Reciprocal Insurance Among Kenyan Pastoralists *

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Abstract

In large areas of low and locally variable rainfall in East Africa, pastoralism is the only viable activity, and cattle are at risk of reduced milk output and even death in dry periods. The herders were nomadic, but following the Kenyan government's scheme of giving titles to group ranches, they are evolving reciprocity arrangements where a group suffering a dry period can send some of its cattle to graze on lands of another group that has better weather. We model such institutions using a repeated game framework. As these contracts are informal, we characterize schemes that are optimal subject to a self-enforcement or dynamic incentive compatibility condition. Where the actual arrangements differ from the predicted optima, we discuss possible reasons for the discrepancy, and suggest avenues for further research.

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1 Introduction

Pastoral herding appeared thousands of years ago when hunter-gatherers domesticated wildlife, selectively breeding livestock that could convert inedible vegetation of previously underutilized arid and semi-arid lands into useable foodstuffs such as milk and meat. The lands are characterized by rainfall that has a low average, but high spatial and temporal variation. Pastoral herders coped with the resulting uncertainty in rangeland productivity by migrating large distances following the rains. Since productive areas were often controlled by resident tribes, the mobility of wandering tribes was constrained. Warrior-enforced encroachment, tempered by informal rules of land tenure as well as wife exchange among tribes, created a variety of mechanisms that fostered short-term sharing by the occupiers of productive lands with those whose land was currently, but temporarily, unproductive.

Generally, the drier the region, the more pastoral herders subsist on foodstuffs derived from livestock. Some of the purest pastoralists – the Maasai, the Samburu, the Turkana and the Boran – live in East Africa where annual rainfall is less than 400 mm. Traditionally, families in these tribes survived mostly on milk from their herds. Milk is virtually a perfect food supplying protein, calories and vitamins. But sufficient production to sustain families depends on herds consuming vegetation from pastures not degraded by excessive livestock grazing and browsing.

Even though no tribe can control enough rangeland to sufficiently reduce the effects of rainfall variability, the pressure to control as large a tract as possible results in rangelands being managed as a common-pool resource. Since the costs of excluding groups arriving from unproductive land are likely to be greater than the gains of defending productive land, rules based on reciprocity and kinship often develop to reduce violence and foster long-term gains of wandering and defending groups. But such relationships are prone to cheating. If reneging for short term gain limits future movement, then staying put during 'bad times' with large herds that were appropriate when times were good is likely to lead to degradation of the land, unless Hardin's idea of 'mutual coercion, mutually agreed upon' is practiced.

The Kenya Government's new land tenure policy during the 1970s (see ILRI 1995) compounded the problem of limited movement and resulting land degradation. When communities received title deeds to exclusive areas, pastoral herders that had relied on transhumance for thousands of years were essentially sedentarized, living on parcels of land that became known as 'group ranches' and which averaged 15,000 hectares (37,000 acres or 60 square miles) in size, much too small to average out rainfall variability within, or even among neighboring clusters of, communities.

Consequently, new methods of organizing livestock movements from areas of low rainfall to high rainfall had to be developed. In theory herders with cash could buy grazing rights on more productive land, a practice that is common in Australia and other areas of the world. Renting land for transferred herds is known as Agistment and relies on trust since receivers of herds are expected to care for them well and senders of the herds are expected to send only easy-to-manage animals (McAllister et al. 2006). But incomes among pastoral herders in East Africa rarely exceed $1 per day, so Agistment practices are rare. Instead, pastoral herders are developing reciprocity arrangements with other communities so that when conditions are poor for one community a fraction of that community's herds can be moved to the more productive lands of the distant partnering community. Unlike arrangements under the Agistment system, transferred livestock by African pastoralists are managed by their owners. For this system to be stable, when the conditions are reversed, communities that previously received herds should be able to send a similar fraction of their herds to the former sending communities. But if the former sending communities renege on their agreement or have not managed their lands well by setting enough rangeland aside to sustain their own returning herds as well as herds of former receivers that will be expected in the future, the payoffs will not be equal and offsetting, and reciprocity arrangements will collapse.

In this paper we consider the viability of such arrangements. For this initial exploration, we use a very simple model. We assume that the groups are symmetric except for the weather realizations: they have the same production and cost functions, and the same marginal probability distributions of weather realizations. When we consider self-enforcing cooperation in Section 5, we will assume that identical periods of this kind repeat indefinitely, ignoring serial correlation of weather and ignoring the dynamics of cattle population through birth and death, and that of land quality through gradual degradation or restoration. While this model serves to yield some useful results and insights, we will later list many dimensions along which it can be generalized; these generalizations are part of our ongoing work.

2 The basic model

Label the two groups 1 and 2. Initially we will introduce variables for either group generically without group labels; then we will bring the groups together and introduce the labels as subscripts. Each group collectively (using its internal structure of governance by the elders or a managing committee) chooses two inputs: the number of cattle x, and the quality of land z. Each input has a cost, either directly monetary, or in terms of some other opportunity foregone. The cost function is

$$C(x, z) = \tfrac{1}{2}\, c\, (x + z)^2 \,. \tag{1}$$

The output, in the form of milk or blood or meat, resulting from these inputs is

$$F(x, z) = A\ x^{\alpha}\ z^{\beta}\,, \tag{2}$$

where α and β are positive, and satisfy a condition that will emerge in the course of the analysis. We expect $\alpha < 1$, as increasing the number of cattle on a given piece and quality of land will suffer from diminishing returns and therefore will not produce proportionately more output. Some idea about the magnitudes of these parameters will emerge from comparisons of the results of the model with reality.

The functional forms are chosen for tractability, and guided by common choices for analogous situations in economics, namely the Cobb-Douglas production function and quadratic cost functions. The key requirements are that the two inputs should be complements in production ($\partial^2 F/\partial x \partial z > 0$) and substitutes in cost ($\partial^2 C/\partial x \partial z > 0$); more general functional forms satisfying these properties should yield qualitatively similar results.

The output generates consumption Y for the group. If the group does not have any reciprocity arrangements with another group, the consumption simply equals output. With such arrangements, Y will denote the group's share of the total output as stipulated in the implicit contract with the other group. The consumption yields utility

$$U(Y) = \begin{cases} \frac{1}{1-\rho}\, Y^{1-\rho} & \text{if } \rho \neq 1 \\ \ln(Y) & \text{if } \rho = 1 \end{cases} \tag{3}$$

where $\rho > 0$ is the Arrow-Pratt coefficient of relative risk-aversion.

The multiplicative constant A depends on the weather conditions. There are two possible conditions, H (good weather) and L (a dry spell). The corresponding values of A are $A_H > A_L$. Let (i, j) denote the state or outcome where group 1 gets weather condition i and group 2 gets weather condition j, for i, $j = H$, L. We denote by p_2 the probability of state (H, H) (both groups get good weather), by p_0 the probability of state (L, L) (both get bad weather), and by p_1 that of each of the states (H, L) and (L, H) (group 1 gets good weather while group 2 gets bad weather, and the other way round). Then

$$p_2 + 2\, p_1 + p_0 = 1\,;$$

the marginal probability for any one group of getting good weather is $p_H \equiv p_2 + p_1$, and that of bad weather is $p_L \equiv p_1 + p_0$.

