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A Note on the Theory of Investment

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In modern theories of macro-economics the ideas regarding the nature and origin of the demand for investment seem to be in a state of confusion. Sometimes the demand for investment is thought of as a result of profit maximization on the part of the producer. Sometimes it is regarded as a result of speculative purchases of new equipment by a certain group, called "investors". Sometimes the demand for investment goods is presented as a function of national income, or of profit, etc. without being derived from any rational theory of behavior at all.

I believe that much of this confusion is due to a vain attempt to squeeze the theory of investment into some static macro-model. To bring out this point I shall first try to illustrate the role of investment in a more or less classical model that is "almost" static.

I consider a closed economy where there is just one homogeneous commodity which can be used either for consumption purposes or for building up the stock of producers' capital (i.e. for investment). (Alternatively one might e.g. consider total output measured in labor units as one commodity. For our argument here such refinements are however not very essential, I think.) Let us introduce the following notations

1) C = total consumer expenditures.

2) R = national income.

3) X = volume of total net output.

4) P = price per unit of X.

5) K = physical stock of capital.

6) I = volume of net investment = the increment of K per unit of time.

7) N = employment.

8) w = wage rate.

9) r = rate of interest.

10) M = stock of money (or liquid assets).

Consumers' expenditure.

We assume that consumers' behavior leads to a consumption function of the following type

$$\frac{C}{p} = f\left(\frac{R}{p}, \frac{M}{p}\right). \tag{1}$$

The production function

$$X = \Phi(N, K). \tag{2}$$

Behavior of producers.

We assume that producers try to adjust themselves to given prices and wages so as to maximize profit. Although society as a whole cannot increase the stock of capital except through a gradual process over time, the individual producers can of course make instantaneous changes in his stock of capital by sales or purchases out of the existing capital stock. There is no reason, from the point of view of profit maximization, why the individual producer should stagger the aquisition of needed capital over time in a continuous fashion that would resemble the "rate of investment" for society as a whole. That "investment per unit of time" is a meaningful concept is due to the fact that production of capital technically is a time-requiring process and not to any characteristic time-requiring process in the "demand for investment". The individual producer can borrow money and buy equipment from others instantaneously, if he pays the required price. Let the aggregate of the profit function of all the producers be

$$PX - wN - rPK$$

where wN is the total wage bill, PK the value of total capital stock and, therefore, rPK the annual interest charges. Maximizing profit with respect to N and K we obtain

(3) $$\frac{\partial \phi}{\partial N} - \frac{W}{P} = 0$$

(4) $$\frac{\partial \phi}{\partial K} - r = 0.$$

<u>The supply of labor.</u>

We assume there is a supply function for labor of the following type:

(5) $$N = N\left(\frac{W}{P}, \frac{M}{P}\right).$$

<u>Liquidity preference.</u>

(6) $$r = L\left(\frac{M}{P}, \frac{R}{P}\right)$$

<u>Supply of money.</u>

(7) M = given quantity of money.

<u>Definitions and market equations.</u>

(8) $$R = PX$$

(9) $$R = C + IP$$

(10) $$\dot{K} \equiv \frac{dK}{dt} = I$$

This system is determinate in the sense that there are just as many equations as unknowns. (Because of (7) the system is <u>not</u> homogeneous in prices and wages).

Is there room for any separate theory of the demand for investment in this model? The answer is that there is a more or less <u>passive</u> demand for investment implicit in the system. It works somewhat as follows: A part of total output, X, is absorbed as consumer goods through the mechanism of the consumption function. The remainder of X is automatically invested.

Why is there no obstacle to selling this remainder ? Because at time t the rate of investment cannot influence profit. It takes time for investment to influence K no matter how high the investment rate is, provided it is finite. At time t it is in fact entirely immaterial to the producers whether or not they absorb the net flow of goods that are available for investment purposes. From the point of view of maximizing profit there is no incentive whatsoever to invest, once the price P is such as to satisfy the system (1) - (10). If the producers, at time t, buy the stream of goods that are available for investment, this is, at time t, a "no profit - no loss" operation. The "almost static" system above is not, however, adequate to bring out this labile character of the demand for investment at the point of equilibrium. But it becomes clear when the system is made really dynamic.

