Identification in Differentiated Products Markets

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Based on

Berry and Haile (2010). "Identification in Differentiated Products Markets Using Market Level Data," *CFDP #1744.*

Berry and Haile (2010). "Nonparametric Identification of Multinomial Choice Demand Models with Heterogeneous Consumers," *CFDP #1718.*

Common Empirical Setting

- market with differentiated goods (cars, schools, mode of transport)
- unit demand on each choice occasion
- data on prices, other characteristics, market shares
- goal: use demand (+ supply) analysis for positive and normative questions.

Differentiated Products Demand (+ Supply)

Empirical studies of...

- market power (Berry, Levinsohn & Pakes, 1995; Nevo, 2001)
- mergers (Nevo, 2000)
- welfare from new technology (Petrin, 2002)
- trade policy (Goldberg, 1995; BLP 1999)
- environmental policy (Goldberg, 1998)
- asymmetric information/insurance (Cardon & Hendel, 2001)
- residential sorting (Bayer, Ferreira & McMillan, 2007)
- school choice (Hastings, Kane & Staiger, 2009)
- media bias (Gentzkow & Shapiro, 2009).

Demand Modeling: Random Utility over Characteristics

- Quandt (1966), McFadden (1974), Hausman-Wise (1978)...
- Berry-Levinsohn-Pakes (1995) “BLP”

$$v_{ijt} = x_{jt}\beta_{it} - \alpha_{it} p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

$$\beta_{it}^{(k)} = \beta_0^{(k)} + \beta_1^{(k)} \theta_{it}^{(k)}$$

$$\epsilon_{ijt} \text{ i.i.d. extreme value,} \quad \theta_{it}^{(k)} \text{ i.i.d. normal}$$

Key features:

1. heterogeneous tastes for characteristics
2. choice/market-specific unobservable ξ_{jt} ($\Longrightarrow$ endogeneity).

Intuition for Identification?

$$\begin{aligned} u_{ijt} &= x_{jt}\beta_{it} - \alpha_{it} p_{jt} + \xi_{jt} + \epsilon_{ijt} \\ &= \delta_{jt} + \mu_{ijt} \end{aligned}$$

- "inversion":given F_μ and mkt shares, unique $\delta_{1t}, \ldots, \delta_{Jt}$
- "regression": $\delta_{jt} = x_{jt}\beta_0 - \alpha_0 p_{jt} + \xi_{jt}$, IV for price
- variation in choice sets⇒info about random coeffs (F_μ)
- supply side restrictions should help (?)

- incomplete
- role of functional form/distributional assumptions?
- econometrics literature doesn't cover model w/key elements:
  - heterogeneous preferences for characteristics
  - choice-specific unobservables/endogeneity.

Some Earlier Work

Identification of Discrete Choice Demand

- taste heterogeneity (semiparametric), no endogeneity, e.g.
  - Ichimura & Thompson (1998)
  - Briesch, Chintagunta & Matzkin (2005)
- endogeneity w/triangular model
  - e.g., Blundell & Powell (2004)
- semiparametric w/ "reduced form" error (e.g., Lewbel 2000)
  - $v_{ij} = x_{ij}\beta + \epsilon_{ij} \quad \epsilon_{ij} \sim F\left(\cdot|x_{ij}\right)$
  - model can't define key primitives (e.g., elasticities)

$\Longrightarrow$ no results even for linear model (semiparametric BLP).

This Project

nonparametric identifiability (supply and demand)

identifiability: Suppose we knew the distribution of the observables. Would this (+ economic model) uniquely determine the primitives/counterfactuals of interest?

Nonparametric Identification: Why?

1. step toward new nonparametric/semiparametric estimators?
2. How should we think about parametric estimates?

   Are functional form/distributional assumptions approximations for estimation or essential maintained hypotheses?

3. What can, in principle, be learned from typical observables?
   - what restrictions on the model are important?
   - what types of variation in the data are important?

Today

Focus talk on "market data" setting

- e.g., BLP (1995)
- observables are:
  - product characteristics
  - market characteristics (e.g., mean income, distn of income)
  - market shares
  - not individual characteristics/choices.

Preview

- nonparametric generalizations of
  - BLP demand model
  - standard oligopoly supply models
- results
  - identification of demand, full random utility model, MC
  - discrimination between oligopoly models
- key sufficient conditions involve
  - index restrictions
  - instrumental variables conditions
- two approaches
  1. nonparametric IV
  2. simultaneous equations/change of variables.