The two groups' weather outcomes are perfectly positively correlated if $p_1 = 0$; in this case reciprocal arrangements will not help. The opposite case of perfect negative correlation corresponds to $p_2 = p_0 = 0$ and $p_1 = \frac{1}{2}$; this is when reciprocal arrangements have the greatest potential. The case of independent outcomes (zero correlation) requires $p_1 = p_H \, p_L = (p_2 + p_1)\,(p_1 + p_0)$, which then simplifies further to $p_1 = \sqrt{p_0 \, p_2}$.

The inputs x and z must be chosen before the weather realization is known. The group's objective is its expected utility

$$EU = p_H \; U(Y_H) + p_L \; U(Y_L) - \tfrac{1}{2}\, c\, (x+z)^2 \tag{4}$$

in obvious notation.

3 One group's optimum

First consider the case where each group is on its own. We assume initially that $\rho \neq 1$; that case can be treated similarly, as we discuss later. We omit group labels, and consider the choice of x and z to maximize expected utility, which in this case becomes

$$\begin{aligned} EU &= p_H \, \frac{1}{1-\rho} \, \left(A_H \, x^\alpha \, z^\beta \right)^{1-\rho} + p_L \, \frac{1}{1-\rho} \, \left(A_L \, x^\alpha \, z^\beta \right)^{1-\rho} - \tfrac{1}{2}\, c\, (x+z)^2 \\ &= \frac{1}{1-\rho} \, \left[p_H \; A_H^{1-\rho} + p_L \; A_L^{1-\rho} \right] \; x^{\alpha(1-\rho)} \; z^{\beta(1-\rho)} - \tfrac{1}{2} \; c\, (x+z)^2 \end{aligned} \tag{5}$$

The first-order conditions for an optimum are

$$\begin{aligned} \frac{1}{1-\rho} \, \left[p_H \; A_H^{1-\rho} + p_L \; A_L^{1-\rho} \right] \; \alpha(1-\rho) \; x^{\alpha(1-\rho)-1} \; z^{\beta(1-\rho)} - c\,(x+z) &= 0 \\ \frac{1}{1-\rho} \, \left[p_H \; A_H^{1-\rho} + p_L \; A_L^{1-\rho} \right] \; x^{\alpha(1-\rho)} \; \beta(1-\rho) \; z^{\beta(1-\rho)-1} - c\,(x+z) &= 0 \end{aligned}$$

or

$$\begin{aligned} \left[p_H \; A_H^{1-\rho} + p_L \; A_L^{1-\rho} \right] \; \alpha \; x^{\alpha(1-\rho)-1} \; z^{\beta(1-\rho)} &= c\,(x+z) \\ \left[p_H \; A_H^{1-\rho} + p_L \; A_L^{1-\rho} \right] \; x^{\alpha(1-\rho)} \; \beta \; z^{\beta(1-\rho)-1} &= c\,(x+z) \end{aligned}$$

when $\rho \neq 1$. Dividing the second of these by the first yields

$$x/\alpha = z/\beta = N, \;\text{ say.} \tag{6}$$

Substituting this into the first-order condition with respect to x yields

$$\left[p_H\ A_H^{1-\rho} + p_L\ A_L^{1-\rho}\right]\ \alpha\ \alpha^{\alpha(1-\rho)-1}\ \beta^{\beta(1-\rho)}\ N^{(\alpha+\beta)(1-\rho)-1} = c\,(\alpha+\beta)\,N\,, \tag{7}$$

and therefore

$$N^{2-(\alpha+\beta)(1-\rho)} = \frac{\alpha^{\alpha(1-\rho)}\ \beta^{\beta(1-\rho)}}{c\,(\alpha+\beta)}\ \left[p_H\ A_H^{1-\rho} + p_L\ A_L^{1-\rho}\right]\,. \tag{8}$$

This yields N in terms of the exogenous variables of the problem, and substituting the solution for N into (6) gives the optimal values of x and z.

These solution expressions remain valid for the case $\rho = 1$, as can be seen by working explicitly with that case when utility is logarithmic. We omit these derivations to save space.

We need the condition

$$(\alpha+\beta)\,(1-\rho) < 2\,. \tag{9}$$

If this fails, the left hand side of (7) becomes an increasing function of N, so the second-order condition for optimization fails. Also, as (8) shows, the condition yields an economically meaningful solution; for example an increase in the cost parameter c reduces N and therefore the optimal x and z.

Substituting from(6) into (5)and using (8), we find the maximized expected utility of each group in isolation:

$$\begin{aligned}
EU^{\text{isol}} &= \frac{1}{1-\rho}\ \left[p_H\ A_H^{1-\rho} + p_L\ A_L^{1-\rho}\right]\ \alpha^{\alpha(1-\rho)}\ \beta^{\beta(1-\rho)}\ N^{(\alpha+\beta)(1-\rho)} - \tfrac{1}{2}\ c\,(\alpha+\beta)^2\ N^2 \\
&= \frac{1}{1-\rho}\ c(\alpha+\beta)\ N^2 - \tfrac{1}{2}\ c\,(\alpha+\beta)^2\ N^2 \\
&= \frac{2-(\alpha+\beta)(1-\rho)}{2\,(1-\rho)}\ c(\alpha+\beta)\ N^2
\end{aligned} \tag{10}$$

4 The full or first-best optimum

The two groups together can achieve better outcomes. If one has the good weather realization H and the other has the bad weather realization L, the total output can be raised by transferring some cattle to graze on the land that is more productive in this weather realization. We emphasize that the "transfer" is not a change of ownership, it is merely a temporary move to better grazing grounds. Some people from the home group travel with the cattle to manage them, and at the end of the season the cattle will return to the home ranch. Also, the fortunate group can share some of the output of its own cattle with the unfortunate group. The good and bad weather conditions fluctuate probabilistically, so these are mutual insurance arrangements and not one-way gifts.

In state (i, j), where the multiplicative constant in group 1's production function is A_i and that in group 2's is A_j, denote the number of cattle transferred from group 2's land to group 1's land by m_{ij}; a negative value of m_{ij} indicates a transfer in the opposite direction. Then the total output is

$$Q_{ij} = A_i \, (x_1 + m_{ij})^\alpha \, z_1^\beta + A_j \, (x_2 - m_{ij})^\alpha \, z_2^\beta \tag{11}$$

Suppose this is split between the groups according to

$$Y_{1,ij} + Y_{2,ij} = Q_{ij} \tag{12}$$

in obvious notation. Then expected utilities of the two groups will be

$$\begin{aligned} EU_1 &= \frac{1}{1-\rho} \left[p_2 \, (Y_{1,HH})^{1-\rho} + p_1 \, (Y_{1,HL})^{1-\rho} + p_1 \, (Y_{1,LH})^{1-\rho} + p_0 \, (Y_{1,LL})^{1-\rho} \right] \\ &\quad - \tfrac{1}{2} \, c \, (x_1 + z_1)^2 \qquad (13) \\ EU_2 &= \frac{1}{1-\rho} \left[p_2 \, (Y_{2,HH})^{1-\rho} + p_1 \, (Y_{2,LH})^{1-\rho} + p_1 \, (Y_{2,HL})^{1-\rho} + p_0 \, (Y_{1,LL})^{1-\rho} \right] \\ &\quad - \tfrac{1}{2} \, c \, (x_2 + z_2)^2 \qquad (14) \end{aligned}$$

In our symmetric setting, the efficient arrangement will maximize the sum of the groups' expected utilities.[1] This is as if a benevolent social planner maximizes social welfare treating the groups equally; therefore we will call the sum of expected utilities social welfare SW. The choice variables in this maximization are the two groups' inputs x_g and z_g chosen before the realization of the weather pattern, and the transfers m_{ij} and the output splits $Y_{g,ij}$ in each weather state.

