New Developments in Auction Theory & Practice

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Introduction

- Developments in auction theory and practice
  - Rapid expansion in theory and practice of multi-item auctions, beginning in 1994 with first FCC auctions for radio spectrum.
- Today's talk
  - Standard clock/SMR auctions
  - What have we learned from practice?
  - New package bidding designs
  - The combinatorial clock auction
- Last part joint with Andy Skrzypacz (& really new!).

Standard clock auction

- Seller gradually raises prices for each type of good.
- Bidders announce demands at current prices.
  - Bidders are generally subject to an activity rule that prevents them from raising demand as auction proceeds.
- Auction ends when there is no excess demand.

Clock/SMR auctions: theory

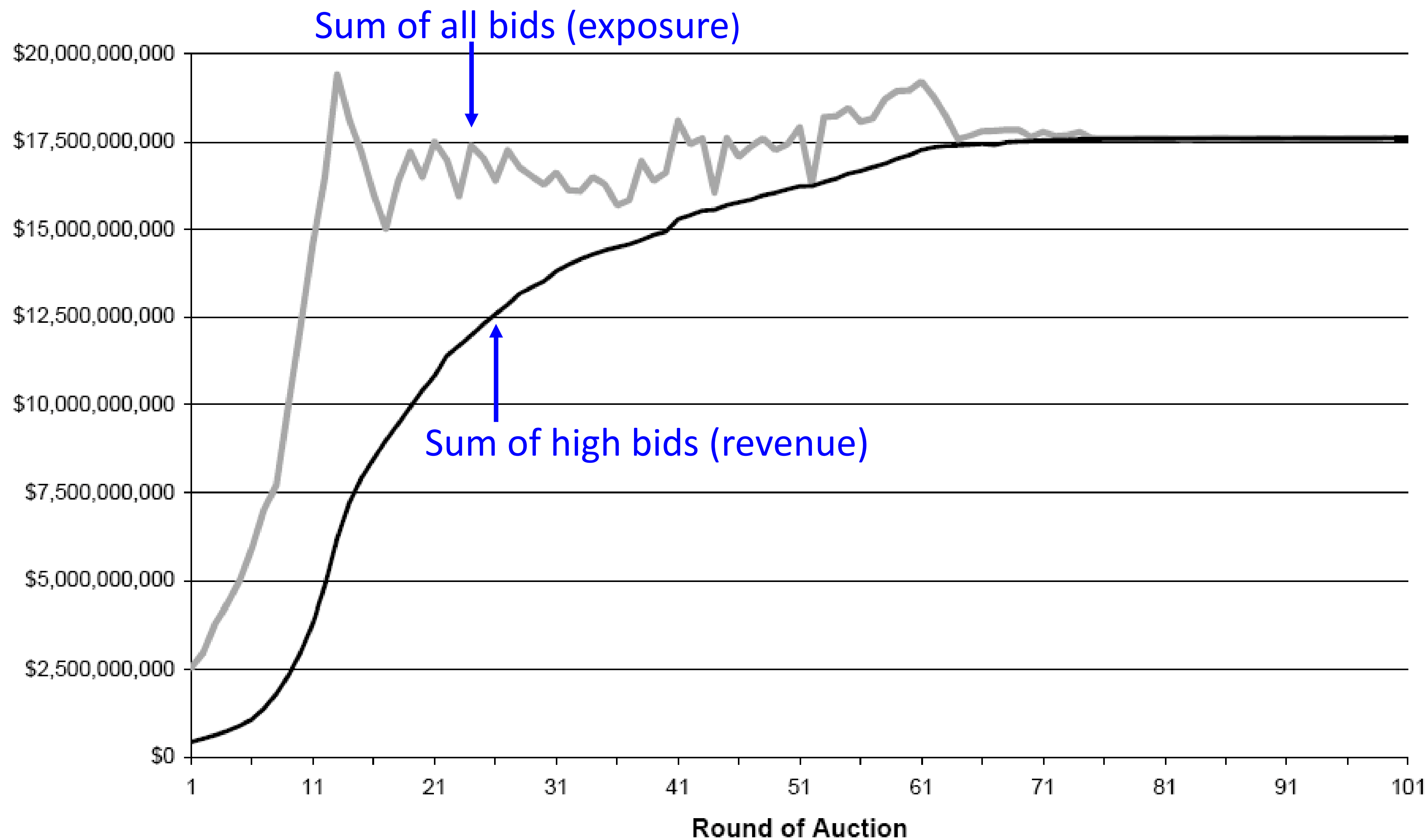
- Rationale for clock/SMR auctions
  - Simultaneous sale of allows substitution and arbitrage
  - Ascending format allows for price & value discovery

- Theory: discover market-clearing prices
  - Suppose bidders view items as ***substitutes*** and bid ***truthfully***, i.e. always bid for most desired set of licenses at current prices.
  - Auction will end at the lowest comp. eqm. prices and hence, at an efficient allocation with equal prices for identical items.
  - Kelso-Crawford (1982), Gul-Stacchetti (2000), Milgrom (2000)

