Dynamics and Stability of Constitutions, Coalitions, and Clubs

Daron Acemoglu (MIT)
Georgy Egorov (Harvard University)
Konstantin Sonin (New Economic School)

MIT

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Motivation

- What makes certain social arrangements (political regimes, constitutions, coalitions, clubs, firms, international peace) emerge in equilibrium and persist?
- Fundamental question for political economy and organizational economics, and pertinent to the study of coalition formation.
- Each specific applied instance leads to different sets of issues.
- Are there general insights?
- How can we apply these results to important political economy problems?
- *This talk*: general framework and applications.
Toward General Insights

- This research project attempts to investigate such general insights.
- Many similarities between different instances (in political economy, organizational economics, club theory etc.).

In particular:

- **Payoffs**: different arrangements imply different payoffs and individuals care about payoffs.
- **Power**: different arrangements reallocate decision-making (political) power and thus affect future evolution of payoffs.

**Strategy**: Formulate a general dynamic framework to investigate the interplay of these two factors in a relatively “detail-free” manner.

- Details useful to go beyond general insights.
Simple Example

- Consider a simple extension of franchise story
- Three states: absolutism $a$, constitutional monarchy $c$, full democracy $d$
- Two agents: elite $E$, middle class $M$

\[
\begin{align*}
w_E(d) &< w_E(a) < w_E(c) \\
w_M(a) &< w_M(c) < w_M(d)
\end{align*}
\]

- $E$ rules in $a$, $M$ rules in $c$ and $d$.
- Myopic elite: starting from $a$, move to $c$
- Farsighted elite: stay in $a$: move to $c$ will lead to $M$ moving to $d$
Naïve and Dynamic Insights

- **Naïve insight**: a social arrangement will emerge and persist if a “sufficiently powerful group” prefers it to alternatives.
- Simple example illustrates: power to change towards a more preferred outcome is not enough to implement change
  - because of further dynamics
- Social arrangements might be stable even if there are powerful groups that prefer change in the short run.
- **Key**: social arrangements change the distribution of political power (decision-making capacity).
- **Dynamic decision-making**: future changes also matter (especially if discounting is limited)
Other Examples

- Members of a club decide whether to admit additional members by majority voting (Roberts 1999)
- Society decides by voting, what degree of (super)majority is needed to start a reform (Barbera and Jackson 2005)
- EU members decide whether to admit new countries to the union (Alesina, Angeloni, and Etro 2005)
- Inhabitants of a jurisdiction determine migration policy (Jehiel and Scotchmer 2005)
- Participant of (civil) war decides whether to make concessions to another party (Fearon 1998, Schwarz and Sonin 2008)
- Dynamic political coalition formation: Junta (or Politburo) members decide whether to eliminate some of them politically or physically (Acemoglu, Egorov, and Sonin 2008)
Voting in Clubs or Dynamic Franchise Extension

- Suppose that individuals \( \{1, \ldots, M\} \) have a vote and they can extend the franchise and include any subset of individuals \( \{M + 1, \ldots, N\} \).
- Instantaneous payoff of individual \( i \) a function of the set of individuals with the vote (because this influences economic actions, redistribution, or other policies).
- Political protocol: majority voting.
- \( \{1, \ldots, M\} \) vote over alternative proposals.
- If next period the franchise is \( \{1, \ldots, M'\} \), then this new franchise votes (by majority rule) on the following period’s franchise etc.
- Difficult dynamic game to analyze.
- But once we understand the common element between this game and a more general class of games, a tight and insightful characterization becomes possible.
Model and Approach

- **Model:**
  - Finite number of individuals.
  - Finite number of states (characterized by economic relations and political regimes).
  - Payoff functions determine instantaneous utility of each individual as a function of state.
  - Political rules determine the distribution of political power and protocols for decision-making within each state.
  - A dynamic game where “politically powerful groups” can induce a transition from one state to another at any date.

- **Question:** what is the **dynamically stable state** as a function of the initial state?
Main Results of General Framework

- An axiomatic characterization of “outcome mappings” corresponding to dynamic game (based on a simple *stability* axiom incorporating the notion of forward-looking decisions).
- Equivalence between the MPE of the dynamic game (with high discount factor) and the axiomatic characterization
- Full characterization: *recursive* and *simple*
- Under slightly stronger conditions, the stable outcome (dynamically stable state) is unique given the initial state
  - but depends on the initial state
- Model general enough to nest specific examples in the literature.
- In particular, main theorems directly applicable to situations in which states can be ordered and static payoffs satisfy single crossing or single peakedness.
A particular social arrangement is made stable by the instability of alternative arrangements that are preferred by sufficiently many members of the society.

- stability of a constitution does not require absence of powerful groups opposing it, but the absence of an alternative stable constitution favored by powerful groups.