Demand Model

- consumer i, market t, products $j \in \mathcal{J}_t$
- "market" $\iff$ choice set
- "product" = "good" = "choice"
- x_{jt}, exogenous observables (product dummies ok)
- $p_{jt} \in \mathbb{R}$, endogenous observables (price)
- $\xi_{jt} \in \mathbb{R}$, market/choice-specific unobservable
- $0 \in \mathcal{J}_t$, "outside good", $J_t = |\mathcal{J}_t| - 1$
- choice set (market) $\iff \left\{ \mathcal{J}_t, \left\{ x_{jt}, p_{jt}, \xi_{jt} \right\}_{j \in \mathcal{J}_t} \right\}$
- $\chi = \text{supp}\left\{ x_{jt}, p_{jt}, \xi_{jt} \right\}$, $\chi^{\mathcal{J}_t} = \text{supp}\left\{ x_{jt}, p_{jt}, \xi_{jt} \right\}_{j \in \mathcal{J}_t}$.

Preferences: Random Utility

population of consumer preferences

- each csr has conditional indirect utility function $u_{it}\left(x_{jt}, p_{jt}, \xi_{jt}\right) : \chi \rightarrow \mathbb{R}$
- random utility: utility is random function on χ
  - probability space $(\Omega, \mathcal{F}, \mathbb{P})$
  - $v_{ijt} = u_{it}\left(x_{jt}, p_{jt}, \xi_{jt}\right) = u\left(x_{jt}, p_{jt}, \xi_{jt}, \omega_{it}\right) \quad \omega_{it} \in \Omega$

Remarks:

- no t on $\mathbb{P}$: all market-specific unobs. heterogeneity is in ξ_{jt}
- completely general heterogeneity in preferences.

Example

Linear Random Coefficients Random Utility Model

$$v_{ijt} = x_{jt}\beta_{it} - \alpha_{it}p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

BLP without parametric independent taste shocks:

$$\begin{aligned} \beta_{it}^{(k)} &= \beta^{(k)}(\omega_{it}) \\ \alpha_{it} &= \alpha(\omega_{it}) \\ \epsilon_{ijt} &= \epsilon_j(\omega_{it}) \end{aligned}$$

more general: $\epsilon_{ijt} = \epsilon_j(x_{jt}, p_{jt}, \omega_{it})$
fully general: $\epsilon_{ijt} = \epsilon_j(x_{jt}, p_{jt}, \xi_{jt}, \omega_{it}) \implies$ our model.

Restrictions on Preferences

- so far completely general, up to scalar ξ_{jt}
- for simplicity:
  - $\left(v_{ijt} - v_{ikt}\right)$ cstly distributed, convex support
  - v_{ijt} strictly decreasing in p_{jt}
- partition $x_{jt} = \left(x_{jt}^{(1)}, x_{jt}^{(2)}\right)$, $x_{jt}^{(1)} \in \mathbb{R}$
- let $\delta_{jt} = x_{jt}^{(1)} + \xi_{jt}$

Assumption 1a. $u_{it}\left(x_{jt}, p_{jt}, \xi_{jt}\right) = u\left(\delta_{jt}, x_{jt}^{(2)}, p_{jt}, \omega_{it}\right)$, with u strictly increasing in δ_{jt}.

- "vertical" ξ_{jt} (testable w/micro data)
- $x_{jt}^{(1)}$ and ξ_{jt} perfect substitutes (can relax)
- e.g., holds in BLP model if:
  - $x_{jt}^{(1)}$ has a degenerate coefficient (standard in practice)
  - or same (positive) RC on $x_{jt}^{(1)}$and ξ_{jt}.

Restrictions on Preferences

For some results (those required for welfare analysis):

Assumption 1b. $u_{it}\left(x_{jt}, p_{jt}, \xi_{jt}\right) = \mu\left(\delta_{jt}, x_{jt}^{(2)}, \omega_{it}\right) - p_{jt}.$

- $\iff \tilde{\mu}\left(\delta_{jt}, x_{jt}^{(2)}, \omega_{it}\right) - \alpha_{it} p_{jt}$, $\alpha_{it} > 0$ $w.p.1$
- with quasilinearity
  - model defines standard aggregate welfare measures
  - map units of probability to units of utilities.

Normalizations

$$v_{ijt} = u\left(\delta_{jt}, x_{jt}^{(2)}, p_{jt}, \omega_{it}\right)$$
$$\delta_{jt} = x_{jt}^{(1)} + \xi_{jt}$$

without loss,

- location of utility: $v_{i0t} = 0 \ \forall i, t$
- scale of utility: only need for results using quasi-linearity
- scale of ξ_{jt} set by linear substitution with $x_{jt}^{(1)}$
- location of ξ_{jt} later.

