Equilibrium Corporate Finance and Macroeconomics

Alberto Bisin (NYU)
Piero Gottardi (Venice)
Guido Ruta (NYU)

Macro Lunch, NYU Stern, March 12, 2009
This is a paper on the theory of General Equilibrium
Introduction

- **This is** a paper on the theory of General Equilibrium
- **One with** two periods, incomplete markets, ......, the whole thing.
Introduction

- **This is** a paper on the theory of General Equilibrium
- **One with** two periods, incomplete markets, ......, the whole thing.
- **When is the last time** you have seen one of those?
Introduction

- **This is** a paper on the theory of General Equilibrium
- **One with** two periods, incomplete markets, ...., the whole thing.
- **When is the last time** you have seen one of those?
- But this is different, we are addressing an interesting economic question.... :-}
Macro with heterogeneous agents and incomplete markets goes a long way without production - with exogenous earning processes; e.g.,

- J. Heathcote, K. Storesletten, and G. Violante (2008) - ”Bewley models”

Production in macro involves, either

- complete markets; or
- investor-entrepreneur models of dynamic contracting

Prototypical examples are, e.g.,:

- U. Jermann (2008) - production based asset pricing literature
- **As a result:** corporate finance as a discipline is almost completely distinct from macroeconomics and standard asset pricing
As a result: corporate finance as a discipline is almost completely distinct from macroeconomics and standard asset pricing.

WHY?

WHY?


WHY?


▷ We follow
The basic model

- Two-period economy, with riskless debt and equity, no asymmetric information
The basic model

- Two-period economy, with riskless debt and equity, no asymmetric information
- Stochastic shocks governed by a Markov structure with transition probability matrix $\Pi$ over the state space $S = \{1, \ldots, S\}$. At $t = 0$, the state of the economy $s_0 \in S$ is known.
The basic model

- Two-period economy, with riskless debt and equity, no asymmetric information
- Stochastic shocks governed by a Markov structure with transition probability matrix $Π$ over the state space $S = \{1, \ldots, S\}$. At $t = 0$, the state of the economy $s_0 \in S$ is known.
- A single type of firm. Production takes place at $t = 1$ according to the function $f(k; s_1)$
The basic model

- Two-period economy, with riskless debt and equity, no asymmetric information
- Stochastic shocks governed by a Markov structure with transition probability matrix $\Pi$ over the state space $S = \{1, \ldots, S\}$. At $t = 0$, the state of the economy $s_0 \in S$ is known.
- A single type of firm. Production takes place at $t = 1$ according to the function $f(k; s_1)$
- $I$ types of consumers, $i = 1, .., I$. Endowment shocks: $w^i_0 = w^i(s_0)$ at date 0 and $w^i(s_1)$ at date 1 in state $s_1$. 
The basic model

- Two-period economy, with riskless debt and equity, no asymmetric information

- Stochastic shocks governed by a Markov structure with transition probability matrix $\Pi$ over the state space $S = \{1, \ldots, S\}$. At $t = 0$, the state of the economy $s_0 \in S$ is known.

- A single type of firm. Production takes place at $t = 1$ according to the function $f(k; s_1)$

- $I$ types of consumers, $i = 1, \ldots, I$. Endowment shocks: $w^i_0 = w^i(s_0)$ at date 0 and $w^i(s_1)$ at date 1 in state $s_1$.

- Perfect competition throughout!
Bond has payoff:

$$\min\{1, \frac{f(k; s_1)}{B}\}$$
Equity has payoff:

$$\max\{f(k; s_1) - B, 0\}$$
The firm’s problem is:

\[ V = \max_{k, B} -k + q(k, B) + p \ B \]  

subject to the solvency constraint (ensuring that the bonds issued are riskfree):

\[ f(k; s_1) \geq B \quad \forall s_1 \]
The firm’s problem is:

\[ V = \max_{k,B} -k + q(k, B) + p B \]  

subject to the solvency constraint (ensuring that the bonds issued are riskfree):

\[ f(k; s_1) \geq B \quad \forall s_1 \]  

Let \( \bar{k}, \bar{B} \) denote the solutions to this problem.
The problem of agent $i$ is:

$$\max_{\theta^i, b^i, c^i} u(c^i_0) + \beta E_{s_0} u(c^i(s_1))$$ (3)

subject to

$$b^i \geq 0, \quad \theta^i \geq 0, \quad \forall i$$ (4)

and

$$c^i_0 = w^i_0 + [ -k + q + p B \theta^i_0 - q \theta^i - p b^i ]$$ (5)

$$c^i(s_1) = w^i(s_1) + [ f(k; s_1) - B \theta^i + b^i ], \quad \forall s_1,$$ (6)

