COWLES COMMISSION DISCUSSION PAPERS, ECONOMICS NO. 234a

The Econometric Approach to Business Fluctuations.

Paper presented at the Cleveland Meeting of the American Economic Association and the Econometric Society, Christmas 1948.

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This paper will proceed from a historical description of successive stages in the development of the econometric approach to business cycle analysis to a brief discussion of some problems requiring further investigation.

Simple theoretical models

The historical origin of the approach can be placed in the construction of mathematical models embodying simple but logically complete business cycle theories. Examples are found in the following references:

Frisch:	Propagation problems and impulse problems in dynamic economics. Essays in honor of Gustav Cassel, London, 1933.
Kalecki:	A macrodynamic theory of business cycles, Econometrica, 1935, pp. 327-344.
Tinbergen:	Suggestions on quantitative business cycle theory, Econometrica, 1935, pp. 241-308.
Samuelson:	Interactions between the multiplier analysis and the principle of acceleration, The Review of Economic Statistics, 1939, pp. 75-78.

The following are the main features of these models.

1. The main objective of the models is to serve as theoretical exercises and experiments.
2. The variables involved are broad aggregates like total consumption, total investment, price indices, etc.

3. The models are <u>logically complete</u>, i.e., they consist of a number of equations equal to the number of variables whose course over time is to be explained.

4. The models are <u>dynamic</u> in that some of the equations connect variables with different timing or (as a limiting case) contain time-derivatives.

5. The models contain at most four kinds of equations, all of which we shall call <u>structural equations</u>:

    (a) <u>Identities</u>, i.e., equations valid by virtue of the definitions of the quantities involved, such as price x quantity = value, savings = investments, etc.

    (b) <u>Institutional rules</u> such as reserve requirements, tax schedules.

    (c) Technological <u>transformation functions</u>.

    (d) <u>Behavior equations</u> representing the joint response of groups of individuals or firms to a common economic environment; examples are demand equations, supply or price-setting equations, profit distribution equation, etc.

The discussion of these models produced considerable clarification as compared with previous "literary" business cycle theories, especially in the following respects.

1. The literary theories were often found to be logically incomplete.

2. A sharper distinction was made between the mechanism by which cyclical movements are propagated, and the shocks and other external (exogenous) influences whereby cyclical movements are

started or modified. For instance, in models employing a system of linear difference equations, it was found that the period and degree of damping of (each of) the cyclical movement(s) permitted by the equation system are determined by the numerical values of the parameters (coefficients and lags) of the structural equations, whereas the amplitude and phase of the cyclical movement (or of each cyclical component) at any given time are determined by initial conditions and by subsequent external (exogenous) influences, systematic or random. It was also realized that in many models turning points do not require specific explanation but come about as a result of the same relationships which describe other phases of cyclical movements.

3. It was found that a mere quantitative change in some behavior parameter could make the difference between a model responding to an external disturbance by a self-propagating cyclical movement and a model monotonically approaching an equilibrium level after each external disturbance.

The "narrowness" of some of the assumptions, like linearity of the equations, discrete integral time lags, etc., may be seen as a price paid temporarily for the sake of clarity and rigor—and well worth paying!

Discussion of policies

Especially Tinbergen developed the use of these models for experimental discussion of the effects of business cycle policies, such as countercyclical public works, price stabilization, measures to reduce speculation, etc. His technique is to associate a given policy with a change in a certain direction in one or more coefficients in the equations, and to study the effect of such changes on the period and the degree of damping of the main cyclical movement permitted by the system. Example: Tinbergen: "On the Theory of Business Cycle Control," Econometrica, 1938, pp. 22-39.

Statistical measurement of equation systems

The realization of the crucial importance of the numerical values of the coefficients and time lags prompted attempts to estimate these parameters from statistical data, largely in the form of time series. This created a completeness requirement of a new sort, which we may call statistical completeness. For each behavior equation it was necessary to specify a sufficient number of relevant variables to give the residual disturbance a random character. This is a much heavier requirement than logical completeness. A logically complete model can purposively disregard quantitatively important factors in the formation of economic decisions, and even omit important types of decision altogether, in order to concentrate on one particular model capable of propagating cyclical movements. A statistically complete model, in order to measure the influence of any particular factor at all, must estimate the effect of all quantitatively important factors. This leads to the introduction of additional explanatory variables, which, because of the requirement of logical completeness, require additional behavior equations (unless they are regarded as exogenous variables). Likewise, a statistically justified model needs to be more flexible in dealing with a complicated reality, than the "narrowness" of assumptions in the purely theoretical models permits.