Three issues in practice

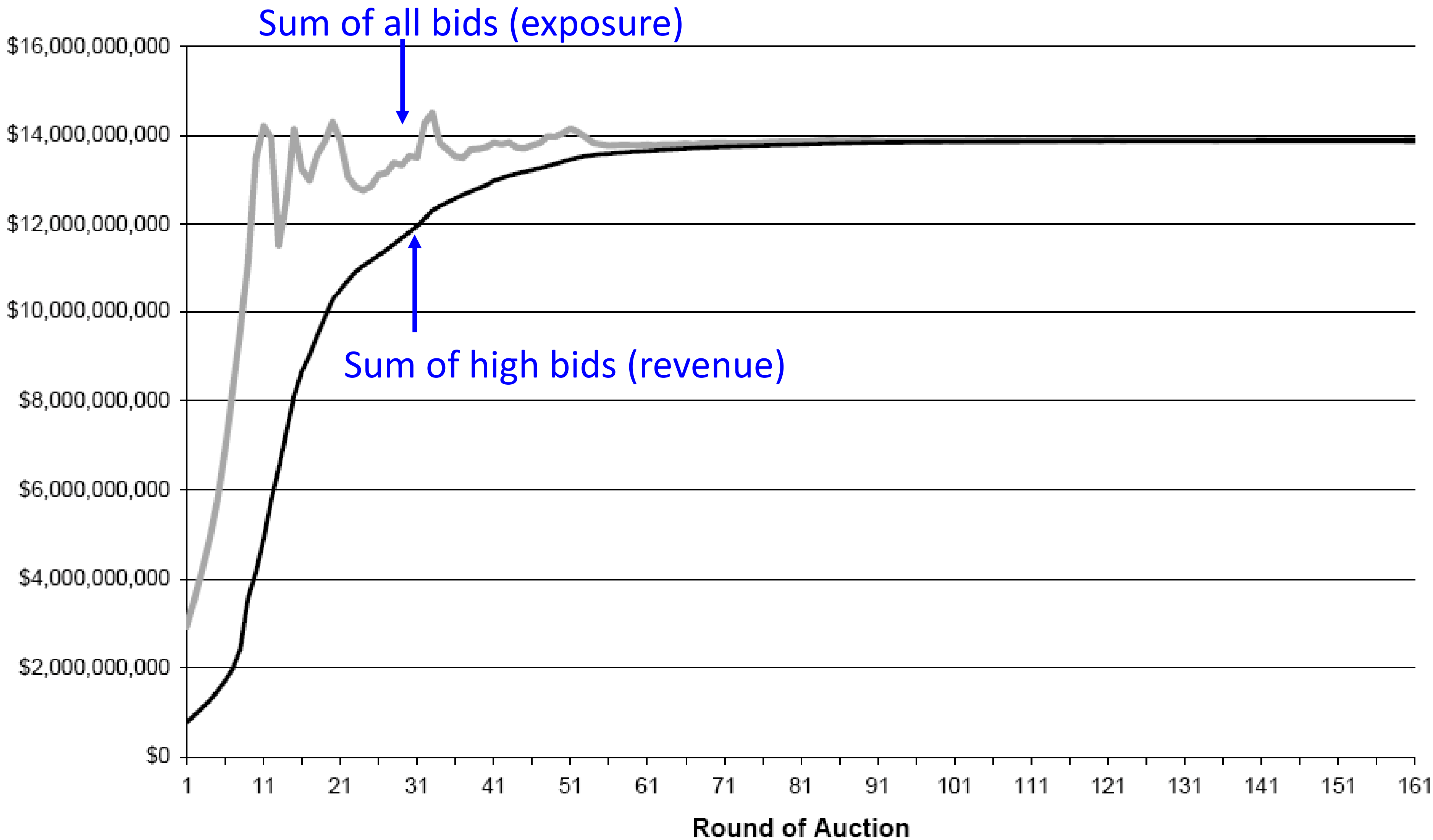
- Bidders may want to assemble packages
  - Package bidders (e.g. new entrants) risk getting some but not all of what they want. But allowing them to drop out entirely when a single price rises means demand can “overshoot” supply.
- Strategic demand reduction
  - Bidders have incentive to reduce demand to keep prices down on infra-marginal units, and opportunities for “market division”
- Bidder budgets can be surprisingly important.
  - Contrary to the theory, bidders often limited by *budgets*, not *values.*

Bidder budgets: FCC auction 35



Bulow, Levin and Milgrom (2009)

Bidder budgets: FCC auction 66



Bulow, Levin and Milgrom (2009)

Activity rule problems

- Activity rule is crucial to make clock/SMR auctions work
  - However, if licenses are different sizes, it may be difficult to substitute back and forth ... relative prices can be unreasonable.

- The FCC's Advanced Wireless Service auction (2006)

Bands	Total MHz	License size	Active rounds	Price per US 10Mhz
A/B/C	50	Small	20+	$1.2 bn
D/E/F	40	Big	1-20	$1.9 bn

Package auction designs

- Researchers have aimed for auction designs that
  - Allow for package bidding to limit exposure risk
  - Limit strategic incentive for demand reduction
  - Allow for substitution to avoid AWS-type outcomes
- Vickrey (VCG) auction is a candidate, but ...
  - Sealed bidding means no “price discovery” (which is especially problematic if bidders have budget constraints!)
  - Prices can be “too low” --- Day & Milgrom (2008) propose to adjust VCG prices up to lowest “core” prices.

Low VCG prices & core adjustment

	Item 1	Item 2	Both
Bidder A	8	0	8
Bidder B	0	8	8
Bidder C	0	0	10

- Vickrey auction: A & B win and each pays 2.
- Outcome isn't in the core: C would offer 10.
- Core adjustment raises each payment to 5.

Combinatorial clock auction

- Clock phase: seller raises prices until no excess demand.
- Bidders can make “supplementary” package bids.
- Then determine allocation and pricing
  - All clock bids and all supplementary bids are considered.
  - Find collection of bids with highest value (max one per bidder)
  - The winning bidders then pay core-adjusted Vickrey prices.

- Idea: package bidding + clock phase for price discovery + VCG pricing for incentives = perfection? (Cramton, 2009)

CCA in practice

- CCA increasingly used in place of clock/SMR auctions
  - Spectrum auctions in UK starting in 2008 and other European countries: e.g. Austria, Denmark, Netherlands, Switzerland.
  - Proposed for other uses such as sale of airport landing slots.
- Rapid transition from theory to practice
  - Ausubel, Cramton, Milgrom (2005); Day-Milgrom (2008).
- Remainder of the talk: look at properties of CCA.

