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<u>SOME EXPERIMENTS IN DEMAND ANALYSIS</u> *

(Section III.)

<u>Significance and Value of Results</u> **

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In this section we shall start with a brief summary of the statistical limitations and drawbacks associated with our investigations; then we shall discuss the relations between our findings and the concepts of economic theory; finally we shall turn to the subject of the general value and usefulness of such results as those obtained in Section II.

1. <u>Statistical limitations.</u>

First, there are a number of deficiencies in the data which must be made quite explicit. The quantity figures in general fail to reflect stock changes adequately; although the Home Consumption Account does allow for changes of bonded stocks, we have no means of measuring stock changes of merchants and retailers. The price series are in most cases compiled from numerous different sources and are not always a true average of different brands, different localities or different seasons of the year. And the deficiencies of using a cost of living index as a price series for competing goods are obvious enough. The income estimates used are those of net national income and there are undoubtedly deficiencies in these statistics, particularly in the early years.(52) Further, the approximation to personal income is very rough and probably has the effect of biassing the estimates of income-elasticity.(53) We have already mentioned the inadequacy of taking total population figures as deflators for such commodities as tobacco and spirits. Finally, we have in all cases ef-

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(52) Cf. Prest, "National Income of the U.K., 1870-1946," <u>Econ. Journal</u>, March 1948, p. [illegible]

(53) Cf. Stone, <u>Journal of the Royal Statistical Society</u>, 1945, p. 391.

* Extract from article to be published in <u>Review of Economics and Statistics</u>.

** See list of equations for explanation of symbols used.

fectively considered the total quantity sold as passing through one large market, consisting of the whole population. Obviously, in the case of spirits and wines, for instance, it would probably be more realistic to confine the analysis to the higher income sections. Lack of data, however, prevents this.

Now these deficiencies are a formidable list and it would be unwise to minimise their importance. Moreover, we can never hope to escape such problems completely, for although we may expect that these difficulties will be relieved in future years as more design is introduced into the collection of economic statistics, the framework of society does change, geographical boundaries do alter and so on. Therefore, we must always be prepared to face difficulties of comparability. Nor is the loophole of taking short periods of time available. For if we confine ourselves to yearly data, we are likely to reduce the amount of independent variation and to increase the danger of unexpected relationships. And if we take refuge in quarterly or monthly data, we have to allow for seasonal adjustments. And even then it must be remembered that owing to serial correlation the number of effective observations is not four times as great in the case of quarterly data as it is with annual data.

Second, the statistical procedure adopted is open to criticism. First of all, there is the problem of formulating the equation. We have already stressed the fact that the choice of equation - whether linear or logarithmic, etc. - is inevitably somewhat arbitrary. And when differing results are obtained from different equations, it may not always be easy to decide which estimates of the parameters are more likely to be near the truth.(54) Then comes the question of the number of independent variables included. Obviously, a good case can be made out for allowing such factors as the distribution of income, the age composition and

(54) The topic of whether there is any unique "truth" will be discussed below.

degree of urbanisation of the population, and the quality of the commodity, to enter all the equations.(55) It might also be thought that advertising expenditure should be included as a separate variable. It must be remembered, however, that much advertisement is competitive between the same firms in an industry and therefore the effect of this factor on the total amount sold by the industry may not be so great. If we include press advertising expenditure as an additional variable in equation (42) of "The Analysis of Market Demand", we have

Original equation:

$$(41) \qquad q = 750.2\, Q^{.072} (p/\Pi)^{-.513}\, e^{.0345t + .0593x}$$

New equation:

$$(42) \qquad q = 611.0\, Q^{.094} (p/\Pi)^{-.488}\, e^{.0540t + .0593x - .0057d}$$

where x = extent of coupon trading;

d = press advertising expenditure.

It appears from this that the effect of advertising is almost negligible, though it must be stressed that the series used refers to press advertising expenditure only and therefore may not be too good an indicator of total expenditure. Probably a more serious problem than advertising expenditure is the correct formulation and interpretation of equations for commodities, where farm or small-holding production and consumption (e. g., milk, vegetables) is at all large. Here we obviously do have a mixed supply and demand relationship and any estimates of income or price elasticity of demand in such cases are likely to be erroneous.