Market Shares

- consumers maximize utility
- $\implies$ market shares:

$$\begin{aligned} s_{jt} &= E_{\mathbb{P}}\left[1\left\{v_{ijt} \geq v_{i\ell t}\ \forall \ell\right\} \middle| \left\{x_{kt}, p_{kt}, \xi_{kt}\right\}_{k \in \mathcal{J}_t}\right] \\ &\equiv s_j\left(\left\{x_{kt}, p_{kt}, \xi_{kt}\right\}_{k \in \mathcal{J}_t}\right). \end{aligned}$$

Assumption 2 (interior mkt shares). $s_j\left(\left\{x_{kt}, p_{kt}, \xi_{kt}\right\}_{k \in \mathcal{J}_t}\right) > 0$ for all$\mathcal{J}_t, j \in \mathcal{J}_t,\ \left\{x_{kt}, p_{kt}, \xi_{kt}\right\}_{k \in \mathcal{J}_t} \in \chi^{\mathcal{J}_t}$.

(no goods with zero market share offered in equilibrium).

Observables

- $\tilde{z}_t$, excluded instruments (e.g., cost shifters)
- M_t = market size
- observe $\left(t, M_t, \mathcal{J}_t, \tilde{z}_t, \{s_{jt}, x_{jt}, p_{jt}\}_{j \in \mathcal{J}_t}\right)$.

Objects of Interest (demand side)

Demand

- i.e., each ξ_{jt} and the function $s_j\left(\{x_{kt}, p_{kt}, \xi_{kt}\}_{k \in \mathcal{J}_t}\right)$
- sufficient for many purposes of demand estimation

The Random Utility Model

- i.e., joint distn of utilities conditional on the choice set
- the primitive of this model
- why of interest: welfare, but need more structure...
- quasilinear specification for these results.

Simplify notation...

- fix $\mathcal{J}_t = \mathcal{J}$
- condition on $x_{1t}^{(2)}, \ldots, x_{Jt}^{(2)}$ and suppress
- let x_{jt} now denote just the scalar $x_{jt}^{(1)}$

$\Longrightarrow$ from now on write:

$$v_{ijt} = u_j\left(\delta_{jt}, p_{jt}, \omega_{it}\right)$$

or

$$v_{ijt} = \mu_j\left(\delta_{jt}, \omega_{it}\right) - p_{jt}.$$

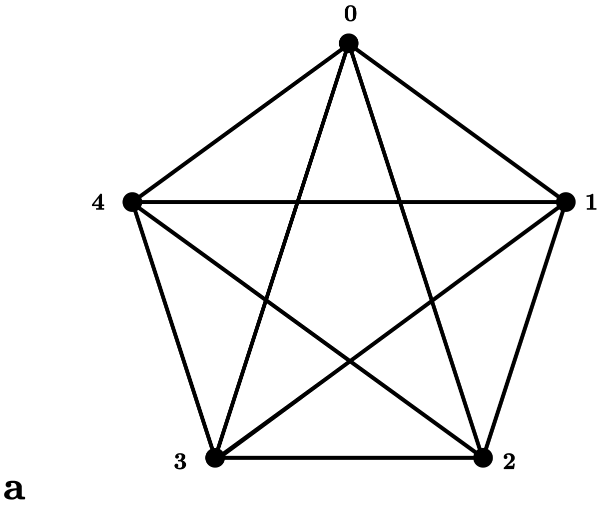
Connected Substitutes (quickly)

Key condition: all products in $\mathcal{J}$ "belong" in one demand system.

Definition. "Product k substitutes to product ℓ at $\{x_{kt}, p_{kt}, \xi_{kt}\}_{k \in \mathcal{J}}$" if $\downarrow \delta_{kt} \implies \uparrow s_{\ell t}$ ($\iff$ if $\uparrow p_{kt} \implies \uparrow s_{\ell t}$).

