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Further Comments on Lawrence R. Klein's Economic Fluctuations in the United States 1921-1941 (Second Draft)

by Carl Christ
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I. Introduction

The objective in preparing these comments is to learn from a study of Klein's sixteen-equation model,[1] so that future model-builders (including I hope myself) will be able to profit by the good work he has done, and avoid whatever shortcomings may be found. I approach this job now with a more charitable spirit than I had when I began. Because I have had time to examine my criticisms of Klein's model, I have come to appreciate the difficulties which led him to do some of the things which are difficult to justify. Concerning several of the criticisms I have positive suggestions to offer; concerning others I have questions; still others I can only mention, and then agree that Klein took the best available compromise. My debt to Domar's Cowles Commission Discussion Paper, Economics 227, will be obvious as we go along.

II. Resume of Klein's Model

Klein's model has sixteen equations, of which four are identities containing no disturbances. There are sixteen endogenous variables, as follows:

C consumer expenditures, in 1934 dollars.

D_1 gross construction expenditure for owner-occupied one-family non-farm housing, in 1934 dollars.

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1. L. R. Klein, Economic Fluctuations in the United States 1921-41, Chicago: Cowles Commission, 1947, unpublished, second draft, Chapter III, pp. 33-76.

(endogenous variables, continued)

D_2 gross construction expenditure for rented non-farm housing, in 1934 dollars.

H inventories at year end, in 1934 dollars.

I net (nonhousing) investment in plant and equipment, in 1934 dollars.

i average corporate bond yield.

K fixed capital at year end, in 1934 dollars.

M_1^D active balances = demand deposits + currency outside banks, in current dollars.

M_2^D idle balances = time deposits, in current dollars.

p general price level

R_1 non-farm rentals, paid or imputed, in current dollars.

r rent index.

v % of non-farm housing units occupied at year end.

W_1 private wages and salaries, in current dollars.

X private output (except housing construction), in 1934 dollars.

Y disposable income, in 1934 dollars.

There are fourteen exogenous variables, as follows:

D_3 gross construction expenditures for farm housing, in 1934 dollars.

D" depreciation on all housing, in 1934 dollars.

$\mathcal{E}$ excise taxes, in current dollars.

$\mathcal{E}_R$ excess bank reserves, in current dollars.

ΔF thousands of new non-farm families.

G government expenditure (except transfers and net government interest) + net exports + net investment of nonprofit institutions, in 1934 dollars.

(exogenous variables, continued)

N^S millions of non-farm housing units at year end.

q price index of capital goods.

q_1 construction cost index.

R_2 farm rentals, paid or imputed, in <u>current</u> dollars.

ρ_o base-year non-farm rent level, in thousands of base-year dollars per annum.

T government revenue - net government interest - transfers + corporate saving,

= net national product - disposable income, in 1934 dollars.

t time, in years.

W_2 government wages and salaries, in <u>current</u> dollars.

The sixteen equations are as follows (I have put them in a different order from Klein's because I wish to group together those which pertain to any given part of the economy):