But including more independent variables may impair the estimates of the

(55) In the case of intermediate goods used in further manufacture, such factors as expected prices, the rate of rise or fall in prices, etc., become highly relevant. But in the case of consumers' goods these points are not very important, for the influence of such forces as pre-budget purchases is likely to be swamped in calendar year data.

parameters in several ways. More disturbances may be introduced - or existing ones given a better chance to exert their influence - or the systematic part of the new variates may be intercorrelated with existing variates (multicollinearity) or with variates on the supply side. In fact, in many cases it is found that these symptoms appear with as few as three independent variates and as a result the estimates of the parameters prove to be inconsistent. In such cases the problem is to reduce the number of variates and not to increase them. Therefore, at all costs the number of influences which we allow to enter our explanatory equation explicitly must be kept small even at the risk of some distortion of the estimates of the parameters.

We must now draw attention to the difficulties of <u>solving</u> the equations. Even with our tools of confluence analysis and first differences we cannot hope to deal fully with errors of sampling and errors of weighting. We have already seen that, if observational errors exist, it can be shown that least-squares estimates of the parameters are biassed. It is highly probable, however, that no simple manner of grappling with these difficulties will be evolved. And certainly in our present state of knowledge, the only possible method is to take every case on its own merits - to examine the particular market conditions under which each commodity is sold and to judge whether the errors in the consumption statistics are random or systematic in character. There is no well-trodden statistical path along which a demand analysis for all the thousand and one commodities sold in everyday life can be pushed without fear of numerous pitfalls.

2. Relations with economic theory.

The first question which calls for discussion is how nearly the price-quantity (or price elasticity) relationships which we evolve in this paper approximate to a theoretical demand curve (or elasticity of demand).

A preliminary point of definition should be cleared up first. We only deal here with price-substitution elasticity and therefore exclude income effects and, further, we deal with reactions to changes in relative prices. Therefore, any estimates of the elasticity of demand in the Marshallian sense would differ from our results on both counts.

Now in principle one cannot construct a theoretical demand (or supply) curve from recorded data at all for the simple reason that these give only one set of observations of quantity and price for each time period, whereas by definition a number of observations relating to the same time period (rather than to the same state of affairs)(56) are required. The only conceivable way of determining a Marshallian demand curve <u>in sensu stricto</u> would be to ask a man how he would regulate his purchases if prices were other than they were but other things were not. And probably he would not really know although, of course, guesses might be made. The difficulty is obviously greater when we wish to draw a demand curve throughout its length than when we simply want to estimate the elasticity of demand at the existing price as the former requirement would mean both estimates of a consumer's reactions to widely differing prices and some assumptions about continuity between the points observed. But even though not so formidable, the problems of estimating demand elasticity are none the less real.(57)

Now if we cannot obtain a Marshallian demand curve direct from recorded data, what is it possible to do? The answer to this seems to depend on what the recorded data can be taken to represent. The kind of point to consider here is

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(56) By our device of eliminating trend, we approximately obtain an unchanged state of affairs (see below p. 7) but clearly this does not exactly fulfill the requirements of pure theory.

(57) We do not touch here on the problems of transition from single consumer to market demand curves. The worst difficulties here concern income-consumption curves (cf. [illegible], "Income Elasticity of Demand", Economic Journal, April 1941) but Morgenstern (cf. "Demand Theory Reconsidered", <u>Quarterly Journal of Economics</u>, February 1948) has pointed out others connected with price and quantity curves.

that the data for any one period will normally be built up from a number of transactions that have taken place during that time. If we assume that all transactions take place in one large market, then we have to consider the fact that the data represent some aggregate or average of purchases which have taken place at a number of different time-points within the period. On the quantity side, there is no trouble, for we only have to sum the various amounts bought. On the prices side, however, we have to take some sort of average. The principle of doing this is clear enough: the average price taken must be such that, if maintained throughout the period, the aggregate quantity consumed would be the same as that corresponding to the series of varying prices which actually did rule. But the practical problem of finding this average is not so easily solved. It can be shown that a simple arithmetic mean is only apposite if the demand curve is a straight line; and in some cases a harmonic mean is needed.(58) A similar problem arises if we assume that all transactions take place simultaneously but in a number of different markets where prices differ, owing to transport costs.(59) And if we consider the joint case of separate markets and transactions taking place at different points in time the problem becomes more complex still.(60)(61)