The implementation of the optimum may be problematic. A group that has a good weather realization may be tempted to renege on its agreement and refuse to accept cattle from the other group that has had a bad weather realization, instead using its greener land for its own herd. And it may be tempted to refuse to share output with the other. If the social planner has enforcement power, or if a formal enforceable contract can be written by the groups, the problem can be solved. We will call this a full or first-best optimum,

[1] More generally, the (Pareto) efficient frontier of negotiation between the two groups will maximize the expected utility of one group for each given level of the expected utility of the other, and the location of the chosen point on this frontier will depend on the relative bargaining strengths of the two.

and characterize it in the rest of this section. But if enforcement power is lacking, the arrangement has to be self-sustaining, based on repeated relationship where the lucky group realizes that some time in the future it may need a return of the favor, and therefore that its short-run gain from reneging has a long-run cost. We will take up this self-enforcing or second-best optimum in the next section.

In the full optimum, x_1, x_2, z_1, z_2, and the m_{ij}, $Y_{1,ij}$ and $Y_{2,ij}$ for i, $j = H$, L are to be chosen to maximize

$$SW = EU_1 + EU_2$$

where the various entities are ultimately defined in terms of the choice variables by (13), (14), (12) and (11). Although the problem looks formidable, it can be solved quite easily in three steps:

Step 1: In each state (i, j), the transfer m_{ij} should be chosen to maximize total output Q_{ij}. The first-order condition for this is

$$\alpha \; A_i \; (x_1 + m_{ij})^{\alpha-1} z_1^{\beta} - \alpha \, A_j \, (x_2 - m_{ij})^{\alpha-1} \; z_2^{\beta} = 0 \, ,$$

i.e. the marginal productivities of cattle on the two plots of land should be equalized. The second-order condition is

$$\alpha < 1 \, , \tag{15}$$

i.e. the marginal products should be decreasing. Then the first-order condition yields

$$\frac{x_1 + m_{ij}}{(A_i \; z_1^{\beta})^{1/(1-\alpha)}} = \frac{x_2 - m_{ij}}{(A_j \; z_2^{\beta})^{1/(1-\alpha)}}$$

so each of these fractions equals the sum of the numerators divided by the sum of the denominators:

$$\frac{x_1 + x_2}{(A_i \; z_1^{\beta})^{1/(1-\alpha)} + (A_j \; z_2^{\beta})^{1/(1-\alpha)}} \, .$$

Then

$$x_1 + m_{ij} = (A_i \; z_1^{\beta})^{1/(1-\alpha)} \; \frac{x_1 + x_2}{(A_i \; z_1^{\beta})^{1/(1-\alpha)} + (A_j \; z_2^{\beta})^{1/(1-\alpha)}} \, .$$

and

$$x_2 - m_{ij} = (A_j \; z_2^{\beta})^{1/(1-\alpha)} \; \frac{x_1 + x_2}{(A_i \; z_1^{\beta})^{1/(1-\alpha)} + (A_j \; z_2^{\beta})^{1/(1-\alpha)}} \, .$$

Therefore

$$m_{ij} = \tfrac{1}{2} \; (x_1 + x_2) \; \frac{(A_i \; z_1^{\beta})^{1/(1-\alpha)} - (A_j \; z_2^{\beta})^{1/(1-\alpha)}}{(A_i \; z_1^{\beta})^{1/(1-\alpha)} + (A_j \; z_2^{\beta})^{1/(1-\alpha)}} - \tfrac{1}{2} \, (x_1 - x_2) \, . \tag{16}$$

and

$$Q_{ij} = (x_1 + x_2)^\alpha \; \left[\; (A_i \; z_1^\beta)^{1/(1-\alpha)} + (A_j \; z_2^\beta)^{1/(1-\alpha)} \; \right]^{1-\alpha} . \tag{17}$$

Observe that m_{ij}, the number of cattle of group 2 moved to graze on group 1's land in state (i, j), is higher if (1) A_i is high relative to A_j, (2) z_1 is high relative to z_2, and (3) x_2 is high relative to x_1. The first of these is the purpose of the reciprocity arrangement: to insure or smooth out fluctuations in income. But the other two can create moral hazard. Each group may be tempted to allow its land to degrade (lower z) and stock more cattle (raise x) beyond the optimum, and then transfer some cattle to benefit from the other's better and less-intensively grazed land. With both groups so tempted, this will turn into a prisoners' dilemma. Since x_i and z_i must be committed before the weather condition is realized, if the two magnitudes are publicly observable, the ability to send cattle can be made contingent on the group having adhered to the optimum, and the moral hazard of cheating on x_i and z_i can be thus overcome. We will throughout assume this to be the case, and in the next section where we consider implementation of the optimum, will focus only on the moral hazard of refusing to accept the other group's cattle (m_{ij}). In the symmetric solution we consider below, the two x's will be equal, as will the two z's, and optimal transfers will depend only on the weather conditions. But in asymmetric situations, monitoring moral hazard will be more problematic. In addition, the issue of allowing transfers to disadvantaged groups for redistributive reasons will have to be considered.

Step 2: In each state, the total output Q_{ij} should be split between the two groups according to (12). When $\rho > 0$, the relevant part of the objective function, namely

$$\frac{1}{1-\rho} \; \left[\; (Y_{1,ij})^{1-\rho} + (Y_{2,ij})^{1-\rho} \; \right]$$

is strictly increasing, strictly concave, and symmetric. Therefore equal division

$$Y_{1,ij} = Y_{2,ij} = \tfrac{1}{2} \; Q_{ij}$$

is optimal. If $\rho = 0$ (risk-neutrality), the division is indeterminate but also irrelevant, so equal division can be chosen without loss of generality. Therefore

$$\left. \begin{aligned} Y_{1,HH} = Y_{2,HH} &= \tfrac{1}{2} \; A_H \; (x_1 + x_2)^\alpha \; \left[z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right]^{1-\alpha} , \\ Y_{1,LL} = Y_{2,LL} &= \tfrac{1}{2} \; A_L \; (x_1 + x_2)^\alpha \; \left[z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right]^{1-\alpha} , \\ Y_{1,HL} = Y_{2,HL} &= \tfrac{1}{2} \; (x_1 + x_2)^\alpha \; \left[\; (A_H \; z_1^\beta)^{1/(1-\alpha)} + (A_L \; z_2^\beta)^{1/(1-\alpha)} \; \right]^{1-\alpha} , \\ Y_{1,LH} = Y_{2,LH} &= \tfrac{1}{2} \; (x_1 + x_2)^\alpha \; \left[\; (A_L \; z_1^\beta)^{1/(1-\alpha)} + (A_H \; z_2^\beta)^{1/(1-\alpha)} \; \right]^{1-\alpha} . \end{aligned} \right\} \tag{18}$$