(1) definition of net national product	$Y + T = C + I + \Delta H + D_1 + D_2 + D_3 - D'' + G$
(2) demand for consumers' goods	$C = \delta_0 + \delta_1 v + \delta_2 t + u_4$
(3) demand for investment	$I = \beta_0 + \beta_1 \frac{pX-\varepsilon}{q} + \beta_2 \left(\frac{pX-\varepsilon}{q}\right)_{-1} + \beta_3 K_{-1} + \beta_4 t + u_2$
(4) demand for inventory	$H = \gamma_0 + \gamma_1 (X - \Delta H) + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 H_{-1} + \gamma_5 t + u_3$
(5) output adjustment	$\Delta X = \mu_0 + \mu_1 (u_3)_{-1} + \mu_2 \Delta p + u_{12}$
(6) demand for owned hsg	$D_1 = \epsilon_0 + \epsilon_1 \frac{r}{q_1} + \epsilon_2 (Y + Y_{-1} + Y_{-2}) + \epsilon_3 \Delta F + u_5$
(7) demand for rental hsg	$D_2 = \zeta_0 + \zeta_1 r_{-1} + \zeta_2 (q_1)_{-1} + \zeta_3 (q_1)_{-2} + \zeta_4 i + \zeta_5 \Delta F_{-1} + u_6$
(8) rent adjustment	$\Delta r = \theta_0 + \theta_1 v_{-1} + \theta_2 Y + \theta_3 \frac{1}{r_{-1}} + u_8$
(9) interest adjustment	$\Delta i = \lambda_0 + \lambda_1 \varepsilon_R + \lambda_2 i_{-1} + \lambda_3 t + u_{11}$
(10) demand for hsg space	$v = \eta_0 + \eta_1 r + \eta_2 Y + \eta_3 t + \eta_4 N^S + u_7$
(11) demand for labor	$W_1 = \alpha_0 + \alpha_1 (pX - \varepsilon) + \alpha_2 (pX - \varepsilon)_{-1} + \alpha_3 t + u_1$
(12) demand for active dollars	$M_1^D = \iota_0 + \iota_1 p(Y + T) + \iota_2 t + \iota_3 p(Y + T)t + u_9$
(13) demand for idle dollars	$M_2^D = \kappa_0 + \kappa_1 i + \kappa_2 i_{-1} + \kappa_3 (M_2^D)_{-1} + \kappa_4 t + u_{10}$
(14) definition of X	$X = Y + T - \frac{1}{p}(W_2 + R_1 + R_2)$
(15) definition of R_1	$R_1 = \frac{1}{200} \rho_0 r (vN^S + v_{-1} N^S_{-1})$
(16) definition of K	$\Delta K = I$

Equation (1) is an identity defining net national product as a sum of demands for consumers' goods, net investment, increase in inventories, housing construction (net), and goods for government use. This sum might be regarded as an aggregate demand; the fact that it is called the definition of net national product indicates that there is implicit in the model an assumption that quantity supplied always equates itself to quantity demanded.

Equations (2) to (5) are related to the market for goods and services, excluding labor and the construction of housing (these two markets will be treated separately immediately below). (2) Demand for consumers' goods is a linear function of income and of trend.

(3) Demand for net investment in plant and equipment is a linear function of (a) present and lagged values of deflated (by capital goods prices) privately produced national-income-at-factor-cost excluding housing, of (b) the stock of plant and equipment at the beginning of the year, and of (c) trend. This function is meant to show the dependence of demand for investment upon (a) anticipated receipts from sales (net of excises) relative to costs, upon (b) existing capital, and upon (c) technological change.

(4) Demand for inventory stocks to hold is a linear function of sales, of expected price change (assumed to be given by a linear combination of current and lagged prices), of the stock of inventories of the year (an inertia factor), and of trend.

Equation (5) expresses the change in private nonhousing output as a linear function of unintended inventory accumulation (assumed to be measured by $(u_3)_{-1}$, the lagged disturbance in the demand-for-inventory-stocks equation (4)), and of the rate of change of general prices. Klein says:

> In the housing and money markets..., we constructed adjustment equations to show the process by which the market is cleared of a a glut.... In both these markets, ... there is much competition so that the classical law of supply and demand operates in its traditional form. But it would be incorrect to assume competitive behavior for the rest of the market. Instead of taking price as the adjustment variable here, we take output.... According to equation [(4)], all inventories held are demanded as a result of a definite behavior pattern except for the amount u_3, which is the random disturbance. We call u_3, undesired or excess inventories which the entrepreneur holds because he misjudged the market. We shall assume, for the economy as a whole, that supply and demand balance except for a random disturbance. Immediately entrepreneurs see <u>undesired</u> inventories accumulating, they decrease production, and <u>immediately</u> they see <u>desired</u> inventories depleted, they step up production.[2]

Equations (6) to (10) pertain to the housing market. (6) Demand for owner-occupied one-family non-farm housing construction, which is purchased by consumers, is a linear function of the real value of rents (where the deflator is construction costs), of accumulated cash balances (assumed to be proportional to the sum of incomes over the three most recent years), and of the increase in number of non-farm families.

(7) Demand for rented non-farm housing construction, which is purchased by entrepreneurs, is a linear function of lagged rents, of anticipated prices of housing (assumed to be given by a linear combination of construction costs lagged one and two years), of corporate bond yield, and of lagged increase in number of non-farm families.

Equation (8) describes the determination of the rent level, which occurs in the housing-construction equations (6) and (7), as a linear function of lagged rents, lagged occupancy rate, and income.