We said that our system so far was "almost static". By this we mean that the only "dynamic" element of the system is that which is brought in by the definition (10). The system presumably has a stationary solution defined by $K \equiv 0$, for all t. The system has in general also non-stationary solutions, where K is not identically zero. This fact already suggests that the possibility of a rational theory of investment requires a theory that involves at least some sort of dynamic elements. Most economists would, however, probably say that the theory above is not "very dynamic", and that the general solution of the system practically speaking is stationary, apart from a "trend" brought in via the gradual growth of the stock of capital. In fact, considering the system as a theory of instantaneous equilibrium at time t one might replace the equation (10 by the statement K_t = a given quantity at time t, and then solve for all the other variables at time t. Regarded in this manner the system shows how equilibrium is established at any point of time. All the "interesting" adjustments take place instantaneously.

I think, however, that the impossibility of a stationary solution where investment is $\neq 0$ indicates something that is much more fundamentally related to economic behavior than is apparent from the system above. To see this, let us consider the profit maximizing equations (3) and (4). How would a producer behave in a situation where these equations are not yet fulfilled? If marginal revenue with respect to labor is larger than the wage he would probably be interested in increasing his expenditure on labor. Similarly, if the marginal revenue of capital were larger than its marginal cost i.e. the interest charges, he would probably want to spend more on capital. These processes of adjustment may conceivably take some time, the speed of adjustment depending on the characteristic behavior of the entrepreneurs.

It would seem then, that if we were to write the equations (3) and (4) not as static equilibrium conditions but as dynamic relations showing the <u>forces</u> directed towards an eventual equilibrium, their form could be approximated as follows:

(3a) $$\frac{d}{dt}(wN) = a_1 \left(P\frac{\partial \phi}{\partial N} - w\right)$$

(4a) $$\frac{d}{dt}(PK) = a_2 \left(P\frac{\partial \phi}{\partial K} - rP\right),$$

where a_1 and a_2 are (positive) constants. (3a) says that the producers will increase their outlay on labor the more rapidly the larger is marginal profit with regard to labor. Similarly, equation (4a) says that the producers will increase their expenditure on capital the more rapidly the larger is marginal profit with regard to capital.

Carrying out the derivation in the left hand members of (3a) and (4a) we have

$$(3a^{*}) \qquad \dot{w}N + w\dot{N} = a_1\left(P\frac{\partial\phi}{\partial N} - w\right)$$

$$(4a^{*}) \qquad \dot{P}K + P\dot{K} = a_2\left(P\frac{\partial\phi}{\partial K} - rP\right).$$

If, in the system of 10 equations above we replace (3) and (4) by ($3a^{*}$) ($4a^{*}$) we have a more highly dynamized system. (We might introduce dynamic elements in the other behavior equation too, but that is irrelevant to our argument here).

The interesting question now is the following: Does the system have a particular solution such that the left hand sides of ($3a^{*}$) and ($4a^{*}$) vanish identically in t, and such that $K \neq 0$.

Obviously this is in general not the case. For suppose that it were true. Then we would be back at our system (1) - (10) above. This system presumably has a solution, $C = \overline{C}(t)$, $R = \overline{R}(t)$, $X = \overline{X}(t)$ etc., for all the 10 variables involved. These solutions contain only one arbitrary constant, which for example may be the value of K at $t = t_0$. Only under special assumptions would these solutions be such that, for all values of t, wN = constant and PK = constant.

Hence, we reach the following conclusions:

a) An expansion, or contraction, of the total wage bill and the value of the capital stock takes place only as long as the point of maximum profit is not yet reached.

b) If, at a point when marginal productivity of labor is equal to the real wage rate, $\dot{N}$ is not zero, this must be due to the fact that $\dot{w}$ is $\neq$ zero.

c) If, at a point when the marginal productivity of capital is equal to the rate of interest, $\dot{K}$ (i.e. real investment) is not zero, this must be due to the fact that $\dot{P}$ is $\neq$ zero.

Equation ($4a^*$) brings out the "high-volatility" nature of the rate of real investment. It depends, at any given moment upon the degree to which profit has been maximized and upon the type of price movements that the system permits. Furthermore, it is likely that a_2 is a rather unstable parameter.

The "high volatility" nature of the demand for investment and its influence upon employment can also be described in another way: The supply of goods depends on the presence of a certain stock of labor and a certain stock of capital and, thus, implies - on the part of the producers - essentially a demand function for N and a demand function for K. But the demand for the output, X, depends on the demand for $\frac{C}{P}$ and the demand for $\dot{K}$. Since the essential parameters of action of the producers are their shares of K and not of $\dot{K}$, an effective demand for all the investment goods that the producers want to supply can be insured only by the presence of some stable element of "inertia" as suggested by the dynamic relation ($4a^*$).