Becker meets Ricardo
Multisector matching with social and cognitive skills

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Introduction

- Individuals participate in education, labor and marriage
  - There is heterogeneity in outcomes across individuals and individual outcomes are correlated across sectors.
  - Cognitive and non-cognitive skills affect individual outcomes

- Evidence on importance of social skills
  - Direct data that market participants value and screen for social skills
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- Evidence on importance of social skills
  - Direct data that market participants value and screen for social skills
  - Using factor models, psychologists and economists show that social factors affect individual outcomes in many sectors
  - In a one factor (cognitive skill) world, Larry Summers would have been president of the United States and George Bush Jr. would have been a nobody.
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1. A model of social skills is one of social interaction where individuals have heterogenous social skills.

2. The model should differentiate cognitive skills from social skills
This Paper

- Builds theory of multisector matching with social and cognitive skills.
  - In each sector, we assume frictionless matching, team production and transferable utility or profit maximization.
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1. The problem of modelling social and cognitive skill interaction becomes: How does cognitive and social skills affect team output?

2. Given efficient team production, the problem of matching within and across sectors is a linear programming problem. This is great for simulation and estimation.
Cognitive and social skills in team production

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- Individuals have different communication costs (social skills)
- Task assignment, based on comparative advantage (Ricardo), takes into account how much remaining time each individual have for task completion.
Results Overview

- Full tasks specialization in labor and education, but partial specialization in marriage
- Many-to-one matching in teams in the labor market, a commonly observed organizational form
- Matching patterns differ across sectors:
  - Labor market: managers and workers sort by cognitive skills
  - Marriage market: spouses sort by both social and cognitive skills
  - Education market: students with different social and cognitive skills attend the same school
Observable Predictions and Simulation Results

Observable predictions:
- Individuals with higher social skills are more likely to become managers/teachers
- Conditional on cognitive skills, managers with better social skills are better paid
- Educational gap between workers and managers/teachers
- Teachers of managers earn higher wages than teachers of workers

Simulations
- Wage distribution qualitatively lognormal, with bivariate uniform skill distribution
- Teachers of future managers/teachers acquire discretely more cognitive skills than teachers of workers
- Wage inequality increases as communication cost decreases
Garicano, Garicano and Rossi Hansberg study how communication costs affect organization design, time use, occupation choice, team matching & human capital investments at work where individuals differ by cognitive skills.

Using a different production technology, we extend them by:

- Add another dimension of individual heterogeneity: communication costs.
- Studying multisector (school, work and marriage) matching.
Model Setup

- Risk-neutral individuals live for two periods
  - Enter education market as students, and then work and marry as adults
  - One unit of time endowment for each sector
  - Free entry of firms and schools

- Individuals are heterogenous in two dimensions
  - (fixed) gross social skill $\eta$, with $\eta \in [\eta, \bar{\eta}]$
  - initial cognitive ability $a$, with $a \in [\underline{a}, \bar{a}]$
  - education transforms $a$ into adult cognitive skill $k$, with $k \in [k, \bar{k}]$

- Individuals’ net payoff: wage ($\omega$) + marriage payoff ($h$) − tuition ($\tau$)
  - individual decision: who to match with in each sector
Output is produced by completion of two tasks, $I$ and $C$

- $\theta^I_i, \theta^C_i$: times $i$ spent on task $I$ and $C$ respectively
- time constraint in each sector: $\theta^I_i + \theta^C_i \leq 1$

Single agent production:

$$\beta k_i \min \left\{ \theta^I_i, \gamma \theta^C_i \right\}, \beta < 1; \gamma > 1$$  \hspace{1cm} \text{(Single)}$$

- no need for coordination: gross social skill $\eta_i$ does not enter production
Team versus Single Agent Production

Single Agent Production

\[ \beta k_i \min \left\{ \theta_i^l, \gamma \theta_i^C \right\}, \beta < 1; \gamma > 1 \]

Team Production

- Consider a two-person team with \((\eta_i, k_i)\) and \((\eta_j, k_j)\)
  - \(\theta_i^l, \theta_j^C\): times \(i\) and \(j\) spend on task \(I\) and task \(C\) respectively
- Specialization needs coordination
  - Individual on task \(C\) bears (one-sided) coordination cost
- Team output:
  \[ \sqrt{k_i k_j \min \left\{ \theta_i^l, \gamma \eta_j \theta_j^C \right\}}, \eta_j < 1 \]  
  (Team)
- Assume team production is always superior to working alone
Let **social skill** $n$: $n \equiv \gamma \eta$. Team output: $\sqrt{k_i k_j \min \{ \theta_i^l, n_j \theta_j^C \}}$