Equation (9) describes the change in corporate bond yield, which occurs in the housing construction equation (7) and in the idle-balances equation (13), as a linear function of excess reserves, of lagged interest rate, and of trend.

(10) Occupancy rate (non-farm), which occurs in the rent adjustment equation (8), is a linear function of rents, of income, of trend, and

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2. Klein, <u>op.cit.</u>, Chapter III, pp. 54-55

of the supply of non-farm dwelling units.

Equation (11) gives the demand for labor, measured by the total wage bill, as a linear function of trend, and of current and lagged values of privately produced national-income-at-factor-cost excluding housing (which is supposed to reflect anticipated receipts from sales, net of excises). Observe that this equation could be omitted without impairing the completeness of the model, because the variable W_1 (wage-bill) does not appear in any other equation; in other words, if this equation were omitted, there would remain a system of 15 equations in 15 variables.

Equations (12) and (13) could similarly be omitted without impairing the completeness of the model, since the variables M_1^D and M_2^D (active and idle balances, respectively) occur in no other equations. (12) Demand for active balances is a nonlinear function of disposable money income and trend. (13) Demand for idle balances is a linear function of current and lagged corporate bond yield, of lagged idle balances, and of trend.

Equations (14) to (16) are definitions containing no disturbances. Equation (14) defines privately produced real output excluding housing construction, which appears in equations (3), (4), (5), and (11).

Equation (15) defines non-farm rentals, which appear in the definition of private nonhousing output, equation (14).

Equation (16) defines stock of capital, which appears in the demand-for-investment equation.

Equations (2), (3), (4), (6), (7), (10), (11), (12), and (13) are demand equations, describing behavior of various economic groups in the population. Equations (5), (8), and (9) are market adjustment equations describing responses of certain market variables to disequilibria. Equations (1), (14), (15), and (16) are identities describing definitional relationships.

III. Comments on Klein's Model.

Most of the following comments are made from an a priori theoretical point of view, without benefit of recourse to time series and the estimates derived from them. Some of the more speculative comments will of course have to be revised in the light of empirical findings.

A. Capacity: Supply and Production Functions.

The traditional economic theory of equilibrium assumes that quantity supplied is equal to quantity demanded, and thatthis equality is brought about by the adjustment of some variable (usually price) which affects one or both quantities. Klein also assumes implicitly, for most markets, that quantity supplied is equal to quantity demanded (the only two exceptions are in inventories, where unintended inventories are assumed to be equal to a random disturbance, and in the market for housing space). But he does not think of the equalization (of quantities supplied and demanded) by the adjustment of some third variable; instead, he thinks of quantity demanded as fixed (except for a random element) by a set of other variables, and then assumes implicitly that quantity supplied is automatically equal to it. This can be seen from the following facts: first, equation (1) defines net national product as a sum of quantities demanded; second, the data used to fit the nine demand equations (2), (3), (4), (6), (7), (10), (11), (12), and (13) are time-series for actual quantities consumed, invested, etc.

This assumption, that the quantity demanded is always obtained because supply is a passive factor with no influence, is objectionable for for two reasons.

First, it provides no limit in the model to the quantity of goods and services which can be produced—no concept of productive capacity, of full-employment-real-income. Such a formulation may not be out of place in a period like the 1920's or the 1930's when there was unused productive capacity, but it is not adequate for a full-employment period like the present.

Second, it produces an asymmetrical model, top-heavy on the side of demand equations. Nine of the sixteen equations are demand equations, three are market adjustment equations, and four are definitions. It is my feeling that, for each sector of the economy (where a sector corresponds to some aggregated quantity such as labor input, goods and services as a whole, consumption, etc.), the demand equation should be balanced by a supply equation or some substitute for it, and that there should also be an equilibrium condition or substitute, so that there will be a kind of completeness about the subset of equations pertaining to each sector. This kind of formalization of the rules of model-building is something I plan to look into further on another occasion.