Now the price data which we have to use are not sufficiently accurate or detailed to enable us to resolve these problems in any very satisfactory manner. Therefore, it must be made quite clear that there is no certainty that the price

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(58) Cf. H. Working, "Factors determining the price of potatoes in St. Paul and Minneapolis", Minnesota Agric. Experimental Station, Technical Bulletin No. 10, 1925.

(59) Cf. H. Working, "The Statistical Determination of Demand Curves", Quarterly Journal of Economics, 1925. This point is obviously not so important in a small country like the United Kingdom. It is likely to be still less important if the practice of fixed resale prices spreads further.

(60) This point is of course quite independent of changes in tastes (affecting the position of the demand schedule) as between different time-points or places.

(61) It will be apparent that the above difficulties do not apply to income elasticities.

and quantity data we use for any one period are co-ordinates of the true theoretical demand curve at all.(62) Nevertheless, there seems to be no very obvious reason why inaccuracies of this sort should be systematic and not random and therefore it may well be that these errors are not a major disaster.

If this point be granted, then we must ask - what do the coefficients of price (or income) elasticity obtained from the regression analysis really represent? At first sight, the answer seems fairly clear. By use of the term in t (or $t + t^2$) we obtain a series of co-ordinates (q, p/Π), (q,Q), etc. which we can approximately regard as representing pairs of observations in the same time period. And if we plot the (p/Π, q|Q) points, say, on double logarithmic paper together with the least-squares line of closest fit (minimising sums of squares in the direction of q) we have a Marshallian-type demand curve and the slope of the line is the price-elasticity of demand (in our sense). But, unfortunately, this is a bit too simple and the meaning of the regression coefficients is not quite so unambiguous. What then is the answer? If we neglect the first-difference approach for the moment, it seems to be that longer period similarities in the movements of the series are reflected in the estimates of the regression coefficients to a greater extent than short period (i. e., year to year) dissimilarities. For the point is that, although our use of a term in t (or $t + t^2$) will enable us to remove secular trends from the price and quantity series, it may well be that both these series still reflect such common changes as those which occur over the course of the trade cycle. And even if there is negative correlation between year to year movements, the magnitude of these short period dissimilarities may be so small compared with the overall variances of the series that the estimates of the multiple correlation coefficient and the

(62) Cf. G. J. Stigler, "The Limitations of Statistical Demand Curves," *Journal of the American Statistical Association*, 1939.

regression coefficients reflect to a much greater degree the common cyclical movement. On the other hand, it would appear that the first difference approach must give much more weight to year to year similarities and dissimilarities in movements.(63) It may well be that measurements of slopes and elasticities derived from the latter method are likely to approximate more to the theoretical concepts. But until much more work has been done in this field it would be unwise to be very dogmatic.(64)

These considerations naturally lead us to some discussion of the phenomenon known as "time-elasticity of demand".(65) The idea here is obvious enough - that consumers' reactions to a price or income change will normally depend on the length of time considered as people do not have perfect knowledge of such changes and as it does take finite time for spending habits to alter. But whether this idea really has anything to do with the elasticity of demand in its original sense is another matter. For the concept of price or income elasticity refers by definition to a given period of time. If this period is long enough for consumers' reactions to work out, then there is no such thing as time-elasticity of demand; if on the other hand we relate consumption changes in period II to price changes in period I, then we are not strictly able to speak of the elasticity of demand at all, for this is a geometrical property of one demand schedule only. But, it may be argued, this is a verbal point and in fact with the time series approach we are essentially postulating that the $(q, p/\Pi)$ points for each time period are co-ordinates of one single demand schedule. How far do we take account of the time-distribution of reactions to price or income changes in such an approach? Now even when no explicit provision is made

(63) This point was first made many years ago. Cf. Hooker, Journal of the Royal Statistical Society, 1905, p. 705.