Step 3: Using the results of steps 1 and 2, social welfare can be expressed in terms of the choice variables x_1, x_2, z_1, z_2:

$$\begin{aligned}
SW &= \frac{2}{1-\rho}\Bigg[p_2 \left\{ \tfrac{1}{2}\, A_H\, (x_1+x_2)^\alpha \left[z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right]^{(1-\alpha)} \right\}^{(1-\rho)} \\
&\qquad + p_1 \left\{ \tfrac{1}{2}\, (x_1+x_2)^\alpha \left[(A_H\, z_1^\beta)^{1/(1-\alpha)} + (A_L\, z_2^\beta)^{1/(1-\alpha)} \right]^{(1-\alpha)} \right\}^{(1-\rho)} \\
&\qquad + p_1 \left\{ \tfrac{1}{2}\, (x_1+x_2)^\alpha \left[(A_L\, z_1^\beta)^{1/(1-\alpha)} + (A_H\, z_2^\beta)^{1/(1-\alpha)} \right]^{(1-\alpha)} \right\}^{(1-\rho)} \\
&\qquad + p_0 \left\{ \tfrac{1}{2}\, A_L\, (x_1+x_2)^\alpha \left[z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right]^{(1-\alpha)} \right\}^{(1-\rho)} \Bigg] \\
&\qquad - \tfrac{1}{2}\, c\,(x_1+z_1)^2 - \tfrac{1}{2}\, c\,(x_2+z_2)^2 \\
&= \frac{2^\rho}{1-\rho}\, (x_1+x_2)^{\alpha(1-\rho)} \\
&\qquad \Big[\left(p_2\, A_H^{1-\rho} + p_0\, A_L^{1-\rho} \right) \left\{ z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \\
&\qquad + p_1 \left\{ (A_H\, z_1^\beta)^{1/(1-\alpha)} + (A_L\, z_2^\beta)^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \\
&\qquad + p_1 \left\{ (A_L\, z_1^\beta)^{1/(1-\alpha)} + (A_H\, z_2^\beta)^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \Big] \\
&\qquad - \tfrac{1}{2}\, c\,(x_1+z_1)^2 - \tfrac{1}{2}\, c\,(x_2+z_2)^2 \qquad (19)
\end{aligned}$$

The first-order conditions are

$$\begin{aligned}
\frac{\partial SW}{\partial x_1} &= 2^\rho\, \alpha\, (x_1+x_2)^{\alpha(1-\rho)-1} \\
&\qquad \Big[\left(p_2\, A_H^{1-\rho} + p_0\, A_L^{1-\rho} \right) \left\{ z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \\
&\qquad + p_1 \left\{ (A_H\, z_1^\beta)^{1/(1-\alpha)} + (A_L\, z_2^\beta)^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \\
&\qquad + p_1 \left\{ (A_L\, z_1^\beta)^{1/(1-\alpha)} + (A_H\, z_2^\beta)^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \Big] \\
&\qquad - c\,(x_1+z_1) = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial SW}{\partial z_1} &= 2^\rho\, (x_1+x_2)^{\alpha(1-\rho)} \\
&\qquad \Big[\left(p_2\, A_H^{1-\rho} + p_0\, A_L^{1-\rho} \right) \left\{ z_1^{\beta/(1-\alpha)} + z_2^{\beta/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)-1} \beta\, z_1^{\beta/(1-\alpha)-1} \\
&\qquad + p_1 \left\{ (A_H\, z_1^\beta)^{1/(1-\alpha)} + (A_L\, z_2^\beta)^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)-1} A_H^{1/(1-\alpha)}\ \beta\, z_1^{\beta/(1-\alpha)-1} \\
&\qquad + p_1 \left\{ (A_L\, z_1^\beta)^{1/(1-\alpha)} + (A_H\, z_2^\beta)^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)-1} A_L^{1/(1-\alpha)}\ \beta\, z_1^{\beta/(1-\alpha)-1} \Big] \\
&\qquad - c\,(x_1+z_1) = 0
\end{aligned}$$

and similarly with respect to x_2, z_2.

In view of the symmetry, we look for a symmetric solution where $x_1 = x_2 = x$ and $z_1 = z_2 = z$. Then the x_1-condition simplifies to

$$\begin{aligned} 2^\rho \, \alpha \, (2\,x)^{\alpha(1-\rho)-1} \; \Big[\; \Big(p_2 \; A_H^{1-\rho} + p_0 \; A_L^{1-\rho} \Big) \; 2^{(1-\alpha)(1-\rho)} \; z^{\beta(1-\rho)} \\ +2\, p_1 \, z^{\beta(1-\rho)} \; \left\{ A_H^{1/(1-\alpha)} + A_L^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)} \Big] \; &= \; c\,(x+z) \end{aligned}$$

or

$$\alpha \, x^{\alpha(1-\rho)-1} \; z^{\beta(1-\rho)} \; \left[p_2 \; A_H^{1-\rho} + p_0 \; A_L^{1-\rho} + 2\, p_1 \; \left\{ \frac{A_H^{1/(1-\alpha)} + A_L^{1/(1-\alpha)}}{2} \right\}^{(1-\alpha)(1-\rho)} \right] = c\,(x+z)$$

Using the abbreviation

$$A_M = \left\{ \frac{A_H^{1/(1-\alpha)} + A_L^{1/(1-\alpha)}}{2} \right\}^{(1-\alpha)} \tag{20}$$

write this as

$$\alpha \, x^{\alpha(1-\rho)-1} \; z^{\beta(1-\rho)} \; \Big[\; p_2 \; A_H^{1-\rho} + 2\, p_1 \; A_M^{1-\rho} + p_0 \; A_L^{1-\rho} \Big] = c\,(x+z)\,. \tag{21}$$

The z_1-condition simplifies to

$$\begin{aligned} & 2^\rho \, (2\,x)^{\alpha(1-\rho)} \Big[\; \Big(p_2 \; A_H^{1-\rho} + p_0 \; A_L^{1-\rho} \Big) \; 2^{(1-\alpha)(1-\rho)-1} \; z^{\beta(1-\rho)-\beta/(1-\alpha)} \; \beta \; z^{\beta/(1-\alpha)-1} \\ + \;\; & p_1 \; \left\{ A_H^{1/(1-\alpha)} + A_L^{1/(1-\alpha)} \right\}^{(1-\alpha)(1-\rho)-1} \; \beta \, z^{\beta(1-\rho)-\beta/(1-\alpha)} z^{\beta/(1-\alpha)-1} \; \left\{ A_H^{1/(1-\alpha)} + A_L^{1/(1-\alpha)} \right\} \Big] \\ = \;\; & c\,(x+z) \end{aligned}$$

and, using the abbreviation introduced above, it becomes

$$\beta \, x^{\alpha(1-\rho)} \; z^{\beta(1-\rho)-1} \; \Big[\; p_2 \; A_H^{1-\rho} + 2\, p_1 \; A_M^{1-\rho} + p_0 \; A_L^{1-\rho} \Big] = c\,(x+z)\,. \tag{22}$$

Dividing (21) by (22) yields

$$x/\alpha = z/\beta = M, \text{ say.} \tag{23}$$

Substituting this in either of the above equations and simplifying, we find

$$M^{2-(\alpha+\beta)(1-\rho)} = \frac{\alpha^{\alpha(1-\rho)} \; \beta^{\beta(1-\rho)}}{c\,(\alpha+\beta)} \; \Big[\; p_2 \; A_H^{1-\rho} + 2\, p_1 \; A_M^{1-\rho} + p_0 \; A_L^{1-\rho} \Big]\,. \tag{24}$$

This yields M in terms of the exogenous variables of the problem, and substituting the solution for M into (23) gives the optimal values of x and z.

