Prices and Investment Incentives

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I. Introduction

We are interested in markets in which

- buyers and sellers first make investments,

- and then trade, realizing values that depend on their (and their partner's) investment and the terms of trade,

- but are subject to contractibility constraints.

The basic questions include:

- How do prices clear the market?

- When do the market prices create efficient investment incentives?

- What kinds of prices are required for efficiency?

- What insights might we gain into market design?

II. An Example: Relationship-Specific Investments

Consider a single buyer and seller. Before trading, each can make an investment, at cost 5, that affects the buyer's value v_b and the seller's cost v_s. In particular,

$$v_b = \begin{cases} 6 & \text{if} \quad \text{no investment} \\ 14 & \text{if} \quad \text{investment} \end{cases}$$

$$v_s = \begin{cases} 8 & \text{if} \quad \text{no investment} \\ 0 & \text{if} \quad \text{investment} \end{cases}$$

The efficient outcome is that both invest.

Suppose that once investments have been made, price is determined as the limiting (as time-period become arbitrarily short) outcome of a complete-information alternating-offers bargaining game between equally patient players.

Then equilibrium prices will be

v_b	v_s	Price
14	8	11
14	0	7
6	8	no trade
6	0	3.

As a result, neither agent will invest.

This “hold-up” problem has been the subject of a large literature, beginning with Williamson.

- If the agents could contract on investments, there would be no problem.

- If the agents could contract on valuations, there would be no problem.

- If the agents were price takers, there would be no problem.

The resulting literature has concentrated on

- designing contracts to make the right agents price-takers at the right time,

- characterizing the inefficiencies that arise, or

- characterizing ways to restore contractibility.

This work has focussed attention on the second-stage market power as the obstacle to obtaining efficient investment incentives, and on the second-stage price-formation process that reflects this market power.

We suspect that

- situations are very common in which people make noncontractible investments before trading,

- market power is not the only obstacle to efficiency in such markets.

We would like to take the issue of market power out of the model by making the second stage competitive, and then study how markets and their prices create (efficient?) investment incentives. Then we can ask

- When will outcomes be efficient, both at the trading stage and the investment stage?

- What is the interaction between prices and investments?

- If prices do not work well, how will investments be affected?

- Do what extent do investment help clear the market?

III. Decision-Specific Investments

Suppose we have

- Continuum of buyers indexed by i.

- Buyer $i \in [0, 1]$ has utility function over labor input and expenditure in the market in question, refereed to here as expenditure on a house, given by

$$u(\ell, ph) + (1 - i)h,$$

where $\ell \in \Re_+$ is the buyer's choice of labor, p is the price of a house, and $h \in \{0, 1\}$ is a binary variable indicating whether the buyer has purchased a house. We will think of the choice of ℓ as the buyer's investment.

Let $\ell^*(p)$ be the optimal level of labor when a buyer buys a house for price p:

$$\ell^*(p) = \arg\max_{\ell} u(\ell, p) + (1 - i) = \arg\max_{\ell} u(\ell, p)$$

so that $\ell^*(p)$ does not depend upon the buyer's index i and satisfies

$$u_1(\ell^*(p), p) = 0$$

Under intuitive assumptions,

$$\frac{d\ell^*(p)}{dp} = -\frac{u_{12}(\ell^*(p), p)}{u_{11}(\ell^*(p), p)} > 0.$$

- Continuum of sellers indexed by j.

- Seller $j \in [0,1]$ has utility function over his labor input and the (possibly 0) proceeds from selling his house

$$v\left(e, p\left(1-h\right)\right) + jh,$$

where $e \in \Re_{+}$ is the seller's choice of labor (or investment), p is the price of the house, $h \in \{0,1\}$ is a binary variable indicating whether the potential seller has sold his house ($h = 1$ indicates that the seller has not sold).

Let $e^*(p)$ be the optimal level of labor when a seller sells his house for a price p:

$$e^*(p) = \arg\max_e v(e,p) + j = \arg\max_e v(e,p),$$

so that $e^*(p)$ satisfies

$$v_1(e^*(p),p) = 0,$$

with

$$\frac{de^*(p)}{dp} = -\frac{v_{12}(e^*(p),p)}{v_{11}(e^*(p),p)} < 0.$$

Once again the optimal labor choice depends upon the anticipated price, but not the seller's index.

Suppose investments and housing transactions are made simultaneously. Then there is a unique "Arrow-Debreu" price p^* that equates quantities supplied and demanded.

