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Corrigendum to "Second Thoughts on Social Welfare Indices," pp. 10-11

by Kenneth Arrow

The example given to show that Scitovsky indifference, as defined on page 10, is not transitive does not, as it stands, accomplish the purpose.[1] The following example is to be substituted.

Let state I be defined by (2,1,2,1), II by (1,2,2,1) and III by (2,2,0,2). Let individual 1 have the following preference scale for bundles of goods: (3,1), (2,2), (4,0), (1,2), and (2,1); and let individual 2 have the scale, (3,0), (1,2), (0,2), (2,1), (2,0) and (1,1).

In state II, each individual is better off or not worse off than in State I, so that it is trivial to say that there exists a redistribution of the goods in state II which will make everybody no worse off than in state I. On the other hand, there are a total of 4 units of commodity 1 and 2 of commodity 2 in state I. Suppose we redistribute the quantities in state I as follows: give 1 unit of commodity 1 and 2 units of commodity 2 to individual 1 and the remainder, 3 units of commodity 1 and 0 units of commodity 2, to individual 2. Then individual 1 is just as well off as in state II, while individual 2 is better off, since he prefers (3,0) to (2,1). Therefore, there is a redistribution of the goods in state I which makes everybody at least as well off as

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1. I am indebted to F. Modigliani, University of Illinois, for pointing this out to me.

in state II, and hence, by the above definition of Scitovsky indifference, states I and II are indifferent.

By looking at their preference scales, it is clear that both individuals are better off in state III than in state II. On the other hand, there are 3 units of each commodity in state II. Redistribute them as follows: (3,1) to individual 1 and (0,2) to individual 2. Then, individual 1 is better off than in III, since he prefers (3,1) to (2,2), while individual 2 is exactly as well off. Therefore, again states II and III are indifferent.

It will now be shown that states I and III are not indifferent. It is again immediate that each individual prefers III to I. In state I, there are 4 units of commodity 1 and 2 of commodity 2; is there any distribution of those units such that everybody is at least as well off as in III?

Let (x,y) be the amount given to individual 1; then individual 2 gets (4 - x, 2 - y). Here, $0 \leq x \leq 4$, $0 \leq y \leq 2$. For simplicity, it will be assumed that the units of the various commodities are indivisible, so that x and y are restricted to integers.

Case I. $x \geq 2$, $y = 2$: Then individual 2 has (2,0), (1,0) or (0,0), all of which are inferior to (0,2), individual 2's status in III.

Case II. $x \leq 2$, $y = 2$: Then individual 1 is worse off than in state III, where he gets (2,2).

Case III. $y = 0$: Then individual 1 has at best (4,0) which is still worse than (2,2), his position in state III.

Case IV. $x \leq 2$, $y = 1$: Then individual 1 has at best (2,1), which is inferior to his position in state III.

Case V. $x \geq 3$, $y = 1$: Then individual 2 gets at best (1,1), which is inferior to (0,2), his position in state III.

Therefore, every possible redistribution in state I leads to a situation in which at least one individual is worse off than in III, so that I and III are not indifferent, and hence Scitovsky indifference is not transitive.

The comparison of states II and III illustrates another difficulty with any form of the compensation principle which does not involve actual payments. It is possible for every individual to be better off in one state than another (here, state III as compared with state II), and yet there is a redistribution of the goods in state II so that everybody is no worse off than in the obviously superior state III. This suggests strongly that unaccomplished redistributions are irrelevant.