Optimal Wage-Tenure Contracts without Search Intensity Commitment

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Abstract

In this paper I explore wage-tenure contracts in a random search framework, where workers search on and off the job for employment opportunities similar to that of Lentz (2010) and Bagger and Lentz (2008). The worker determines the frequency by which employment opportunities arrive through a costly choice of search intensity, which is unobserved by the firm and cannot be directly contracted upon. Firms differ in the marginal productivity by which they employ workers. The model also includes worker human capital heterogeneity governed by a learning by doing process. Wages are set by sequential auction as in Postel-Vinay and Robin (2002). The exercise is closely related to Burdett and Coles (2003, 2010), and to make the problem interesting I follow their assumption of risk averse agents and imperfect capital markets. Optimal wage-tenure contracts are shown to generally be back loaded in order to discourage the worker from generating outside wage pressure on the existing match. The framework allows a decomposition of wage growth into three components: human capital accumulation, search capital accumulation, and backloading. From this point of view, the model is also closely related to Bagger et al. (2013), where wage dynamics are estimated allowing for human capital and search capital accumulation but where wage contracts are assumed to be flat unless renegotiated.

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1 Introduction

In this paper I explore wage-tenure contracts in a random search framework, where workers search on and off the job for employment opportunities. The worker determines the frequency by which employment opportunities arrive through a costly choice of search intensity. which is unobserved by the firm and cannot be directly contracted upon. Firms differ in the marginal productivity by which they employ workers. Worker human capital grows according to a learning by doing process. Wages are set by sequential auction as in Postel-Vinay and Robin (2002). Here, wages are renegotiated upon arrival of an outside offer as the two firms compete over the worker. The firms will compete in an auction fashion by submitting utility promises. In a reflection of efficient separation, the most productive firm will win and will deliver a minimum utility promise to the worker equal to that associated with full surplus extraction with the least productive firm. Depending on the wage mechanism, the worker's incentives to generate outside meetings include extracting additional rents from her current employment relationship, which can cause search to be inefficiently high (as viewed by the contracting parties) relative to the joint worker-firm match surplus maximizing level. The worker cannot commit to a given search intensity level, which imposes an incentive compatibility constraint on the mechanism design problem. Furthermore, both worker and firm cannot commit to maintain the relationship if their values fall below their outside options.

The wage contract is the mechanism that maximizes firm profits subject to the lifetime utility promise to the worker and subject to the constraint that worker search satisfies the worker's incentive compatibility constraint. The exercise is closely related to Burdett and Coles (2003, 2010) and Menzio and Shi (2010), and to make the problem interesting I follow their assumption of risk averse agents and imperfect capital markets that eliminate the option of side payments.

The Postel-Vinay and Robin (2002) framework has the feature that employers extract all rents associated with a match in excess of the total surplus of a worker's current match. Hence, if the worker is either unemployed or extracting full surplus in a given match, the worker will receive none of the additional rents associated with meeting a more productive employer. Consequently, in a variable search intensity framework where search is costly, such workers will not be contributing to match creation. If worker search is not an essential good in match production one can still rationalize strictly positive unemployment to employment flows through employer effort. However, the contracting framework in this paper also admits the possibility of ex ante rent extraction due to constraints on the contract or the shape of the utility function. The presence of a minimum wage is one example which is also studied in Flinn and Mabli (2009). A utility function that has the feature that $\lim_{w \to \underline{w}} u'(w) = -\infty$ will also effectively impose a minimum wage $\underline{w}$ on the problem. Finally, the arbitrary constraint of for example a flat wage profile can also by itself result in ex ante rent extraction. In all these cases, it is possible that employers will voluntarily give the worker a strictly higher lifetime utility than what is required by the worker's outside option. Consequently, there will be strictly positive rents to unemployed search and to search where the worker is extracting full surplus from her current match. The paper allows both non-essential worker search and ex ante rent extraction.

The analysis focuses on the case where workers are ex ante homogenous, but due to a stochastic learning by doing process, workers will be ex post heterogenous. It is a straightforward extension to allow for ex ante worker productivity heterogeneity as well.

It is shown that whenever the search intensity incentive compatibility constraint is binding (which is generally the case whenever the worker is not extracting the full surplus of the match), the optimal wage-tenure contract will backload wages in order to reduce outside wage pressure on the match. In this case, wages increase within a job through three channels; the contracted upon growth rate in wages, the occasional arrival of an outside job opportunity resulting in a renegotiation of the terms of the contract, and the contracted upon response to an increase in the workers general human capital. For the case of an interior solution, the wage of the worker will not jump in response to a jump in the worker's human capital, but the wage growth rate as well as the worker's expected lifetime utility may. The worker's lifetime utility increases steadily over an employment spell, be it within or between jobs, but as in Postel-Vinay and Robin (2002) actual wages may decrease between jobs. In contrast to Postel-Vinay and Robin (2002) but consistent with Burdett and Coles (2010), the optimal wage-tenure contract also generally implies a decreasing job separation hazard in job duration because the worker's search intensity is declining with duration and limits to the jointly efficient level at the point where the worker is extracting all the surplus from the match. In the Burdett and Coles (2010) framework the declining separation hazard is a result of the worker turning down more outside offers as she moves up the wage/tenure profile, which is then associated with an increase in the incidence of inefficient job separation. It is worth noting that in contrast with the Burdett and Coles (2010) framework, all separation is efficient in this paper. Instead search inefficiencies are related to the intensity margin.

The paper provides steady state equilibrium characterization and numerical examples to illustrate the optimal wage-tenure mechanism. The paper contributes to (at least) two existing research efforts in the literature. One is the study of optimal wage contracts in models with labor market frictions such as Burdett and Coles (2003, 2010). Obviously the study of wage determination in models with friction is a much larger literature related to the Nobel prize winning research by Peter Diamond, Dale T. Mortensen, and Chris Pissarides. However, this work is typically very strong on predictions on the division of rents from match creation rather than the particular path of payments during the match, which is often pinned down with the ultimately arbitrary assumption of flat wage profiles.

The backloading of wage contracts in this paper is related to that of the wage posting framework in Burdett and Coles (2003, 2010) in that in both cases backloading addresses outside competition for the worker. However, in a more subtle way the two frameworks differ quite substantially. In the current paper backloading derives primarily from an attempt by the employer to discourage the worker from engaging in costly rent seeking behavior in the current match. In the wage posting setting in Burdett and Coles (2003, 2010) wages are backloaded to discourage the worker from moving to other firms, even if these moves are efficient.

The paper is also related to the empirical study of wage dynamics with contributions such as Altonji and Shakotko (1987); Topel (1991); Yamaguchi (2010); Bagger et al. (2013); Kambourov and Manovskii (2009). As in Yamaguchi (2010) and Bagger et al. (2013) the framework allows wages to respond to two key accumulation processes: human capital and search capital. But in contrast to these papers, the current framework does not impose a flat piece rate wage contract, but allows the contract to be set optimally by the firm. Estimates in Kambourov and Manovskii (2009) suggest a substantial role for human capital destruction associated with occupational moves which is also a key presumption in the unemployment persistence work in Alvarez and Shimer (2011). Currently, the framework makes the assumption that layoffs are purely associated with destruction of search capital. It is however straightforward to allow layoffs to be associated with human capital changes as well. Currently, the contract does not allow the employer to insure the worker against any of such layoff related events. There is to my knowledge no empirical work that attempts to distinguish backloading as a separate source of wage growth from accumulation processes in search and human capital. The framework I present in this paper has the great advantage of being sufficiently tractable that it is at least from a technical point of view conceivable that it can be used in such an exercise.

2 The model

Time is continuous, and firms and workers discount time according to rate r. A worker dies according to Poisson rate d. The mass of workers is normalized at one, which implies that the flow of newly born workers into the economy also equals d. The worker's instantaneous utility function is given by $u(w)$, where w is the instantaneous wage. $u(\cdot)$ is strictly increasing and concave. Workers all enter the economy with human capital level h_0. A worker's human capital can increase according to a learning-by-doing process such that whenever the worker is working, her human capital increases from h_{i-1} to h_i according to Poisson rate γ_{i-1}, $i = 1, \ldots, N$, where $h_{i-1} < h_i$. Human capital is bounded above by h_N and by definition, $\gamma_N = 0$. There is no human capital depreciation.

Vacancies are heterogeneous in a single dimensional productivity index $p \in (b, \bar{p})$. A match between a skill level h worker and a productivity p firm produces revenue $R(p, h) = p + h$. The productivity of a match is constant over time. The distribution of productivity levels in the vacancy pool is represented by the cumulative distribution function $F_p(\cdot)$ which is assumed constant over time. Jobs are destroyed at exogenous rate δ. Workers can be either employed or unemployed. An unemployed skill level h worker receives income stream $b + h$. Regardless of the state, a worker can engage in costly search for job opportunities. Specifically, at cost $c(\lambda)$ the worker can affect the Poisson arrival rate $\lambda + \underline{\lambda}$ of meetings with vacancies, where $\lambda \geq 0$. It is assumed that $c(\cdot)$ is increasing and convex. It is assumed that workers meet firms at rate $\underline{\lambda}$ even if they do no search at all. Active search by the worker acts on top of this base offer arrival rate. One can think of $\underline{\lambda}$ as a result of active firm search for workers and a matching technology where each side's search effort is not an exclusive good.