- Efficiency-enhancing changes are often resisted because of further social changes that they will engender.
  - Pareto inefficient social arrangements often emerge as stable outcomes.
Applications

- Voting in clubs.
- Dynamic taxation with endogenous franchise.
- Stability of constitutions.
- Political eliminations.
- From a follow-up paper: dynamics of political selection
  - a small amount of incumbency advantage can lead to the emergence and persistence of very incompetent/inefficient governments (without asymmetric information)
  - a greater degree of democracy does not necessarily ensure better governments
  - but, a greater degree of democracy leads to greater flexibility and to better governments in the long run in stochastic environments.
Related Literature

- Papers mentioned above as applications or specific instances of the general results here.
- Dynamic economic interactions with transferable utility—Gomes and Jehiel (2005).
- Dynamic inefficiencies with citizen candidates—Besley and Coate (1999).
Model: Basics

- Finite set of individuals $\mathcal{I}$ ($|\mathcal{I}|$ total)
  - Set of coalitions $\mathcal{C}$ (non-empty subsets $X \subset \mathcal{I}$)
- Each individual maximizes discounted sum of payoffs with discount factor $\beta \in [0, 1)$.
- Finite set of states $\mathcal{S}$ ($|\mathcal{S}|$ total)
- Discrete time $t \geq 1$
- State $s_t$ is determined in period $t$; $s_0$ is given
- Each state $s \in \mathcal{S}$ is characterized by
  - Payoff $w_i(s)$ of individual $i \in \mathcal{I}$ (normalize $w_i(s) > 0$)
  - Set of winning coalitions $\mathcal{W}_s \subset \mathcal{C}$ capable of implementing a change
  - Protocol $\pi_s(k)$, $1 \leq k \leq K_s$: sequence of agenda-setters or proposals ($\pi_s(k) \in \mathcal{I} \cup \mathcal{S}$)
Winning Coalitions

Assumption

(Winning Coalitions) For any state $s \in S$, $\mathcal{W}_s \subseteq \mathcal{C}$ satisfies two properties:
(a) If $X, Y \in \mathcal{C}$, $X \subseteq Y$, and $X \in \mathcal{W}_s$ then $Y \in \mathcal{W}_s$.
(b) If $X, Y \in \mathcal{W}_s$, then $X \cap Y \neq \emptyset$.

- (a) says that a superset of a winning coalition is winning in each state
- (b) says that there are no two disjoint winning coalitions in any state
- $\mathcal{W}_s = \emptyset$ is allowed (exogenously stable state)
- Example:
  - Three players 1, 2, 3
  - $\mathcal{W}_s = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$ is valid (1 is dictator)
  - $\mathcal{W}_s = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$ is valid (majority voting)
  - $\mathcal{W}_s = \{\{1\}, \{2, 3\}\}$ is not valid (both properties are violated)
Dynamic Game

1. Period \( t \) begins with state \( s_{t-1} \) from the previous period.

2. For \( k = 1, \ldots, K_{s_{t-1}} \), the \( k \)th proposal \( P_{k,t} \) is determined as follows. If \( \pi_{s_{t-1}}(k) \in S \), then \( P_{k,t} = \pi_{s_{t-1}}(k) \). If \( \pi_{s_{t-1}}(k) \in I \), then player \( \pi_{s_{t-1}}(k) \) chooses \( P_{k,t} \in S \).

3. If \( P_{k,t} \neq s_{t-1} \), each player votes (sequentially) yes (for \( P_{k,t} \)) or no (for \( s_{t-1} \)). Let \( Y_{k,t} \) denote the set of players who voted yes. If \( Y_{k,t} \in \mathcal{W}_{t-1} \), then \( P_{k,t} \) is accepted, otherwise it is rejected.

4. If \( P_{k,t} \) is accepted, then \( s_t = P_{k,t} \). If \( P_{k,t} \) is rejected, then the game moves to step 2 with \( k \mapsto k+1 \) if \( k < K_{s_{t-1}} \). If \( k = K_{s_{t-1}} \), \( s_t = s_{t-1} \).

5. At the end of each period (once \( s_t \) is determined), each player receives instantaneous utility \( u_i(t) \):

\[
u_i(t) = \begin{cases} 
  w_i(s) & \text{if } s_t = s_{t-1} = s \\
  0 & \text{if } s_t \neq s_{t-1}
\end{cases}
\]
Define binary relations:

- states \( x \) and \( y \) are payoff-equivalent

\[
x \sim y \iff \forall i \in \mathcal{I} : w_x(i) = w_y(i)
\]

- \( y \) is weakly preferred to \( x \) in \( z \)

\[
y \succeq_z x \iff \{ i \in \mathcal{I} : w_y(i) \geq w_x(i) \} \in \mathcal{W}_z
\]

- \( y \) is strictly preferred to \( x \) in \( z \)

\[
y \succ_z x \iff \{ i \in \mathcal{I} : w_y(i) > w_x(i) \} \in \mathcal{W}_z
\]

Notice that these binary relations are not simply preference relations.