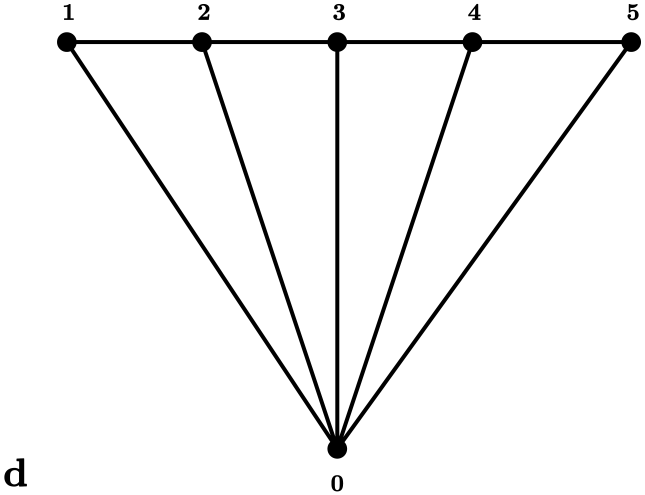
Assumption 3 ("Connected Substitutes") At all $\{x_{kt}, p_{kt}, \xi_{kt}\}_{k \in \mathcal{J}} \in \chi^{\mathcal{J}}$, the directed graph of $\Sigma(\mathcal{J})$ is strongly connected.

Example: MN Probit, MN Logit, Mixed Logit, etc.

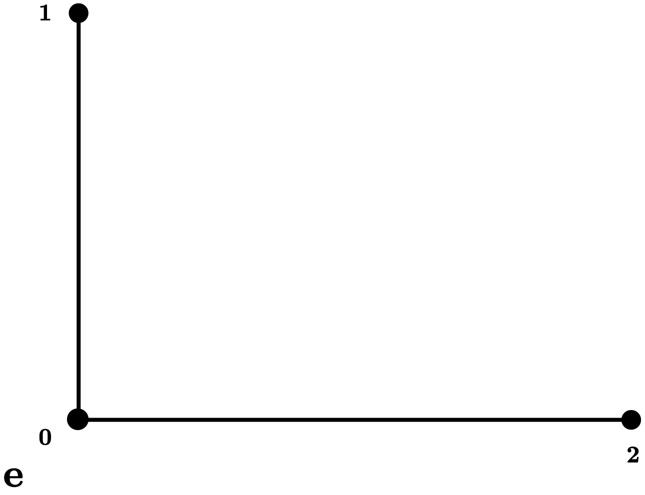


Local Competition + Outside Good

(e.g., Rochet-Stole)



Example: Independent Goods + Outside Good



Key Implication

- suppose price falls (or "quality index" δ_{jt} rises) for all products $j \in \mathcal{I} \subset \mathcal{J}$, and no others
- then total market share $\sum_{j \in \mathcal{I}} s_j$ increases.

Invert Market Shares

- let $x_t = (x_{1t}, \ldots, x_{Jt})$, $p_t = (p_{1t}, \ldots, p_{Jt})$, $\delta_t = (\delta_{1t}, \ldots, \delta_{Jt})$
- rewrite market share equation

$$\begin{aligned} s_{jt} &= E_{\mathbb{P}}\left[1\left\{u_j\left(\delta_{jt}, p_{jt}, \omega_{it}\right) \geq u_k\left(\delta_{kt}, p_{kt}, \omega_{it}\right)\ \forall k\right\}\right] \\ &\equiv \sigma_j\left(\delta_t, p_t\right). \end{aligned}$$

Lemma 1. For any price vector p and any market share vector $s = (s_1, \ldots, s_J)'$ on the interior of $\triangle^J$, there is at most one vector δ such that $\sigma_j(\delta, p) = s_j\ \forall\ j$.

- Berry (1994), Berry and Pakes (2008) for linear model
- Gandhi (2008) for NP model (w/equivalent condition)
- existence of inverse σ_j^{-1}, not characterization.

Approach 1: Nonparametric IV Conditions

Review: Nonparametric Regression

Newey & Powell (2003)

- $y_i = \Gamma(x_i) + \epsilon_i$
- IV conditions
  1. "exclusion" : $E[\epsilon_i | z_i] = 0$ a.s.
  2. "completeness": $E[B(x_i) | z_i] = 0$ a.s. implies $B(x_i) = 0$ a.s.
     - given 1, 2 is necessary and sufficient for identification of Γ
     - nonparametric analog of standard rank condition.

Identification of Demand

Strategy: adapt Newey-Powell (2003) to our setting

- inversion $\Longrightarrow$ $\delta_{jt} = \sigma_j^{-1}(s_t, p_t) \quad \forall j$, where $\delta_{jt} \equiv x_{jt} + \xi_{jt}$
- $\Longrightarrow$ $\xi_{jt} = -x_{jt} + \sigma_j^{-1}(s_t, p_t) \quad \forall j$

Assumption 4. (mean independence) $E[\xi_{jt} | x_t, \tilde{z}_t] = 0$ a.s. (incorporates location normalization)

Assumption 5. (completeness) For all functions $B(s_t, p_t)$ with finite expectation, if $E[B(s_t, p_t) | x_t, \tilde{z}_t] = 0$ a.s. then $B(s_t, p_t) = 0$ a.s.