Firms post vacancies and compete over workers as in Postel-Vinay and Robin (2002). If a firm meets a currently employed worker the two firms will engage in an auction over the worker. To be specific, say that it is an English or second price sealed bid auction. A bid in this auction is a promise of an employment contract that delivers a minimum lifetime utility to the worker. As a result of the greater revenue stream, a more productive firm is able to offer a worker a greater lifetime utility value. Consequently, the result of the auction between any two given firms is that the most productive firm wins the worker and will employ the worker with a promise to provide the worker with lifetime utility no less than the most utility the other firm is willing to offer. If an unemployed worker meets a vacancy, that firm will hire the worker with an employment contract that offers at a minimum the utility value of unemployment.

The contractual environment is one of limited commitment: The worker's search intensity choice is not observable by the firm and the worker cannot commit to any particular search choice. Furthermore, it is assumed that the firm cannot commit to not match outside offers. Postel-Vinay and Robin (2004) explore the firm's option to commit to not match outside offer. The firm can commit to an employment contract that specifies the worker's wage and search intensity at each point in time. Both the worker and the firm can at any point choose to terminate the match in favor of their respective outside options, which for the worker is unemployment and for the firm a zero value. Hence, the contract must at any point respect these two participation constraints or it would result in a quit or a lay off. By the assumption that there is no commitment in the search intensity choice, the contract must satisfy an incentive compatibility constraint such that the worker does not at any point want to deviate to a more preferable search intensity choice.

2.1 Contractable search intensity benchmark.

[Lentz (2010), Bagger and Lentz (2010) work with this contract. Optimal contract will be flat.] TBC.

2.2 The worker's lifetime utility

A skill level h worker who is matched with a type p firm will at a given point in time, t, anticipate a future discounted lifetime stream of utility which has utility value V_t. The particular realization of V_t is a result of the worker's employment history, which includes human capital growth realizations. The support of the worker's utility realization is given by $V_t \in \left[\tilde{L}(h,p), M_h(p)\right]$, where $\tilde{L}_h(p)$ is the least lifetime utility a type p firm will give a skill level h worker regardless of previous employment history, and $M_h(p)$ is the most lifetime utility it is willing to provide subject to its participation constraint. Rather than refer to a firm by its type p, it will in the following often be useful to refer to a firm's type by $M_h(p)$ instead. The expression for $M_h(p)$ will be derived later. For now, it is enough to state that $M_h(p)$ is monotonically increasing in p, a property that will follow trivially once $M_h(p)$ is derived. Define the distribution of maximum utility promises for a skill level h worker in the vacancy pool by $F_h(M) = F_p(p_h(M))$, where $p_h(M) = M_h^{-1}(M)$ is the inverse of the $M_h(p)$ function. $p_h(M)$ is increasing and monotone in M. Also, adopt the shorthand $L_h(M) = \tilde{L}_h(p_h(M))$ as the minimum utility value offered by a firm type M.

Now, consider a skill level h worker who is employed with a type p firm at a point in time t with an employment contract that any point in time specifies $\{w(s(t)), \lambda(s(t))\}$, that is a wage and worker search intensity conditional on past history, which is defined by $s(t) = \{o(\tau), h(\tau)\}_{\tau=0}^{t}$, where $o(\tau)$ specifies the type of any outside vacancy meeting at time τ. If no vacancy is met, then $o(\tau) = U$. $h(\tau)$ specifies the human capital of the worker at time τ. Suppressing contract and firm type arguments in the value function, the worker's time t lifetime utility valuation can be written recursively by,

$$
\begin{aligned}
(r+\delta) V(s(t)) - \dot{V}(s(t)) = & u(w(s(t))) - c(\lambda(s(t))) + \delta U + \\
& [\lambda(s(t)) + \underline{\lambda}] \left[\int_{V(s(t))}^{M_h(p)} [M' - V(s(t))] \, dF_h(M') + \right. \\
& \left. \int_{M_h(p)}^{\bar{M}_h} \left[\max\left[M_h(p), L_h(M')\right] - V(s(t)) \right] dF_h(M') \right] \\
= & u(w(s(t))) - c(\lambda(s(t))) + \delta U + \\
& [\lambda(s(t)) + \underline{\lambda}] \Bigg[M_h(p) F(M_h(p)) - V(s(t)) F(V(s(t))) - \\
& \int_{V(s(t))}^{M_h(p)} F_h(M') dM' - [F(M_h(p)) - F(V(s(t)))] V(s(t)) + \\
& \bar{F}(M_h(p)) [M_h(p) - V(s(t))] \Bigg] \\
= & u(w(s(t))) - c(\lambda(s(t))) + \delta U + \\
& [\lambda(s(t)) + \underline{\lambda}] \int_{V(s(t))}^{M_h(p)} \bar{F}_h(M') dM'. \qquad (1)
\end{aligned}
$$

The rate of change over time includes the possibility of the worker's human capital growing. When the worker gets an outside offer, which happens at rate λ_t, if the outside firm is of type less than $V(s(t))$, it is of no use to the worker since it cannot force a renegotiation of the existing contract which is already offering more than the outside firm can match. If the outside firm is of type $M' \in [V(s(t)), M_h(p)]$, then Bertrand competition between the outside firm M' and the worker's current firm M results in the worker staying with firm M which wins the worker at price M', which it delivers to the worker through a renegotiated contract. The expected gain from such meetings is expressed in the first integral on the right hand side of equation (1). If the worker meets a firm a firm of type $M' > M_h(p)$, the worker will leave and go to the outside firm where she will receive lifetime utility value no less than M, which is her old firm's bid for her to stay. However, it is possible that the new firm's minimum contract value is such that $L_h(M') > M_h(p)$, in which case she will obtain $L_h(M')$ from the new firm. The expected gains from such meetings are expressed in the second integral on the right hand side of equation (1). The value of unemployment is given by,

$$
rU_h = \max_{\lambda} \left[u(b+h) - c(\lambda) + [\lambda + \underline{\lambda}] \int_{U_h}^{\bar{M}_h} \max[0, L_h(M) - U_h] \, dF_h(M) \right]. \qquad (2)
$$

2.2.1 Minimum wages and ex ante rent extraction

So far the analysis has left open the possibility of ex ante rent extraction by the worker, which is tied to the possibility of a minimum wage constraint like $w \geq \underline{w}$ where $\underline{w}$ is some minimum wage imposed on the problem by for example labor market legislation. A minimum wage can also be effectively imposed on the problem by the behavior of the utility function if for example $\lim_{w \to \underline{w}} u'(w) = -\infty$. Cases like these make it possible that $\tilde{L}_h(p') > M_h(p)$ for some $p' > p \geq b$, meaning that if a worker meets a type p' firm, she would receive rents in excess of full rent extraction with her current employer (of if unemployed, the value of unemployment) even though the p' firm does not strictly speaking have to offer her more than $M_h(p)$ to make her accept employment with the firm. The possibility of ex ante rent extraction due to minimum wages is discussed extensively in Flinn and Mabli (2009). In the case of this paper, the issue is exacerbated by the backloading of wage profiles. For simplicity, the main part of the analysis of the paper will assume away lower wage bounds, which will eliminate the complication of finding a fixed point in $\tilde{L}_h(p)$. The analysis will then return to the issue later in the paper. So, for the main part of the analysis it will be assumed that there is no lower wage bound and that the utility function is such that $u'(\cdot) \in [\underline{u}', \bar{u}']$. This will imply that $\tilde{L}_h(p) = U_h$ for all h and p.

2.3 Optimal contract design

In what follows consider a type p firm's problem of designing a profit maximizing employment contract. It does so subject to a constraint that it must provide the worker with some minimum lifetime utility value $\underline{V}$ that is a result of either competition with another firm or the worker's unemployed outside option. As mentioned above, the firm may ultimately choose to provide more than that, but $\underline{V}$ is the fundamental promise constraint on the contract. Following the literature on dynamic contracting, for example Atkeson and Lucas (1992) and in a related application, Menzio and Shi (2010), the optimal wage contract is analyzed in a recursive dynamic programming framework using that the problem can be written using worker lifetime utility as an auxiliary state variable in the firm's profit maximization problem.

Consider a type p firm's time t valuation of an employment contract,

$$\mathcal{C}(t) = \{w(s(\tau)), \lambda(s(\tau))\}_{\tau=t}^{\infty}$$

and associated worker utility value $\{V(s(\tau))\}_{\tau=t}^{\infty}$. To keep notation as simple as possible, the firm type M is suppressed as an argument in the value functions below.

$$\begin{aligned}
[r+\delta]\,\Pi_t\left(s\left(t\right),V\left(s\left(t\right)\right)\right) &= p+h\left(t\right)-w\left(s\left(t\right)\right)-\left[\lambda\left(s\left(t\right)\right)+\underline{\lambda}\right]\hat{F}_{h(t)}\left(V\left(s\left(t\right)\right)\right)\Pi_t\left(V\left(s\left(t\right)\right)\right) \\
&\quad+\left[\lambda\left(s\left(t\right)\right)+\underline{\lambda}\right]\int_{V(s(t))}^{M_h(p)}\Pi\left(M'\right)dF_{h(t)}\left(M'\right)+\dot{\Pi}_t\left(V\left(s\left(t\right)\right)\right) \\
&= p+h\left(t\right)-w\left(s\left(t\right)\right)+\dot{\Pi}_t\left(V_t\right) \\
&\quad+\left[\lambda\left(s\left(t\right)\right)+\underline{\lambda}\right]\int_{V(s(t))}^{M_h(p)}\Pi'\left(M'\right)\hat{F}_h\left(M'\right)dM', \qquad (3)
\end{aligned}$$

where by definition of $M_h(p)$, $\Pi\left(M_h(p)\right)=0$. The firm has an instantaneous profit flow of $p+h(t)-w(s(t))$. The match is destroyed for exogenous reasons at rate δ. At rate $\lambda(s(\mathrm{t}))$ the worker meets an outside firm and with probability $\hat{F}_{h(t)}(V_t)$ the meeting triggers a termination of the current contract. If the outside firm is of type $M'\in\left[V(s(\mathrm{t})),M_h(p)\right]$ the worker and the firm will enter into a new contract with a minimum utility promise of M' which has profit value $\Pi(M')$ to the firm. If the outside firm is of type $M'>M$, the worker leaves and the match terminates.

