Jacob Marschak: *The Rationale of the Demand for Money and Money Illusion.*

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**Definitions, Assumptions and Results**

Plan of the a-th Person (a=1,...,A) for the n-th Good (n=1,...,N).
(Subjected to a, n omitted.) Time unit = length of interval between two successive marketing dates. Planning date, t=0. Horizon (latest marketing) date, t=T.

<table>
<thead>
<tr>
<th>Restrictions on Signs</th>
<th>Stocks Drought</th>
<th>Stocks Retained</th>
<th>Stocks Sold</th>
<th>At Price</th>
<th>Consumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥ 0,</td>
<td>≥ 0,</td>
<td>&gt; -∞,</td>
<td>&gt; 0,</td>
<td>&lt; ∞</td>
<td>≥ 0</td>
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<table>
<thead>
<tr>
<th>Date of Marketing</th>
<th>Period of Consumption</th>
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<tbody>
<tr>
<td>0</td>
<td>x₀</td>
</tr>
<tr>
<td>1</td>
<td>x₁</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
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<tr>
<td>t</td>
<td>xₜ</td>
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<td>...</td>
<td>...</td>
</tr>
<tr>
<td>T-1</td>
<td>x₁</td>
</tr>
<tr>
<td>T</td>
<td>xₜ</td>
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</table>

Find values for sets \(\{yₜ^n\}, \{yₜ^n\}, \{xₜ^n\}\), \(n=1,...,N, t=0,...,T-1\),

that would maximize the

\[ u_{(a)}(\{xₜ^n\}, \{yₜ^n\}) \]

subject to above restrictions on signs and to

1. Budget restrictions \(\sum_{n, na} xₜ^n pₜ^{t-n} = 0\),
2. Market imperfection \(pₜ^{t-n} = fₜ^{t-n}(zₜ^{t-n})\), \(n=1,...,N; t=0,...,T\), where the N-th good is numeraire and legal tender: \(pₜ^{t-n} = fₜ^{t-n} = 1\) for all \(a, t\).

The given are: initial stocks \(\{y₀^n\}\); functions \(u_{(a)}\), \(\{fₜ^n\}\).
The set \( \{ r_n^t \} \) is restricted by "clear-the-market" condition:

\[ \sum_{a} x_{na}^t = 0 \text{ for all } n. \]

Marginal utilities:

\[ u_n^T = \frac{\partial u(a)}{\partial y_n^T}; u_n^t = \frac{\partial u(a)}{\partial x_n^t}, t = 0, \ldots, T-1; \text{ all non-negative.} \]

Illiquidities:

\[ s_{na}^t = \frac{d r_{na}^t}{d s_{na}^t}, t = 0, \ldots, T; \text{ all non-positive.} \]

Special, mutually independent, assumptions (omitting subscript \( a \)).

(I) Static case: \( T=0 \); or \( T=1 \), \( u_n^T = 0 \), implying \( x_n^T = y_n^T = 0 \) for all \( n \).

(II) Paper money: \( u_n^t = 0, t = 0, \ldots, T, \) implying \( y_n^t = y_n^t \) for all \( n, t \).

(III) Perfect markets: \( s_{na}^t = 0 \) (perfect liquidity) for all \( n, t \).

Method. Express sign restrictions (see PLAN) by equations

\[ \frac{d x_n^t}{d t} - (r_n^t)^2 = y_n^T - (r_n^T)^2 = x_n^T - (s_n^t)^2, t = 0, \ldots, T-1, \]

where \( r_n^t, r_n^T, s_n^t \) are real. \( \lambda, \mu, \nu \) are Lagrange multipliers.

General result (non-static, imperfect markets):

\[ (5) \quad u_n^t + \mu_n^t = \lambda_n^t (p_n^t + s_n^t g_n^t) = \lambda_n^t (p_n^t + s_n^t g_n^t) + \Lambda_n^t, \]

\[ (6) \quad \mu_n^t = y_n^t s_n^t = 0 = \nu_n^T - \lambda_n^T x_n^T, t = 0, \ldots, T. \]

Result for Special Cases:

(I), (II) Static, perfect: \( p_n^0 = u_n^0 / u_n^T; x_n^0 \geq 0 \) ("classical case").

(I), (II), (III) Static, perfect, with paper money: \( x_n^0 = 0 = y_n^T \) ("money of account").

(III) Non-static, perfect: If prices change \( (p_n^{t+1} \neq p_n^t \) for all \( n \) then all stocks but one are "unloaded", down to consumption needs of next period.

(III), (II) Non-static, perfect, with paper money: If prices change then money stock is either zero or absorbs all resources except those needed for current consumption.

Money Illusion. Solve (1), (2), (4)-(6) for all supplies (demands) of the \( a \)-th Person, \( x_{n, a}^t \). In case of perfect markets (III) each \( x_{n, a}^t \) is a function of all "absolute" prices -- \( p_n^0, p_n^1, \ldots, p_n^{T-1}, n = 1, \ldots, N-1 \); not of the "relative" prices -- \( p_n^0 / p_1^0, p_n^1 / p_1^1, \ldots, p_n^{T-1} / p_1^{T-1}, n = 2, \ldots, N-1 \). (With imperfect markets, the givens are \( \{ t_{na}^t \}, \text{not}\{p_{na}^t\}; \text{but the statement remains true.}\)

Generalisations not yet discussed: Uncertainty; Production; Borrowing.