Associated with the price p^* is the marginal buyer

$$i(p^*) \equiv 1 + u(\ell^*(p^*), p^*) - u(\ell^*(0), 0)$$

and marginal seller

$$j(p^*) \equiv v\left(e^*\left(p\right), p^*\right) - v\left(e^*(0), 0\right) = i(p^*).$$

The set of buyers who purchase is given by $[0, i(p^*)]$ and the set of sellers who sell is given by $[0, j(p^*)]$.

The efficient outcome is for buyers and sellers in the intervals $[0, i(p^*)]$ and $[0, j(p^*)]$ to undertake investments $\ell^*(p^*)$ and $e^*(p^*)$ and to trade the good at price p^*, with other buyers and sellers choosing investments $\ell^*(0)$ and $e^*(0)$.

Now suppose agents make their labor choices *before* entering the housing market, correctly predicting the subsequent market-clearing price.

For example, a buyer chooses the labor before the price for houses is realized, while the purchase decision is made after the price is realized.

In equilibrium, L_i^* is (weakly) decreasing and E_i^* increasing.

These gives us ex post reservation prices, which for buyers are strictly increasing in ℓ and strictly decreasing in i. For sellers they are strictly decreasing in e and strictly increasing in j.

Ex post demand and supply are given by:

$$\begin{aligned} D(p, L) &= \lambda\{i | u(L_i, p) + 1 - i \geq u(L_i, 0)\} \\ S(p, E) &= \lambda\{j | v(E_j, p) \geq v(E_j, 0) + j\}. \end{aligned}$$

A (possibly random) price function is given by

$$p(\omega) : [0, 1] \rightarrow \Re_+.$$

A *rational expectations equilibrium* is a price function, $p^* : [0, 1] \longrightarrow \Re_+$, and labor and effort choices, L^* and E^*, such that, for all $\omega \in [0, 1]$,

$$D(p^*(\omega), L^*) = S(p^*(\omega), E^*, \omega),$$

and buyers and sellers are choosing labor and effort optimally, given p^*.

Given a random price, let γ denote the induced measure over prices, with

$$\begin{aligned} \underline{p}(\gamma) &= \min \{p \in supp\,(\gamma)\} \\ \overline{p}(\gamma) &= \max \{p \in supp\,(\gamma)\}. \end{aligned}$$

In any rational expectations equilibrium, there is an index $i(\gamma)$ such that all buyers $i < i(\gamma)$ are willing to trade at any price in $[\underline{p}(\gamma), \overline{p}(\gamma)]$, while no buyer $i > i(\gamma)$ is willing to trade at any price in the interval.

Similarly, there is an index $j(\gamma) = i(\gamma)$ such that all sellers $j < j(\gamma)$ are willing to trade at any price in $[\underline{p}(\gamma), \overline{p}(\gamma)]$, while no seller $j > j(\gamma)$ is willing to do so.

$\underline{p}(\gamma)$ is positive, as is the volume of trade.

Proposition 1

(1.1) There is a unique deterministic rational expectations equilibrium of the certain economy, given by the Arrow-Debreu market-clearing price p^**. This equilibrium is efficient.*

(1.2) Equilibria with random prices exist. In such equilibrium, the quantity of trade on the housing market is the same for all realized prices.

$$\underline{p}(\gamma) < p^* < \overline{p}(\gamma),$$

and

$$i(\gamma) < i(p^*),$$

and hence any equilibrium with random prices is inefficient.

Why should we expect prices to be random? Why are we interested in equilibria with random prices?

More generally, the coincident discontinuities in demand and supply seem hopelessly special. Wouldn't even tiny perturbations eliminate the coincidence?

Perturb the economy slightly by adding in a random amount of "noise sellers." This will smooth out supply and demand and give a unique equilibrium for every realized quantity of random noise traders. Let the quantity of noise sellers have support $[0, \varepsilon]$, with realization ω.

Lemma 1 *For almost all ω, the ex post market-clearing price $p(\omega)$ is unique. The ex post market-clearing price is strictly decreasing in ω.*

Proposition 2 *A rational expectations equilibrium exists for every* $\epsilon > 0$*.*

Proposition 3

(3.1) Suppose $\varepsilon \longrightarrow 0$*. Then any converging subsequence of equilibrium prices* p_ε *converges weakly to a sunspot equilibrium distribution of the unperturbed economy.*

(3.2) The allocation induced by the limiting prices $\lim_{\varepsilon \to 0} p_\varepsilon$ *is inefficient.*

As a result, the inefficient outcome is the robust one, and the efficient outcome the coincidence.