Klein's position can be defended on two grounds. One, mentioned above, is the existence of excess capacity during most of the period 1921-41 for which his model was made. The other is that if both supply and demand equations are used, a choice between two unsatisfactory alternatives presents itself: (1) An equation stating that quantity supplied equals quantity demanded can be used as an equilibrium condition, after the manner of Walras. This has the disadvantage of being somewhat unrealistic: equilibrium in the real world is <u>not</u> something which exists continuously. (2) A market adjustment equation specifying a response of some kind to any discrepancy between amounts supplied and demanded can be used instead of an equilibrium condition. This has the disadvantage that it is difficult if not impossible to measure quantity supplied and quantity demanded as two separate magnitudes if they are not equal, because one observes only the <u>actual</u> transactions of various economic units and not their <u>intentions</u>. (Klein did not and could not use this as a defense of his procedure because he introduces market adjustment equations himself, in order to explain the price variables which he uses in some of his demand equations. But it is possible, as Colin Clark has done in a model submitted for publication in Econometrica, to use no supply or adjustment equations and no price variables.)

Klein in a recent paper[3] defends his explicit omission of supply equations on a third ground, namely that if demand functions for factors of production are known, then the supply function for output is determined because of the technical relation between input and output (i.e., the production function). This is quite true. But there is no production function in Klein's model. And it is not obvious that one may justify the omission of one equation (the supply equation) by pointing to its dependence on a second equation (the production function) if this second equation is also omitted.

Klein would defend his model against this criticism by his statement that "The technical relationship, the production function, has been, so to speak, solved out of the system in constructing the models, but it has not been neglected."[4] When Klein says the production function has been "solved out of the system" he refers to the following process (which is made explicit in Chapter II of his second draft). First a profit function and a production function are postulated:

(17) $x = \phi(n, d, h)$

(18) $\pi = f(x, n, d, h, p)$

where x = real output; n, d, h = real inputs of labor, fixed capital, and

3. L. R. Klein, "The Use of Econometric Models as a Guide to Economic Policy," Econometrica, XV, pp. 136-7, April, 1947. Reprinted as Cowles Commission Paper, New Series, No. 23.

4. Ibid, p. 136 (italics mine).

inventories respectively; π = profits; and $\underline{p}$ is a vector whose elements are prices and lagged quantities. Second the production function is substituted into the profit function to obtain what I shall call the g-function:

$$\pi = f\left[\phi(n, d, h), n, d, h, \underline{p}\right] = g(n, d, h, \underline{p}). \tag{19}$$

Third, the g-function is maximized with respect to n, d, and h, and the resulting three equations are solved simultaneously to obtain the demand-for-factor equations:

$$n = \psi_1(\underline{p}) \tag{20}$$

$$d = \psi_2(\underline{p}) \tag{21}$$

$$h = \psi_3(\underline{p}) \tag{22}$$

Then the profit and production functions are omitted from the model and are not referred to again. (This presentation of Klein's procedure neglects aggregation, and neglects the transformation from anticipated to actual profits and prices, but my point can be made just as well either way.)

Klein's claim, that the production function has not been neglected, can be true if and only if the production function is uniquely determined when the demand-for-factor equations and the profit function are given: If the production function is not so determined, then the supply function is not determined either (because the supply function can be deduced as noted above from the production function and the demand-for-factor equations), and thus an important part of the model is neglected. So the question is whether the demand-for-factor equations (20)-(22) and the profit equation (18) determine the production function (17) uniquely: if yes, Klein's claim is upheld; if no, it is not, and we should introduce explicitly the production function (or else something from which it can be derived).

To examine this problem, consider the demand-for-factor equations (20)-(22). We might integrate each of these with respect to the variable on its left side. This gives us three equations, but they can all be made to look alike by appropriate assignment (in each equation) of those "constants of integration" containing variables other than the one with respect to which integration was performed, so that we have really only one equation. Then we might think that this one equation was the "true" g-function, i.e., the function we would get if we substituted the "true" production function into the "true" profit function.[5] But we have no guarantee that the "true" g-function is of such a form that the set of equations

$$(23)\qquad \frac{\partial g}{\partial n} = 0, \qquad \frac{\partial g}{\partial d} = 0, \qquad \frac{\partial g}{\partial h} = 0$$

will be like (20)-(22), i.e., already solved for n, d, and h. It may be that the equations (23) must be solved simultaneously in order to get the demand equations into the form (20)-(22),[6] and in this case the "true"