- they encode information about preferences and political power.
Preferences and Acyclicity

Assumption

**Payoffs** Payoff functions \( \{ w_i(\cdot) \} \) satisfy:

(a) For any sequence of states \( s_1, \ldots, s_k \) in \( S \),

\[
 s_{j+1} \succ s_j \quad \text{for all} \quad 1 \leq j \leq k - 1 \implies s_0 \not\succ s_k.
\]

(b) For any sequence of states \( s, s_1, \ldots, s_k \) in \( S \) with \( s_j \succ s s \) (for all \( 1 \leq j \leq k \)),

\[
 s_{j+1} \succ s s_j \quad \text{for all} \quad 1 \leq j \leq k - 1 \implies s_0 \not\succ s s_k.
\]

(a) rules out cycles of the form \( y \succ z, z \succ y, z \succ x, x \succ y \),

(b) rules out cycles of the form \( y \succ s z, z \succ s y, z \succ s x \),

Weaker than transitivity of \( \succ s \).

These assumptions cannot be dispensed with in the context of a general treatment because otherwise Condorcet-type cycles emerge.
We will also strengthen our results under:

**Assumption**

**Comparability**  For $x, y, z \in S$ such that $x \succ z y$, $y \succ z y$, and $x \sim y$, either $y \succ z x$ or $x \succ z y$.

This condition sufficient (and “necessary”) for uniqueness.
Approach and Motivation

- **Key economic insight:** *with sufficiently forward-looking behavior, an individual should not wish to transition to a state that will ultimately lead to another lower utility state.*

- Characterize the set of allocations that are consistent with this insight—without specifying the details of the dynamic game.
  - Introduce three simple and intuitive axioms.
  - Characterize set of mappings $\Phi$ such that for any $\phi \in \Phi$, $\phi : S \rightarrow S$ satisfies these axioms and assigns an **axiomatically stable state** $s^\infty \in S$ to each initial state $s_0 \in S$ (i.e., $\phi (s) = s^\infty \in S$ loosely corresponding to $s_t = s^\infty$ for all $t \geq T$ for some $T$).

- Interesting in its own right, but the main utility of this axiomatic approach is as an input into the characterization of the (two-strategy) MPE of the dynamic game.
Axiom 1

(Desirability) If $x, y \in S$ are such that $y = \phi(x)$, then either $y = x$ or $y \succneq_x x$.

- A winning coalition can always stay in $x$ (even a blocking coalition can)
- A winning coalition can move to $y$
- If there is a transition to $y$, a winning coalition must have voted for that
Axiom 2

(Stability) If \( x, y \in S \) are such that \( y = \phi(x) \), then \( y = \phi(y) \).

- Holds “by definition” of \( \phi(\cdot) \): \( \exists T : s_t = \phi(s) \) for all \( t \geq T \); when \( \phi(s) \) is reached, there are no more transitions

- If \( y \) were unstable (\( y \neq \phi(y) \)), then why not move to \( \phi(y) \) instead of \( y \)
Axiom 3

(Rationality) If \( x, y, z \in S \) are such that \( z \succ_x x \), \( z = \phi(z) \), and \( z \succ_x y \), then \( y \neq \phi(x) \).

- A winning coalition can move to \( y \) and to \( z \)
- A winning coalition can stay in \( x \)
- When will a transition to \( y \) be blocked?
  - If there is another \( z \) preferred by some winning coalition
  - If this \( z \) is also preferred to \( x \) by some winning coalition (so blocking \( y \) will lead to \( z \), not to \( x \))
  - If transition to \( z \) is credible in the sense that this will not lead to some other state in perpetuity
States \( s \in \mathcal{S} \) is \( \phi \)-stable if \( \phi(s) = s \) for \( \phi \in \Phi \).

Set of \( \phi \)-stable states: \( \mathcal{D}_\phi = \{ s \in \mathcal{S} : \phi(s) = s \text{ for } \phi \in \Phi \} \).

We will show that if \( \phi_1 \) and \( \phi_2 \) satisfy the Axioms, then
\[ \mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D} \]

- Even if \( \phi \) is non-unique, notion of stable state is well-defined.
- But \( \phi_1(s) \) and \( \phi_2(s) \) may be different elements of \( \mathcal{D} \).
Axiomatic Characterization of Stable States

Theorem

Suppose Assumptions on Winning Coalitions and Payoffs hold. Then:

1. There exists mapping $\phi$ satisfying Axioms 1–3.
2. This mapping $\phi$ may be obtained through a recursive procedure (next slide).
3. For any two mappings $\phi_1$ and $\phi_2$ that satisfy Axioms 1–3 the sets of stable states of these mappings coincide (i.e., $D_{\phi_1} = D_{\phi_2} = D$).
4. If, in addition, the Comparability Assumption holds, then the mapping that satisfies Axioms 1–3 is “payoff-unique” in the sense that for any two mappings $\phi_1$ and $\phi_2$ that satisfy Axioms 1–3 and for any $s \in S$, $\phi_1 (s) \sim \phi_2 (s)$. 
Theorem (continued)