The expected utility EU of each group in the symmetric solution is $\frac{1}{2}\,SW$. Using (19), we have

$$\begin{aligned} EU &= \tfrac{1}{2}\frac{2^\rho}{1-\rho}\,(2\,x)^{\alpha(1-\rho)}\,\Big[\,\Big(p_2\;A_H^{1-\rho}+p_0\;A_L^{1-\rho}\Big)\;2^{(1-\alpha)(1-\rho)}\;z^{\beta(1-\rho)} \\ &\qquad +2\;p_1\;\Big\{\,A_H^{1/(1-\alpha)}+A_L^{1/(1-\alpha)}\,\Big\}^{(1-\alpha)(1-\rho)}\;\;z^{\beta(1-\rho)}\,\Big]-\tfrac{1}{2}\,c\,(x+z)^2 \end{aligned}$$

Simplifying this and using (23) and (24) gives the expected utility of each group in the full optimum:

$$\begin{aligned} EU^{\text{fullopt}} &= \frac{1}{1-\rho}\,\Big[\,p_2\;A_H^{1-\rho}+2\,p_1\;A_M^{1-\rho}+p_0\;A_L^{1-\rho}\Big]\;\alpha^{\alpha(1-\rho)}\;\beta^{\beta(1-\rho)}\;M^{(\alpha+\beta)(1-\rho)} \\ &\qquad -\tfrac{1}{2}\;c\,(\alpha+\beta)^2\;M^2 \\ &= \frac{1}{1-\rho}\;c(\alpha+\beta)\;M^2-\tfrac{1}{2}\;c\,(\alpha+\beta)^2\;M^2 \\ &= \frac{2-(\alpha+\beta)(1-\rho)}{2\,(1-\rho)}\;c(\alpha+\beta)\;M^2 \qquad (25) \end{aligned}$$

4.1 Gains from reciprocity arrangement

We can compare (10) from Section 3 with (25) above, to show the gains from the reciprocity arrangement to transfer cattle when one group has good weather and the other has bad weather. Consider the case $\rho < 1$. Then

$$EU^{\text{fullopt}} > EU^{\text{isol}} \quad \text{if and only if} \quad M^2 > N^2\,, \qquad (*)$$

i.e. if and only if $M^{2-(\alpha+\beta)(1-\rho)} > N^{2-(\alpha+\beta)(1-\rho)}$, i.e.

$$\begin{aligned} p_2\;A_H^{1-\rho}+2\,p_1\;A_M^{1-\rho}+p_0\;A_L^{1-\rho} &> p_H\;A_H^{1-\rho}+p_L\;A_L^{1-\rho} \\ &= (p_2+p_1)\;A_H^{1-\rho}+(p_1+p_0)\;A_L^{1-\rho}\,, \end{aligned}$$

i.e.

$$p_1\;\left[\;A_M^{1-\rho}-\frac{A_H^{1-\rho}+A_L^{1-\rho}}{2}\;\right]>0\,.$$

If $p_1 = 0$, then the left hand side of this expression is identically zero, so the two expected utilities are equal. Thus no gains from the reciprocal arrangement are possible if weather conditions for the two groups are perfectly positively correlated. If $p_1 > 0$, using (20) the inequality becomes

$$\left\{\frac{A_H^{1/(1-\alpha)}+A_L^{1/(1-\alpha)}}{2}\right\}^{(1-\alpha)(1-\rho)} > \frac{A_H^{1-\rho}+A_L^{1-\rho}}{2}$$

Since $\rho < 1$, this is equivalent to

$$\left\{ \frac{A_H^{1/(1-\alpha)} + A_L^{1/(1-\alpha)}}{2} \right\}^{(1-\alpha)} > \left\{ \frac{A_H^{1-\rho} + A_L^{1-\rho}}{2} \right\}^{1/(1-\rho)} . \tag{**}$$

Since $\alpha < 1$, we have

$$\frac{1}{1-\alpha} > 1 > 1 - \rho ,$$

therefore (**) is true by Jensen's inequality.

If $\rho > 1$, each of the two steps leading to (*) and (**) reverses the direction of the inequality. With this even number of reversals, the same final result remains valid.

4.2 Other properties of the full optimum

When weather outcomes of the two groups are different, let m denote the number of cattle transferred from a group with the L weather condition, to graze on the land of the group with the H condition. Using (16) in the symmetric optimum, we have an expression for the fraction of the herd transferred:

$$\frac{m}{x} = \frac{(A_H/A_L)^{1/(1-\alpha)} - 1}{(A_H/A_L)^{1/(1-\alpha)} + 1} . \tag{26}$$

Remarkably, this is independent of other parameters: the exponent β of land quality and the degree of risk aversion ρ (although the result does depend on these entities being constants, that is, on the Cobb-Douglas production function and constant relative risk-aversion).

Table 1: Fraction of herd transferred to better-weather land

	α				
A_H/A_L	0.1	0.3	0.5	0.7	0.9
1.5	0.222	0.282	0.385	0.589	0.966
2.0	0.367	0.458	0.600	0.820	0.998
5.0	0.713	0.818	0.923	0.991	1.000
10.0	0.856	0.928	0.980	0.991	1.000

Table 1 shows the values of m/x corresponding to different combinations of A_H/A_L and α. This fraction rises with A_H/A_L, which is quite intuitive since bigger productivity difference between good and bad weather lands should trigger a larger transfer. It also rises with α; the explanation is that a higher *alpha* means that diminishing returns set in more slowly to cattle grazing on a given piece of land, so more can be transferred without lowering the marginal product too much.

In reality we typically find around 90% of herds moved in bad weather conditions. Therefore the combinations $A_H/A_L = 2$, $\alpha = 0.7$, and $A_H/A_L = 5$, $\alpha = 0.5$, seem reasonable. This will guide our numerical calculations in what follows.