Since the worker's search intensity is not observable, the optimal contract design is subject to the incentive compatibility problem that the worker does not want to deviate from the prescribed $\lambda(s(t))$ in the contract. Specifically, if the employment contract implies a lifetime utility value of $V(s(t))$ to the worker, the first order condition for the worker's optimal search choice is,

$$c'\left(\lambda\left(s\left(t\right)\right)\right)=\int_{V\left(s\left(\mathrm{t}\right)\right)}^{M_h(p)}\hat{F}_h(M')dS', \qquad (4)$$

which follows directly from equation (1) and the assumption on utility implying $\tilde{L}_h(p)=U_h$. If the right hand side of equation (4) is greater than the left hand side for the prescribed search intensity, the marginal gains to an increase in search intensity would be greater than the marginal cost, and the worker would want to deviate in the direction of increased search. And vice versa. Conditional on an outside offer of value greater than V, with probability $\frac{F_h(M_h(p))-F_h(V)}{1-F_h(V)}$, the worker extract rents from the current employer as opposed to extraction of rents from an outside employer. The optimal contract will in part be seeking to address this source of search incentives.

With the participation constraints that both worker and firm are always free to leave the contract without penalty and with the general assumption of no side payments, the firm will design a contract to solve the problem (in the following p has been suppressed in the functional expressions for notational simplicity),

$$
\begin{aligned}
(r+\delta+\gamma_n)\,\Pi_n(V) = \max_{w,\lambda,\dot{V},V_{n+1}} \Big[& p + h_n - w + [\lambda + \underline{\lambda}] \int_V^{M_n} \Pi_h'(M')\,\hat{F}_n(M')\,dM' \\
& + \gamma_n \Pi_{n+1}(V_{n+1}) + \Pi_n'(V)\,\dot{V} \Big] \\
s.t.:\ c'(\lambda) = & \int_V^{M_n} \hat{F}_n(M')dM' \\
(r+\delta+\gamma_n)\,V = & \, u(w) - c(\lambda) + [\lambda + \underline{\lambda}] \int_V^{M_n} \hat{F}_n(M')\,dM' + \gamma_n V_{n+1} + \delta U + \dot{V} \\
M_{n+1} \geq & \, V_{n+1} \geq U_{n+1} \\
\dot{V}_n(M_n) \leq & \, 0
\end{aligned}
$$

The last two constraints follow from the participation constraints that the contract cannot deliver less value than each agent's readily available outside option. This convenient translation of a value function boundary into a state space boundary is also done in Wright and Wong (2011). The following proposition summarizes the results of the optimization problem.

Proposition 1. *The optimal wage contract offered by a type p firm to a skill level h_n worker subject to a worker lifetime utility promise $V \in [U_n, M_n(p)]$ satisfies,*

$$\Pi_n'(V) = \frac{-1}{u'(w_n(V))} \tag{5}$$

$$\Pi_{n+1}'(V_{n+1}(V)) = \Pi_n'(V) - \frac{r+\delta+\gamma_n}{\gamma_n}\eta_n(V) \tag{6}$$

$$c'(\lambda_n(V)) = \int_V^{M_n} \hat{F}_n(M')dM' \tag{7}$$

$$
\begin{aligned}
\dot{V}_n(V) &= \frac{-[r+\delta]\,\mu_n(V)\,\hat{F}_n(V)}{\Pi''(V)} \\
&= \frac{\hat{F}_n(V)\,(u'(w(V)))^2 \int_V^M \Pi'(M')\,\hat{F}_n(M')\,dM'}{u''(w(V))\,w'(V)\,c''(\lambda(V))},
\end{aligned} \tag{8}
$$

where $\mu_n(V) \geq 0$ is the Lagrange multiplier on the incentive compatibility constraint and $\eta_n(V)$ is the Lagrange multiplier on the $V_{n+1} \geq U_{n+1}$ constraint. For skill levels h_n $n = 0, \ldots, N-1$, and at the *upper utility promise bound, $V = M_n(p)$, the contract satisfies,*

$$w_n(M_n) = p + h_n + \gamma_n \Pi_{n+1}(V_{n+1}(M_n)) \tag{9}$$

$$\lambda_n(M_n) = 0 \tag{10}$$

$$\dot{V}_n(M_n) = 0 \tag{11}$$

$$M_n = \frac{u(w_n(M_n)) + \gamma_n V_{n+1}(M_n) + \delta U_n}{r + \delta + \gamma_n}. \tag{12}$$

At the highest skill level, h_N, and at the upper utility promise bound, $V = M_N(p)$, the contract satisfies,

$$w_N(M_N) = p + h_N \tag{13}$$

$$\lambda_N(M_N) = 0 \tag{14}$$

$$\dot{V}_N(M_N) = 0 \tag{15}$$

$$M_N = \frac{u(p + h_N) + \delta U_N}{r + \delta}. \tag{16}$$

Proof. To solve for the optimal contract set up the Lagrangian for the problem where ν, μ, and η are the respective Lagrange multipliers,

$$\begin{aligned} L_n\left(V, w, \dot{V}, \lambda, V_{n+1}\right) = & \frac{p + h_n - w + [\lambda + \underline{\lambda}] \int_V^{M_n} \Pi'(M') \hat{F}_n(M') \, dM' + \gamma_n \Pi_{n+1}(V_{n+1}) + \Pi_n'(V) \dot{V}}{r + \delta + \gamma_n} + \\ & \nu_n(V) \left\{ u(w) - c(\lambda) + [\lambda + \underline{\lambda}] \int_V^{M_n} \hat{F}_n(M') \, dM' + \delta U_n + \gamma_n V_{n+1} + \dot{V} - \right. \\ & \left. (r + \delta + \gamma_n) V \right\} + \mu_n(V) \left[c'(\lambda) - \int_V^{M_n} \hat{F}_n(M') dM' \right] + \\ & \eta_n(V) \left[V_{n+1} - U_{n+1} \right] \end{aligned}$$

The problem ignores the constraints $V \leq M$ and $V \geq U$ which will turn out to not be binding. The first order conditions are,

$$\begin{aligned} \nu_n(V) &= \frac{1}{u'(w_n(V))(r + \delta + \gamma_n)} \\ \nu_n(V) &= \frac{-\Pi_n'(V)}{r + \delta + \gamma_n} \\ \mu_n(V) c''(\lambda_n(V)) &= \nu_n(V) \left[c'(\lambda_n(V)) - \int_V^{M_n} \hat{F}_n(M') \, dM' \right] - \frac{\int_V^{M_n} \Pi_n'(M') \hat{F}_n(M') \, dM'}{r + \delta + \gamma_n} \\ \eta(V) &= -\gamma_n \nu_n(V) - \frac{\gamma_n \Pi_{n+1}'(V_{n+1}(V))}{r + \delta + \gamma_n} \end{aligned}$$

And the associated slackness conditions

$$0 = \mu_n(V)\left[\int_V^{M_n} \hat{F}_n(M')dM' - c'(\lambda_n(V))\right]$$

$$0 = \nu_n(V)\Big\{u(w_n(V)) - c(\lambda_n(V)) + \lambda_n(V)\int_V^{M_n} \hat{F}_n(M')\,dM' +$$

$$\delta U_n + \gamma_n V_{n+1}(V) + \dot{V}_n(V) - (r+\delta+\gamma_n)V\Big\}$$

$$0 = \eta_n(V)\left[V_{n+1}(V) - U_{n+1}\right]$$

It must be that $\nu_n(V) > 0$ which then provides a key relationship between marginal utility and the slope of the firm's profit function,

$$-\Pi_n'(V)\,u'(w_n(V)) = 1,$$

which is equation (5). For any utility promise $V \leq M_n$ the search intensity level $\lambda_n(V)$ and the shadow cost on the IC constraint are given by,

$$c'(\lambda_n(V)) = \int_V^{M_n} \hat{F}_n(M')dM'$$

$$\mu_n(V) = \frac{-\int_V^{M_n} \Pi_n'(M')\,\hat{F}_n(M')\,dM'}{c''(\lambda_n(V))\left[r+\delta+\gamma_n\right]}.$$