Revealed preference activity rule

- Activity rule needed to make clock round meaningful.

- If bidder demands X at prices p, then for any $Y \neq X$

$$B(Y) \leq B(X) + p \bullet (Y-X)$$

- Encourage bidders to bid according to true preferences
  - "Truthful" in clock phase - i.e. bid true demand curve.
  - "Truthful" in supplementary phase - i.e. bid full valuation.

A simple example

- One unit of divisible good to be allocated
- Two symmetric bidders with linear demand curves
- Marginal values: $u(x) = 1 - x$
- Total values: $U(x) = \int_0^x u(z)dz = x(1 - x/2)$

- Efficient outcome: $x_1 = x_2 = 1/2$
- Market clearing price: $p = 1/2$.

- Vickrey payment: $U(1) - U(1/2) = 1/8$

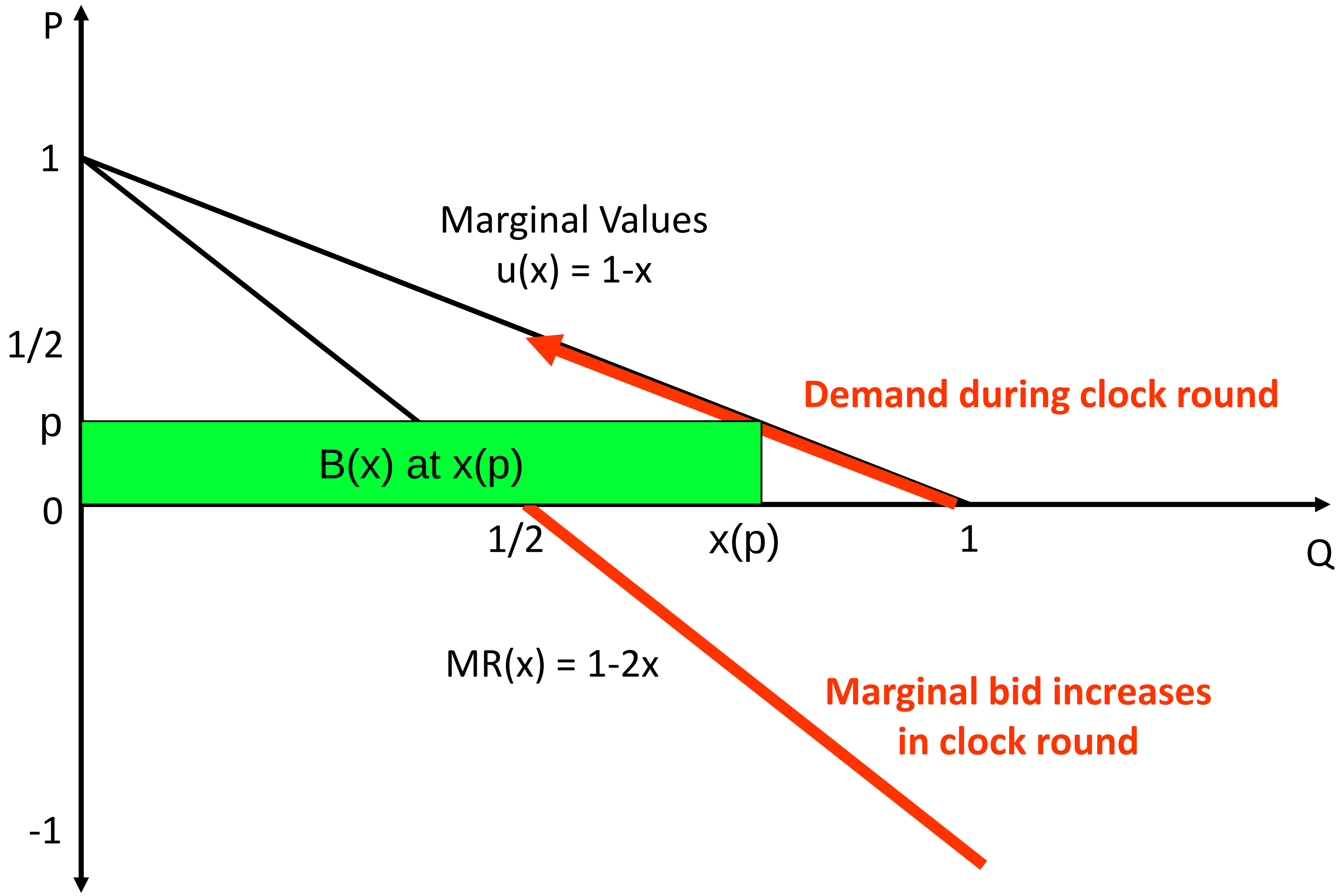
CCA with straightforward bidding

- As clock price rises, each bidder demands $x(p) = 1-p$
  - Market clears at $p=1/2$ with $x_1 = x_2 = 1/2$