Does the reciprocal arrangement, by reducing the risk of large losses, enable each group to maintain a larger herd size? It turns out that the answer depends on the degree of risk-aversion. From (23), (6), (24) and (8), we have

$$\frac{x^{\text{fullopt}}}{x^{\text{isol}}} = \frac{M}{N} = \left[\frac{p_2\ A_H^{1-\rho} + 2\,p_1\ A_M^{1-\rho} + p_0\ A_L^{1-\rho}}{p_H\ A_H^{1-\rho} + p_L\ A_L^{1-\rho}} \right]^{1/[2-(\alpha+\beta)(1-\rho)]}$$

Table 2 shows this ratio for various A_H/A_L and ρ. Again remarkably, the qualitative behavior of the ratio is largely independent of other parameters like the probabilities and α, β. (The numbers shown are for the case $p_0 = p_1 = p_2 = 0.25$ and $\alpha = 0.75$, $\beta = 0.25$.)

Table 2: Ratio of herd size in full optimum to that in isolation

	ρ					
A_H/A_L	0.0	0.5	1.0	1.5	2.0	20
1.5	1.028	1.011	1.000	0.992	0.985	0.968
2.0	1.069	1.027	1.000	0.979	0.962	0.968
5.0	1.201	1.087	1.000	0.931	0.887	0.968
10.0	1.265	1.127	1.000	0.897	0.847	0.968

We see that if $\rho < 1$ (low risk-aversion) the ratio is > 1 and rises with A_H/A_L, and if $\rho > 1$ (high risk-aversion) the ratio is < 1 and falls as A_H/A_L rises. This numerical finding can be proved rigorously using some complicated algebra. The result goes against the intuition stated above: optimum herd sizes under the reciprocal arrangement are smaller, not larger, when traders are highly risk-averse. Also, the ratio is not monotonic in ρ at high end, but the intuition for that is unclear.

4.3 When is the full optimum self-enforcing?

Suppose group 1 gets the good weather realization A_H while group 2 has the bad one A_L. Group 1 may refuse to accept the assigned transfer of cattle m_{HL}, and consume its own output $A_H\, x_1^\alpha\, z_1^\beta$ instead of its assigned share $Y_{1,HL}$ in the joint output, where all these quantities are for the full optimum. It will thereby gain utility

$$\Delta U^{\text{renege}} = \frac{1}{1-\rho} \left[\left(A_H\, x_1^\alpha\, z_1^\beta \right)^{1-\rho} - (\, Y_{1,HL}\,)^{1-\rho} \right] .$$

This reneging will have long-term costs. Make the usual trigger strategy assumption that the reciprocity arrangement will collapse after any incident of cheating. Then the expected utility cost for each subsequent period is

$$EU^{\text{fullopt}} - EU^{\text{isol}} .$$

Suppose r is the interest rate, so a low r indicates higher concern for future costs and benefits, or a high r indicates more impatience. Then the condition for the full optimum to be self-enforcing is that the one-period utility gain not exceed the capitalized value of the subsequent flow of expected utility costs, that is,

$$\frac{1}{1-\rho} \left[\left(A_H\, x_1^\alpha\, z_1^\beta \right)^{1-\rho} - (\, Y_{1,HL}\,)^{1-\rho} \right] \leq \frac{1}{r} \left[EU^{\text{fullopt}} - EU^{\text{isol}} \right]$$

or

$$EU^{\text{fullopt}} - EU^{\text{isol}} - \frac{r}{1-\rho} \left[\left(A_H\, x_1^\alpha\, z_1^\beta \right)^{1-\rho} - (\, Y_{1,HL}\,)^{1-\rho} \right] \geq 0 \,, \qquad (27)$$

where x_1 and z_1 are the full-optimum values x and z (common to the two groups because of symmetry) found from (23) and (24), and $Y_{1,HL}$ is found from (18). This condition places an upper bound $\overline{r}$ on the interest rate (on the degree of impatience) for which the full optimum is self-enforcing. All of this analysis pertains to group 1's non-reneging. Similar analysis for group 2 would yield a condition exactly like (27), with x_1, z_1 replaced by x_2, z_2 and $Y_{1,HL}$ by $Y_{2,LH}$; because of symmetry this imposes the same upper bound on $\overline{r}$.

Table 3 presents numerical calculations of this threshold for plausible parameter values. We fix $c = 1$ and $A_L = 1$ without loss of generality; among the cost parameters only the ratio A_H/A_L matters. Consistent with the observation above about the fraction of herds moved, we consider two cases. (1) $\alpha = 0.75$, $\beta = 0.25$ and $A_H = 2$, (2) $\alpha = 0.5$, $\beta = 0.5$ and $A_H = 5$. For the probabilities, we take $p_2 = 0.5$, $p_1 = 0.2$ and $p_0 = 0.1$. Then the probabilities for any one group are $p_H = 0.7$ and $p_L = 0.3$. This is roughly consistent with the recent observation that each group suffers a dry year about once every three years. (The frequency of dry seasons has increased in recent years, possibly because of global climate change.) Also, $p_1 \approx p_H\, p_L$, so the weather outcomes are approximately uncorrelated across groups, which is the roughly neutral case for achieving gains from reciprocity.

Table 3: Upper bound on r for self-enforcement of full optimum

ρ	Case (1) $\alpha = 0.75,\ \beta = 0.25,\ A_H = 2$ $\bar{r}$	Case (2) $\alpha = 0.5,\ \beta = 0.5,\ A_H = 5$ $\bar{r}$
0.0	0.2771	0.1685
0.5	0.3687	0.3299
1.0	0.4767	0.5845
2.0	0.7505	1.5560
5.0	2.3637	30.4185

The full optimal degree of reciprocity is self-enforcing if the actual interest rate faced by the groups is below this upper bound $\bar{r}$. We see that as risk aversion increases, the bound increases, increasing the chances of the condition being fulfilled; this accords with intuition. But the numerical results present an ambiguous picture about the likelihood of it being met in reality. We have some evidence about the magnitude of the actual interest rates r for these herders. In principle, they can borrow from cooperatives and banks at rates in the range of 12 to 20 percent per year. But the availability of such loans is quite constrained, so the implied or shadow interest rates are significantly higher. Second, there is some anecdotal evidence that farmers in neighboring areas borrow to buy equipment only if the loans pay back in one season, suggesting a rate of around 100%. (But this may contain an option value component.) Thus the likely range of actual values of interest rates overlaps with the range of upper bounds we have calculated.

If the condition (27) is not met, fully optimal reciprocity cannot be sustained on the basis of the groups' long-run self-interest. More limited reciprocity can be sustained, and we will examine such constrained or second-best solutions in the next section. But the groups may also attempt to sustain the full optimum by cultivating ties such as intermarriage that lead them to take the other group's welfare directly into account in their own benefit-cost calculation. Such ties do exist, and it will be interesting to see whether they are selectively more prominent in situations where (27) is less likely to be fulfilled.