By the slackness condition and the first order condition on $\lambda_n(V)$, it must be that $\mu_n(M_n) = 0$. That is, when the worker is extracting all surplus the incentive constraint is not binding (the firm's and worker's incentives are perfectly aligned here) and that the optimal search intensity level satisfies, $c'(\lambda_n(M_n)) = 0$. If the boundary condition on the $V_{n+1}(V)$ choice is not binding (that is, $\eta_n(V) = 0$) we have that,

$$\Pi_{n+1}'(V_{n+1}(V)) = \Pi_n'(V)$$

$$\Updownarrow$$

$$w_{n+1}(V_{n+1}(V)) = w_n(V).$$

Thus, if the boundary condition is not binding, upon realizing an increase in skill, the worker's wage will continue smoothly from its previous point. If the boundary condition is binding,

$\eta_n(V)$ is given by,

$$\eta_n(V) = \frac{\gamma_n}{r+\delta+\gamma_n}\left[\Pi_n'(V) - \Pi_{n+1}'(U_{n+1})\right].$$

By the envelope condition, the first derivative of the profit function is,

$$\begin{aligned}\Pi_n'(V) =& \frac{-[\lambda+\underline{\lambda}]\,\Pi_n'(V)\,\hat{F}_n(V) + \Pi_n''(V)\,\dot{V}_n(V)}{r+\delta+\gamma_n} - \nu_n(V)\left[[\lambda+\underline{\lambda}]\,\hat{F}_n(V) + (r+\delta+\gamma_n)\right] \\ &+ \mu_n(V)\,\hat{F}_n(V) \\ =& \frac{-[\lambda+\underline{\lambda}]\,\Pi_n'(V)\,\hat{F}_n(V) + \Pi_n''(V)\,\dot{V}_n(V)}{r+\delta+\gamma_n} \\ &+ \frac{\Pi_n'(V)}{r+\delta+\gamma_n}\left[[\lambda+\underline{\lambda}]\,\hat{F}_n(V) + (r+\delta+\gamma_n)\right] + \mu_n(V)\,\hat{F}_n(V).\end{aligned}$$

This implies,

$$\begin{aligned}\frac{-\Pi_n''(V)\,\dot{V}_n(V)}{r+\delta+\gamma_n} &= \mu_n(V)\,\hat{F}_n(V) \\ &\Updownarrow \\ \dot{V}_n(V) &= \frac{-\mu_n(V)\,\hat{F}_n(V)\,(r+\delta+\gamma_n)}{\Pi_n''(V)} = \frac{\hat{F}_n(V)\int_V^{M_n}\Pi_n'(M')\,\hat{F}_n(M')\,dM'}{c''(\lambda_n(V))\,\Pi_n''(V)} \\ &= \frac{u'(w_n(V))^2\,\hat{F}_n(V)\int_V^{M_n}\Pi_n'(M')\,\hat{F}_n(M')\,dM'}{c''(\lambda_n(V))\,u''(w_n(V))\,w_n'(V)} \geq 0,\end{aligned}$$

where the last equality follows from, $\Pi_n''(V) = u''(w_n(V))\,w_n'(V)/u'(w_n(V))^2$. As long as $w_n'(V) > 0$ we then get that $\dot{V}_n(V) \geq 0$ always. This states that the optimal wage growth rate is strictly positive as long as the incentive compatibility constraint is binding, $\mu_n(V) > 0$. Thus, wages are backloaded in direct response to the worker's incentive to search for and consequently bring outside wage pressure to the match.

When the worker is extracting all the surplus, that is $\mu_n(M_n) = 0$, it follows that,

$$\dot{V}_n(M_n) = 0. \tag{17}$$

By the definition of M_n it must be that $\Pi_n(M_n) = 0$, which then in combination with equation (17) implies,

$$w_n(M_n) = p + h_n + \gamma_n \Pi_{n+1}(V_{n+1}(M_n)),$$

and the expression for M_n follows,

$$M_n = \frac{u(w_n(M_n)) + \gamma_n V_{n+1}(M_n) + \delta U_n}{r + \delta + \gamma_n}.$$

The firm participation constraint imposes $V \leq M$. This turns out to not be binding. At $V = M$, the optimal contract that delivers $V = M$, implies $\dot{V} = 0$ and so the unconstrained problem above satisfies the firm participation constraint. The worker participation constraint imposes $V \geq U$. Any wage contract is made conditional on an initial utility promise $V \geq U_n$. Since the optimal wage contract has $\dot{V}(V) \geq 0$ for any $V \in [U_n, M_n]$, the unconstrained problem above satisfies the worker participation constraint. □

Equation (8) in Proposition 1 shows that the optimal wage contract is backloaded in direct response to a binding incentive compatibility constraint on the worker's search intensity choice. Generally, for any utility promise $V < M_n$ and when there is positive mass in the offer distribution between V and M_n, the incentive compatibility constraint will be binding, that is $\mu_n(V) > 0$. This is because in this case part of the worker's gains to search is rent extraction from her current firm, and that this rent transfer mechanism between the firm and the worker is inefficient from the point of view of the contracting problem. In order to induce the rent transfer, the worker incurs real search costs, and the value of the transfer is diminished because it is delivered in a non-smooth fashion. Hence, the worker is searching too much relative to the jointly efficient level, and consequently the firm uses backloading in order to induce the worker to search less.

To illustrate the inefficiency of the rent transfers associated with contract renegotiations consider the case where the worker and firm can contract directly on the worker's search intensity, $\tilde{\lambda}_n(V)$, but the firm cannot commit to not match outside offers. In this case, the search intensity is determined by the first order equation,

$$\begin{aligned} c'\left(\tilde{\lambda}_n(V)\right) &= \int_V^{M_n} \hat{F}_n(M')\, dM' - \frac{\int_V^{M_n} \Pi_n'(M')\hat{F}_n(M')\, dM'}{\Pi_n'(V)} \\ &= \frac{\int_V^{M_n} \left[\Pi_n'(V) - \Pi_n'(M')\right]\hat{F}_n(M')\, dM'}{\Pi_n'(V)} \leq 0. \end{aligned}$$

However, this runs into the boundary condition that $\tilde{\lambda}_n(V) \geq 0$. Hence, the efficient choice is $\tilde{\lambda}_n(V) = 0$. From the point view of the match, gains to search come from extraction of rents from outside firms that are better than the worker's current firm. However, absent minimum wages, there is no ex ante rent extraction from outside firms and hence, the jointly efficient search level is zero. If there were gains to be had from contacts with outside firms, such gains would be modified negatively by the fact that the arrival of outside offers is also associated with the occasional renegotiation of the employment contract between the firm and the worker. This results in a non-smooth wage path which is why the right hand side of the above expression is strictly negative for $V < M_n$.

2.4 Equilibrium

The equilibrium is stated in terms of the fundamental firm productivity heterogeneity so that for a given vacancy offer distribution $\tilde{F}(p)$, an equilibrium is a collection,

$$\left\{U_n, \left\{\tilde{L}_n(p), M_n(p), \left\{w_n(V,p), \lambda_n(V,p), \dot{V}_n(V,p), \Pi_n(V,p)\right\}_{V=\tilde{L}(p)}^{M_n(p)}\right\}_{p=b}^{\bar{p}}\right\}_{n=1}^{N}$$

that satisfies the following equations,

$$U_n = \frac{u(b+h_n)}{r} \tag{18}$$

$$M_n(p) = \frac{u(w_n(M_n,p)) + \gamma_n V_{n+1}(M_n,p) + \delta U_n}{r+\delta+\gamma_n}. \tag{19}$$

$$F_n(M) = \tilde{F}\left(M_n^{-1}(M)\right) \tag{20}$$

$$\hat{F}_n(M) = 1 - F_n(M) \tag{21}$$

$$\tilde{L}_n(p) = U_n \tag{22}$$

$$\Pi_n'(V,p) = -1/u'(w_n(V,p)) \tag{23}$$

$$c'(\lambda_n(V,p)) = \int_V^{M_n(p)} \hat{F}_n(M')dM' \tag{24}$$

$$\begin{aligned}(r+\delta+\gamma_n)V = {} & u(w_n(V,p)) - c(\lambda_n(V,p)) + [\lambda_n(V,p) + \underline{\lambda}]\, c'(\lambda_n(V,p)) \\ & + \delta U_n + \dot{V}_n(V,p) + \gamma_n V_{n+1}(V,p)\end{aligned} \tag{25}$$

$$\begin{aligned}(r+\delta+\gamma_n)\Pi_n(V,p) = {} & p + h_n - w_n(V,p) + [\lambda_n(V,p) + \underline{\lambda}] \int_V^{M_n(p)} \Pi_n'(M',p)\, \hat{F}_n(M')\, dM' \\ & + \Pi_n'(V,p)\, \dot{V}_n(V,p) + \gamma_n \Pi_{n+1}(V_{n+1}(V,p))\end{aligned} \tag{26}$$

$$\dot{V}_n(M_n(p), p) = 0 \tag{27}$$

$$\Pi_n(M_n(p), p) = 0. \tag{28}$$

In the case of strictly positive ex ante rent extraction, say because of minimum wages, then existence boils down to a fixed point argument in $\tilde{L}(p)$. As it is currently, existence is simple solution of a differential equation system that is initialized at $V = M_n(p)$.

Remark 1. Proof of existence to follow. In the no ex ante rent extraction case, this is pretty trivial.