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5. For example if the demand for factors is given by

$$(20')\qquad \alpha_1 + \beta_1 n + \gamma_1 \underline{p} = 0$$

$$(21')\qquad \alpha_2 + \beta_2 d + \gamma_2 \underline{p} = 0$$

$$(22')\qquad \alpha_3 + \beta_3 h + \gamma_3 \underline{p} = 0$$

then integration yields

$$\alpha_1 n + \tfrac{1}{2}\beta_1 n^2 + \gamma_1 \underline{p} n + f_1(d,\ h,\ \underline{p}) = 0$$

$$\alpha_2 d + \tfrac{1}{2}\beta_2 d^2 + \gamma_2 \underline{p} d + f_2(n,\ h,\ \underline{p}) = 0$$

$$\alpha_3 h + \tfrac{1}{2}\beta_3 h^2 + \gamma_3 \underline{p} h + f_3(n,\ d,\ \underline{p}) = 0$$

These three equations must all be identical, and they are all equivalent to the following g-function:

$$(19')\qquad (\alpha_1 + \gamma_1 \underline{p})n + (\alpha_2 + \gamma_2 \underline{p})d + (\alpha_3 + \gamma_3 \underline{p}) + \tfrac{1}{2}\beta_1 n^2 + \tfrac{1}{2}\beta_2 d^2 + \tfrac{1}{2}\beta_3 h^2 + f_4(\underline{p}) = g(n,\ d,\ h,\ \underline{p})$$

6. In fact it is certain, if the production function contains all the variables n, d, and h, because then each partial derivative of the g-function will contain them too.

g-function is obtained not by integrating (20)-(22) as they stand but by integrating some set of three equations of which (20)-(22) are the simultaneous solution. Since there is an infinity of such sets, it seems obvious that there is no unique g-function corresponding to given demand equations (20)-(22).[7] There is another restriction, however, on the choice of such a set of equations to integrate, besides the fact that (20)-(22) must be a solution of the set: the g-function obtained by integration of the set must be such that it could be obtained from the given profit function (18) and some production function which has in it only real quantities (i.e. which contains no prices or costs). It may be that there is only one set of equations which satisfies this restriction; this appears unlikely but I have not yet explored the point carefully[8] -- if it turns out that there is only one set, then Klein is justified in omitting the production function (although he agreed when I saw him last month, before I had written this paper, that a production function should be included).

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7. Another reason why a g-function obtained from the demand-for-factor equations (20)-(22) is not likely to be unique is this: in a set of equations, of which (20)-(22) are the simultaneous solution, some of the equations will contain more than one of the variables n, d, and h, and therefore we might not know which variable to use in integrating which equation. Therefore we might be unable to choose between g-functions derived from must one of the infinity of sets of equations of which (20)-(22) are the solution, let alone between g-functions derived from different sets.

8. A convincing, though not rigorous, indirect argument against the possibility of deriving the production function uniquely from given demand-for-factor and profit equations is this: if the production function could be so derived, then it and the demand-for-factor equations would together determine uniquely the supply-of-product equation, or in other words the demand-for-factor equations (with a given profit equation) would imply the supply-of-product equation! This seems completely absurd, and therefore it must be impossible to derive the production function uniquely from the demand-for-factor and profit equations.

B. Inventories

The treatment of inventories brings up again the problem of whether to assume that equilibrium is continuously maintained between quantities supplied and demanded. The idea of "undesired" or "unintended inventories" comes up if occasional disequilibrium is assumed to be possible, but under perpetual equilibrium it has no meaning, because equilibrium (nicely defined by Friedman as "the position which if attained would be maintained") means precisely that no one would rather that he had acted differently under the given conditions outside his control, like technical constraints and other people's demand and supply curves. Klein would rather admit the possibility of disequilibrium in most markets, and therefore he must deal with unintended inventories. So far I am with him.

In each of Klein's twelve behavior equations (2) to (13) there is a random disturbance, which is supposed to indicate that behavior is not exactly described by these equations because of various small disturbing causes which produce an unpredictable fluctuation. But in the demand-for-inventories equation (4) the disturbance u_3 is assumed to represent unintended inventories; the lagged value of this u_3 appears in the market adjustment equation (5), which describes the behavior of entrepreneurs in changing their output in response to disequilibrium. This seems to mean that, stripped of its disturbance u_3, equation (4) expresses exactly the intentions of people in holding inventories, while all the other behavior equations stripped of their disturbances describe people's intentions only approximately.

Klein's procedure here is objectionable for two reasons: First, it requires the nonrandom terms of equation (4) to account completely for intended inventories, as mentioned already, so that (4) is not a stochastic equation like the others. Second, it asks unintended inventories to be random instead of correlated with the cycle in some way.