5 Self-enforcing second-best

If the full optimum is not automatically self-enforcing, we can find the second-best optimum that explicitly imposes constraints on the choice variables to ensure that neither group wants to renege on its obligation to accept transferred cattle on its land when it gets good weather and the other group gets bad weather. The constraints are just like (27) above, except that instead of the expected utility in the full optimum EU^{fullopt}, we must use the expression for expected utility as a function of the variables being chosen. Therefore the problem is to maximize the sum of expected utilities

$$
\begin{aligned}
SW &= EU_1 + EU_2 &(28)\\
&= \frac{1}{1-\rho}\left[\, p_2\,(Y_{1,HH})^{1-\rho} + p_1\,(Y_{1,HL})^{1-\rho} + p_1\,(Y_{1,LH})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho}\right]\\
&\quad + \frac{1}{1-\rho}\left[\, p_2\,(Y_{2,HH})^{1-\rho} + p_1\,(Y_{2,LH})^{1-\rho} + p_1\,(Y_{2,HL})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho}\right]\\
&\quad -\tfrac{1}{2}\,c\,(x_1+z_1)^2 - \tfrac{1}{2}\,c\,(x_2+z_2)^2 &(29)
\end{aligned}
$$

subject to the constraints that no more can be given to the two groups than the total available output in each of the four states:

$$
A_i\,(x_1+m_{ij})^\alpha\, z_1^\beta + A_j\,(x_2-m_{ij})^\alpha\, z_2^\beta \geq Y_{1,ij} + Y_{2,ij} \tag{30}
$$

for $i\,, j = H\,, L$, and the two conditions ruling out reneging:

$$
\begin{aligned}
&\frac{1}{1-\rho}\left[\, p_2\,(Y_{1,HH})^{1-\rho} + p_1\,(Y_{1,HL})^{1-\rho} + p_1\,(Y_{1,LH})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho}\right] - EU^{\text{isol}}\\
&- \frac{r}{1-\rho}\left[\left(A_H\,x_1^\alpha\,z_1^\beta\right)^{1-\rho} - (\,Y_{1,HL}\,)^{1-\rho}\right] \geq 0\,, &(31)
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{1}{1-\rho}\left[\, p_2\,(Y_{2,HH})^{1-\rho} + p_1\,(Y_{2,LH})^{1-\rho} + p_1\,(Y_{1,HL})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho}\right] - EU^{\text{isol}}\\
&- \frac{r}{1-\rho}\left[\left(A_H\,x_2^\alpha\,z_2^\beta\right)^{1-\rho} - (\,Y_{2,LH}\,)^{1-\rho}\right] \geq 0\,. &(32)
\end{aligned}
$$

Note that now all the x's, z's, m's and Y 's are to be determined afresh in the new constrained optimum; the previous solutions of the full optimum are not to be used here.

Observe that the only place m_{ij} appear is on the left hand side of (30), and it is obviously optimal to choose them to make that side, namely total output in that state, as large as possible. This is exactly as in Step 1 of the work on the full optimum in Section 4. Therefore the transfer rules (16) and the output expression (17) derived there remain valid, and (30) becomes

$$(x_1 + x_2)^{\alpha} \ \left[\ (A_i \ z_1^{\beta})^{1/(1-\alpha)} + (A_j \ z_2^{\beta})^{1/(1-\alpha)} \ \right]^{1-\alpha} \geq Y_{1,ij} + Y_{2,ij} \tag{33}$$

The fact that the transfer rule is not affected by the imposition of the self-enforceability constraint has a useful implication. Our earlier result (26) on the fraction of herds transferred in the full optimum remains valid for the constrained optimum. Therefore so does our inference about the plausible values of α, A_H etc. based on observations for the fractions transferred in reality.

Substituting the outcome of Step 1, we are left with the choice of x_1, z_1, x_2, z_2 and the $Y_{g,ij}$ for groups $g = 1, 2$ and states $i\,, j = H\,, L$. This a Kuhn-Tucker nonlinear programming problem with inequality constraints. Form its Lagrangian

$$\begin{aligned}
\mathcal{L} \ = \ & \frac{1}{1-\rho} \ \left[\ p_2\,(Y_{1,HH})^{1-\rho} + p_1\,(Y_{1,HL})^{1-\rho} + p_1\,(Y_{1,LH})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho} \right] \\
& \quad + \frac{1}{1-\rho} \ \left[\ p_2\,(Y_{2,HH})^{1-\rho} + p_1\,(Y_{2,LH})^{1-\rho} + p_1\,(Y_{2,HL})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho} \right] \\
& \quad - \tfrac{1}{2} \ c\,(x_1 + z_1)^2 - \tfrac{1}{2} \ c\,(x_2 + z_2)^2 \\
& + \sum_{i\,,j=H\,,L} \lambda_{ij} \ \left\{ \ (x_1 + x_2)^{\alpha} \ \left[\ (A_i \ z_1^{\beta})^{1/(1-\alpha)} + (A_j \ z_2^{\beta})^{1/(1-\alpha)} \ \right]^{1-\alpha} - Y_{1,ij} - Y_{2,ij} \ \right\} \\
& + \mu_1 \ \left\{ \ \frac{1}{1-\rho} \ \left[\ p_2\,(Y_{1,HH})^{1-\rho} + p_1\,(Y_{1,HL})^{1-\rho} + p_1\,(Y_{1,LH})^{1-\rho} + p_0\,(Y_{1,LL})^{1-\rho} \right] - EU^{\text{isol}} \right. \\
& \quad \left. - \ \frac{r}{1-\rho} \ \left[\ \left(A_H\, x_1^{\alpha}\, z_1^{\beta} \right)^{1-\rho} - (\,Y_{1,HL}\,)^{1-\rho} \ \right] \ \right\} \\
& + \mu_2 \ \left\{ \ \frac{1}{1-\rho} \ \left[\ p_2\,(Y_{2,HH})^{1-\rho} + p_1\,(Y_{2,LH})^{1-\rho} + p_1\,(Y_{2,HL})^{1-\rho} + p_0\,(Y_{2,LL})^{1-\rho} \right] - EU^{\text{isol}} \right. \\
& \quad \left. - \ \frac{r}{1-\rho} \ \left[\ \left(A_H\, x_2^{\alpha}\, z_2^{\beta} \right)^{1-\rho} - (\,Y_{2,LH}\,)^{1-\rho} \ \right] \ \right\}
\end{aligned}$$

The first-order conditions for the $Y_{g,ij}$ are

$$\begin{aligned}
\partial\mathcal{L}/\partial Y_{1,HH} \ &= \ (1 + \mu_1)\,p_2 \ (Y_{1,HH})^{-\rho} - \lambda_{HH} = 0 \\
\partial\mathcal{L}/\partial Y_{1,HL} \ &= \ [\,(1 + \mu_1)\,p_1 + r\,\mu_1\,] \ (Y_{1,HL})^{-\rho} - \lambda_{HL} = 0 \\
\partial\mathcal{L}/\partial Y_{1,LH} \ &= \ (1 + \mu_1)\,p_1 \ (Y_{1,LH})^{-\rho} - \lambda_{LH} = 0 \\
\partial\mathcal{L}/\partial Y_{1,LL} \ &= \ (1 + \mu_1)\,p_0 \ (Y_{1,LL})^{-\rho} - \lambda_{LL} = 0
\end{aligned}$$

for group 1, and similarly for group 2. In the symmetric solution we will have $\mu_1 = \mu_2 = \mu$ and $\lambda_{HL} = \lambda_{LH} = \lambda_M$, say. Also

$$Y_{1,HL} = Y_{2,LH} = \overline{Y}, \quad Y_{1,LH} = Y_{HL} = \underline{Y}, \quad \text{say.}$$

Then

$$\frac{\overline{Y}}{\underline{Y}} = \left[\frac{(1+\mu)\, p_1 + r\, \mu}{(1+\mu)\, p_1} \right]^{1/\rho} > 1\,. \tag{34}$$

Thus the lucky group is allowed to keep a fraction of total output greater than the 50% it would get in the full optimum, just enough to offset its temptation to renege.