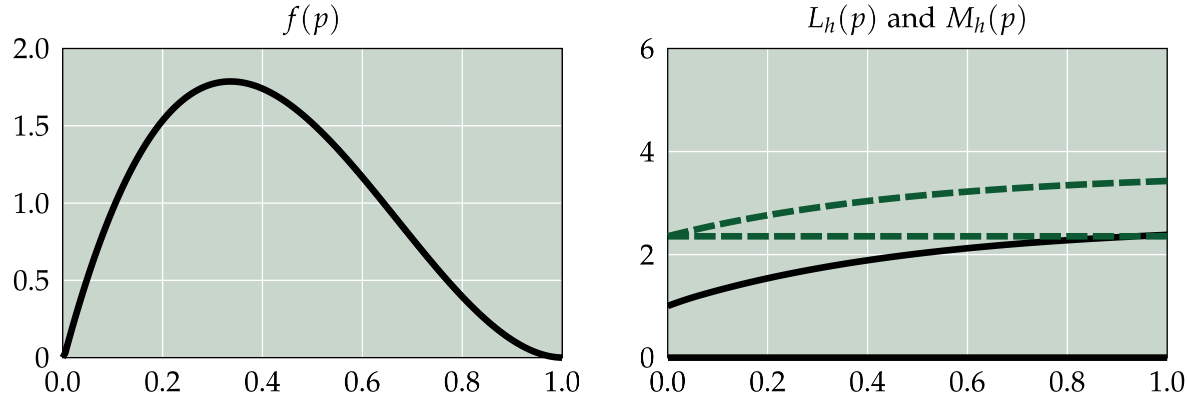
With ex ante rent extraction there will be a question of existence of a fixed point in a $\tilde{L}(p)$ mapping, but again, not clear why existence would be a problem here.

Remark 2. Uniqueness: At first glance, not so obvious. There are some interesting positive feedbacks where an increase in $\tilde{L}(p)$ for some type will tend to put upward pressure on $\tilde{L}(p')$ of other types p'.

2.5 Solution algorithm

The equilibrium solution algorithm is a fixed point search in a function $L = TL$, where $T : \mathcal{F} \to \mathcal{F}$ maps from the space of of single dimensional functions, $\mathcal{F}$, into itself. T is defined via the equations in the equilibrium definition above. For a given function $\tilde{L}(p)$, T maps into $\hat{L} = T\tilde{L}$ as follows. Given $\tilde{L}$ equations (18), (19), and (24) provide the implied functions U, $M(p)$, and $\lambda_M(V)$. The remaining objects are obtained by solving the differential equation (3) from the initialization at $\Pi_M(M) = 0$ and $\Pi'_M(M) = -1/u'(p(M))$ toward $V = L(M)$, for each firm type $M(p)$. For a given firm type, $M(p)$, and for the given initialization at state space point $V_M^0 = M$, go to the next state space point at $V_M^1 = M - \varepsilon$. Make a guess at a solution $\Pi'_M(V_M^1)$, which then immediately provides $w_M(V_M^1)$ and $\dot{V}_M(V_M^1)$ according to equations (23) and (25). Using an appropriate interpolation between $\Pi'_M(V_M^0)$ and $\Pi'_M(V_M^1)$ to evaluate the integral on the right hand side of equation (3), one can then obtain the implied value for $\Pi_M(V_M^1)$. By the definition of the derivative $\Pi'_M(V_M^1) = \lim_{\varepsilon \to 0}\left(\Pi_M(V_M^0) - \Pi_M(V_M^1)\right)/\varepsilon$, this then reduces to a simple fixed point search in $\Pi'_M(V_M^1)$ to find the value such that $\Pi'_M(V_M^1) = \left(\Pi_M(V_M^0) - \Pi_M(V_M^1)\right)/\varepsilon$, where the right hand side is determined as a function of $\Pi'_M(V_M^1)$ as described above. Once a solution is found, move on to the next state space point $V_M^2 = M - 2\varepsilon$ and repeat. Continue this process either until $V_M^i = U$ or $\Pi'_M(V^i) = 0$, whichever comes first. This then determines $L(M) = V^i$. Do this for all firm types $M(p)$ and this yields an updated function $L(M)$. Equilibrium is found when the updated $\tilde{L}(p)$ is equal to the initial guess $\tilde{L}(p)$. As a practical matter, simple iteration on $\tilde{L}(p)$ turns out to work quite well as a search method.

Figure 1: Offer distribution and firm type conditional value promise support bounds.



Note: $L_0(p)$ and $M_0(p)$ in black solid lines. $L_1(p)$ and $M_1(p)$ in green dashed lines.

2.6 A simple example

To provide an illustration of numerical solutions of the model equilibrium make the following specifications.

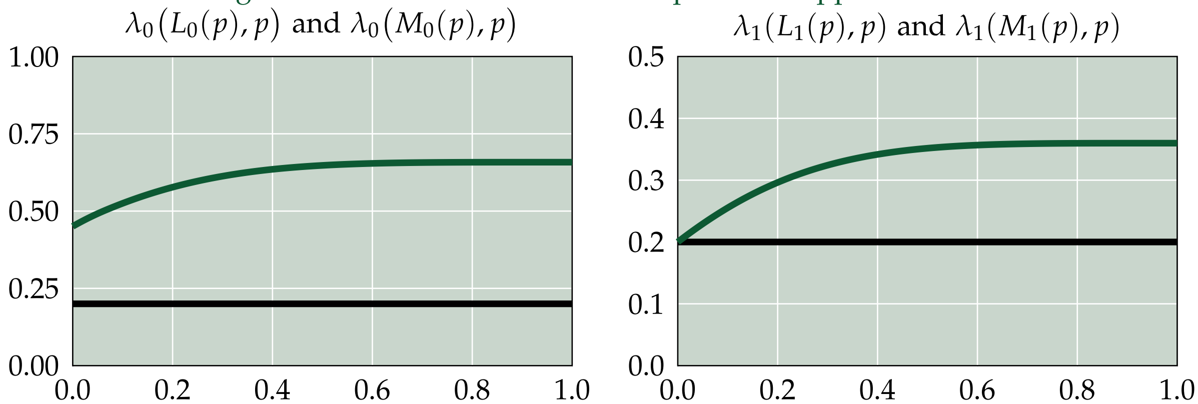
$$u(w) = \frac{1 - \exp(-\alpha_0 w)}{\alpha_0}$$
$$c(\lambda) = \frac{(c_0 \lambda)^{1+c_1}}{1 + c_1}$$

Assume $F_p(\cdot)$ is a beta distribution with parameters (β_0, β_1). The following solution is obtained for the parameterization

$$\begin{aligned} \theta &= (\alpha, c_0, c_1, \beta_0, \beta_1, r, \delta, b, \bar{p}, \underline{\lambda}, h_0, h_1, \gamma_0, d) \\ &= (2, 2, 1, 2, 3, 0.07, 0.2, 0, 1, 0.2, 0, 0.2, 0.2, 0.02). \end{aligned}$$

That is, the offer distribution is right skewed, constant absolute risk aversion utility, a quadratic search cost function, discount factor at 0.05 with the implication that rates are stated at the annual level. The job destruction rate is set at 0.2. Figure 1 shows a number of firm type conditional equilibrium objects. The right panel in Figure 1 shows $M(p)$ and $L(p)$. The value of unemployed search is in the equilibrium $U_0 = 0$ and $U_1 = 2.35$. In this version of the model without any lower wage bounds, we have full ex ante rent extraction by the firms and consequently $L_0(p) = U_0$ and $L_1(p) = U_1$ for all p. With minimum wages or other features that generate ex ante rent extraction, it is possible that $L_h(p) > U_h$ for some p. Simulations have consistently produced

Figure 2: Offer arrival rate at value promise support bounds.



