Identification of the Parameters of Linear Stochastic Difference Equations and Asymptotic Properties of their Maximum - and Quasi-Maximum - Likelihood Estimates

Herman Rubin

December 16, 1948

Let $A_{ux} x_t' = u_t'$ be a complete system of linear stochastic difference equations. Suppose that $u_t' = \rho u_{t-1}' + v_t'$, v_t independent for different t. For this system to be identified, it is necessary that the structural system $A_{ux} x_t' = u_t'$ be identified. However, this condition is not sufficient. We can have either finite or continuous indentifiability (Hurwicz's terminology). If all identification restrictions are closed restrictions* on A_{ux} ; then identifiability at a point implies local identifiability, and hence asymptotic properties of the estimate of A_{ux} and ρ (which is always identified whenever A_{ux} is identified) are simple corollaries of the corresponding properties of systems with non-autocorrelated disturbances.

If we break x into the jointly dependent component y and the predetermined component z, we obtain $y_t' = \Pi_{yz} z_t' + u_t^{\sim\prime}$, $u_t^{\sim\prime} = A_{uy}^{-1} u_t'$, the quasi-reduced form. Then $u^{\sim}$ satisfies a similar equation to u, namely,

$$u_t^{\sim\prime} = \rho^{\sim} u_{t-1}^{\sim\prime} + v_t^{\sim\prime},\quad v_t^{\sim\prime} = A_{uy}^{-1} v_t',\quad \rho^{\sim} = A_{uy}^{-1} \rho A_{uy}.$$

We may also obtain the reduced form, $y_t' = \Pi_{yz} z_t' + \rho^{\sim} y_{t-1}' - \rho^{\sim} \Pi_{yz} z_{t-1}' + v_t^{\sim\prime}$. This is always identified. We can then see that the original system is identified if and only if the quasi-reduced form is identified and the structural system is identified. Some results on the first of those problems have been obtained.

---

*The restrictions confine Π_{yz} to a closed set.

$$\mathcal{E}(y_t' \mid z_t) = \Pi_{yz} z_t' + \rho^{\sim}(y_{t-1}' - \Pi_{yz} z_{t-1}')$$