(64) I owe the ideas in this paragraph to Dr. G. H. Orcutt.

(65) Cf. H. Smith, "A Note on Time Elasticity of Demand", Economica, 1937.

for lags in the determining variates, some account is nevertheless taken of preceding values by virtue of the continuity properties of most economic series. But to test the importance of this point more fully, two lines of thought were explored. First, an experiment was tried using five-yearly data instead of annual series for the All Drinks series. The results were:

$$(43) \qquad \log (q/n) = 1.5475 + .8513 \log (Q/n) - .6455 \log (p/\Pi) - .0030\, t \log e$$

$$\bar{R}^2_{1.234} = .968$$

compared with estimates of .771, -.671 and -.013 (66) for b, c and r, respectively from equation (24).

But obviously this is not sufficient, for it might well be argued that there is a "hangover" of consumers' reactions from one five-year period to another. The theoretical points stand whether we take monthly or ten-yearly data. Therefore, a second experiment was made by adding a lagged price series to (25). As a result we had

$$(44) \qquad \log (q/n) = 1.3436 + .2599 \log (Q/n)_0 + .6664 \log (Q/n)_{-1} - .3497 \log (p/\Pi)_0 - .2567 \log (p/\Pi)_{-1} - .0161\, t \log e$$

$$\bar{R}^2_{1.23456} = .972.$$

How can all this be interpreted? Even in theory, there is no such thing as "the" elasticity of demand, if we abandon the simplifying assumption of no "hangover". Therefore, all that one can hope to find from statistical investigations is the relative influence of past and present values of the determining variates on people's behavior, neither of these being "the" price or income

(66) An explanation of the lower coefficient of t in (43) may be that the estimate is biassed due to the increase in the continuity of the explanatory series when five-yearly averages are taken.

elasticity. And further, these values may differ somewhat according to the time classification of the data. Finally, we must remember that the value of any series at time t is dependent on the values at t_{-1}, t_{-2}, ..., and therefore estimates of the relative importance of past and current values of a determining variate are normally blurred.

The last topic to discuss in this sub-section is the implications of _ceteris paribus_ clauses in demand theory and analysis. The first point to note is that, if one is interested in obtaining an estimate of one of the parameters in the demand relationship (e. g., price elasticity), then the estimate obtained will depend (except where we have a logarithmic relationship) on the level at which the other variables are held constant - whether at the level at the beginning or the end of the period, for instance. In our particular statistical technique, the parameters are such that the sums of squares of the residuals are minimised and it must be made clear that we should not expect to obtain the same parameters if other estimating procedures were used. But it must also be stressed that this degree of indeterminacy is just as much a weakness of the theoretical concept as of the statistical technique.

This brings us to the further consideration of whether it is logical or useful to insist on a _cet. par._ clause in all cases. Is it logical or useful to derive the price elasticity for margarine on the assumption of constant butter prices? For the price of butter itself depends on the production and price of margarine. Marshall drops a hint on this subject (67) and in fact it may be true that nothing less than the complex techniques of, e.g., Girschick and Haavelmo. (68) will suffice in dealing with such close interrelations as this.

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(67) _Principles of Economics_, p. 100 n. and p. 106 n.

(68) "Statistical Analysis of the Demand for Food," _Econometrica_, April, 1947.

3. General value and usefulness of results.

We now come to the final section of this article and our task is to assess the general value of such results as we have obtained in our investigations.

First we shall take the case where we have a close agreement between the observed and calculated values of the dependent variable. One result which emerges is that in most cases it would appear (69) that changes in quantity are associated with changes in income and prices over a long period of time. Now this at least confirms the *a priori* postulates of theory. (70) Further, it shows that there are persistent relationships in economic affairs and, as we take as long a span as sixty-five years, there are some grounds for believing that these relationships are enduring. Although it might be possible to find evidence of association between say, income and quantity changes over a short period without all this trouble, by definition the only method of testing whether such an association is likely to be enduring is to take a long period of *time* rather than a long series of *observations*. Where we not only have a good fit but the estimates of the parameters survive the confluence tests, then we have results which may be of interest for individual commodities. It must be made clear, however, that any estimates of elasticities (abstracting for the moment from the ambiguities mentioned in sub-section (2)) are only applicable to the demand schedule for the commodity as a whole and not to that for the products of individual producers. (71) Even so, such investigations may be of profit to monopolistically

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(69) We must be careful here to specify precisely that we cannot *prove* that such an association exists and therefore we use the phrase "would appear." The most we can do by statistical analysis is to *disprove* that such an association does *not* exist. Cf. Tinbergen, *Statistical Testing of Business Cycle Theories*, Vol. I, p. [illegible].