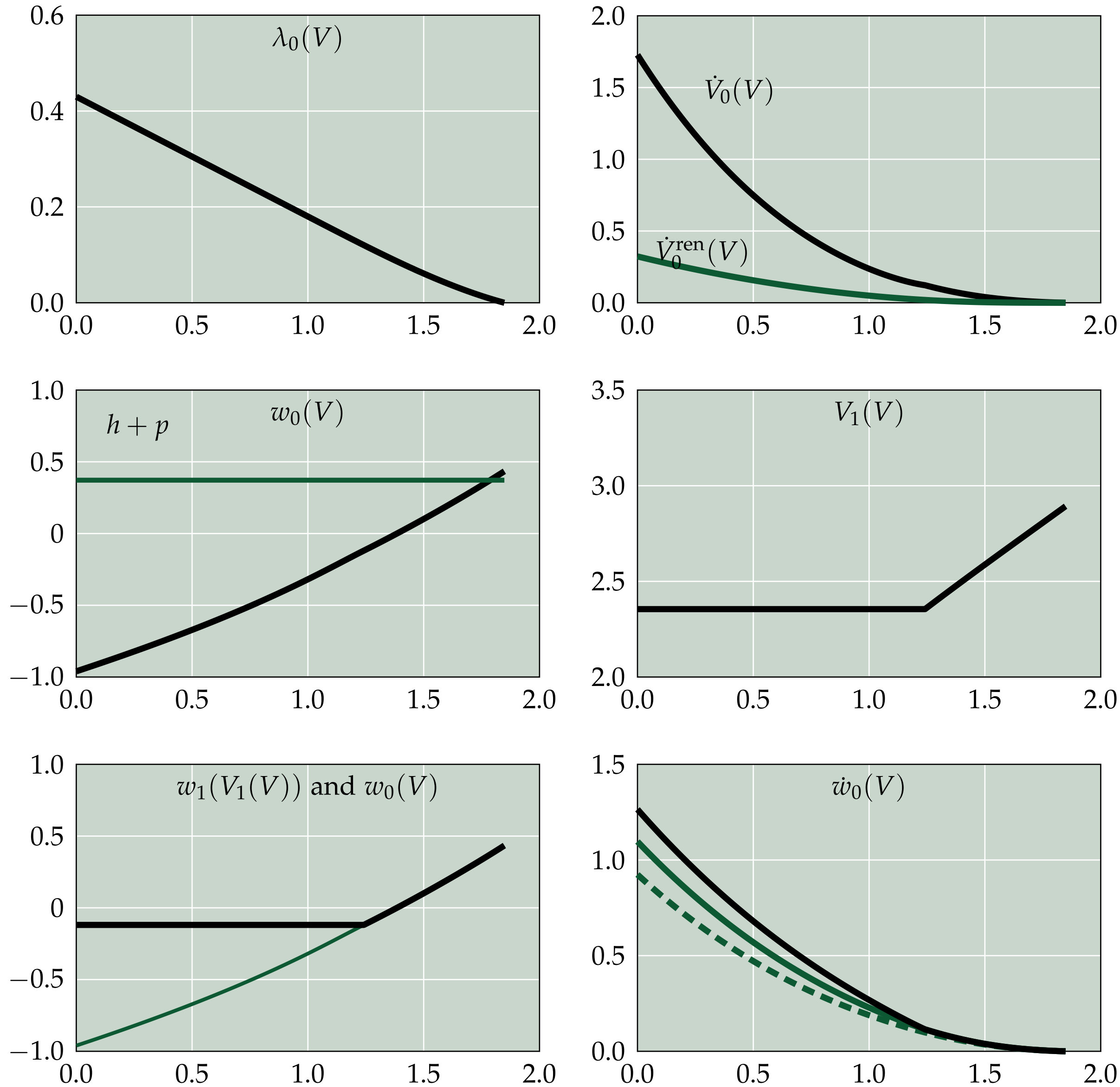
Note: Offer arrival rate at lower value support bound in green and at upper support bound in black.

$L_h(p)$ weakly increasing in p in these cases, but it is not a feature I have a general proof for. The left panel shows the offer density distribution that the simulation is produced with.

Figure 2 shows the offer arrival rates conditional on firm type at the upper and lower value support bounds. At full surplus extraction any type of worker chooses not to search and outside offers arrive purely from the exogenous arrival process. The search intensity of a newly hired worker right out of unemployment is increasing in the firm's type. In the version of the model with ex ante rent extraction this is not necessarily true, however, when $L_h(p)$ is constant in p, the gains to search are monotonically increasing in p given a current utility promise of $L_h(p)$. Hence, the feature that a newly hired worker out of unemployment will search more intensely for outside offers the higher up the ladder she has landed. While not efficient, it is a natural feature of a model where high wage realizations require a meeting with not one, but at a minimum two high productivity firms. Once a meeting with one high productivity firm is in hand, the incentive to generate a meeting with another goes up. Because of income effects, newly hired out of unemployment low skill workers search more intensely at a given rung on the ladder than high skill workers do.

Figure 3 shows the median firm type employment contract for a low skill worker. The worker will start employment with the firm at some initial utility promise depending on the worker's labor market history. Conditional on no arrival of outside employment opportunities, from there the contract will evolve according to the law of motion $\dot{V}(V)$ as shown in upper right hand panel. If the worker meets an outside firm, it is possible that the contract renegotiation will result in a jump in the utility value with the firm to some higher level V' in which case employment will

Figure 3: Median firm employment contract, low skill



Note: Lower right panel: Solid black line shows total within job wage growth. The solid green line shows within job wage growth excluding growth due to skill growth. The dashed green line shows wage growth attributable to the current wage contract, only.

continue from that point under the new contract. The left top panel shows the worker's search intensity as a function of the current utility promise. The worker's incentive to search for outside options are proportional to the possible rents the worker has yet to extract from the current match. Once at full surplus extraction, the worker ceases to search regardless of her current firm's position in the ladder. That search is zero at $M_h(p)$ is a particular feature of the no ex ante rent extraction case. The right top panel shows the two sources of utility promise growth within the worker's current job. $\dot{V}(V)$ is the contracted upon change in utility over time. $\dot{V}_n^{\text{ren}}(V)$ is the expected change in the worker's lifetime utility due to contract renegotiation caused by outside offer arrival. It is defined by,

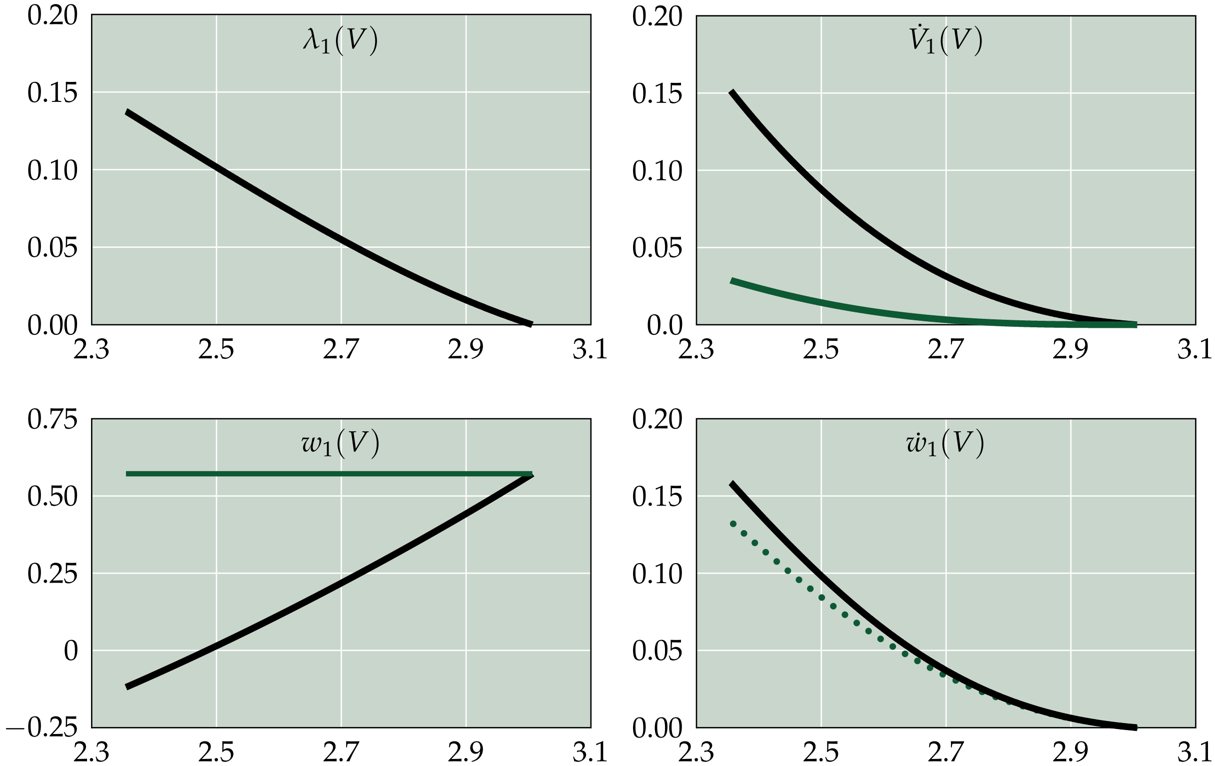
$$\begin{aligned}\dot{V}_n^{\text{ren}}(V,p) &= [\lambda_n(V,p)+\underline{\lambda}]\int_V^{M_n(p)}[M'-V]\,dF_n(M') \\ &= [\lambda_n(V,p)+\underline{\lambda}]\left[\int_V^{M_n(p)}\hat{F}_n(M')\,dM'-\hat{F}(M_n(p))[M_n(p)-V]\right].\end{aligned}$$

In a wage contract without back loading such as (Postel-Vinay and Robin, 2002) all of within job wage growth comes from the $\dot{V}_n^{\text{ren}}(V)$ channel. However, by backloading wages, the firm can deliver the same utility promise at greater profits by smoothly increasing the value of the contract and thereby over time move the worker to a lower search intensity state. It is worth pointing out that for a given utility promise, the particular design of the contract does not change the worker's search intensity at the point in time in question. The backloading of wages is a mechanism that lowers the worker's search intensity going forward as the terms of the contract become more favorable. In the current example, the contract is delivering more wage growth through contracted upon backloading than through the renegotiation channel. Ideally, the firm would like to completely eliminate renegotiation through aggressive backloading, but the curvature of the utility function limits this instrument.