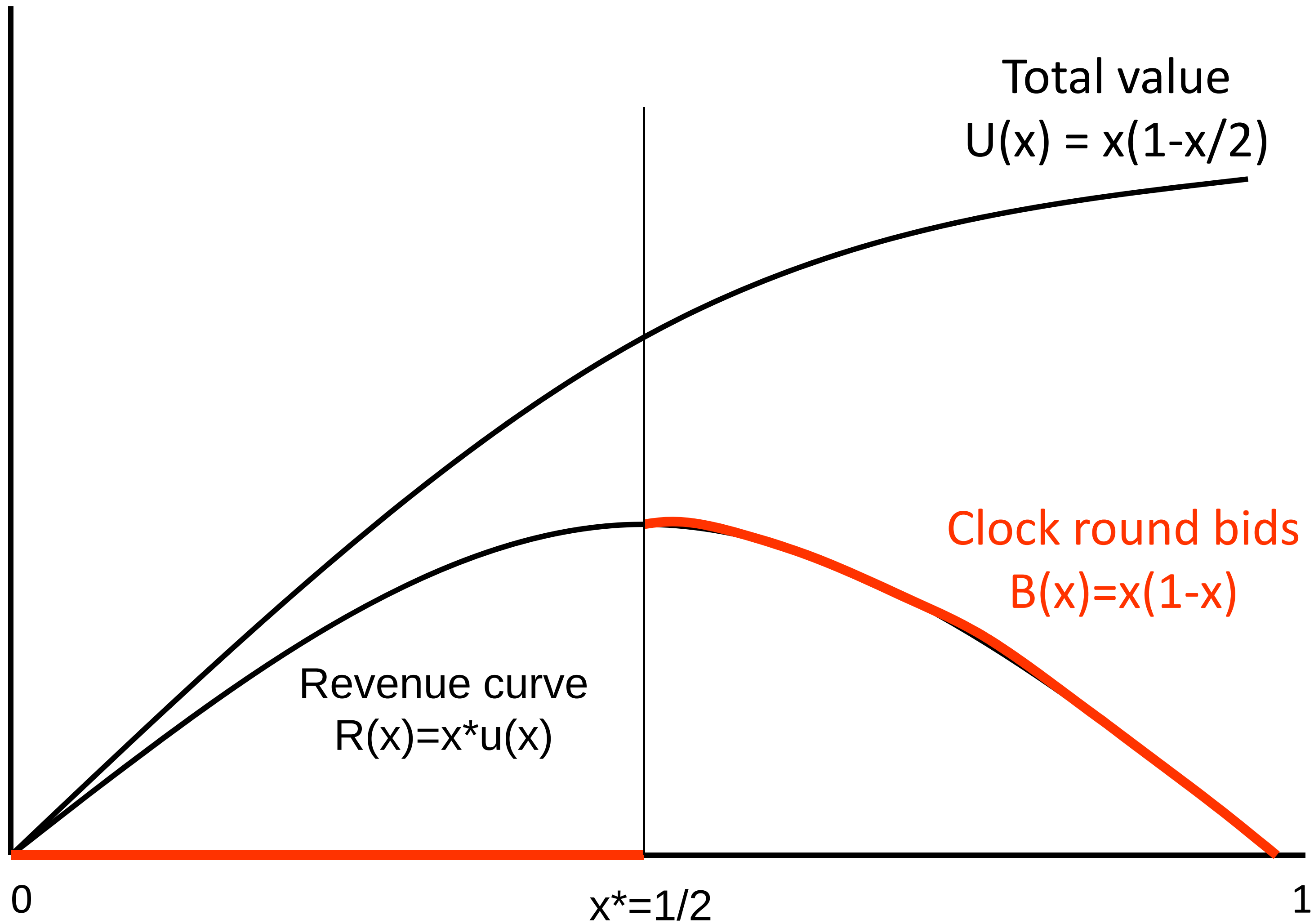
- Supplementary bids: if truthful, $B(x) = U(x) = x(1-x/2)$
  - Each gets $1/2$ and pays $1/8$ => **Vickrey**!

- But what if there are no supplementary bids?
  - Same allocation, but each bidder pays **zero**!

Demand and bids



Another look



Optimal supplementary bids

Result 1. For *any* supplementary bids, the final allocation is unchanged and equals the clock allocation: $x_1^* = x_2^* = 1/2$.

Pf. Follows from revealed preference

$$\text{If } x < 1/2, \text{ then} \quad B(x) \leq B(1/2) + p^*(1/2 - x)$$

$$\text{If } x > 1/2, \text{ then} \quad B(x) \leq B(1/2) + \int_{1/2}^{x} p(z)dz < p^*(x - 1/2)$$

So for any feasible allocation (x_1, x_2) not equal to (1/2, 1/2)

$$B_1(x_1) + B_2(x_2) < B_1(1/2) + B_2(1/2)$$

Result (also stated in Cramton, 2009) depends on clock round ending with no excess supply, but not on details of the example.

Optimal supplementary bids, cont.

Result 2. Each bidder is indifferent across *all possible* supplementary bid strategies.

Proof. Allocation is fixed, so *i* will get $x_i^* = 1/2$ regardless. Furthermore, *i*'s payment depends only on *j*'s final bids.

Result 3. Any pair of supplementary bid strategies is a continuation equilibrium of the CCA auction.

Proof. Follows immediately from complete indifference.

