Essential interest-bearing money

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The Lagos-Wright Model

- Leading framework in contemporary monetary theory

- Models individuals exposed to idiosyncratic risk generated by the random arrival of opportunities to produce and consume output over time

- Risk is modeled as the outcome of random search in a decentralized market; or as the outcome of random shocks to preferences and technologies in a centralized market

- Either way, the key simplifying property of the framework is quasi-linear preferences, which removes the distribution of wealth as an endogenous state variable
Optimal Monetary Policy (Standard Approach)

- Assume lump-sum tax instrument (society has coercive power)

- Then Friedman rule is implementable
  - via deflation (use tax to contract the money supply); or
  - via interest-bearing money (use tax to finance interest obligation)

- Interest-bearing money is not essential

- Absent lump-sum taxes, constrained-efficient allocation achieved with zero intervention
My Paper

- Examine optimal policy design in a Lagos-Wright model where

  1. trade is competitive among agents

  2. *all* trade is voluntary (including trade between agents and government)
Results

• I identify a class of incentive-feasible policies that improve welfare beyond what is achievable with zero intervention

• Any policy in this class necessarily entails a non-negative inflation rate and a strictly positive nominal interest rate

• Despite the absence of a lump-sum tax instrument, there exists an incentive-feasible policy that implements the first-best allocation
Environment

- Preferences and resource constraints

\[ E_0 \sum_{t=0}^{\infty} \beta^t \{x_t(i) + \pi [u(c_t(i)) - h(y_t(i))] \} \]

\[ \int x_t(i)di \leq 0 \]
\[ \int c_t(i)di \leq \int y_t(i)di \]

- First-best is \( y^* \) satisfying \( u'(y^*) = h'(y^*) \) and \( \{x_t(i)\} \) satisfying \( E_t x_t(i) = 0 \)
Market structure, timing, and policy

- Government intervention occurs at the beginning of each day, prior to day-market trading.

- Agent who enters with money $m$ has an option of transforming $m$ into $\mathcal{R}m - T$ units of money.

- Note: money is like an interest-bearing bond subject to a redemption fee.

- Day and night markets competitive; let $(v_1, v_2)$ denote price of money.
Decision-making

- Day budget constraint

\[ x = v_1 [\omega(Rm_1 - T) + (1 - \omega)m_1 - m_2] \]

or

\[ x = \omega(Rq_1 - \tau) + (1 - \omega)q_1 - \phi q_2 \]

- \((q_1, q_2) \equiv (v_1 m_1, v_2 m_2)\) real money balances, day and night

- \(\omega \in [0, 1]\) prob. of exercising redemption option

- \(\phi \equiv v_1/v_2\) and \(\tau \equiv v_1 T\) real redemption fee
Day market

\[ D(q_1) \equiv \max_{\omega, q_2} \{ \omega (Rq_1 - \tau) + (1 - \omega)q_1 - \phi q_2 + N(q_2) \} \]

\[ \omega = 1 \quad \text{if} \quad (R - 1)q_1 > \tau \]
\[ \omega \in [0, 1] \quad \text{if} \quad (R - 1)q_1 = \tau \]
\[ \omega = 0 \quad \text{if} \quad (R - 1)q_1 < \tau \]

\[ \phi = N'(q_2) \]

\[ D'(q_1) = \begin{cases} 
R & \text{if} \quad (R - 1)q_1 > \tau \\
1 & \text{if} \quad (R - 1)q_1 < \tau 
\end{cases} \]
Night market

\[ C(q_2) \equiv \max_{c, q_1^+} \left\{ u(c) + \beta D(q_1^+) : q_1^+ = (v_1^+/v_1)\phi(q_2 - c) \geq 0 \right\} \]

\[ I(q_2) \equiv \beta D((v_1^+/v_1)\phi q_2) \]

\[ P(q_2) \equiv \max_{y, q_1^+} \left\{ -h(y) + \beta D(q_1^+) : q_1^+ = (v_1^+/v_1)\phi(q_2 + y) \right\} \]

[A1] Assume that debt-constraint binds tightly for consumers (hence, will not exercise redemption)

[A2] Assume inactive agents exercise redemption option (so producers will too)
Government

- Let $M$ denote money supply; $M^-$ is “previous” period’s money supply.

- [A1] implies $M^-$ is held entirely by producers and inactives at beginning of day.

- [A2] implies producers and inactives will find it optimal to exercise redemption.

