Remark on Properties of Nonnegative Matrices

T. W. Anderson

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[The following is a quotation from a letter dated September 24, 1951 received by Koopmans]

I have noticed a number of Cowles Commission Discussion Papers on properties of nonnegative matrices. It seems to me that many of these results follow immediately from properties of transition probability matrices in the theory of finite Markov chains. Let $A = (a_{ij})$ with $a_{ij} \geq 0$ and $\max_j \sum_i a_{ij} = 1$. Let $b = (b_i)$ with $b_i = 1 - \sum_j a_{ij}$. Then

$$C = \begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix}$$

is a transition probability matrix. Each characteristic root of $A$ is a root of $C$ because if

$$Ax = \lambda x$$

then

$$C \begin{pmatrix} x \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} x \\ 0 \end{pmatrix}.$$