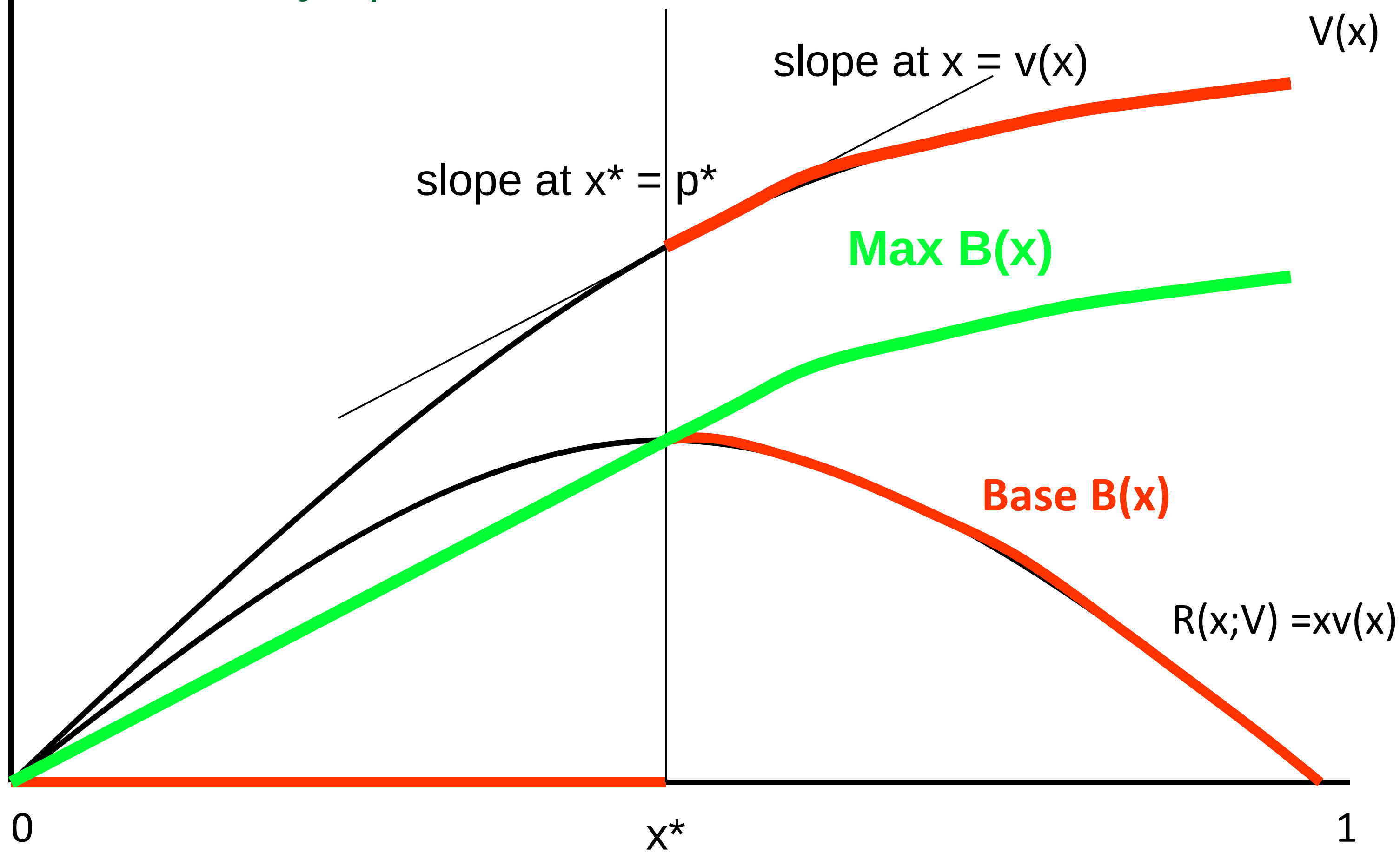
Supplementary bids in general can move the prices *up* or *down*.

Modeling clock strategies

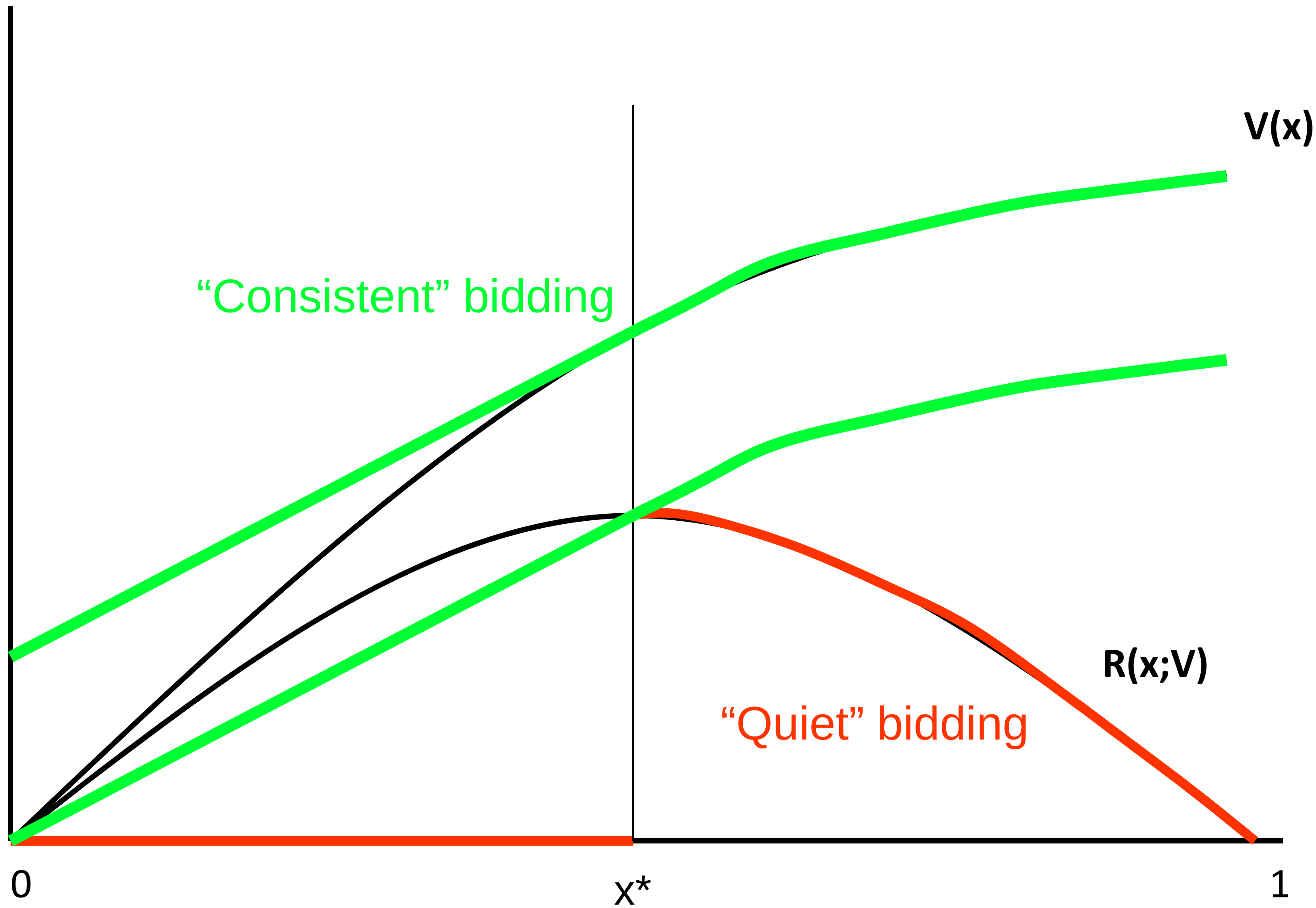
- Model clock phase with "proxy" bidding
  - Each bidder submits value function *V(x)* to proxy
  - Demand *v(x)=V'(x)* must be "regular" (decreasing & MR ↓)
  - Proxy bids according to $x(p;V) = argmax_x\ V(x) - px$
- Why proxy? Eliminates multiplicity of equilibria that can arise due to flexibility in clock strategies - "simplification".