The middle left panel shows the low skill worker wage as a function of the utility promise. It also shows the output of the match $h+p$. Not surprisingly, wages are increasing in the utility promise. In anticipation of profits associated with human capital growth, the optimal contract actually pays the worker a wage in excess of the match output for high utility promises. The right middle panel shows the utility promise continuation as the worker moves from low skill to high skill. For low utility promises, the choice of $V_1(V)$ is at the lower bound, $V_1(V)=U_1$. The implication for the wage path as the worker moves from low skill to high skill are shown in the lower left panel. The solid green line is the wage for the low skilled worker and the solid black line the wage the worker receives immediately following becoming high skill. When the choice of $V_1(V)$ is not in a corner, it is seen that the wage does not change as the worker becomes high skill, but in the corner the wage jumps up.

The black solid line in the lower right panel shows the total within job wage growth of a low skill worker. The solid green line shows within job wage growth conditional on no skill increase

Figure 4: Median firm employment contract, high skill



Note: Lower right panel: Solid black line shows total within job wage growth rate. Dotted green line shows wage growth attributable to the current contract, only.

event arrival. Finally, the dashed green line further excludes wage growth due to renegotiation and consequently shows only the wage growth that is dictated directly by the current contract and conditional on no change in human capital. In the example, most of the within job wage growth comes directly from backloading and growth through contract renegotiation and skill improvements is secondary. The difference between the solid green and black lines is an artifact of the discrete and stochastic jumps in skill which makes the boundary condition bind and wages jump. It can likely be eliminated with a smooth and deterministic growth process in skill. Of course wage growth that is attributed to human capital growth would remain, but it is built into the optimal choice of $\dot{V}_0(V)$.

Figure 4 shows the median firm employment contract for a high skill worker. Backloading of wages is also a central feature of the contract for a high skill worker, but less so in this case where the income effect has decreased the elasticity of the worker's search intensity response to the utility promise. Furthermore, human capital is at its peak and wage growth is not affected by this channel either. Consequently, wage growth is substantially less in the high skilled state.

The lower right panel compares total within job wage growth with that from the contract, only. As can be seen wage growth due to contract renegotiation happens to be a minor source of wage growth also for the high skilled state in the example.

3 Comparison With a Flat Wage Contract

In the interest of comparing the results with a perfectly flat wage contract (but allow a value jump in case of a skill jump), consider the firm's problem of providing a given utility promise V subject to $\dot{V} = 0$. Denote the firm's profits subject to this constraint by, $\Pi^F(V)$,

$$\begin{aligned}
(r+\delta+\gamma_n)\,\Pi_n(V) = & \max_{w,\lambda,V_{n+1}} \Big[p + h_n - w + [\lambda + \underline{\lambda}] \int_V^{M_n} \Pi_h'(M')\,\hat{F}_n(M')\,dM' \\
& + \gamma_n \Pi_{n+1}(V_{n+1}) \Big] \\
s.t.:\ c'(\lambda) = & \int_V^{M_n} \hat{F}_n(M')dM' \\
(r+\delta+\gamma_n)\,V = & \ u(w) - c(\lambda) + [\lambda + \underline{\lambda}] \int_V^{M_n} \hat{F}_n(M')\,dM' + \gamma_n V_{n+1} + \delta U_n \\
M_{n+1} \geq & \ V_{n+1} \geq U_{n+1}.
\end{aligned}$$

The contract that delivers a utility promise V is characterized by,

$$\begin{aligned}
\lambda_n^F(V) &= c'^{-1}\left(\int_V^{M_n} \hat{F}_n(M')dM' \right) \\
w_n^F(V) &= u^{-1}\left((r+\delta+\gamma_n)\,V + c\left(\lambda_n^F(V)\right) - \left[\lambda_n^F(V) + \underline{\lambda}\right] \int_V^{M_n} \hat{F}_1(M')\,dM' - \delta U_1 - \gamma_n V_{n+1} \right) \\
\Pi_n^F(V) &= \frac{p + h_1 - w_1^F(V) + \left[\lambda_1^F(V) + \underline{\lambda}\right] \int_V^{M_n} \Pi_1^{F\prime}(M')\,\hat{F}_1(M')\,dM' + \gamma_n \Pi_{n+1}(V_{n+1})}{r+\delta+\gamma_n} \\
\eta(V) &= -\gamma_n \nu_n(V) - \frac{\gamma_n \Pi_{n+1}'(V_{n+1}(V))}{r+\delta+\gamma_n} \\
\nu_n(V) &= \frac{1}{u'(w_n(V))\,(r+\delta+\gamma_n)}
\end{aligned}$$

Hence, for the skill increase utility promise, if the constraint is not binding, it is characterized by,

$$\Pi_{n+1}'(V_{n+1}(V)) = \frac{-1}{u'(w_n(V))}.$$

Otherwise,

$$V_{n+1}(V) = U_{n+1}.$$

The Lagrangian is,

$$\begin{aligned} L_n\left(V, w, \dot{V}, \lambda, V_{n+1}\right) = & \frac{p + h_n - w + [\lambda + \underline{\lambda}] \int_V^{M_n} \Pi'(M') \hat{F}_n(M')\, dM' + \gamma_n \Pi_{n+1}(V_{n+1}) + \Pi_n'(V) \dot{V}}{r + \delta + \gamma_n} + \\ & \nu_n(V) \left\{ u(w) - c(\lambda) + [\lambda + \underline{\lambda}] \int_V^{M_n} \hat{F}_n\left(M'\right) dM' + \delta U_n + \right. \\ & \left. \gamma_n V_{n+1} + \dot{V} - (r + \delta + \gamma_n) V \right\} + \mu_n(V) \left[c'(\lambda) - \int_V^{M_n} \hat{F}_n(M') dM' \right] + \\ & \eta_n(V) \left[V_{n+1} - U_{n+1} \right] \end{aligned}$$

By the envelope condition,

$$\Pi_n^{F\prime}(V) = \frac{-\hat{F}_n(V) \int_V^{M_n} \Pi_n^{F\prime}(M') \hat{F}_n(M')\, dM'}{c''\left(\lambda_n^F(V)\right) \left[\left[\lambda_1^F(V) + \underline{\lambda}\right] \hat{F}_n(V) + r + \delta + \gamma_n \right]} - \frac{1}{u'\left(w_n^F(V)\right)}.$$

So, in this case where the firm is constrained to offer a flat contract, it is possible that the slope of the profit function becomes positive if the Lagrange multiplier on the IC constraint is sufficiently large. In which case the firm would prefer to make the worker a higher utility promise. In this case, even in the absence of a lower wage bound, there is the possibility of an equilibrium with ex ante rent extraction.

4 Efficiency

TBC

5 Steady state

There is a measure one of workers. Denote by u_n and e_n the measures of skill h_n unemployed and employed workers, respectively, which in steady state are given by,

$$
\begin{aligned}
u_0\left(\underline{\lambda}+d\right) &= e_0\delta + d \\
u_1\left(\underline{\lambda}+d\right) &= e_1\delta \\
e_0\left(\delta+d+\gamma_0\right) &= u_0\underline{\lambda} \\
e_1\left(\delta+d\right) &= u_1\underline{\lambda} + e_0\gamma_0.
\end{aligned}
$$

This implies,

$$
\begin{aligned}
u_0 &= \frac{d\left(\delta+d+\gamma_0\right)}{d\left(\delta+d+\gamma_0\right)+\left(d+\gamma_0\right)\underline{\lambda}} \\
e_0 &= \frac{d\underline{\lambda}}{d\left(\delta+d+\gamma_0\right)+\left(d+\gamma_0\right)\underline{\lambda}} \\
e_1 &= \frac{\underline{\lambda}}{\delta+d+\underline{\lambda}}\cdot\frac{\left(d+\underline{\lambda}\right)\gamma_0}{d\left(\delta+d+\underline{\lambda}\right)+\left(d+\underline{\lambda}\right)\gamma_0} \\
u_1 &= \frac{\delta}{\delta+d+\underline{\lambda}}\cdot\frac{\underline{\lambda}\gamma_0}{d\left(\delta+d+\underline{\lambda}\right)+\left(d+\underline{\lambda}\right)\gamma_0}.
\end{aligned}
$$

It is convenient to describe the match distribution by the cumulative distribution function $G_n\left(V,p\right)$ which is the fraction of employed skill h_n workers that are matched with an employer of type p or lower and is currently employed at an expected lifetime utility of V or less. The steady state condition on the match distribution requires that rate of change over time in $G_n\left(V,p\right)$ be zero, which can be expressed as,

$$
\begin{aligned}
u_0\underline{\lambda}\tilde{F}_0\left(p\right) = e_0\Bigg\{&\hat{F}_0\left(V\right)\int_b^p\int_U^{\min[M_0(p'),V]}\left[\lambda_0\left(V',p'\right)+\underline{\lambda}\right]dG_0\left(V',p'\right)+ \\
&\left(\delta+d+\gamma_0\right)G_0\left(V,p\right)+\int_{M_0^{-1}(V)}^{p}\dot{V}_0\left(V,p'\right)g_0\left(V,p'\right)dM'\Bigg\}
\end{aligned}
$$

$$\Updownarrow$$

$$
\begin{aligned}
\left[\delta+d+\gamma_0\right]\tilde{F}_0\left(p\right) = &\int_b^p\int_U^{\min[M_0(p'),V]}\left[\delta+d+\gamma_0+\hat{F}_0\left(V\right)\left[\lambda_0\left(V',p'\right)+\underline{\lambda}\right]\right]dG_0\left(V',p'\right) \\
&+\int_{M_0^{-1}(V)}^{p}\dot{V}_0\left(V,p'\right)g_0\left(V,p'\right)dM',
\end{aligned}
\tag{29}
$$