(70) In most cases, the expectations of signs of the parameters were fulfilled.

(71) Considerations of an altogether different kind come in here, e.g., for the individual firm it is usually more important to increase (or preserve) its share of the existing market.

controlled industries, trade associations or taxation authorities. And even when the estimates of the individual parameters are imprecise, this of course does not mean that the analysis is useless for prediction purposes. Provided the intercorrelation between, say, income and time persists, it may be possible to make reasonable estimates of future consumption. However, prediction is a tricky subject which raises a number of awkward points and it is not our intention to discuss them here.(72)

Probably more striking than the above results, however, is the fact that in some cases we were only able to explain a small part of the variance by the "economic" variates, income and relative prices, and the major part of the explanation was to be found in the omnibus variate, time. Now it may, of course, be true that time is closely associated with other "economic" variates. Experiments were made to test this (e.g., tobacco prices were inserted in the explanation of beer) but it was still not possible to attribute a much greater proportion of the variance to their influence. It is, of course, true that we could not test all possible "economic" influences, but nevertheless it does seem that a major part of the explanation must be attributed to the general nexus of social and psychological forces we group under the heading "time". This seems to have two main implications. Is there any point in formulating a demand theory which says that relative prices and income are the major determinants of the quantities demanded if in fact their influence is very small? And should not our practical investigations lay much more emphasis on a study of particular markets and communities and also be directed to some examination of the psychology of tastes?

Second, we must consider the case where no good fit was obtainable at all,

(72) A short treatment is given by Stone, <u>Consumers' Expenditure in the United Kingdom 1920-38</u>, Chapter 13. (to be published shortly)

or at least where the estimates of the parameters were such as to be wildly improbable. Examples of this were coffee, imported wines and coal.[73] Now clearly these "failures" may have been due to poor data, lack of independent variation in the sample of observations available, incorrect formulation of the equation, and so on. But a number of different approaches were tried and it appears strange that none of them proved adequate. Whether our statistical approach was adequate or not, the point still stands- should we not be prepared to face the fact not only that human behaviour may not be rational in all cases (in the economic sense of maximising gains) but also that it may not even be consistent in economic affairs? Although we may hope to find some enduring relationships, we should not perhaps expect that these will cover the whole field of economic life.

But perhaps the most important lesson which it is possible to learn from these investigations is that it is quite impossible to speak of "the" elasticity of demand at all. Over sixty years ago, Alfred Marshall wrote:[74]

> "I believe that inductions with regard to the elasticity of demand and deductions based on them have a great part to play in economic science."

Now it is clear that this high hope has not yet been fulfilled and it seems reasonable to argue that it never will be until the lesson is digested that "the" elasticity of demand is a mirage which never did and never will exist. Any numerical estimate which does not specify the market under survey, the length of time and the independent variates considered is completely and utterly meaningless.

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(73) Personal consumption only.

(74) "On the Graphic Method of Statistics", *Journal of the Royal Statistical Society*, Jubilee Volume, 1885.

Equations in "Some Experiments in Demand Analysis"

Common Symbols

q = quantity consumed

Q = real income (Deflated National income at factor cost)

n = population

P = price of commodity under analysis

II = other prices

t = time

e = base of natural logs.

