td>
<td>34.22</td>
<td>11.52</td>
<td>8.0</td>
</tr>
<tr>
<td>$x_n$</td>
<td></td>
<td>8.14</td>
<td>10.09</td>
<td>12.78</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>Shippers' Forecast</td>
<td></td>
<td>14.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Regression Forecast</td>
<td></td>
<td>5.41</td>
<td>7.33</td>
<td>9.49</td>
</tr>
<tr>
<td></td>
<td>Using $x_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Using Shippers' Forecasts</td>
<td></td>
<td>23.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_n$</td>
<td>Naive Forecast</td>
<td>3.98</td>
<td>0.71</td>
<td>1.14</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>Regression Forecast</td>
<td></td>
<td>3.36</td>
<td>0.73</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>Using $x_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Using Shippers' Forecasts</td>
<td></td>
<td>0.78</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
teresting to note that, in the cement industry data, both the naive and regression forecasts using the shippers' forecasts are much less accurate than the corresponding forecast using observed sales.

In the case of the price forecasts, the comparison is much less favorable to the regressions. This is to be expected since the price regressions are dominated by the variable that is used as the naive forecast, but the coefficients in the regressions are not chosen in such a way as to yield the best regression forecast by the criterion used in the table. In only one case, southern pine lumber, does the regression forecast more accurately than the naive forecast. In the other three industries, the naive forecast has a slight, but presumably nonsignificant, edge. In the cement industry, the forecast which uses the shippers' forecasts is again worse than all comparable forecasts.

6. Conclusions

The conclusions of this paper can be stated very briefly. The production regression, (4), gives quite satisfactory results in each application, with the possible exception of department store shoes. Whether the rather unsatisfactory results in this case are caused by the use of seasonally adjusted series, by other data deficiencies, or by the inapplicability of the model to retailing, it is not possible to say.

The price regression, (5), gives higher $R^2$'s on the whole, but less satisfactory estimates. The high $R^2$'s appear to result mainly from the fact
that prices tend to move in the same direction for several successive periods. With the partial exception of southern pine lumber, the price regressions fail to measure any substantial effects of the sales, output and inventory variables on price movements. With the same partial exception, the price regressions fail conspicuously to predict turning points, and provide no more accurate forecasts of price than does a simple extrapolation of lagged price.

There are several factors which undoubtedly contribute to this result. The first is data deficiencies. The price series, in particular, are suspect. In one case (tires), there is a known discrepancy between the coverage of the price and physical volume series. There may be similar discrepancies in the other series. Also, many recorded price indexes are known to be extremely inflexible. Partly, this is because some price changes, such as hidden discounts, are often not recorded in the index. In many cases, it is because the very agency which compiled the index did so as part of the administration of industry-wide price agreements and such agreements tend to be insensitive to short run changes in market conditions. Thus, the available sample of price series is biased in favor of those products for which oligopolistic factors are important. Finally, it also appears to be true that pricing is intrinsically a more complicated decision than production. There are several considerations which support this assertion. One consideration is the noteworthy fact that there exists a vast operations research literature on production planning, but almost none on price
planning. This suggests that the latter is much harder to reduce to a set of rules than the former. Another consideration is the fact that production planning is much more of a professional matter in firms than is price planning. There are many production engineers but few price engineers. This also suggests that production is more amenable than price to the use of simple rules and formulas. Last, and perhaps least, is the fact that the price decision rule in the model underlying (4) and (5) is much more complicated in form than the production decision rule.
References