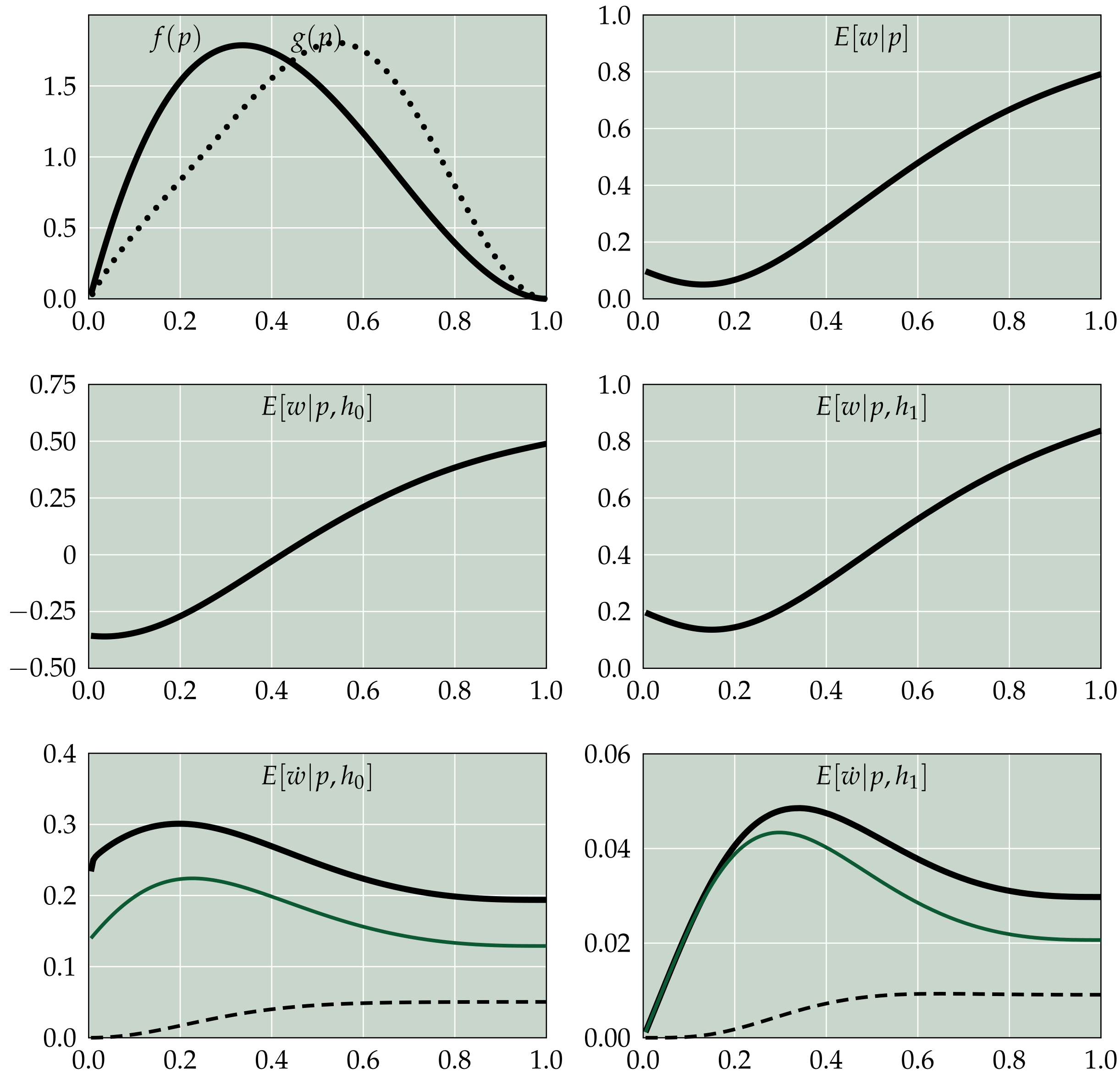
where $g_n(M, V)$ is the probability density function associated with $G_n(V, p)$. The steady state distribution for the high skill workers is,

$$\begin{aligned}
u_1 \underline{\lambda} \tilde{F}_1(p) = e_1 \Bigg\{ & \hat{F}_1(V) \int_b^p \int_U^{\min[M_1(p'),V]} \left[\lambda_1(V', p') + \underline{\lambda}\right] dG_1(V', p') + (\delta + d) G_1(V, p) \\
& + \int_{M_1^{-1}(V)}^p \dot{V}_1(V, p') g_1(V, p') dM' \Bigg\} \\
& - e_0 \gamma_0 \int_b^p \int_U^{M_0(p')} I\left[V_1(V', p') \leq V\right] dG_0(V', p') \\
& \Updownarrow \\
\frac{\delta \underline{\lambda}}{d + \underline{\lambda}} \tilde{F}_1(p) = & \int_b^p \int_U^{\min[M_1(p'),V]} \left[\delta + d + \left[\lambda_1(V', p') + \underline{\lambda}\right] \hat{F}_1(V)\right] dG_1(V', p') \\
& + \int_{M_1^{-1}(V)}^p \dot{V}_1(V, p') g_1(V, p') dM' \\
& - \frac{e_0 \gamma_0}{e_1} \int_b^p \int_U^{M_0(p')} I\left[V_1(V', p') \leq V\right] dG_0(V', p')
\end{aligned}$$

With the steady state match distribution $G(M, V)$ in hand, one can then work out the steady state wage distribution $H(w) = \int I\left[w_{M'}(V') \leq w\right] dG(M', V')$.

The following figures show some of the characteristics of the steady state. The top left panel in Figure 5 shows the firm type offer distribution $f(p)$ and the steady state match distribution over firm types $g(p)$, which stochastically dominates $f(p)$ as it must in this model. The top right panel and the two middle panels show the firm type conditional average wage in steady state. The two middle panels show the firm type conditional average wage also conditional on worker skill level. Not surprisingly worker wages are lower for low skill workers conditional on firm type due to the smaller worker contribution to match output, but also because low skill workers tend to be younger and have accumulated less search capital which translates into a less favorable bargaining position. Regardless of worker skill level, the average wage is non-monotone in firm type, although broadly increasing. The non-monotonicity is a general feature of the Postel-Vinay and Robin (2002) type wage setting process and if anything the backloading in this setting amplifies the feature. Consider the case of a high skill worker employed by a type $p = b = 0$ firm. The outside option of unemployment is equal to the maximal utility promise $M_1(b) = U_1$ that the firm can offer. Consequently, the wage contract is simply $w = R(b, h_1) = b + h_1 = 0.2$. Now, consider the wages in matches with a somewhat more productive firm type, $p = 0.1$. If a worker happened to obtain a bargaining position equal to the maximal utility it is willing to give the worker, $M_1(0.1)$, the worker would again be receiving a flat profile exactly equal to match

Figure 5: Steady State



output, $w = R(0.1, h_1) = 0.3$, which of course exceeds that of the $p = b$ type firm. However, this is not the typical kind of worker that is employed in this firm. The match distribution has more mass towards a bargaining position of unemployment because the arrival of outside offers most often result in the worker leaving the firm rather than staying with a renegotiated contract. Given a bargaining position of unemployment, wages are substantially lower in anticipation of higher wages with future firms *and* as part of the backloading of the wage contract with the current firm.

The two lower panels in Figure 5 show the average firm type conditional *within job* wage growth rates for low and high skill workers. This is average observed growth rate of a randomly selected individual from a given firm conditional on the worker staying with the firm. The bold line shows the total wage growth rate. The thin solid line is the wage growth rate built into the contract, and the dashed thin line is the growth rate in wages that is contributed to renegotiation of wages due to outside offer arrivals. For the high skill workers the bold line is the sum of the growth rates of the two thin lines. For low skill workers, wages can jump due to skill increase, but only if their utility promise as a low skill worker is less than the value of unemployment as a high skill worker, in which case the firm's choice of continuation utility promise is at a corner. Otherwise, the wage path will be flat over a skill jump. Consequently, the expected total wage growth rate of a low skill worker exceeds the sum of the contractual wage growth rate and growth due to renegotiation. This is true more so for low type firms than high type firms due to the greater likelihood that wages must jump when the worker's skill jumps for these firms.

In a flat wage contract all within job wage growth would be attributed to contract renegotiation due to outside offer arrival. In the above example, growth due to renegotiation is a secondary source of wage growth. The credible threat that the worker can summon outside wage pressure induces the firm to grow wages even in the absence of the realization of such pressure. This presents an identification problem in that it is not enough to estimate actual offer arrival rates and offer distributions in order to determine wage pressure. One must also know the sensitivity of the search intensity response to utility promises. The greater the sensitivity, the greater the backloading.

6 Empirical implications

The model obviously has implications for wage growth within and between jobs. It can produce substantial wage variation just like any other on the job search model with firm heterogeneity. The mean-min ratio as a measure of wage dispersion is in this case a bit uninteresting since with something like log utility, the min wage can be zero, or whatever the subsistence income level in the utility function is set at. The model also produces negative duration dependence in job-to-job separations in line with the data. Perhaps most importantly, it is a fairly tractable model that allows rich heterogeneity on both worker and firm sides. The model allows a measurement of the

7 Estimation of Wage Dynamics

TBC.

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