Any $\phi$ that satisfies Axioms 1–3 can be recursively computed as follows. Construct the sequence of states $\{\mu_1, \ldots, \mu_{|\mathcal{S}|}\}$ with the property that if for any $l \in (j, |\mathcal{S}|]$, $\mu_l \not\succ_{\mu_j} \mu_j$. Let $\mu_1 \in \mathcal{S}$ be such that $\phi(\mu_1) = \mu_1$. For $k = 2, \ldots, |\mathcal{S}|$, let

$$\mathcal{M}_k = \{s \in \{\mu_1, \ldots, \mu_{k-1}\} : s \succ_{\mu_k} \mu_k \text{ and } \phi(s) = s\}.$$ 

Define, for $k = 2, \ldots, |\mathcal{S}|$,

$$\phi(\mu_k) = \begin{cases} \mu_k & \text{if } \mathcal{M}_k = \emptyset \smallskip \mathcal{M}_k \ni z \ni x \in \mathcal{M}_k \text{ with } x \succ_{\mu_k} z & \text{if } \mathcal{M}_k \neq \emptyset. \end{cases}$$

(If there exist more than one $s \in \mathcal{M}_k$: $\exists z \in \mathcal{M}_k$ with $z \succ_{\mu_k} s$, we pick any of these; this corresponds to multiple $\phi$ functions).
Extension of Franchise Example

- Get back to the simple extension of franchise story
- Three states: absolutism \( a \), constitutional monarchy \( c \), full democracy \( d \)
- Two agents: elite \( E \), middle class \( M \)

\[
\begin{align*}
w_E(d) &< w_E(a) < w_E(c) \\
w_M(a) &< w_M(c) < w_M(d)
\end{align*}
\]

\[
\mathcal{W}_a = \{ \{E\}, \{E, M\} \}, \mathcal{W}_c = \{ \{M\}, \{E, M\} \}, \mathcal{W}_d = \{ \{M\}, \{E, M\} \}
\]

Then: \( \phi(d) = d, \phi(c) = d \), therefore, \( \phi(a) = a \)

- Indeed, \( c \) is unstable, and among \( a \) and \( d \) player \( E \), who is part of any winning coalition, prefers \( a \)
- Intuitively, if limited franchise immediately leads to full democracy, elite will not undertake it
Example (continued)

- Assume $\mathcal{W}_c = \{ \{ E, M \} \}$ instead of $\mathcal{W}_c = \{ \{ M \}, \{ E, M \} \}$
- Then: $\phi (d) = d$, $\phi (c) = c$, and, $\phi (a) = c$
- $a$ became unstable because $c$ became stable

Now assume $\mathcal{W}_a = \mathcal{W}_c = \mathcal{W}_d = \{ \{ E, M \} \}$ and

$$w_E (a) < w_E (d) < w_E (c)$$
$$w_M (a) < w_M (c) < w_M (d)$$

- $a$ is disliked by everyone, but otherwise preferences differ
- Then: $\phi (d) = d$, $\phi (c) = c$, and $\phi (a)$ may be $c$ or $d$
- In any case, $\mathcal{D} = \{ c, d \}$ is the same
Proof of Theorem (1)

- Take sequence \( \{\mu_1, \ldots, \mu_{|S|}\} \) such that

\[
\text{if } 1 < j < l < |S|, \text{ then } \mu_l \not\succ_{\mu_j} \mu_j.
\]

- Assumption (Payoffs)a implies “acyclicity”. Thus, for any nonempty collection of states \( Q \subset S \), there exists state \( z \in Q \) such that for any \( x \in Q \), \( x \not\succ_z z \).

- Apply to \( Q = S \); obtain \( \mu_1 = z \)
- Apply to \( Q = S \setminus \{\mu_1\} \); obtain \( \mu_2 \ldots \)
Proof of Theorem (2)

- Let $\phi(\mu_1) = \mu_1$
  - This is the only possibility given Axiom 1 (desirability)
- For $k \geq 2$, let
  
  $$\mathcal{M}_k = \{s \in \{\mu_1, \ldots, \mu_{k-1}\} : s \succ_\mu_k \mu_k \text{ and } \phi(s) = s\}$$

  - Set of “higher” states which are preferred to $\mu_k$ and $s$
  - If $\phi(\mu_k) \neq \mu_k$, then $\phi(\mu_k) \in \mathcal{M}_k$ by desirability and stability axioms
- Define
  
  $$\phi(\mu_k) = \begin{cases} 
  \mu_k & \text{if } \mathcal{M}_k = \emptyset \\
  z \in \mathcal{M}_k : \forall x \in \mathcal{M}_k \text{ with } x \succ_\mu_k z & \text{if } \mathcal{M}_k \neq \emptyset 
  \end{cases}$$

  - Such $z$ exists by Assumption (Payoffs)b applied to $\mathcal{M}_k$
  - This ensures that rationality axiom holds
Proof of Theorem (3)