Identification of Demand

Theorem 1. *Under Assumptions 1a and 2–5, demand is identified.*

Theorem 1

Assumption 1a. (utility)

Assumption 2. (interior market shares)

Assumption 3. (connected substitutes)

Assumption 4. (mean independence)

Assumption 5. (completeness)

Sketch of proof (mostly Newey-Powell)

- $\xi_{jt} = -x_{jt} + \sigma_j^{-1}(s_t, p_t)$
- $$E\left[\xi_{jt}|x_t, \tilde{z}_t\right] = E\left[\sigma_j^{-1}(s_t, p_t)\Big| x_t, \tilde{z}_t\right] - x_{jt}$$ $$= 0 \text{ a.s. by mean independence}$$
- suppose $E\left[\tilde{\sigma}_j^{-1}(s_t, p_t)\Big| x_t, \tilde{z}_t\right] - x_{jt} = 0$ a.s.
- $\implies E\left[B(s_t, p_t)|x_t, \tilde{z}_t\right] = 0$ a.s.
    - where $B(s_t, p_t) = \sigma_j^{-1}(s_t, p_t) - \tilde{\sigma}_j^{-1}(s_t, p_t)$
- completeness $\implies \tilde{\sigma}_j^{-1} = \sigma_j^{-1}$ a.s. $\implies \sigma_j^{-1}$ identified
- $\xi_{jt} = -x_{jt} + \sigma_j^{-1}(s_t, p_t)$
- $\implies$ identification of demand: $s_j\left(\{x_{kt}, p_{kt}, \xi_{kt}\}_{k \in \mathcal{J}}\right)$
    - shares observed
    - all arguments of s_j now known.

Identification of demand follows from the same IV conditions required to identify regression models with additive errors.

Key: adequate instruments

- cost shifters or proxies
- + exogenous product characteristics ("BLP IV").

Full Identification of the Random Utility Model

add quasilinearity, $v_{ijt} = \mu_j(\delta_{jt}, \omega_{it}) - p_{jt}$, and large support:

Assumption 6. supp $p_t | \{x_{jt}, \xi_{jt}\}_{j \in \mathcal{J}} \supset$ supp $(\mu_1(\delta_{1t}, \omega_{it}), \ldots, \mu_J(\delta_{Jt}, \omega_{it}))_t \,|\, \{x_{jt}, \xi_{jt}\}_{j \in \mathcal{J}}$.

Theorem 2. Under Assumptions 1b and 2–6, the joint distribution of $(v_{i1t}, \ldots, v_{iJt})$ conditional on any $(\{x_{kt}, p_{kt}, \xi_{kt}\}_{k \in \mathcal{J}}) \in \chi^{\mathcal{J}}$ identified.

Sketch of Proof (standard argument)

- recall: $v_{ijt} = \mu_j(\delta_{jt}, \omega_{it}) - p_{jt}$
- market share of the outside good given p_t, x_t, ξ_t :

$$Pr\left(\mu_1(\delta_{1t}, \omega_{it}) \leq p_{1t}, \ldots, \mu_J(\delta_{Jt}, \omega_{it}) \leq p_{Jt}\right)$$

- each ξ_{jt} (and $\therefore$ δ_{jt}) already identified (Thm 1)
- large support$\longrightarrow$joint distn of $\mu_1(\delta_{1t}, \omega_{it}), \ldots, \mu_J(\delta_{Jt}, \omega_{it})$
- since $v_{ijt} = \mu_j(\delta_{jt}, \omega_{it}) - p_{jt}$ we are done.

(note: no "identification at infinity").

Adding a Supply Side

Adding a Supply Side

Idea: generalize parametric empirical approach based on solution of first-order conditions for marginal costs (Rosse, 1970; Bresnahan, 1982; BLP, 1995; etc.).

Assumption 7. $\sigma_k(\delta_t, p_t)$ is continuously differentiable wrt p_ℓ $\forall k, \ell \in \mathcal{J}$.