Klein's procedure successfully avoids the problem, however, of measuring the amount of inventories which people intend to hold as distinct from those they actually hold -- and this is a problem which has to be avoided, it seems to me, because it has no solution within the framework of existing available data.

Economic theory frequently uses variables which cannot be measured, chiefly in the categories of intentions and expectations. In purely theoretical work this is not a serious difficulty, or even a difficulty at all, but in econometrics it is. One way of escape is to assume outright a certain relation between the required unobservable variable and certain observable variables, on the ground that presumably there are tangible bases for the determination of these unobservable variables. This is the kind of thing involved in Hicks' elasticity of expectation[8] and Metzler's coefficient of expectation.[9] Klein also makes use of it in transforming from anticipated to actual values of prices, profits, etc.[10]

According to this method, whenever an unobservable variable appears in an equation, it is replaced by that function of observable variables to which it is assumed to be equal. For instance, Klein replaces anticipated rate of change of price by a linear combination of price change from year -1 to year 0 and price change from year -2 to year -1.[11]

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8. Hicks, Value and Capital (London: Oxford University Press, 1939) p. 205.

9. Metzler, "The Nature and Stability of Inventory Cycles," Review of Economic Statistics, XXIII (August, 1941), p. 119.

10. Klein, Economic Fluctuations, chapter II, p. 7 and passim.

11. Klein, op.cit., chapter II, p. 7, equations (2.1.6a), (2.1.9), (2.1.6b).

This device might be used to get rid of the unobservable variable "intended (or unintended) inventories" in favor of observable variables. For example, in equation (1) the intended net inventory investment ΔH occurs as a component of income (we grant for a moment the assumption that the demand equations alone are sufficient to determine income, but recall part A of these comments). We will eliminate this variable in two steps, first expressing it in terms of observable variables only and then substituting this observable expression for it in equation (1). Intended inventory holdings H can be expressed on a priori grounds as a linear function of two unobservable variables, namely expected price change and expected sales:

(24) $H = \alpha_1 + \alpha_2 \text{ an } \Delta p + \alpha_3 \text{ an } s$

If now it is assumed that (25) expected price change is a linear combination of the two previous price changes, and that (26) expected sales are equal to last year's sales, we then have (27) intended inventories expressed in terms of observable variables only:

(25) $\text{an } \Delta p = \beta_1 + \beta_2 \Delta p + \beta_3 \Delta p_{-1}$

(26) $\text{an } s = s_{-1}$

(27) $H = \alpha_1' + \beta_2 \Delta p + \beta_3 \Delta p_{-1} + \alpha_3 s_{-1}$

Therefore we can substitute (27) for H in equation (1), and then forget about equations (25) to (27). In this way the unobservable variables an Δp, an s, and H are eliminated. The result of such a process is a model containing only observable variables, and this is just what we want in order to test the hypotheses implied in its equations.

But observe that hypotheses like equations (25) to (27) are not tested directly, because there are no observations of an Δp, an s and H with which to compare their calculated values. The same is true of equation (1), as long as it contains the unobservable H explicitly instead of some function of observable variables like (27). No equation can be tested directly which contains unobservable variables explicitly. Such an equation can be tested only indirectly, in combination with another (other) such equation(s) of a form so that they all together yield an equation which has unobservable variables eliminated and which is itself therefore directly testable. This means that if no confirmation is obtained (i.e. if there is a poor fit or poor prediction) for an equation which has been formed by eliminating unobservable variables from two or more other equations, then it is not possible to get any empirical evidence (from what has been done so far) on the question of which of the two or more other equations is at fault and which is not. The only thing to do is to change one of them and try all over again, and the results may indicate whether that change was getting hotter or colder -- whether it was an improvement or not.

I have said nothing about disturbances in connection with this device for eliminating unobservable variables. If it is desired to find a calculated value for any of the unobservable variables which have been eliminated, such as an Δp or an s or H,[12] then it is necessary that there

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12. This might indeed be desirable in evaluating the performance of a model, for while there are no data with which to compare the calculated values, there may well be accepted theoretical beliefs concerning the behavior of anticipations, or of other unobservables. A high or low correspondence between calculated and "believed-in" values of some variable might bolster or undermine one's confidence in a model.

be no disturbance in the equation expressing that variable in terms of observable variables.