- $\omega(n_l, k_l)$ is total earnings of type $l$ employee with skills $(n_l, k_l)$.
- Let a firm hires $i$ and $j$ to produce $\delta$ units of output where $\delta$ is small.
- Need to allocate $\delta(k_i k_j)^{-\frac{1}{2}}$ units of effective time to tasks $I$ and $C$.
- Let $\theta_i^l$ and $(\delta(k_i k_j)^{-\frac{1}{2}} - \theta_i^l)$ be times $i$ and $j$ allocate to task $I$ respectively. Then $\theta_i^C = (\delta(k_i k_j)^{-\frac{1}{2}} - \theta_i^l) n_i^{-1}$ and $\theta_j^C = \theta_i^l n_j^{-1}$ are times $i$ and $j$ allocate to task $C$ respectively.
Firm pays for time used. Leftover time sold elsewhere.

The expenditure of the firm’s allocations is:

$$\omega(n_i, k_i) (\theta_i^l + \left( \frac{\delta}{\sqrt{k_i k_j}} - \theta_i^l \right) \frac{1}{n_i}) + \omega(n_j, k_j) \left( \frac{\delta}{\sqrt{k_i k_j}} - \theta_i^l + \theta_i^l \frac{1}{n_j} \right)$$

The firm chooses $\theta_i^l$ to minimize above.

**Proposition.** Full task specialization is optimal, i.e., an individual is assigned to task $I$ or $C$.

**Corollary.** (Ricardo): Specialization is by comparative advantage. Let $i$ do $I$ if

$$\frac{\omega(n_i, k_i)}{\omega(n_j, k_j)} \leq \frac{1 - n_j^{-1}}{1 - n_i^{-1}}$$
Many-to-one matching in teams: one member (manager) on task $C$ and several members (workers) on task $I$

Manager social skill: span of control or leadership

Workers’ social skills have no value for team production. So $\omega(n_i, k_i) = w(k_i)$.

**Proposition.** There is a cutoff $\hat{n}(k)$ such that a type-$(n, k)$ individual does task $C$ if and only if $n \geq \hat{n}(k)$.

- Managers are individuals with better social skills
- Occupation choice is based on cognitive and social skills
Consider a type- \((n_m, k_m)\) manager who chooses \(n_m\) workers of type \((n_i, k_i)\) at wage \(\omega(k_i)\) to maximize

\[
\max_{(k_1, \ldots, k_{nm})} \sum_{i=1}^{n_m} \left[ \sqrt{k_m k_i} - \omega(k_i) \right]
\]

- In optimum, workers have the same \(k_w\)
- Can be rewritten as \(\phi(k_m) = \max_{k_w} \left[ \sqrt{k_m k_w} - \omega(k_w) \right]\)
- Define equilibrium matching \(\mu(k_m) \in \arg \max_{k_w} \left[ \sqrt{k_m k_w} - \omega(k_w) \right]\)

**Proposition.** (Becker) Equilibrium exhibits positive assortative matching (PAM) along cognitive skills: \(\mu'(k) > 0\)
Due to free entry of firms, a type- \((n, k)\) manager earns \(n\phi(k)\) with

\[
\phi(k) = \max_{k'} \left[ \sqrt{kk'} - \omega(k') \right]
\]

- First-order condition

\[
\left. \frac{d\omega(k')}{dk'} \right|_{k' = \mu(k)} = \frac{1}{2} \sqrt{\frac{k}{k'}} \iff \frac{d\omega(k)}{dk} = \frac{1}{2} \sqrt{\frac{\mu^{-1}(k)}{k}}
\]

- Envelope condition

\[
\frac{d\phi(k)}{dk} = \frac{1}{2} \sqrt{\frac{\mu(k)}{k}}
\]
Monogamy between $i$ and $j$ means they spend all their time with each other (**cannot sell time to outside market**)

- **Assuming that it is not efficient to each produce alone**, choose $\theta_i^l$ ($\theta_i^c = 1 - \theta_i^l$) and $\theta_j^l$ ($\theta_j^c = 1 - \theta_j^l$) to maximize marital output

$$\sqrt{k_i k_j \left( \min \left\{ \theta_i^l, n_j (1 - \theta_j^l) \right\} + \min \left\{ \theta_j^l, n_i (1 - \theta_i^l) \right\} \right)}$$

- Optimal solution

$$\theta_i^l = \frac{n_i n_j - n_j}{n_i n_j - 1}, \text{ and } \theta_j^l = \frac{n_i n_j - n_i}{n_i n_j - 1}.$$ 

- Optimal marital output

$$\sqrt{k_i k_j \frac{2n_i n_j - n_j - n_i}{n_i n_j - 1}}.$$
Proposition. Full specialization is not optimal; equilibrium exhibits PAM along both $n$ and $k$.