Marginal Costs

Index structure:

- $mc_j\left(q_{jt}, z_{jt}^{(1)} + \eta_{jt}, z_{jt}^{(2)}\right)$
  - $q_{jt} = M_t s_{jt}$
  - unobserved cost shock η_{jt}
  - observed exogenous cost shifters $\left(z_{jt}^{(1)}, z_{jt}^{(2)}\right)$, $z_{jt}^{(1)} \in \mathbb{R}$
  - restriction: linear substitution between $z_{jt}^{(1)}$ and η_{jt}
- let $\zeta_{jt} = z_{jt}^{(1)} + \eta_{jt}$
- fix (and suppress) $z_t^{(2)} = \left(z_{1t}^{(2)}, \ldots, z_{Jt}^{(2)}\right)$
- let z_{jt} now denote $z_{jt}^{(1)}$.

First-Order Conditions

Assumption 8 (marginal cost and FOC). For all j
(i) $mc_j(q_{jt}, \zeta_{jt})$ is strictly monotonic in ζ_{jt};
(ii) there exists a function ψ_j (possibly unknown) such that for any equilibrium value of (s_t, p_t),

$$mc_j(M_t s_{jt}, \zeta_{jt}) = \psi_j\left(s_t, M_t, D_t\left(s_t, p_t\right), p_t\right)$$

where $D_t\left(s_t, p_t\right)$ is the $J \times J$ matrix of partial derivatives $\left\{ \frac{\partial \sigma_k\left(p_t, \sigma^{-1}(s_t, p_t)\right)}{\partial p_\ell} \right\}_{k,\ell}$

We show: part (ii) holds for standard oligopoly models under the assumptions already made.

Inverting the Supply Side

$$mc_j(M_t s_{jt}, \zeta_{jt}) = \psi_j\left(s_t, M_t, D_t\left(s_t, p_t\right), p_t\right)$$

Lemma 2. For any market size M_t and any (s_t, p_t) there is exactly one $(\zeta_{1t}, \ldots, \zeta_{Jt}) \in \mathbb{R}^J$ consistent with Assumption 8.

Identification of Marginal Costs

- fix M_t
- inversion $\implies \zeta_{jt} = z_{jt} + \eta_{jt} = \pi_j^{-1}(s_t, p_t) \quad \forall j$
- π_j^{-1} and each η_{jt} identified using same "Newey-Powell" argument (with x_t, z_t as IV)
- recall $mc_j(q_{jt}, \zeta_{jt}) = \psi_j(s_t, D_t(s_t, p_t), p_t)$
- if ψ_j known, then mc_j is identified

Theorem 3. Suppose that Assumptions 1a, 2-5, 7 and 8 hold. Then, for all j (i) each η_{jt} is identified and (ii) if ψ_j is known, $mc_j(q_{jt}, \zeta_{jt})$ is identified on the support of (q_{jt}, ζ_{jt}).

Approach 2: Change of Variables

One Motivation

1. completeness condition hard to interpret, check
2. argument not constructive

Change of Variables Approach

- constructive
- close link to classical arguments for supply and demand
  - system of equations
  - standard exclusion and support conditions
- limitation: more structure.

System of Equations

Recall, inverted equilibrium relations:

$$\begin{aligned} x_{jt} + \xi_{jt} &= \sigma_j^{-1}(s_t, p_t) \quad \forall j \\ z_{jt} + \eta_{jt} &= \pi_j^{-1}(s_t, p_t) \quad \forall j. \end{aligned}$$

Location normalizations for ξ_t, η_t

- take any $\left(x^0, z^0\right)$ and any $\left(s^0, p^0\right)$ in $\text{supp}(s_t, p_t) \mid \left(x^0, z^0\right)$
- $\forall j$ let $\sigma_j^{-1}\left(s^0, p^0\right) - x_j^0 = \pi_j^{-1}\left(s^0, p^0\right) - z_j^0 = 0$.

Change of Variables

Assumption 9 (unique prices). There is a unique vector of equilibrium prices associated with any (δ, ζ).

- FOC have unique soln *for prices,* given (δ, ζ)
- or unique selection for each (δ, ζ)
    - no random eqm selection
    - no eqm selection on x_{jt} or ξ_{jt} instead of their sum, etc.

Assumption 10. (densities) $(\xi_1, \ldots, \xi_J, \eta_1, \ldots, \eta_J)$ have a positive joint density $f_{\xi,\eta}$ on $\mathbb{R}^{2J}$.

Assumption 11. (Jacobian) The vector function $(\sigma_1^{-1}, \ldots, \sigma_J^{-1}, \pi_1^{-1}, \ldots, \pi_J^{-1})'$ has continuous partial derivatives and nonzero Jacobian determinant.