- We have constructed mapping $\phi$ using the recursive procedure.
- By construction, $\phi$ satisfies Axioms 1–3.
- Any such mapping could be obtained by this procedure:
  - The only degree of freedom we had was choosing $z \in \mathcal{M}_k$.
  - Different choice of $\phi(\mu_k)$ does not affect $\mathcal{M}_l$ for $l > k$.
  - Any choice of sequence $\{\mu_1, ..., \mu_{|S|}\}$ such that $\mu_l \not\sim \mu_j$ whenever $1 < j < l < |S|$ would lead to the same (comprehensive) set of mappings.
- This proves parts 1 and 2.
Proof of Theorem (4)

- **Part 3:** For any two mappings \( \phi_1 \) and \( \phi_2 \) that satisfy Axioms 1–3 the sets of stable states of these mappings coincide.
  - This holds because \( \phi(\mu_k) = \mu_k \) if and only if \( M_k = \emptyset \), and this does not depend on \( \phi \).

- **Part 4:** If, in addition, Assumption (Comparability) holds, then for any two mappings \( \phi_1 \) and \( \phi_2 \) that satisfy Axioms 1–3 and for any \( s \in S \), \( \phi_1(s) \sim \phi_2(s) \).
  - Assumption: For \( x, y, z \in S \) such that \( x \succ_z z \), \( y \succ_z z \), and \( x \sim y \), either \( y \succ_z x \) or \( x \succ_z y \).
  - If \( \phi_1(s) \sim \phi_2(s) \), then either \( \phi_1(s) \succ_s \phi_2(s) \) or \( \phi_2(s) \succ_s \phi_1(s) \).
  - Both options would violate Rationality axiom.
Back to Dynamic Game

Assumption

(Agenda-Setting and Proposals) For every state $s \in S$, one (or both) of the following two conditions is satisfied:

(a) For any state $q \in S \setminus \{s\}$, there is an element $k : 1 \leq k \leq K_s$ of sequence $\pi_s$ such that $\pi_s(k) = q$.

(b) For any player $i \in I$ there is an element $k : 1 \leq k \leq K_s$ of sequence $\pi_s$ such that $\pi_s(k) = i$.

- Exogenous agenda, sequence of agenda-setters, or mixture.
- This assumption ensures that all proposals will be considered (or all agenda-setters will have a chance to propose)

Definition

(Dynamically Stable States) State $s^\infty \in S$ is a dynamically stable state if there exist a protocol $\{\pi_s\}_{s \in S}$, a MPE strategy profile $\sigma$ (for a game starting with initial state $s_0$) and $T < \infty$, such that in MPE $s_t = s^\infty$
Slightly Stronger Acyclicty Assumption

**Assumption (Stronger Acyclicity)** For any sequence of states $s, s_1, \ldots, s_k$ in $S$ such that $s_j \sim s_l$ (for any $1 \leq j < l \leq k$) and $s_j \succ_s s$ (for any $1 \leq j \leq k$)

$$s_{j+1} \sim s_j \text{ for all } 1 \leq j < k - 1 \implies s_1 \nprec_s s_k.$$  

Moreover, if for $x, y, s$ in $S$, we have $x \succ_s s$ and $y \nprec_s s$, then $y \nprec_s x$.

- Stronger version of part (b) of Payoffs Assumption.
- First part: $\sim$-acyclicity as opposed $\succ$-acyclicity
- Second part: slightly stronger than acyclicity
  - but weaker than transitivity within states, i.e., $x \succ_s s$, $y \nprec_s s$, then $y \nprec_s x$, whereas transitivity would require $x \succ_s s$, $s \succ_s y$, then $x \succ_s y$, which implies our condition, but is much stronger.
- Alternative (with equivalent results): voting yes has a small cost.
Noncooperative Characterization

Theorem

**Noncooperative Characterization** Suppose Assumptions on Winning Coalitions and Payoffs hold. Then there exists $\beta_0 \in [0, 1)$ such that for all $\beta \geq \beta_0$, the following results hold.

1. For any mapping $\phi$ satisfying Axioms 1–3 there is a protocol $\{\pi_s\}_{s \in S}$ and a MPE $\sigma$ of the game such that $s_t = \phi(s_0)$ for any $t \geq 1$; that is, the game reaches $\phi(s_0)$ after one period and stays in this state thereafter. Therefore, $s = \phi(s_0)$ is a dynamically stable state.
Theorem

... Moreover, suppose that Stronger Acyclicity Assumption holds. Then:

2. For any protocol \( \{ \pi_s \}_{s \in S} \) there exists a MPE in pure strategies. Any such MPE \( \sigma \) has the property that for any initial state \( s_0 \in S \), it reaches some state, \( s^\infty \) by \( t = 1 \) and thus for \( t \geq 1 \), \( s_t = s^\infty \).
   Moreover, there exists mapping \( \phi : S \rightarrow S \) that satisfies Axioms 1–3 such that \( s^\infty = \phi \left( s_0 \right) \). Therefore, all dynamically stable states are axiomatically stable.