C. Consumption

Klein's consumption function (2) depends only on income and a small trend. Income is certainly the most important single variable underlying consumption, but it is too much to ask that the relation between consumption and income be so stable as to be given by such a simple equation.

Income is important in the consumption function because it provides the budget constraint within which consumers must operate. Two refinements suggest themselves here. First, it might be useful to include capital gains in income for this purpose, or at least realized capital gains, because they give a person a little more elbow-room within his constraint. Since the effect of capital gains on consumption is likely to be different from that of ordinary income, it will be preferable to insert capital gains directly into the consumption function as a separate variable with its own coefficient.[13]

Second, people may spend more than their incomes through the use of consumer credit. This has to be paid back, of course, and so in the aggregate the change in consumer credit outstanding would presumably be a helpful variable in the consumption function. Change in consumer credit outstanding is a kind of hybrid involving changes both in the budget constraint facing households and in the behavior of households within a constraint. Probably the best way to untangle these two is to regard as exogenous the conditions under which consumer credit is granted (requirements

13. This suggestion is due to Koopmans. Klein uses a similar procedure in model I in his Economic Fluctuations, chapter III, page 2, when he introduces wage-income and non-wage-income as separate variables in the consumption function.

specifying minimum permissible down payments, such as Regulation W etc.) and to regard as endogenous the actual amount of consumer credit outstanding.[14]

The question of whether real cash balances influence consumption has received a good deal of attention but in the interwar years no clearcut answer was provided by the data, because real balances changed only according to aquite smooth trend and so their influence could not be tested. On an a priori basis it seems to me to be worth checking, now that we have the war and early postwar data, to see whether the inclusion of such a variable might not improve the equation. I would guess that the important thing is not the absolute value of real balances, but the deviation from some "normal" value which might be given by a time trend. But if trend is included in the consumption function anyway then the results would be the same using deviations as using absolute values.

It has often been said that cross-section budget studies probably do not give a good picture of the consumption function because a person whose income changes will not immediately settle down to the consumption pattern of his new income-class. Time-series studies should take this fact into account too, by including some lagged variable(s) in the consumption function. Klein tells me that in his current model for Canada they are getting good results using lagged consumption as a variable for this purpose.

D. Characteristics of the Production Function.

Though no production function appears in Klein's model, his demand-for-factor equations are derived from one (see part A), and

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14. The present form of this suggestion is also due to Koopmans.

therefore it is important to examine it:[15]

$$(28) \quad x = An^{\alpha_1} \; d^{\alpha_2} \; e^{\alpha_3 t + \alpha_4 t^2} \; u_1$$

If such an equation is to describe actual output, the services-of-capital variable d must depend on the amount of capital actually in use, and not (as in Klein's work) simply on the existing stock of capital.[16] The required distinction between employed and existing capital might be made on the basis of figures on percentage of capacity in use, if one doesn't mind occasionally using more than 100% of capacity, and if such figures are available either for the economy as a whole or for an exhaustive set of sectors of it.

New capital is presumably more efficient than old. Klein makes d a function of gross investment in the current year and of capital stock at the beginning of the year, thus allowing for a difference between capital less than a year old and all other capital, but this is as far as he goes. It is desirable to construct a production function in which a given physical quantity of equipment -- say one ten-inch screw-cutting lathe -- would contribute more to output if it were newer. The problem might be solved even with Klein's production function (28), I should think, by multiplying each individual quantity of capital by some number between zero and one (one for brand new capital, and progressively smaller numbers for progressively older capital) and then adding them all up to get the "total stock of capital." In this way, one physical unit of newer capital would contribute more to the "stock of capital" than one physical unit of older capital, so that the marginal

15. This is equation (2.1.1a) on page 6 of chapter II of Klein's Economic Fluctuations; n is labor input, d is services of capital, t is time, u_1 is a random disturbance, and A and $\alpha_1 \ldots \alpha_4$ are parameters.

16. For very short periods during which the level of employment of capital does not change, the two measures of capital are proportional and it does not matter which is used, but for any period covering more than just one stage of a cycle it matters a good deal.

product of a physical unit of capital would be greater for newer capital, which is what we want. Now if the number by which we multiply each quantity of capital is made equal to the percentage of its original value which is left after depreciation has been deducted, we can translate our scheme into conventional bookkeeping terms. The "stock of capital" will then be given by a sum, from minus infinity to the present, of gross investment minus depreciation, i.e. of net investment. This is the figure which is conventionally used for stock of capital. The marginal product of any given physical piece of capital, as computed from the production function under this scheme, would then be the same as the marginal product of a new piece of capital equal in quantity to the depreciated quantity of the given piece of capital.