Specific Symbols See under individual commodity

Annual Series are used in all cases except Drinks equation (43) where 5 yearly series are used

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Beer (1870-1938)

(9) $\log (q/n) = 3.5592 + 0.2183 \log (Q/n) - 1.0190 \log (P/II)$

$$\overline{R^2}_{1.23} = .881$$

(10) $\log (q/n) = 2.0288 + .6761 \log (Q/n) - .7012 \log (P/II) - .0085\, t \log e$

$$\overline{R^2}_{1.234} = .921$$

(11) $\log (q/n) = 2.4665 + .3489 \log (Q/n) - .5922 \log (P/II) - (.0053 + .00012\, t)\, t \log e$

$$\overline{R^2}_{1.2345} = .944$$

(14) $(\log (q/n)_t - \log (q/n)_{t-1}) = -.00076 + .2333 (\log (Q/n)_t - \log (Q/n)_{t-1}$

$- .6634 (\log (P/II)_t - \log (P/II)_{t-1}) - .000059 (2t + 1)$

$$\overline{R^2}_{1.234} = .605 \qquad \sigma_{b_{12}} = .121 \qquad \sigma_{b_{13}} = .084$$

$$\overline{S^2}_{1.234} = .0134$$

or (15) $$(q/n) = a' (Q/n)^{.2333} (P/\Pi)^{.6634} e^{-.0018t - .00014 t^2}$$

Spirits (1870-1938)

(16) $$\log (q/n) = 1.4222 + 1.0996 \log (Q/n) - .8365 \log (P/\Pi) - .0219 t \log e$$

$$\overline{R^2}_{1.234} = .976$$

(17) $$\log (q/n) = 1.6948 + .6963 \log (Q/n) - .6675 \log (P/\Pi) - (.0211 + .00029t) t \log e$$

$$\overline{R^2}_{1.2345} = .988$$

(19) $$q/n = .1369 + .7605 (Q/n) - .0145 (P/\Pi) - (.0243 + .0001t)t$$

$$\overline{R^2}_{1.2345} = .955$$

(20) $$(\log (q/n)_t - \log (q/n)_{t-1}) = -.0062 + .7355 (\log (Q/n)_t - \log (Q/n)_{t-1}) - .8612 (\log (P/\Pi)_t - \log (P/\Pi)_{t-1}) - .00011 (2t + 1)$$

$$\overline{R^2}_{1.234} = .714 \qquad \sigma_{b_{12}} = .150 \qquad \sigma_{b_{13}} = .079$$

$$\overline{S^2}_{1.234} = .022$$

or (21) $$(q/n) = a' (Q/n)^{.7355} (P/\Pi)^{-.8612} e^{-.0143t - .00025 t^2}$$

Spirits (1920 -1938)

(22) $$q = a' Q^{.602} (P/\Pi)^{-.569} e^{-.033t}$$

Drinks (1870-1938)

(24) $$\log (q/n) = 1.7755 + .7705 \log (Q/n) - .6714 \log (P/\Pi) -.0135 t \log e$$

$$\overline{R^2}_{1.234} = .965$$

(25) $$\log (q/n) = 1.4089 + .1521 \log (Q/n)_0 + .7328 \log (Q/n)_{-1} - .6974 \log (P/\Pi) -.0157 t \log e$$

$$\overline{R^2}_{1.2346} = .971$$

(26) $\log(q/n) = 1.4251 + .871 \log(Q/n)_{-1} - .5921 \log(P/\Pi) - .0157\, t \log e$

$$\overline{R}^2_{1.345} = .971$$

(44) $\log(q/n) = 1.3436 + .2599 \log(Q/n)_o + .6664 \log(Q/n)_{-1} - .3497 \log(P/\Pi)_o - .2567 \log(P/\Pi)_{-1} - .0161\, t \log e$

$$\overline{R}^2_{1.23456} = .972$$

(43) [Five Yearly Series] $\log(q/n) = 1.5475 + 8513 \log(Q/n) - .6455 \log(P/\Pi) - .0030\, t \log e$

$$\overline{R}^2_{1.234} = .968$$

Tobacco (1870-1938) [x is discontinuity variate: values 0 to 1914, 1 after 1920.]