Identification: Demand Side

Assumption 12. (full independence) $(x_t, z_t) \perp\!\!\!\perp (\xi_t, \eta_t)$.

Assumption 13. (large support 2) $\text{supp}(x_t, z_t) = \mathbb{R}^{2J}$.

Theorem 4. Under Assumptions 1a, 2, 3 and 8–13, demand is identified.

Theorem 4

Assumption 1a. (utility)

Assumption 2. (interior market shares)

Assumption 3. (connected substitutes)

Assumption 8. (marginal cost and FOC)

Assumption 9. (unique prices)

Assumption 10. (densities)

Assumption 11. (Jacobian)

Assumption 12. (full independence)

Assumption 13. (large support 2)

Sketch of Proof

- observe joint density of market shares and prices$|x_t, z_t$
  $f_{s,p}\left(s_t, p_t | x_t, z_t\right) = \ldots$
  $f_{\xi,\eta}\left(\sigma_1^{-1}\left(s_t, p_t\right) - x_{1t}, \ldots, \pi_J^{-1}\left(s_t, p_t\right) - z_{Jt}\right) \left|\mathbb{J}(s_t, p_t)\right|$
- for any $\left(\hat{s}, \hat{p}, \hat{x}, \hat{z}, x, z\right)$ we can construct the ratio

$$\frac{f_{\xi,\eta}\left(\sigma_1^{-1}\left(\hat{s}, \hat{p}\right) - x_1, \ldots, \pi_J^{-1}\left(\hat{s}, \hat{p}\right) - z_J\right) \left|\mathbb{J}\left(\hat{s}, \hat{p}\right)\right|}{f_{\xi,\eta}\left(\sigma_1^{-1}\left(\hat{s}, \hat{p}\right) - \hat{x}_1, \ldots, \pi_J^{-1}\left(\hat{s}, \hat{p}\right) - \hat{z}_J\right) \left|\mathbb{J}\left(\hat{s}, \hat{p}\right)\right|}$$

- Jacobian determinants cancel

Sketch of Proof

- observe joint density of market shares and prices$|x_t, z_t$
  $f_{s,p}\left(s_t, p_t | x_t, z_t\right) =$
  $f_{\xi,\eta}\left(\sigma_1^{-1}\left(s_t, p_t\right) - x_{1t}, \ldots, \pi_J^{-1}\left(s_t, p_t\right) - z_{Jt}\right) \left|\mathbb{J}(s_t, p_t)\right|$
- for any $\left(\hat{s}, \hat{p}, \hat{x}, \hat{z}, x, z\right)$ we can construct the ratio

$$\frac{f_{\xi,\eta}\left(\sigma_1^{-1}\left(\hat{s}, \hat{p}\right) - x_1, \ldots, \pi_J^{-1}\left(\hat{s}, \hat{p}\right) - z_J\right)}{f_{\xi,\eta}\left(\sigma_1^{-1}\left(\hat{s}, \hat{p}\right) - \hat{x}_1, \ldots, \pi_J^{-1}\left(\hat{s}, \hat{p}\right) - \hat{z}_J\right)}$$

- Jacobian determinants cancel
- repeated integration $\longrightarrow F_{\xi_j}\left(\sigma_j^{-1}\left(\hat{s}, \hat{p}\right) - x_j\right)$ for any $\hat{s}, \hat{p}, x_j$
- by normalization, $F_{\xi_j}\left(\sigma_j^{-1}\left(s^0, p^0\right) - x_j^0\right) = F_{\xi_j}\left(0\right)$
- standard arguments $\Longrightarrow$ σ_j^{-1} identified $\Longrightarrow$ ξ_{jt} identified
- repeat for all j, t $\longrightarrow$identification of demand.

Further Results with COV Approach

- identification of η_{jt} and marginal costs
  - same COV proof strategy$\rightarrow$identification of $\pi_j^{-1}(\cdot)$ and η_{jt}
  - if supply model (i.e. $\psi_j(\cdot)$) known, identification of MC follows as before
- full identification of the RU model
  - as before with quasilinearity & large support.

Falsifiability of the Supply Model

Context

- old literature in IO: (e.g., Bresnahan 1982, 1989)
  - infer firm "conduct" from equilibrium supply behavior
  - test→choose among possible models
- *general intuition:* "rotations of demand"
- *limited formal results* (Lau, 1982)
  - nonstochastic models, homogeneous goods
  - "conjectural variations" models.