Examples of studies of this kind

Author	Title	Country and Period	Type of Data	No. of Equations[1]
J. Tinbergen	An Econometric Approach to Business Cycle Problems, Paris, 1937	Netherlands 1923-35	Annual	22

1. Including identities.

Author	Title	Country and Period	Type of Data	No. of Equations
E. A. Radice	"A Dynamic Scheme for the British Trade Cycle," *Econometrica*, 1939, pp. 47-56.	U.K. 1929-37	Annual	4
J. Tinbergen	*Business Cycles in the U.S.A.* Geneva, 1939.	U.S.A. 1919-32	Annual	48
L. Klein	*Economic Fluctuations in the United States*, to be published by the Cowles Commission.[2]	U.S.A. 1921-41	Annual	16
Colin Clark	A System of Equations Explaining the U.S. Trade Cycle, 1921-41, manuscript.	U.S.A. 1921-41	Quarterly	6

Development of statistical methods

Tinbergen's statistical estimation procedures were essentially single-equation procedures based (implicitly) on the notion of one-way causation within a given behavior equation. The variable representing the response or decision of a group was taken as "dependent" variable in least squares estimation, the variables entering as factors in that decision as "independent" or (better) "determining" variables. However, this notion of a one-way street of causation exists only in the minds of the particular group taking the particular decision considered. For a considerable time econometricians (including myself) did not see that the simultaneous existence of many relationships calls for statistical procedures recognizing the simultaneous joint determination of all endogenous variables by the intersection of many behavior schedules and other structural relationships. This point was brought out clearly in Haavelmo, "The Statistical Implications of Simultaneous Equations," *Econometrica*, 1943, pp. 1-12.

As a result of this article, a revision of statistical methods has been undertaken, largely by various members of the Cowles Commission, to

2 See also L. Klein, "The Use of Econometric Models as a Guide to Policy," *Econometrica*, 1947, pp. 111-151.

take into account the co-existence of all equations of a model. This revision has two main aspects, neither of which can be regarded as in any way completed:

(a) A discussion of the identifiability of each structural equation or parameter.

(b) Actual estimation procedures for those equations or parameters that are found identifiable.

To illustrate the concept of identifiability, let us consider a model specifying only linear equations, and specifying for each equation only the set of variables allowed to occur in it. Denote by S and V the set of all equations, and the set of all variables, respectively, specified by the model. Denote by E one particular structural equation, by V_E the variables allowed in it. The set S-E of all other equations then contains the variables $V-V_E$ plus, in general, a set W_E consisting of those of the variables V_E that are common to E and S-E.

Equations	S	E	S-E
Variables	V	V_E	$V-V_E+W_E$

Now attempt to eliminate from the "other equations" S-E the variables $V-V_E$ not occurring in E. This may or may not be possible, but if possible, this elimination produces at least one linear relationship E' between variables W_E all of which are allowed in E. It is clear that in this case no number of observations will permit estimation of the coefficients of the variables W_E in the equation E. Indeterminacy is produced by the existence of a second relationship between the same variables, this second relationship being implied by the other equations (S-E) of the model. This situation, recognized and discussed originally by Frisch,[3] is now described by saying that the coefficients of variables W_E in E, and the equation E itself, are not identifiable. Criteria for identifiability in linear models have been developed.[4]

With respect to estimation problems (b) it seemed for a while as if the combined principles of logical and statistical completeness were becoming an obsession to those trying to compute estimates. It seemed that for consistent[5] estimation of even a single behavior parameter, it would be necessary to devise and estimate a complete system of equations covering all aspects of the economy. However, more flexible estimation methods have now been developed by Anderson and Rubin,[6] whereby it is, for instance, possible consistently to estimate a given structural equation while specifying only

1. the "dependent" variables occurring in that equation,
2. the "predetermined" variables occurring in that equation,

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3. R. Frisch, Statistical Confluence Analysis by Means of Complete Regression Systems, Oslo, 1934.

4. The author has submitted for publication elsewhere an expository article "Identification Problems in Economic Model Construction" which contains full references to the original work on identification.

5. An estimate of a parameter is called consistent if, in the sampling distribution of the estimate, the probability of a deviation from the true value of the parameter larger than some given amount (however small) tends to zero as the number of observations is increased indefinitely.

6. H. Rubin and T. W. Anderson, "Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations," Annals of Mathematical Statistics, March, 1949.

3. a number of other "predetermined" variables occurring in the remainder of the system, this number to be sufficient to ensure identifiability of the equation to be estimated.

Here "predetermined" variables means either exogenous variables or lagged values of endogenous variables, and "dependent" variables means values of endogenous variables not subject to time lags.