(27) $\log(q/n) = 3.7070 - .7993 \log(Q/n) - .0652 \log(P/\Pi) + .0101\, t \log e$

$$\overline{R}^2_{1.234} = .939$$

(28) $\log(q/n) = 1.4295 + .4087 \log(Q/n) - .1209 \log(P/\Pi) + .0090\, t \log e + .322\, x \log e$

$$\overline{R}^2_{1.2345} = .969$$

(29) $\log(q/n) = 2.0170 + .2111 \log(Q/n) - .2247 \log(P/\Pi) + (.0097 + .00015t)\, t \log e + .193\, x \log e$

$$\overline{R}^2_{1.23456} = .991$$

(30) $\log(q/n) = 1.9769 + (.0099 + .00016\, t)\, t \log e + .158\, x \log e$

$$\overline{R}^2_{1.456} = .987$$

(32) $(\log(q/n)_t - \log(q/n)_{t-1} = .0055 \text{ (or } .0067) + .1416 (\log(Q/n)_t - \log(Q/n)_{t-1}) - .3065 (\log(P/\Pi)_t - \log(P/\Pi)_{t-1}) + .000068\,(2t + 1)$

$$\overline{R}^2_{1.234} = .390 \qquad \overline{S}^2_{1.234} = .00006 \qquad \sigma_{b_{12}} = .065 \qquad \sigma_{b_{13}} = .055$$

Note: The reason why we have 2 estimates of the scale constant in (32) is that in (29) we have a discontinuity factor x which has values of 0 to 1914 and 1 in later years. An approximation to including this factor in (32) is to estimate two values, one for the years 1870-1914, the other for 1920-38. The difference between these two estimates is a measure of the influence of x.

or (33) $$(q/n) = a' \, (Q/n)^{.1416} \, (P/\Pi)^{-.3065} \, e^{.0127t + .00016t^2 + .0028x}$$

Tobacco (1920-38) [d is advertising expenditure; x is coupon trading expenditure]

(41) $$q = 750.2 \; Q^{.072} \, (P/\Pi)^{-.513} \, e^{.0345t + .0593x}$$

(42) $$q = 611.0 \; Q^{.094} \, (P/\Pi)^{-.488} \, e^{.0340t + .0595x - .0057d}$$

Tea (1870-1938)

(34) $$\log (q/n) = 1.8953 + .0977 \log (Q/n) - .0526 \log (P/\Pi) + .0099\, t \log e + .123\, x \log e$$

$$\bar{R}^2_{1.2345} = .9839$$

Margarine (1900 - 38) [P_1 = price of margarine; P_2 = price of butter; x = discontinuity variate = 0 to 1920, 1 from 1920-3, 0 from 1924-38]

(37) $$\log (q/n) + .43 \log (Q/n) = 4.3140 - 1.8320 \log (P_1/P_2) + 1.0964 \log (P_1/\Pi) + .0250\, t \log e + .465\, x \log e$$

$$\bar{R}^2_{1.2345} = .927$$

Potatoes (1900-38)

(38) $$\log (q/n) = 2.5631 + .3880 \log (Q/n) - .6031 \log (P/\Pi) - .0104\, t \log e$$

$$\bar{R}^2_{1.234} = .444$$

(39) $$\log (q/n) = 2.8872 + .5425 \log (Q/n) - .9056 \log (P/\Pi) - .0122\, t \log e$$

$$\bar{R}^2_{1.234} = .562$$

Note: Imports excluded from (39)

Soap 1900-38

(40) $$\log (q/n) = 1.3348 + .3649 \log (Q/n) - .0355 \log (P/\Pi) + .0111\, t \log e$$

$$\bar{R}^2_{1.234} = .947$$

Comparison of Social and Economic Factors

Basic equation $\sigma_{q_c}^2 = \sigma_E^2 + \sigma_s^2 + 2^r \sigma_E \sigma_s$

where σ_{q_c} = standard deviation of calculated (q/n)

σ_E = standard deviation of calculated $(b \log (Q/n) + c \log (P/\Pi))$

σ_S = standard deviation of calculated $(r + st)\, t \log e$

r = correlation coefficient of E and S series.

Beer $\sigma_{q_c}^2 = .0014 + .0035 + .0031$

Spirits $\sigma_{q_c}^2 = .0068 + .0453 + .0234$

Tobacco $\sigma_{q_c}^2 = .0001 + .0132 - .0016$

Tea $\sigma_{q_c}^2 = .0001 + .0108 + .0022$