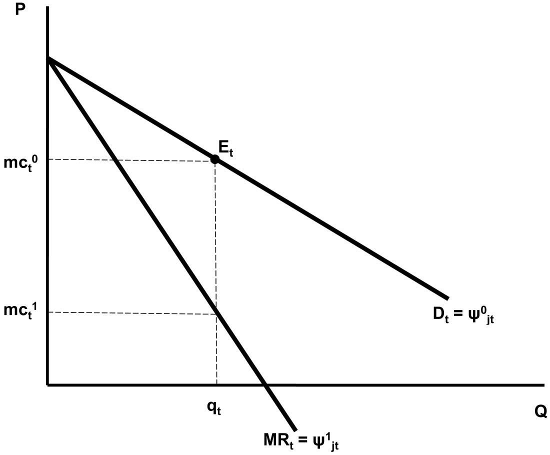
Falsifiability

recall: $mc_j\left(q_{jt},\zeta_{jt}\right)=\psi_j\left(s_t,M_t,D_t\left(s_t,p_t\right),p_t\right)$

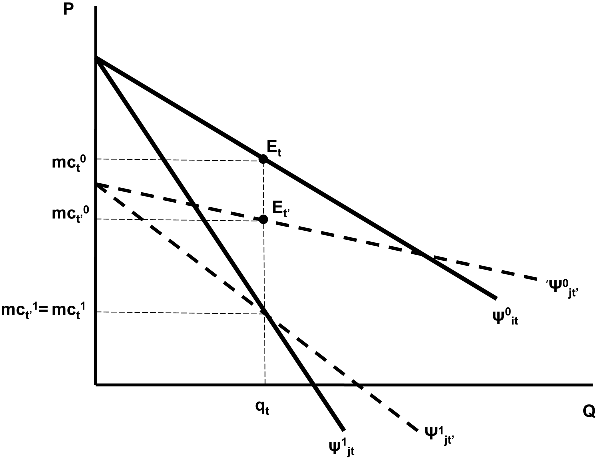
Remark 1. Suppose
(i) $t'\neq t$ and/or $j'\neq j$,
(ii) $mc_j\left(\cdot\right)=mc_{j'}\left(\cdot\right)$, and
(iii) $\left(q_{jt},\zeta_{jt}\right)=\left(q_{j't'},\zeta_{j't'}\right)$.
Then $\psi_j\left(s_t,D_t\left(s_t,p_t\right),p_t\right)=\psi_{j'}\left(s_{t'},D_{t'}\left(s_{t'},p_{t'}\right),p_{t'}\right)$.

Detecting a false oligopoly model

- fix ζ_j (remember: these identified w/o knowing ψ_j)
- suppose "residual MR function" ψ_j "rotates" under the true model.



Different marginal costs rationalize E_t, depending on model.



Rotation of ψ_j rules out false null.

General Intuition

- generalize to any pair of models
- rotation of ψ_j under true model rules out false null unless rotation of ψ_j under truth implies rotation under null too (for all products)
- similar to intuition in Bresnahan (1982, 1989), Lau (1982) but
  - allow randomness, product differentiation, non-CV models
  - clarifies role of "residual MR function" ψ_j.

Conclusion

My interpretation: mostly positive overall message

- with limited structure, identification holds under standard conditions
- key requirement: adequate instruments
- old insights generalize for "tests" of supply model

Before: some doubts in profession about identification, even for semiparametric BLP model

After: less restrictive models are NP identified, under same conditions required for elementary models (e.g., regression).

Some Qualifications

1. index restrictions
   - less restrictive than prior models, but important
   - can relax: $\delta_{jt} = \delta_j(x_{jt}, \overset{+}{\xi}_{jt})$ instead of $\delta_{jt} = x_{jt} + \xi_{jt}$
2. BLP instruments (x_{-j}) alone not enough(?)
   - cost shifters (or proxies)
   - AND exogenous product characteristics.

Micro Data

- key data: individual-product “match”, e.g.
  - distance to hospital, school, retailer
  - family size $\times$ car size
  - household exposure to product advertising
  - consumer-specific predicted Rx drug plan cost
- results
  - identification with more general utility & specification of ξ_{jt}

    $$v_{ijt} = \beta_{it} s_{ijt}^{(1)} + \mu\left(x_{jt}, p_{jt}, s_{ijt}^{(2)}, \xi_{jt}\left(s_{ijt}^{(2)}\right), \omega_{it}\right), \quad \beta_{it} > 0$$
  - testable restrictions
  - BLP instruments alone are adequate
  - other possible IV available.