This does not seem to be the minimum information necessary for consistent estimation, but it is the minimum amount for which satisfactory estimation methods have been worked out. Similar procedures have been devised by Rubin for simultaneous estimation of a subset of a complete set of equations. It is a general principle of these estimation methods based on incomplete specification of the model that estimates are both more accurate (in large samples) and more costly to compute, the greater the extent to which the entire model is correctly specified, with respect to the variables entering into each equation, and with respect to the functional form of the equations. The equations of Klein's model have all been estimated by the "limited information method" developed by Anderson and Rubin.

Nature of the results

The expression "statistical testing of business cycle theories" has been used in connection with the construction of models. In one respect this expression claims too much, in another respect (as Klein has pointed out) it claims too little. The term "statistical screening of business cycle theories" would convey better that unique answers are often not obtained from the limited data available. The term "statistical construction of business cycle theories" would express better the contribution that work with the data makes to the formulation of hypotheses. But the latter term would fail to indicate the importance and indeed indispensability of hypotheses concerning economic behavior, which are not derived from the time series studied. "Business cycle theories statistically screened and inspired" seems the best single phrase to describe the nature of the results obtained or aimed for by econometric model construction.

As a very simple example of the screening function of econometrics let us consider the well-known acceleration principle as an explanation of fluctuations in investment activity. This principle has been given a prominent role in theoretical models by Harrod,[7] Samuelson (l.c.) and others. In its strict formulation the principle says that net investment fluctuates in such a manner that the stock of producers' goods is made to maintain a constant proportion to the total rate of production.[8] This proposition has a strong appeal to the theorist, but its validity hinges on the implied assumption that productive capacity is at all times in substantially full use. By a simple comparison of time series of the rate of increase in production and the rate of investment, for some industries and for the whole economy of various countries, Tinbergen established[9] that the correlation between these series is far

7. R. F. Harrod, The Trade Cycle, Oxford, 1936.

8. Many authors write here "rate of production of consumers' goods" instead of "total rate of production," but this is not even good theory, since a certain portion of the stock of producers' goods serves for the production of producers' goods.

9. J. Tinbergen, "Statistical Evidence on the Acceleration Principle," Economica, May, 1938.

from close, and, to the extent that it exists, the amplitude of the fluctuations in capacity increase is about half of that required by the strict acceleration principle. This result defeated any claim for the acceleration principle as indicating the main factor determining investment activity, and stimulated the formulation of alternative hypotheses, by which investment responds to profits or profit rates as the main basis of profit expectations.

As an example of a situation where the screening is inconclusive, let us place ourselves in the position of 1945 and consider the important question of the effect of liquid assets on consumption expenditure out of a given income. This question derives its importance from the circumstance that, since World War II, liquid assets held by consumers are much larger than they have previously been. But this very fact, combined with the circumstance that even at their lower level of inter-war years liquid assets showed only moderate fluctuations around a smoothly rising trend, makes for a very wide margin of error in the statistical estimate of the response coefficient of consumption to liquid assets from inter-war data,[10] i.e., from the only data available in 1945.

In this example the statistical screening is inconclusive because a relevant variable has stepped far outside its previous range of variation. Other instances of inconclusiveness are met when a priori considerations admit to a given behavior equation a number of variables between which other relationships also exist. The discussion of identification problems is designed to trace this type of indeterminacy systematically to its source.

10. See L. Klein, "The Use of Econometric Models as a Guide to Policy," Econometrica, 1947, pp. 111-151, especially pp. 22-23.

In such cases, the degree of indeterminacy in results can sometimes be diminished, by the use of data other than time series (e.g., interview data, cross section data). Another possible device toward improvement of identifiability is disaggregation of a decision variable as a means of introducing explanatory factors specific to the decisions of smaller groups of economic agents. In the article referred to in footnote 4 I have commented on the use made of this device by Ezekiel[11] to obtain identifiability of the investment equation.

It cannot be held against the econometric approach, I believe, that in certain circumstances it may leave important questions unanswered. Its true calling is not to answer all questions. Rather, its task can be described as follows:

1. To formulate all relevant hypotheses to which the available data may conceivably make an answer possible.

2. To extract from these data all information bearing on those hypotheses.

3. To select from the set of competing hypotheses the one hypothesis best supported, or the set of those hypotheses equally well supported, by the data.

4. To evaluate in some way the degree of confidence which can be placed in the rejection of the hypotheses not so selected.