- Hence, $(R - 1)M^-$ interest obligation, offset in part by redemption fee revenue $(1 - \pi)T$. 
• Government can also create new money at rate $\mu$; so GBC is

$$(R - 1)M^- = M - M^- + (1 - \pi)T$$

• Using $v_1M \equiv \phi q_2$, GBC expressed in real terms is

$$\tau = (R/\mu - 1)(1 - \pi)^{-1}\phi q_2$$

**Defn:** An *incentive-feasible policy* $(R, \mu, \tau)$ satisfies GBC and conditions [A1] and [A2].
Stationary competitive eqm (conditional on a given I-F policy)

\[ \delta \beta \pi u'(y) = [1 - \delta \beta (1 - \pi)]h'(y) \]

where \( \delta \equiv R/\mu \)

\[ \phi = \pi u'(y) + (1 - \pi)h'(y) \]

\[ \tau = (\delta - 1)(1 - \pi)^{-1}\phi \]

\[ F(q_1) = \begin{cases} 
\pi & \text{for } 0 \leq q_1 < (\phi/\mu)y \\
1 - \pi & \text{for } (\phi/\mu)y \leq q_1 < (\phi/\mu)2y \\
1 & \text{for } (\phi/\mu)2y \leq q_1 < \infty 
\end{cases} \]
Zero Intervention

- $R = \mu = 1$ and $\tau = 0$ is trivially an incentive-feasible policy

- Implies $\delta = 1$; which, by eqm condition above determines an equilibrium level of output $0 < y_0 < y^*$

- In a monetary equilibrium, the debt-constraint for consumers will bind tightly so that [A1] holds

- Condition [A2] holds trivially as well
• Note: there exists a class of incentive-feasible policies \((R, \mu)\) satisfying 
\[ \delta = \frac{R}{\mu} = 1 \text{ and } R > 1 \] 
that implements the zero intervention allocation \(y_0\) as an equilibrium.

• Money is superneutral when it is introduced in the form of interest.
Welfare-improving I-F policies

- Restrict attention to policies that satisfy

\[ 1 < \delta < \frac{1}{\beta} \]

- Note that any such policy necessarily entails \( \tau > 0 \) (exclusive money finance is not possible)

- Need to check whether [A2] holds: will inactive agents exercise redemption?
• They enter the day with \( q_1 = (\phi / \mu) y \) and will exercise iff \( (R - 1)q_1 > \tau \);
or iff
\[
\left( \delta - \frac{1}{\mu} \right) \phi y > \left( \frac{\delta - 1}{1 - \pi} \right) \phi y
\]
which implies
\[
\mu > \left[ \frac{1 - \pi}{1 - \delta \pi} \right] > 1 \text{ for } \delta > 1
\]

• So deflation is not incentive-feasible; implies \( R > 1 \) is necessary

• Need to check whether [A1] holds: are consumers debt-constrained?

• Answer is yes if \( \delta < 1 / \beta \); which I have assumed
**Proposition 1** Under the range of incentive-feasible policies $1 < \delta < 1/\beta$ and $\mu > 1$, there exists a stationary monetary equilibrium with an allocation $y_0 < \hat{y}(\delta) < y^*$ characterized by

$$\delta \beta \pi u'(\hat{y}) = [1 - \delta \beta (1 - \pi)] h'({\hat{y}})$$

- Note: $\hat{y}(\delta)$ is strictly increasing in $\delta$ and ex ante welfare is strictly increasing in $\hat{y}$ over range $1 < \delta < 1/\beta$

- Corollary: policy $\delta \nearrow 1/\beta$ implements first-best
Relation to literature

- **Berentsen, Camera and Waller** (JET 2007) also make a case for interest-bearing money

- Introduce a “bank” in the day-market that pays interest on deposits of cash from producers, redirecting these funds to consumers in the form of interest-bearing loans

- For this solution to work, the bank must be endowed with at least a limited record-keeping technology (seems reasonable, but not necessary if policy is designed correctly)
• **Hu, Kennan and Wallace** (JPE 2009) study a LW model and report that first-best implementation is possible with zero intervention; at least, if agents are sufficiently patient

• They assume pairwise meetings in one of the subperiods

• This grants “maximum freedom” in designing trading protocols conducive to efficient implementation

• The search friction places limits on coalition formation; that is, it effectively imposes a communication barrier between the members of a match and the rest of the community
• As the size of a meeting is increased (say, by replicating the buyer-seller pair in each meeting), the core converges to a competitive equilibrium

• HKW result will fail to hold when trade among individuals is competitive; see also Tsu and Wallace (JET 2007)

• **Kocherlakota** (JET 2003) also makes a case for an (illiquid) interest-bearing government asset

• Advantage: linear mechanism

• Disadvantage: requires trading restriction
Conclusion

• The modeling choice of centralized versus decentralized trade appears to have important policy implications

• I identify a class of incentive feasible policies that improve welfare beyond what achievable with zero intervention when trade is centralized

• Any such policy in this class requires a strictly positive nominal interest rate and a non-negative inflation rate

• “Banking” is not essential here; what is missing?