3. If, in addition, Assumption (Comparability) holds, then the MPE is essentially unique in the sense that for any protocol \( \{ \pi_s \}_{s \in S} \), any MPE strategy profile in pure strategies \( \sigma \) induces \( s_t \sim \phi \left( s_0 \right) \) for all \( t \geq 1 \), where \( \phi \) satisfies Axioms 1–3.
Proof of Theorem (I)

Part 1

- For any state $s$, choose $\pi_s(\cdot)$ such that $\pi_s(K_s) = \phi(s)$ (last proposal is $\phi(s)$)
- Let everyone who weakly prefers $\phi(s)$ to $s$ ($w_i(\phi(s)) \geq w_i(s)$) vote for $\phi(s)$
  - These constitute a winning coalition in $s$
- In previous votings these block any transition to any other state as no other state is preferred by a winning coalition
- As a result, $\phi(s)$ will be implemented.
- A similar argument for any $\pi_s(\cdot)$ establishes existence.
  - In particular, truncate the game with continuation payoffs given by the axiomatic characterization from a certain point onwards, use backward induction and verify that strategies are Markovian.
Proof of Theorem (II)

Part 2

- Suppose that \( s \) leads to \( \chi(s) \)
- Let \( \psi(s) = \chi^{S}(s) \)
- Suppose that in an equilibrium, if state is \( s \), then proposals at stages \( k_1, k_2, \ldots, k_j \) are accepted
- Let these proposals be \( P_{k_1}, P_{k_2}, \ldots, P_{k_j} \)
- We immediately have
  \[ \psi(P_{k_1}) \succeq_s \psi(P_{k_2}) \succeq_s \cdots \succeq_s \psi(P_{k_j}) \succeq_s \psi(P_{k_1}) \]
- Stronger acyclicity assumption implies
  \[ \psi(P_{k_1}) \sim \psi(P_{k_2}) \sim \cdots \sim \psi(P_{k_j}) \]
Proof of Theorem (III)

Part 2 (continued)

- Stability Axiom for $\psi$: satisfied by definition
- Desirability Axiom: otherwise players would wait until next period instead of accepting $\psi(P_{kj})$
- Rationality Axiom: otherwise some other proposal would be accepted in between
- Transition in one step: if this proposal is made, it is accepted

Part 3

- Follows from Part 2 and Axiomatic Characterization
Dynamic vs. Myopic Stability

Definition
State $s^m \in S$ is *myopically stable* if there does not exist $s \in S$ with $s \succ_{s^m} s^m$.

Corollary
1. State $s^\infty \in S$ is a (dynamically and axiomatically) stable state only if for any $s' \in S$ with $s' \succ_{s^\infty} s^\infty$, and any $\phi$ satisfying Axioms 1–3, $s' \neq \phi(s')$.
2. A myopically stable state $s^m$ is a stable state.
3. A stable state $s^\infty$ is not necessarily myopically stable.

E.g., state $a$ in extension of franchise story
Inefficiency

Definition

(Inefficiency) State \( s \in S \) is (strictly) Pareto inefficient if there exists \( s' \in S \) such that \( w_i(s') > w_i(s) \) for all \( i \in \mathcal{I} \).
State \( s \in S \) is (strictly) winning coalition inefficient if there exists a winning coalition \( \mathcal{W}_s \subset \mathcal{I} \) in \( s \) and \( s' \in S \) such that \( w_i(s') > w_i(s) \) for all \( i \in \mathcal{W}_s \).

Clearly, if a state \( s \) is Pareto inefficient, it is winning coalition inefficient, but not vice versa.

Corollary

1. A stable state \( s^\infty \in S \) can be (strictly) winning coalition inefficient and Pareto inefficient.
2. Whenever \( s^\infty \) is not myopically stable, it is winning coalition inefficient.
Applying the Theorems in Ordered Spaces

- The characterization theorems provided so far are easily applicable in a wide variety of settings.
- In particular, if the set of states is ordered and static preferences satisfy single crossing or single peakedness, all the results provided so far can be applied directly.
- Here, for simplicity, suppose that $\mathcal{I} \subset \mathbb{R}$ and $\mathcal{S} \subset \mathbb{R}$ (more generally, other orders on the set of individuals and the set of states would work as well)
Single Crossing and Single Peakedness

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w.(\cdot)$. Then, single crossing condition holds if whenever for any $i, j \in \mathcal{I}$ and $x, y \in \mathcal{S}$ such that $i < j$ and $x < y$, $w_i(y) > w_i(x)$ implies $w_j(y) > w_j(x)$ and $w_j(y) < w_j(x)$ implies $w_i(y) < w_i(x)$.