The next comment is not so much directed at Klein as at economics: the variables entering theproduction function are supposed to be real quantities, not value quantities. But in order to add up real quantities to get aggregates like real income or product or capital etc., it is necessary to make them commensurate with each other in some way, and this is conventionally done by translating them into value terms. This would be satisfactory, except that the relative values do not stay constant, nor do the proportions of different kinds of goods measured in physical units. One continues to use index numbers and aggregates, because the constraints within which economists operate do not at the moment permit anything better, but one is uncomfortable about it.

The Cobb-Douglas type of production function might be replaced by systems of linear production functions, but I believe the latter technique is necessarily too non-aggregated for this kind of work.

E. <u>The Labor Market.</u>

If equation (11), demand for labor, were dropped from Klein's model, then total private wage-bill W_1 would be dropped too because it appears in no other equation. Thus the model is complete and determinate without any variables or equations relating to the labor market. This seems unreasonable to me, to regard the labor market as an appendage which has no effect on the remainder of the economy. I think it would be better to include a wage index and the level of employment[17] as endogenous variables, with additional equations as an integral part of the model.

F. <u>Money and Credit.</u>

In Klein's model the money market is in almost the same position as the labor market -- it is chiefly something tacked on afterwards. If equations (12) and (13), demand for active and idle balances, are dropped, then the stock-of-money variables M_1^D and M_2^D disappear also because they appear in no other equations of the model. Thus the model is complete and determinate without any money-market variables other than interest rate i.

Interest rate is completely determined in Klein's model by exogenous and predetermined variables, according to equation (9).[18] In the stripped-down model without the money equations (12) and (13), the only other equation containing the interest rate is (7), the demand for construction of rental housing. Thus even the interest rate does not enter very deeply into the economic theory underlying Klein's model.

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17. These two variables do not appear in Klein's model.

18. This means that equation 9 is self-sufficient in the sense that it can be treated alone without reference to any other variables or equations, because it is not affected by what happens in the rest of the model.

It is my intuitive feeling that monetary phenomena exert a very important influence on economic fluctuations -- this seems to me to be obvious, and the only reason I mention it is because Klein's model does not reflect this feeling. Cash balances might be expected to influence consumption and inventory-investment, and also prices and thereby wages, profits, and investment. The interest rate might be expected to influence not only the construction of rental housing, but also investment in inventories and in plant and equipment. It might even affect savings.

G. Time Lags in Production.

As observed in part A, the quantity demanded is regarded in Klein's model as being automatically produced. There is no provision for a lag between the taking of an investment decision and the flow of output as a result of it. This might be taken care of by (1) a production function in which output is determined by lagged investment and lagged stock-of-capital, or by (2) an investment function in which actual investment depends on lagged investment decisions (and hence on "lagged expectations," meaning the values which were regarded last year as being expected to occur this year), or by (3) a combination of both. The amounts of such lags might well be different for different sectors of the economy and will have to be determined empirically by trying to fit equations with various lags.

H. Exogenous Variables.

Klein has classified capital-goods price q and construction cost q_1 as exogenous, recognizing that this is a compromise. The definition of an exogenous variable, on which the estimation techniques are based, requires that it be statistically independent of the disturbances to the equations of the model, and there is no reason to believe that q and q_1 meet this requirement. Likewise, probably farm residential construction D_3 and farm rentals R_2 should not be exogenous.

The variable T, composed chiefly of government revenue, is classed as exogenous. Of course the thing which should really be exogenous is the tax-rate structure, which might be described by two or three parameters in an institutional-constraint equation expressing government revenue as a function of income. Then government revenue would be endogenous.

J. Other Comments.

Several other comments have suggested themselves to me, or have been suggested to me. Since they are almost all included in Domar's Cowles Commission Discussion Paper, Economics, 227, and since I have not yet worked out specific plans for meeting them, I will simply let the matter rest with the statement that they can be found in Domar's paper.

In other words this paper is intended to include not all the comments which I think appropriate to Klein's model, buy only those on which I have at this point something positive to say.