Possible final bids under RP rule

These bids hold $B(x^*)$ fixed. Can also raise $B(x^*)$ arbitrarily, and eg can bid *any* upward translation.



Quiet and consistent strategies



Best responses

Result. If opponents are “consistent”, BR is “truthful” + _____.

Proof (for two bidders).

- Suppose B bids $V(x)$ and then is consistent in supp. phase.
- If A wins x units, it will have to pay $V(1)-V(1-x)$.
- So A’s marginal price for its xth unit is $v(1-x)$.
- So A optimally should buy x_A units where $u(x_A)=v(1-x_A)$
- A can achieve this *for any V* by bidding truthfully in the clock phase because clock round will end with $u(x_A)=p=v(x_B)=v(1-x_A)$.

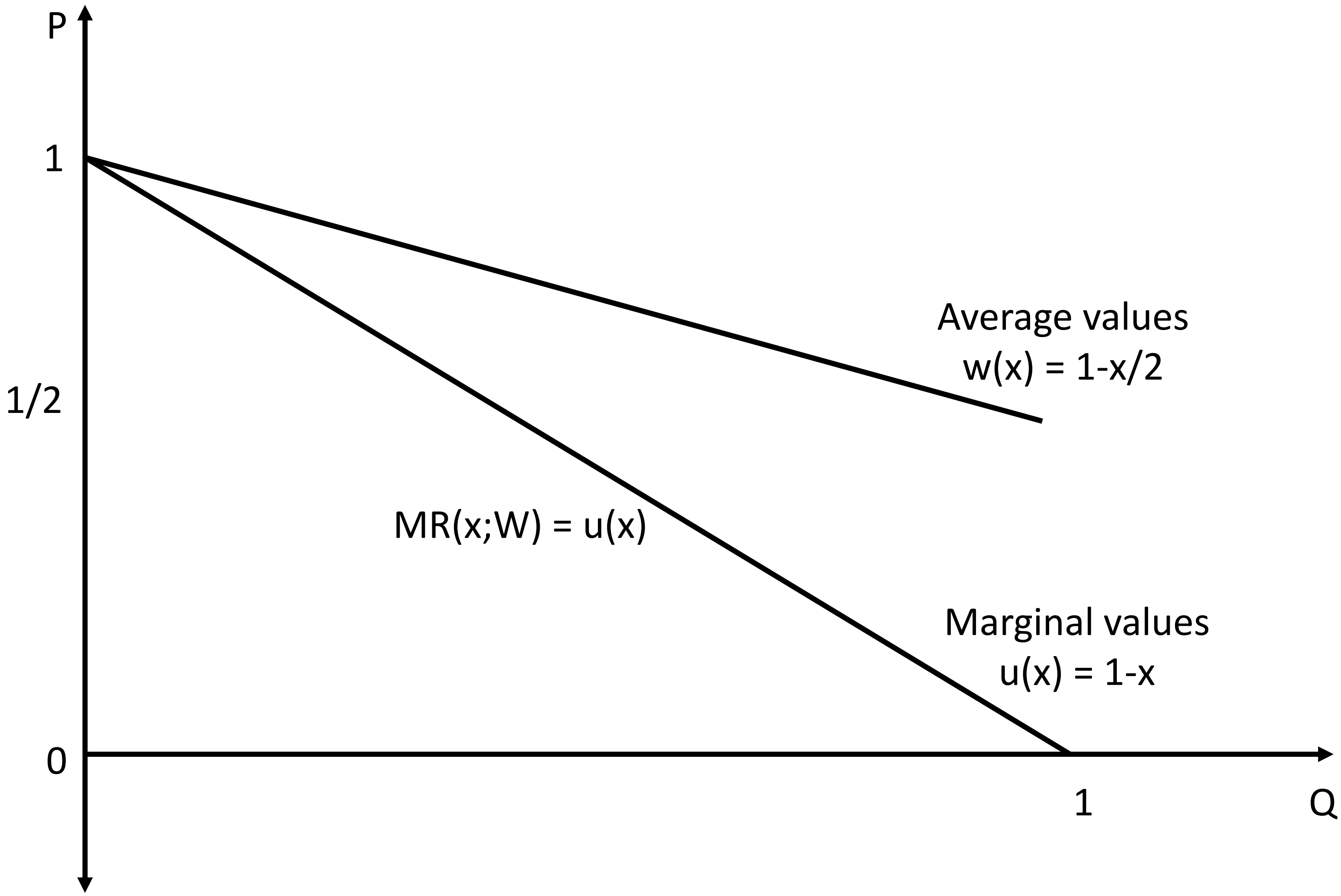
Best responses

Result. If opponents are "quiet", BR is "average value" + ____.