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w.(\cdot)$. Then, single-peaked preferences assumption holds if for any $i \in \mathcal{I}$ there exists state $x$ such that for any $y, z \in \mathcal{S}$, if $y < z \leq x$ or $x \geq z > y$, then $w_i(y) \leq w_i(z)$. 
Generalizations of Majority Rule and Median Voter

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, state $s \in \mathcal{S}$, and set of winning coalitions $\mathcal{W}_s$ that satisfies Assumption on Winning Coalitions. Player $i \in \mathcal{I}$ is called quasi-median voter (in state $s$) if $i \in X$ for any $X \in \mathcal{W}_s$ such that $X = \{j \in \mathcal{I} : a \leq j \leq b\}$ for some $a, b \in \mathbb{R}$.

- That is, quasi-median voter is a player who belongs to any “connected” winning coalition.
- Denote the set of quasi-median voters in state $s$ by $M_s$ (it will be nonempty)

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$. The sets of winning coalitions $\{\mathcal{W}_s\}_{s \in \mathcal{S}}$ has monotonic quasi-median voter property if for each $x, y \in \mathcal{S}$ satisfying $x < y$ there exist $i \in M_x$, $j \in M_y$ such that $i \leq j$. 
A Weak Genericity Assumption

Let us say that preferences \( w(\cdot) \), given the set of winning coalitions \( \{W_s\}_{s \in \mathcal{S}} \), are \textit{generic} if for all \( x, y, z \in \mathcal{S} \), \( x \succeq_z y \) implies \( x \succeq_z y \) or \( x \sim y \).

This is (much) weaker than the comparability assumption used for uniqueness above.

- In particular, it holds generically.
Theorem on Single Crossing and Single Peakedness

Theorem

Suppose the Assumption on Winning Coalitions holds.

1. If preferences satisfy single crossing and the monotonic quasi-median voter property holds, then Assumptions on Payoffs above are satisfied and the axiomatic characterization (Theorem 1) applies.

2. If preferences are single peaked and all winning coalitions intersect (i.e., \(X \in \mathcal{W}_x\) and \(Y \in \mathcal{W}_y\) imply \(X \cap Y \neq \emptyset\)), then Assumptions on Payoffs are satisfied and Theorem 1 applies.

3. If, in addition, in part 1 or 2, preferences are generic, then the Stronger Acyclicity Assumption is satisfied and the noncooperative characterization (Theorem 2) applies.

Note monotonic median voter property is weaker than the assumption that \(X \in \mathcal{W}_x \land Y \in \mathcal{W}_y \implies X \cap Y \neq \emptyset\).
Voting in Clubs

- $N$ individuals, $\mathcal{I} = \{1, \ldots, N\}$
- $N$ states (clubs), $s_k = \{1, \ldots, k\}$
- Assume single-crossing condition

  \[
  \text{for all } l > k \text{ and } j > i, \ w_j(s_l) - w_j(s_k) > w_i(s_l) - w_i(s_k)
  \]

- Assume “genericity”:

  \[
  \text{for all } l > k, \ w_j(s_l) \neq w_j(s_k)
  \]

- Then, the theorem for ordered spaces applies and shows existence of MPE in pure strategies for any majority or supermajority rule.
- It also provides a full characterization of these equilibria.
Voting in Clubs

- If in addition only odd-sized clubs are allowed, unique dynamically stable state.
- Equilibria can easily be Pareto inefficient.
- If “genericity” is relaxed, so that $w_j(s_l) = w_j(s_k)$, then the theorem for ordered spaces no longer applies, but both the axiomatic characterization and the noncooperative theorems can still be applied from first principles.
- Comparison to Roberts (1999): much simpler analysis under weaker conditions, and more general results (existence of pure-strategy equilibrium, results for supermajority rules etc.)
- Also can be extended to more general structure of clubs
  - e.g., clubs on the form $\{k - n, \ldots, k, \ldots, k + n\} \cap I$ for a fixed $n$ (and different values of $k$).
An Example of Elite Clubs

- Specific example: suppose that preferences are such that
  \[ w_j(s_n) > w_j(s_{n'}) > w_j(s_{k'}) = w_j(s_{k''}) \]
  for all \( n' > n \geq j \) and \( k', k'' < j \)
  - individuals always prefer to be part of the club
  - individuals always prefer smaller clubs.

- Winning coalitions need to have a strict majority (e.g., two out of three, three out of four etc.).
- Then,
  - \( \{1\} \) is a stable club (no wish to expand)
  - \( \{1, 2\} \) is a stable club (no wish to expand and no majority to contract)
  - \( \{1, 2, 3\} \) is not a stable club (3 can be eliminated)
  - \( \{1, 2, 3, 4\} \) is a stable club

- More generally, clubs of size \( 2^k \) for \( k = 0, 1, \ldots \) are stable.
- Starting with the club of size \( n \), the equilibrium involves the largest club of size \( 2^k \leq n \).
Example: Taxation

- Suppose there are \( k \) individuals \( 1, 2, \ldots, k \), and \( k \) states \( s_1, s_2, \ldots, s_k \), where \( s_j = \{1, 2, \ldots, j\} \).
- Suppose winning coalition is a simple majority rule of players who are enfranchised:

\[
\mathcal{W}_{s_j} = \{X \in \mathcal{C} : \# (X \cap s_j) > j/2\}.
\]

- Suppose player \( i \)'s payoff is

\[
w_i(s_j) = (1 - \tau_{s_j}) A_i + G_{s_j}
\]

where \( A_i \) is player \( i \)'s productivity; \( G_{s_j} \) and \( \tau_{s_j} \) are the public good and the tax rate voting franchise is \( s_j \).