What lies behind this inefficiency?

- We have many agents in the ex post economy, but perhaps we have too few types.

- Let us consider instead a model with many *heterogeneous* agents in the ex post market.

IV. Match-Specific Investments

Let there be

Unit measure of buyers, with types $\beta \in [0, 1]$.
Unit measure of sellers, with types $\sigma \in [0, 1]$.

Each buyer β chooses investment $b \in [0, \bar{b}]$ at cost $c_B(b, \beta)$.

Each seller σ chooses investment $s \in [0, \bar{s}]$ at cost $c_S(s, \sigma)$.

A match of a buyer of investment b with a seller of investment s results in a gross surplus of

$$v(b, s) = h_B(b, s) + h_S(b, s) \geq 0,$$

where $h_B(b, s)$ and $h_S(b, s)$ are the buyer and seller *premuneration* values.

Assumptions:

1. Surplus v is $\mathcal{C}^2$, increasing in b and s, and strictly supermodular:

$$\frac{\partial^2 v(b,s)}{\partial b \partial s} > 0.$$

2. Cost function c_B is $\mathcal{C}^2$, strictly increasing, convex in b, satisfies

$$c_B(0,\beta) = \frac{\partial c_B(0,\beta)}{\partial b} = 0,$$

and single-crossing:

$$\frac{\partial^2 c_B(b,\beta)}{\partial b \partial \beta} > 0.$$

Analogously for c_S.

3. Given a price p for the match of $b - s$, buyer's payoff is

$$h_B(b, s) - p,$$

while seller's payoff is

$$h_S(b, s) + p.$$

4. Premuneration values $h_B(b, s)$ and $h_S(b, s)$ are $\mathcal{C}^2$, weakly increasing, with

$$\frac{d^2 h_B(b, s)}{dbds} > 0 \qquad \text{a}nd \qquad \frac{d^2 h_S(b, s)}{dbds} \geq 0;$$

Information:

- Seller investments are public, and hence prices can condition on seller investments.

- We will be interested in the case in which buyer investments are also public, in which case prices can be condition on both buyer and seller investments (can be *personalized*) as well as the case in which prices cannot be conditioned on buyer investments (and so are *uniform*).

Feasible Matching:

The second stage begins with investment functions $\mathbf{b} : [0,1] \longrightarrow [0,\bar{b}]$ and $\mathbf{s} : [0,1] \longrightarrow [0,\bar{s}]$.

The (closures of) the set sets of investments are:

$$\begin{aligned} \mathcal{B} &= \mathsf{cl}(\mathbf{b}([0,1])) \\ \mathcal{S} &= \mathsf{cl}(\mathbf{s}([0,1])). \end{aligned}$$

Intuitively (e.g., in equilibrium), a feasible matching is a measure preserving bijection $\tilde{b} : \mathcal{S} \longrightarrow \mathcal{B}$. Its inverse is $\tilde{s}$.

A personalized price function is a function $p_P : \mathcal{B} \times \mathcal{S} \to \mathbb{R}$.

A personalized-pricing equilibrium is a feasible outcome $(\mathbf{b}, \mathbf{s}, \tilde{b})$ and a personalized price function p_P such that "no seller or buyer has a profitable deviation."

Properties of Equilibrium:

1. Personalized price equilibria exist (a straightforward argument).

2. In every personalized pricing equilibrium, $\mathbf{b}$ and $\mathbf{s}$ are increasing, and matching is positively assortative in investments.

3. Premuneration values are irrelevant.

4. Constrained efficiency: For any type $\sigma = \beta \equiv \phi$,

$$v(\mathbf{b}(\phi), \mathbf{s}(\phi)) - c_B(\mathbf{b}(\phi), \phi) - c_S(\mathbf{s}(\phi), \phi)$$

equals

$$\max_{b \in \mathcal{B}, s \in \mathcal{S}} v(b, s) - c_B(b, \phi) - c_S(s, \phi), i.e.,$$

the underinvestment incentive and matching externality cancel.

Effectively, the market is competitive.

5. Possible (unconstrained) inefficiency: Inefficient personalized price equilibria exist, in which coordination failures lead to inefficient investments in attributes.

6. Rationing. A personalized price equilibrium is a uniform rationing price equilibrium if

$$p_P(b, s) = p_P(\tilde{b}(s), s) \ \forall b \geq \tilde{b}(s).$$

Every personalized price equilibrium can be supported by a uniform rationing price.