Proof (again, for two bidders).

- Suppose B bids *V(x),* then is quiet.
- If A wins *x* units, it will pay R(1;V)-R(1-x;V)
- So A's marginal price for its *x*th unit is *MR(1-x;V)*.
- So A optimally should buy x_A units where $u(x_A)=MR(1-x_A;V)$.
- A can guarantee this *for any V* by bidding a *W* such that $MR(x;W)=u(x)$. The clock round will end with $w(x_A)=v(1-x_A)$, which implies that $MR(x_A;W) = u(x_A)=MR(1-x_A;V)$.

Average value bidding

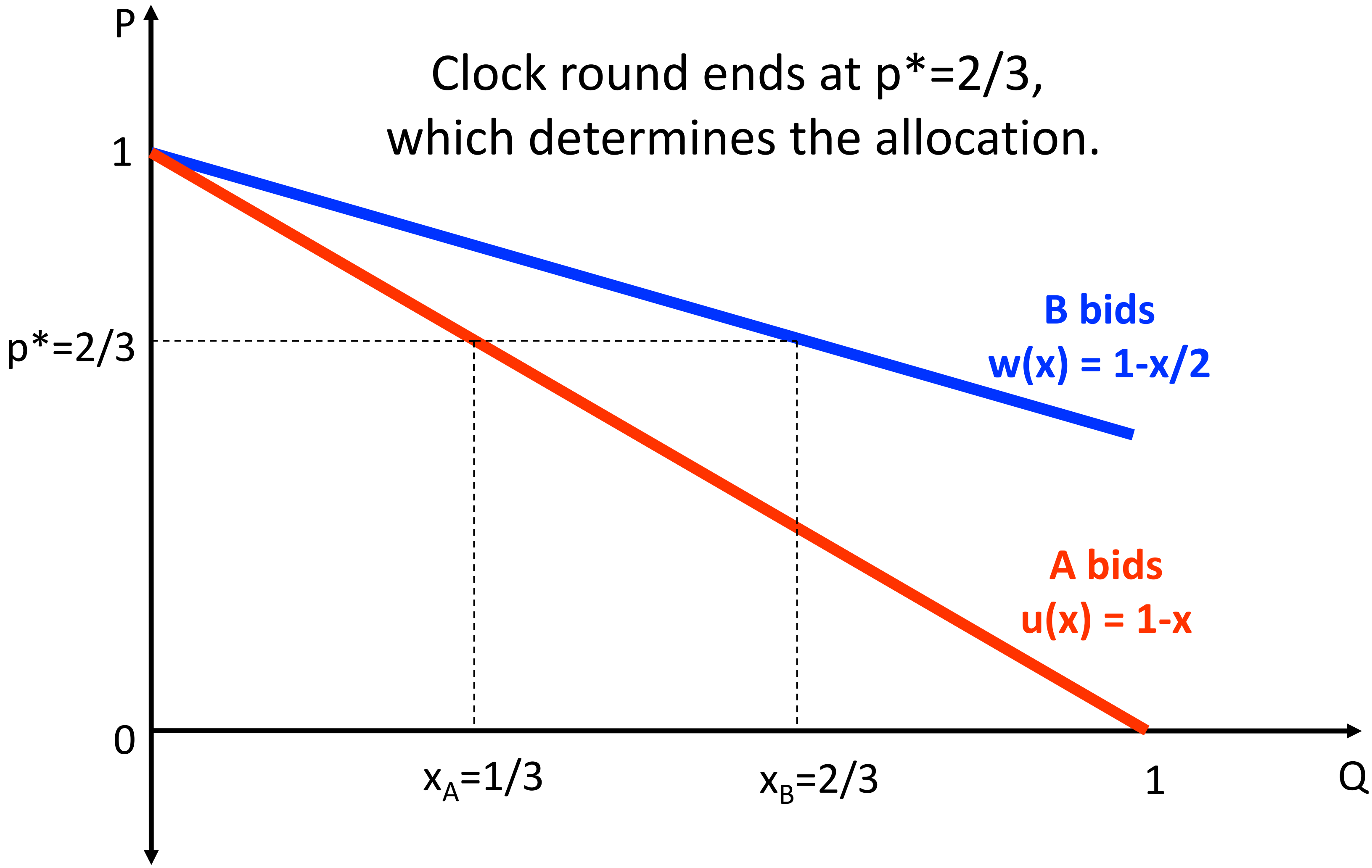


Efficient and inefficient equilibria

- Both bidders are truthful + consistent
  - In clock round, both demand x=1-p, so allocation is efficient, and supplementary bids raise prices to Vickrey level.
- Both bidders are average value + quiet
  - In clock round, both demand x=1-2p, so allocation is efficient, and clock bids alone result in Vickrey prices
- A is truthful + quiet; B is average value + consistent
  - Inefficient: A demands x=1-p and B demands x=1-2p.

Note: for these equilibria, it doesn't matter whether or not A/B know each other's valuations - informationally insensitive.