- Assume \( A_i > A_j \) for \( i < j \), so the first players are the most productive ones.
Example: Taxation (continued)

- $\tau_{s_j}$ is the tax rate determined by the median voter in the club $s_j$ (or by one of the two median voters with equal probability in case of even-sized club).
- The technology for the production of the public good is

$$G_{s_j} = H \left( \sum_{i=1}^{k} \tau_{s_j} A_i \right),$$

where $H$ is strictly increasing and concave.
Example: Taxation (continued)

- In light of the previous theorem, to apply our results, it suffices to show that if \( i, j \in s_k, s_{k+1} \), then
  \[
  w_j(s_{k+1}) - w_j(s_k) > w_i(s_{k+1}) - w_i(s_{k+1})
  \]
  whenever \( i < j \).

- This is equivalent to
  \[
  (1 - \tau_{s_{k+1}}) A_j - (1 - \tau_{s_k}) A_j \geq (1 - \tau_{s_{k+1}}) A_i - (1 - \tau_{s_k}) A_i,
  \]

- Since \( A_j < A_i \), this is in turn is equivalent to
  \[
  \tau_{s_{k+1}} \geq \tau_{s_k}.
  \]

- This can be verified easily, so the theorem for order spaces can be applied.
Stable Constitutions

- $N$ individuals, $\mathcal{I} = \{1, \ldots, N\}$
- In period 2, they decide whether to implement a reform ($a$ votes are needed)
- $a$ is determined in period 1
- Two cases:
  - Voting rule $a$: stable if in period 1 no other rule is supported by $a$ voters
  - Constitution $(a, b)$: stable if in period 1 no other constitution is supported by $b$ voters
- Preferences over reforms translate into preferences over $a$
  - Barbera and Jackson assume a structure where these preferences are single-crossing and single-peaked
  - Motivated by this, let us assume that they are strictly single-crossing
- Stable voting rules correspond to myopically (and dynamically) stable states
- Stable constitutions correspond to dynamically stable states
Political Eliminations

- The characterization results apply even when states do not form an ordered set.
- Set of states $\mathcal{S}$ coincides with set of coalitions $\mathcal{C}$
- Each agent $i \in \mathcal{I}$ is endowed with political influence $\gamma_i$
- Payoffs are given by proportional rule

$$w_i(X) = \begin{cases} \frac{\gamma_i}{\gamma_X} & \text{if } i \in X \\ 0 & \text{if } i \not\in X \end{cases}$$

where $\gamma_X = \sum_{j \in X} \gamma_j$

and $X$ is the “ruling coalition”.

- this payoff function can be generalized to any function where payoffs are increasing in relative power of the individual in the ruling coalition
Political Eliminations (continued)

- Winning coalitions are determined by weighted (super)majority rule 
  \( \alpha \in [1/2, 1) \)

\[
W_X = \left\{ Y : \sum_{j \in Y \cap X} \gamma_j > \alpha \sum_{j \in X} \gamma_j \right\}
\]

- Genericity: \( \gamma_X = \gamma_Y \) only if \( X = Y \)
- Assumption on Payoffs is satisfied and the axiomatic characterization applies exactly.
- If players who are not part of the ruling coalition have a slight preference for larger ruling coalitions, then Stronger Acyclicity Assumption is also satisfied.
Other Examples

- Inefficient inertia
- The role of the middle class in democratization
- Coalition formation in democratic systems
- Commitment, (civil or international) conflict and peace
Concluding Remarks

- A class of dynamic games potentially representing choice of constitutions, dynamic voting, club formation, dynamic coalition formation, organizational choice, dynamic legislative bargaining, international or civil conflict.
- Common themes in disparate situations.
- This paper: a framework for general analysis and tight characterization results.
- *Simple implications*: social arrangements are unstable **not** when some winning coalition (e.g., majority) prefers another social arrangement, but when it prefers another **stable** social arrangement
- We show that this gives rise to inefficiencies: a Pareto dominated state may be stable, even if discount factor is close to 1
Open Questions

- Beyond acyclicity...
- More on stochastic shocks.
- Different winning coalitions to implement different social arrangements/political changes

**Most important:** model dynamic games with more limited foresight and richer dynamics so as to understand the dynamic evolution of constitutions, coalitions, clubs, governments, organizations,...

- gradual extension of the franchise, constitutional cycles, slow improvements in the quality of governance...