Now suppose prices cannot be conditioned on buyers' investments.

A uniform price function is a function $p_U : \mathcal{S} \to \mathbb{R}$.

A uniform-pricing equilibrium is a feasible outcome $(\mathbf{b}, \mathbf{s}, \tilde{b})$ and a uniform price function p_U such that "no seller or buyer has a profitable deviation."

Properties of Equilibrium:

1. Uniform price equilibria exist (a tedious and convoluted argument).

2. In every uniform pricing equilibrium, $\mathbf{b}$ and $\mathbf{s}$ are increasing, and matching is positively assortative in investments.

3. Premuneration values matter.

4. Uniform-price equilibria are in general inefficient, even in the absence of coordination failures.

The origins of inefficiency (for premuneration values that nearly allow efficinecy):

- Sellers who set their equilibrium personalized prices would attract low-quality buyers whom they can no longer ration out.

- In response, sellers raise their prices to deter such buyers, but also increase their investments to retain good buyers.

- Buyers reduce their investments.

- Seller payoffs increase, buyer payoffs decrease.

For less fortuitous premuneration values, all agents are worse off, and trade can collapse completely.

When can (an efficient) personalized price equilibrium be achieved as a uniform price equilibrium?

In a uniform-pricing equilibrium, $\mathbf{b}$, $\mathbf{s}$, and the matching function $\tilde{b}$ (or $\tilde{s}$), solve

$$\max_{b,s} h_B(b,s) - p_U(s) - c_B(b,\beta), \quad \text{and}$$
$$\max_{s} h_S(\tilde{b}(s),s) + p_U(s) - c_S(s,\sigma).$$

Implied first order conditions:

$$\frac{\partial h_B(b,s)}{\partial b} - \frac{\partial c_B(b,\beta)}{\partial b} = 0,$$
$$\frac{\partial h_B(b,s)}{\partial s} - \frac{dp_U(s)}{ds} = 0,$$
$$\frac{\partial h_S(\tilde{b}(s),s)}{\partial b}\frac{d\tilde{b}(s)}{s} + \frac{\partial h_S(b,s)}{\partial s} + \frac{dp_U(s)}{ds} - \frac{\partial c_S(s,\sigma)}{\partial s} = 0.$$

These can be rewritten to eliminate the buyer's premuneration value and price function as:

$$\begin{aligned}
\frac{\partial v(b,s)}{\partial b} - \frac{\partial h_S(b,s)}{\partial b} - \frac{\partial c_B(b,\beta)}{\partial b} &= 0, \\
\frac{\partial h_S(\tilde{b}(s),s)}{\partial b}\frac{d\tilde{b}(s)}{s} + \frac{\partial v(b,s)}{\partial s} - \frac{\partial c_S(s,\sigma)}{\partial s} &= 0,
\end{aligned}$$

Because personalized price equilibria are constrained efficient:

$$\begin{aligned}
\frac{\partial v(b,\tilde{s}(b))}{\partial b} - \frac{\partial c_B(b,\beta)}{\partial b} &= 0 \quad \text{at} \quad b = \mathbf{b}(\beta), \\
\frac{\partial v(\tilde{b}(s),s)}{\partial s} - \frac{\partial c_S(s,\sigma)}{\partial s} &= 0 \quad \text{at} \quad s = \mathbf{s}(\sigma).
\end{aligned}$$

This leads to:

Proposition 4 *A personalized-pricing equilibrium allocation can be achieved in a uniform-pricing equilibrium if, and (essentially) only if, the sellers' premuneration values do not depend on the level of the buyer's attribute.*

V. Implications—what have we learned?

1. The absence of market power in the ex post market does not ensure efficient investment incentives.

2. Efficient investment incentives require two features

- agents “must” face fixed prices, and

- agents must fact the right prices.

3. In our first model, eliminating market power does not ensure that agents face the right prices. The difficulty here appears to be too little heterogeneity in the ex post market.

4. In our second model, heterogeneity in the ex post market can lead to efficient investment incentives, but only if prices are sufficiently rich.

5. In general, the second model gives us a trade-off between the costs of personalized prices and the inefficiencies of uniform prices.

6. This trade-off can be mitigated if premuneration values are appropriately designed. This gives rise to scope for market design.

Extensions -

1. We would like a model in which sellers can endogenously decide whether to bear the costs of personalization. Such a model has proven surprisingly elusive.

2.

The End