It may be added, as has often been pointed out, that the logical form of these hypotheses is often a conditional one, such as "given the

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[11] M. Ezekiel, "Saving, Consumption and Investment II," *American Economic Review*, June 1942, pp. 272-307.

occurrence of variables x_1, x_2, x_3 in a certain behavior equation" the hypothesis states "that variable x_4 does not influence the type of decisions involved."

The conditional form of these hypotheses leaves full scope for the use of general theoretical knowledge as well as incidental historical information. In fact, the quantitative measurement of general behavior patterns can be seen as a framework against which incidental departure from the pattern can be more easily perceived. For instance, how shall we determine from post-World War II data whether the higher volume of liquid assets has indeed stimulated consumption expenditure, if no estimate is available as to what consumption would be if the former degree of liquidity prevailed? In his paper earlier in this session, Professor Gordon mentioned, as a difficulty of the historical method, the problem of assigning relative weights to the various factors operative in a given historical situation. The econometric method estimates relative response coefficients representing the average weights with which various factors enter in a relationship during the whole of a certain period. A large unexplained residual found for a given year from such an analysis is evidence either that one of the factors has had considerably more (or less) than its average influence in that year, or that a special factor not generally operative has exerted considerable influence in that year. Thus incidental factors are found (unless two or more of them cancel each other's effects) by incidental failure of the econometric explanation. The econometric approach thus appears not as a competitor of the historical approach, but as an important instrument of it.

Problems for further investigation

Among the most important problems for further inquiry I would place what might be called the economics of model construction. The "output" of model construction is (conditional) knowledge concerning the behavior patterns of economic agents and the implications of these patterns for the dynamic properties of the economy as a whole. The "inputs" are efforts of research workers and resources devoted to data collection and computation. A choice between "methods of production" is involved in decisions regarding the degree of aggregation of variables as affecting the number of equations in the model, regarding the number of variables in each equation, regarding the functional form of the various equations, etc.

Models may either be "general-purpose" models, to extend our knowledge of the generation of business cycles, or models designed specifically to help answer a particular policy problem, such as assessing the effects of countercyclical tax policy. For a "specific-purpose" model, the choice of the various dimensions of the model is guided by a balancing of marginal cost against presumed added precision in answering the policy question raised. A somewhat systematic enumeration and description of policy problems that could be brought nearer solution by econometric models would therefore be a helpful guide to model-builders. Such an undertaking would have two aspects: the formulation of policy objectives (such as full employment with efficient use of resources) and the description of the main instruments of policy (such as tax schedules, social security rates, reserve requirements).

In the construction of a "general-purpose" model intuitive considerations necessarily have a larger role to play. But even here we meet the technical production problem of a balancing of two ways in which the quality of the "product" is affected by varying the dimensions of the model. Many criticisms of the econometric approach can in principle be met by further refinements of the model used. Change in behavior parameters can, for instance, be introduced by describing the mode of change by other parameters. But clearly, no model however detailed can claim to accommodate all somehow relevant aspects of a complicated and changing reality. Suppose, however, for the sake of argument, that the specification of the model (i.e., the definition of each equation and of the variables in it, the specification of the functional form of each equation, possibly including gradual change of its coefficients, the choice of the nature of the distribution of random elements, etc.) were our only problem, because some benevolent deity has undertaken to supply, for each such model that we may devise, those numerical values of the unknown coefficients and other parameters specified in the model, that most closely approximate reality. Then disaggregation and added detail would always be rewarded by added knowledge. In fact no such deity is operating, and the parameters of each model need to be estimated from observations spanning a finite time-period. A point may therefore be reached where the potential benefit from further detail is defeated by less accurate estimation of any given parameter.[12] Thus, for each body of data, there is such a thing as an optimum degree of detail,[13] although we are far from knowing at which point, in various directions, this optimum degree is reached.

It is highly probable, that the substitution of quarterly data for annual data would lead to improvements in knowledge well worth the added

12. The mathematical reason for this effect is known as "loss of degrees of freedom in estimation."

13. That is, even without counting the research effort as a cost. The optimum will involve less detail if such costs are counted heavily, although I feel that, in view of the potential value to society of the added accuracy of results, such costs should not weigh heavily.

cost. As a preliminary step in that direction, the Cowles Commission is now investigating models permitting serially correlated random elements.[14]

It may be emphasized again that any results reached are conditional upon hypotheses not derived from or tested by the time series studied. Another important group of problems for further inquiry is the elaboration and consolidation of the theory of economic behavior of populations of individuals or firms under conditions of economic change and uncertainty. The econometric approach is not a substitute for theory, but one of the servants of theory.

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14. See the communications by H. Rubin and H. Chernoff in the session of the Institute of Mathematical Statistics of December 27, 1948, Cleveland, Ohio.