Other than this general result, algebraic calculations do not provide much insight about the solution of the constrained optimum problem. Therefore we present a table of typical numerical calculations. These are for the same set of parameters as for Case (1) of Table 3, namely $\alpha = 0.75$, $\beta = 0.25$, $\rho = 0.5$, $p_2 = 0.5$, $p_1 = 0.2$ and $p_0 = 0.1$. Table 4 shows the results.

Table 4: Sample numerical solution for constrained optimum

r	0.3687	0.4000	0.6000	0.8000	1.0000
μ	0.00000	0.00656	0.02299	0.02469	0.02339
$\overline{Y}\,/\,(\,\overline{Y} + \underline{Y}\,)$	0.5000	0.5065	0.5326	0.5459	0.5539
x	0.7519	0.7518	0.7515	0.7514	0.7513
EU	1.5077	1.5077	1.5073	1.5069	1.5066

The first column is for the value of r exactly at the upper bound that is consistent with the full optimum being self-enforcing. Therefore the multiplier μ on the self-enforcement constraint is zero. For higher values of r the constraint does affect the solution, but remarkably little. Even when r is substantially above the upper bound, the Lagrange multiplier on the constraint is quite small. (In fact μ decreases slightly as r increases to very high levels, but as (34) shows, the product $r\,\mu$ has an independent influence that keeps substantive magnitudes like the output share monotonic.) Only a little more than 50% of the output suffices to keep the lucky group in line. The size of the herd decreases very slightly, as does the expected utility.

In our context, giving a larger share of output to the hosting group may need to be managed in subtle ways. The herds are transferred over large distances, as much as 100 kilometers. It is impractical to send any of the milk back to the owner group's home ranch. Some members of that group have traveled with the herd to manage it, and they can consume the milk. They can also sell some milk locally on the host groups' land, but probably have to do so at an unfavorable price. Thus the hosting group may de facto get a large share of the milk. That may overfulfill the host group's no-reneging condition, but may call into question the owner group's incentive to send animals. In fact there are other dimensions of output, namely blood, meat, and any calves born during the stay at the host ranch. Herders from the owner group that have traveled with the herd to manage it can decide whether to draw blood and how much, and how many cattle (if any) to kill for meat, so they can ensure that more goes back to the owner group with the cattle at the end of the dry spell. Also, the owner group retains rights to calves. Then a suitable combination of these four dimensions of output can be constructed to meet the relevant no-reneging condition (31) or (32) with equality, even though the single dimension of milk may not be capable of being split up in just the right proportions.

Another and perhaps stronger reason for sending to a ranch with better rainfall may be to improve the prospects for survival of the animals themselves. A proper treatment of that aspect requires a richer dynamic model; that is a part of our future research plans.

6 Concluding comments

We have developed a model of the reciprocal arrangements that enable Kenyan cattle herders to cope with weather fluctuations across their group ranches. The key mechanism is repeated interaction - the short term gains from reneging on your promise to take in a less-fortunate partner group's cattle must be weighed against the long term costs from collapse of the mutual insurance arrangement. We made many special assumptions to simplify or ignore other aspects of the situation and to produce a tractable model. Even this extremely simple model yields some insights. Some key parameters can be calibrated by comparing the results with observations. Then it appears that the degree of patience required for successful self-sustaining reciprocity is right in the range of the rates of time discounting that the herders face. Therefore we should expect to see success in some instances and not others. In the latter cases the groups may create supplementary supporting mechanisms such as intermarriage to improve the prospects of cooperation, or they may modify the scheme to reduce the temptation to renege. We find that the optimal modification is to give the host group a larger share of the milk produced by the transferred cattle, and argued that this may happen naturally because of the difficulty of transporting milk back for consumption by the owner group.

Thus the model appears to be a promising start, but many features must be added for better and deeper understanding. These are among our plans for future work.

Dynamics

Successive periods in our simple model are linked only by the repeated game. In reality there are many other links. The quantity of cattle is not a matter of totally independent choice each period, but evolves as a state variable. New purchases and births add to the stock, and sales and deaths reduce the stock. The births and deaths can be functions of the quantity and quality of land in relation to the size of the herd, and also the weather outcome. The quality of land is also a state variable, increased by better maintenance effort and degraded by grazing, which depends on the size of the herd that grazes on the land. Weather can also be correlated over time. These modifications will turn our repeated game into a dynamic game, which is far harder to analyze.

Unequal sizes

We assumed the two groups to be identical (except of course in the actual realizations of weather outcomes in any one period), and found symmetric solutions. In reality, land endowments of groups differ widely. Recognition of these asymmetries will alter the analysis in several ways. Smaller groups generally have bigger incentives to renege, making self-enforcement harder. If one group has land of naturally better quality or permanently better weather conditions than the other, we will have to consider ethical issues of whether the unfortunate group should somehow be given a redistributive transfer from the fortunate group's output, and if so, the practical policy issues of how such transfers can be implemented.

Multiple groups

We considered only two groups. In reality the region has several groups and group ranches. Each has ties with many other groups and can in principle have multiple reciprocity agreements in place. This can however make it harder to sustain any one such agreement. If group A can renege on promise to accept B's cattle, but then use a separate arrangement with C when the need arises, this threatens the viability of the arrangement with B. The system needs multilateral punishments whereby C will refuse to deal with A if A has previously reneged on its arrangement with B. Theoretical analyses as well as case studies of such arrangements exist, for example Kandori (1992), Greif (1993), and Dixit (2004), but Kenyan herders may not have the necessary multilateral communication, norms, and sanctions to sustain them.

Other insurance

In recent years, international organizations have developed and experimented with more formal insurance schemes, based on objective indexes of weather and rangeland conditions, to cover ranchers against livestock mortality caused by droughts (Mude, 2010). In future research we will study how these relate to the relation-based informal and self-enforcing arrangements examined here.

Empirical research and evidence

This work has already involved some useful interaction between theoretical modeling and empirical evidence, for example the evidence concerning the actual fraction of herds transferred helped us pin down the plausible ranges of the parameters α and A_H, and the theoretical results on the fraction of output that had to be given to the host group allowed us to infer the likelihood of survival of the reciprocal arrangement. More links of this kind can be exploited to improve our understanding and analysis, and this is one line of our continuing research on this topic. Our plans for such research include the following:
(1) Conducting questionnaire and experimental studies to estimate r, ρ etc.
(2) Gathering data for systematic statistical estimation of α, β, A_H, A_L
(3) Relating the success or failure of such arrangements of individual groups or pairs to their specific circumstances including the interest rates they face, whether they have made supporting arrangements like intermarriage, etc.

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