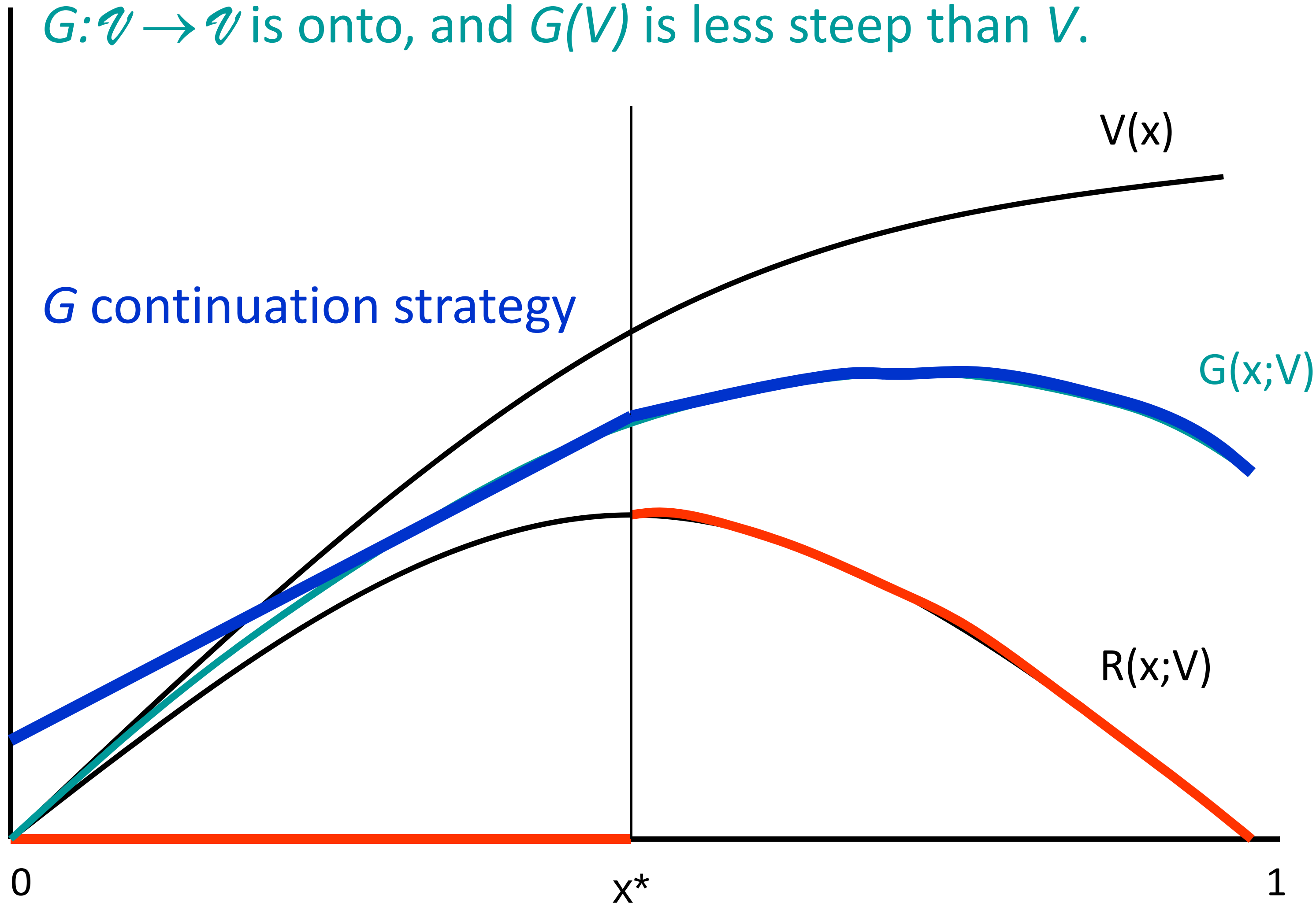
Example of inefficient equilibrium



Non-contingent strategies

Bidders have "regular" valuations drawn from $\mathscr{V}$.

G: $\mathscr{V} \rightarrow \mathscr{V}$ is onto, and *G(V)* is less steep than *V*.



Non-contingent equilibria

- Characterization of non-contingent equilibria
  - Suppose A believes that for any V that B bids in the clock round, it will play a G-continuation strategy. Then A's best response is to bid W in clock round where $U_A(x)=G(x;W)$.

(Idea: pay vickrey prices against G(V), but pick allocation against V)

  - Fix G_A,G_B. There is an equilibrium where *A* bids $G_B^{-1}(U_A)$ in clock round and $G_A(G_B^{-1}(U_A))$ in supp. round., & conversely for B.
  - The equilibrium leads to an efficient outcome (and Vickrey prices) if $G_A=G_B$, but otherwise no reason to expect efficiency.

Again - all these equilibria are informationally insensitive.

“Contingent” equilibria

- Bidders can use their supplementary bids to drive opponent prices up or down at zero cost to themselves.
  - Raise B(x) to V(x) => winners pay clinching prices.
  - Raise B(x*) only (and a lot) => winners pay zero.

- Flexibility allows for a large number of “contingent” equilibria, including zero-price “collusive” equilibria.

Relaxing the activity rule?

- Original ACM paper suggested a relaxed activity rule, which would change some results, e.g. the strict indifferences.
- However the UK 10-40 GHz auction did have a more relaxed rule, and bidders did some very odd things (Jewitt & Li, 2009)
  - **Arqiva**: final clock bid was for 3 licenses. It raised this bid to £1.600m in the supplementary round, but also bid £1.599m for 2 licenses.
  - **BT**: final clock bid was for 3 licenses. It raised this bid to £1.001m in the supplementary round, but also bid £1.001m for 1 license.
  - **Faultbasic**: final clock bid was for 2 licenses. It raised this bid to £350K in the supplementary round, but also bid £750K for 1 license.
  - **Orange**: final clock bid was for 3 licenses. It raised this bid to £2.999m in the supplementary round, but also bid £2.999m for 2 licenses.

Summary

- Last 15 years has seen a lot of work on multi-item auctions.
- Combinatorial clock auction appears to have emerged as a favorite - but it has some surprising properties.
  – With RP activity rule, if clock round ends with market clearing, bidders are completely indifferent across supplementary bids.
  – Yet optimal strategies depend crucially on beliefs about how opponents will resolve this indifference.
  – Leads to vast number of equilibria, some efficient, others not.
- These issues arise even in simplest substitutes setting, which seems to highlight some pitfalls of complex auction design.