Monogamy limits specialization

Equilibrium sorts in two dimensions: individuals marry their own type

Equilibrium total marital output for a $(n, k, n, k)$ marriage:

$$\frac{2n}{n + 1}^k$$
Task assignment is exogenous
- teachers do task $C$
- students do task $I$

Team production function: $\sqrt{a_ik_t} \min \{\theta'_i, n_t\theta^C_t\}$
- in equilibrium, a type-$(n_t, k_t)$ teacher can manage $n_t$ students
- input: student’s initial cognitive skill $a_i$
- output: student’s adult cognitive skill $k_i$

Tuition and teacher wage
- tuition $\tau(k_t)$ depends on school quality – teacher’s cognitive skill $k_t$
- teacher wage: $n_t\tau(k_t)$
Equilibrium Education Choice

- Education choices maximize future net payoff

\[
\max_{k_t} \left\{ n_s \phi(\sqrt{a_s k_t}), \omega(\sqrt{a_s k_t}), n_s \tau(\sqrt{a_s k_t}) \right\} + \frac{n_s}{n_s + 1} \sqrt{a_s k_t} - \tau(k_t)
\]

- Conditional on occupation choice: equilibrium exhibits PAM
  - future managers/teachers choose higher \( k_t \) if \( a_s \) and \( n_s \) are larger: endogenous positive correlation between social and cognitive skills
  - future workers choose more \( k_t \) if \( a_s \) and \( n_s \) are larger

- Proposition. There is an educational gap: a student who has marginally more \( a_s \) or \( n_s \) and switches from being a worker to being a teacher/manager will discretely increase his or her schooling investment
General Equilibrium

- All markets clear
  - labor market clears: equilibrium wages for workers, managers, and teachers equal demand with supply for each type of adult
  - marriage market clears: trivial with a sex ratio of one
  - education market clears: school tuitions are set such that available slots in schools equal the total supply for each type of student

- Equilibrium equivalent to a utilitarian social planner solving a linear programming problem
Numerical Simulation: Education Choice
Numerical Simulation: Equilibrium Wage
Numerical Simulation: Wage Distribution
Social planner chooses number (measure) of \((n_m, k_m, n_w, k_w)\) firms and number of \((n_t, k_t, n_s, a_s)\) schools to maximize:

\[
\sum_{\text{firm types}} \# \text{ firm type } (n_m, k_m, n_w, k_w) \times \left( n_m \sqrt{k_m k_w} \right)
\]

\[
+ \sum_{\text{marriage types}} \# \text{ marriage type } (n, k, n, k) \times \left( \frac{2n}{n+1} k \right)
\]

subject to, for each adult type \((n, k)\),

demand by firms + schools \(\leq\) supply of adults

and for each student type \((n, a)\),

school slots for students \(\leq\) supply of students

wages and student payoffs: multipliers attached to the constraints

infinite dimensional, dual program
Related Literature (Partial List)

- Importance of non-cognitive (including social) skills
  - Almlund, Duckworth, Heckman and Kautz (2011), Heckman (2011) ...

- Frictionless transferable utility model of marriage
  - one factor: Becker (1973, 1974) ...

- Task assignment and hierarchies
  - Roy (1951), Sattinger (1975) ...
  - Lucas (1978), Rosen (1978, 1982), Eeckhout and Kircher (2011) ...

- Communication costs, organization design, time use, occupational choice, human capital investment, one factor labor market matching

- Linear programming model of frictionless multifactor marriage matching model
  - Chiappori, McCann and Nesheim (2010)
Two sided communication costs

- Let $i$ and $j$ spend $\theta_i^l$ on task $I$ and $\theta_j^C$ on task $C$ respectively. Team output:
  \[ \sqrt{k_i k_j} \min \left\{ \eta_i \theta_i^l, \gamma \eta_j \theta_j^C \right\} \]

- $N_w m$ is a set of workers managed by manager $m$ where
  \[ \sum_{N_w m} \eta_w = \gamma \eta_m \]

- Then the manager solves:
  \[ \phi(k_m) = \max_{N_w m} \sum_{N_w m} \eta_w \sqrt{k_m k_w} - \omega(\eta_w, k_w) \]

- Some results:
  \[ \omega(\eta_w, k_w) = \eta_w \omega(k_w) \]
  \[ \omega(\eta_m, k_m) = \eta_m \phi(k_m) \]

- No matching between social skills of manager and her workers.
- Future workers will also make schooling investments based on their social and cognitive abilities.
We present a tractable framework for multisector matching:

- All three sectors share qualitatively the same team production function.
  - Team production function incorporates specialization and task assignment.
  - Specify an explicit role for social skills in production.

Capture matching patterns in each of the three sectors.

Generate predictions consistent with empirical observations.

A first pass theory of social and cognitive skills:

- Many possible extensions.