where $\theta^i_0$ is some predetermined stock-holdings agent $i$ enters the economy with (and $\sum_i \theta^i_0 = 1$).
The problem of agent $i$ is:

$$\max_{\theta^i, b^i, c^i} u(c^i_0) + \beta E_{s_0} u(c^i(s_1)) \quad (3)$$

subject to

$$b^i \geq 0, \quad \theta^i \geq 0, \quad \forall i \quad (4)$$

and

$$c^i_0 = w^i_0 + [-k + q + p B] \theta^i_0 - q \theta^i - p b^i \quad (5)$$

$$c^i(s_1) = w^i(s_1) + [f(k; s_1) - B] \theta^i + b^i, \quad \forall s_1 \quad (6)$$

where $\theta^i_0$ is some predetermined stock-holdings agent $i$ enters the economy with (and $\sum_i \theta^i_0 = 1$).

Let $\bar{\theta}^i, \bar{b}^i, \bar{c}^i_0, (\bar{c}^i(s_1))_{s_1 \in S_1}$ denote the solutions of this problem.
The problem of agent $i$ is:

$$\max_{\theta^i, b^i, c^i} u(c^i_0) + \beta E_{s_0} u(c^i(s_1))$$

subject to

$$b^i \geq 0, \quad \theta^i \geq 0, \quad \forall i$$

and

$$c^i_0 = w^i_0 + [-k + q + p B] \theta^i_0 - q \theta^i - p b^i$$

$$c^i(s_1) = w^i(s_1) + [f(k; s_1) - B] \theta^i + b^i, \quad \forall s_1,$$

where $\theta^i_0$ is some predetermined stock-holdings agent $i$ enters the economy with (and $\sum_i \theta^i_0 = 1$).

Let $\bar{\theta}^i, \bar{b}^i, \bar{c}^i_0, (\bar{c}^i(s_1))_{s_1 \in S_1}$ denote the solutions of this problem.

Let $MRS^i(s_1)$ denote the marginal rate of substitution between consumption in states $s_0$ and $s_1$ for consumer $i$ evaluated at his optimal consumption choice $\bar{c}^i$. 
In equilibrium, the equity price map faced by the firm must satisfy the following consistency condition:

\[ q(k, B) = \max_i E_{s_0} \left[ MRS^i(s_1)(f(k; s_1) - B) \right] \text{ for all } k, B \]
Equilibrium; cont.ed

- In equilibrium, the equity price map faced by the firm must satisfy the following consistency condition:

\[ q(k, B) = \max_i E_{s_0} \left[ MRS_i^i(s_1)(f(k; s_1) - B) \right] \text{ for all } k, B \]

- In equilibrium, also,

- \[ q = q(\bar{k}, \bar{B}) \].

- Markets clear:

\[
\begin{align*}
\sum_i c_0^i + k & \leq \sum_i w^i \\
\sum_i c_i(s_1) & \leq \sum_i w_i(s_1) + f(k; s_1), \text{ for all } s_1
\end{align*}
\]

or equivalently:

\[
\begin{align*}
\sum_i \theta_i & \leq 1 \\
\sum_i b_i & \leq B
\end{align*}
\]
In Dreze (1974), the consistency condition takes the following form:

$$q(k, B) = E \sum_i \bar{\theta}^i M R S^i(s_1) [f(k; s_1) - B] \text{ for all } k, B;$$
Objective function of the firm

- In Dreze (1974), the consistency condition takes the following form:

\[ q(k, B) = E \sum_i \bar{\theta}^i MRS^i(s_1) [f(k; s_1) - B] \text{ for all } k, B; \]


\[ q(k, B) = E \sum_i \theta_0^i MRS^i(s_1) [f(k; s_1) - B] \text{ for all } k, B; \]
Objective function of the firm

- In Dreze (1974), the consistency condition takes the following form:

\[ q(k, B) = E \sum_i \bar{\theta}^i MRS^i(s_1) [f(k; s_1) - B] \text{ for all } k, B; \]


\[ q(k, B) = E \sum_i \theta_0^i MRS^i(s_1) [f(k; s_1) - B] \text{ for all } k, B; \]

- In this paper (following Makowski ...):

\[ q(k, B) = \max_i EMRS^i(s_1) [f(k; s_1) - B] \text{ for all } k, B; \]
Proposition: At a competitive equilibrium, shareholders unanimous support the production and financial decisions of firms $\bar{k}, \bar{B}$. That is every agent $i$ holding a positive initial amount $\theta_i^0$ of equity of the representative firm will be made -weakly - worse off by any other choice $k', B'$ of a firm (for which, obviously the firm’s value will be -weakly - lower).
Welfare

- **Definition:** A competitive equilibrium is *constrained Pareto efficient* if its allocations are:

1. feasible: there exists a production plan $k$ of firms such that:

   $$\sum_i c_i^0 + k \leq \sum_i w^i$$

   $$\sum_i c_i^i(s_1) \leq \sum_i w^i(s_1) + f(k; s_1) \text{ for all } s_1$$

2. attainable with the existing asset structure: that is there exists $B$ and, for each consumer’s type $i$, a pair $\theta^i, b^i$ such that:

   $$c_i^i(s_1) = w^i(s_1) + [f(k; s_1) - B] \theta^i + b^i, \quad \forall s_1$$

3. if we cannot find another feasible and attainable allocation which is Pareto improving.
Definition: A competitive equilibrium is constrained Pareto efficient if its allocations are:

1. feasible: there exists a production plan $k$ of firms such that:

$$
\sum_i c^i_0 + k \leq \sum_i w^i
$$

$$
\sum_i c^i(s_1) \leq \sum_i w^i(s_1) + f(k; s_1) \quad \text{for all } s_1
$$

2. attainable with the existing asset structure: that is there exists $B$ and, for each consumer’s type $i$, a pair $\theta^i, b^i$ such that:

$$
c^i(s_1) = w^i(s_1) + [f(k; s_1) - B] \theta^i + b^i, \quad \forall s_1
$$

3. if we cannot find another feasible and attainable allocation which is Pareto improving.

Proposition: Competitive equilibria are constrained Pareto efficient.
Proposition: At a competitive equilibrium, the capital structure of each individual firm is indeterminate, with the only, possible exception of the case where the optimal choice obtains at a point where the no default constraint binds. On the other hand, the equilibrium capital structure of all firms in the economy is, at least partly, determinate:

- either $B$ is such that all equityholders are also bondholders (in which case $B$ lies in an interval, or
- $B = f(k; s_1)$. 
Proposition: At a competitive equilibrium, the capital structure of each individual firm is indeterminate, with the only, possible exception of the case where the optimal choice obtains at a point where the no default constraint binds. On the other hand, the equilibrium capital structure of all firms in the economy is, at least partly, determinate:

- either $B$ is such that all equityholders are also bondholders (in which case $B$ lies in an interval, or
  - $B = f(k; s_1)$.

Why ”all equityholders are also bondholders”? Otherwise equityholders is at constraint where he would go short on the bond, which he could achieve by leveraging the firm.
Parametric exercise

- Two types of consumers; both with utility \( u = \frac{c^{1-\gamma}}{1-\gamma} \), for \( \gamma = 2 \) and \( \beta = 0.95 \).
Parametric exercise

- Two types of consumers; both with utility $u = \frac{c^{1-\gamma}}{1-\gamma}$, for $\gamma = 2$ and $\beta = 0.95$.
- Production technology: $f(k; s_1) = a(s_1)k^\alpha$, with $\alpha = 0.75$. 
Parametric exercise

- Two types of consumers; both with utility $u = \frac{c^{1-\gamma}}{1-\gamma}$, for $\gamma = 2$ and $\beta = 0.95$.
- Production technology: $f(k; s_1) = a(s_1)k^\alpha$, with $\alpha = 0.75$.
- Two states with the following structure of endowment and productivity shocks:

<table>
<thead>
<tr>
<th>$w^1$</th>
<th>$s$</th>
<th>$\bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>2.1429</td>
<td>4.7143</td>
</tr>
</tbody>
</table>
Parametric exercise

- Two types of consumers; both with utility $u = \frac{e^{1-\gamma}}{1-\gamma}$, for $\gamma = 2$ and $\beta = 0.95$.

- Production technology: $f(k; s_1) = a(s_1)k^\alpha$, with $\alpha = 0.75$.

- Two states with the following structure of endowment and productivity shocks:

<table>
<thead>
<tr>
<th></th>
<th>$s$</th>
<th>$\bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^1$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$w^2$</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>$a$</td>
<td>2.1429</td>
<td>4.7143</td>
</tr>
</tbody>
</table>

- We consider the case where at date 0 the state is also recession, i.e. $w^i_0 = w^i(\bar{s})$ for all $i$, and $\pi(\bar{s}) = .8$, $\pi(\bar{s}) = .2$, or the persistence of the shocks is fairly high.
Parametric exercise; cont.ed

Stockholdings for 1: $\theta^1$
Stockholdings for 2: $\theta^2$

Bondholdings for 1: $b^1$
Bondholdings for 2: $b^2$

1’s willingness to pay for the stock: $wp^1$
2’s willingness to pay for the stock: $wp^2$

1’s willingness to pay for the bond: $wp^1$
2’s willingness to pay for the bond: $wp^2$
Parametric exercise; cont.ed

Value of the single firm \((-k+q+pB)\), at the MRSs of the candidate equilibrium \((X)\)
Risky Debt

Value of the single firm \((-k + q + pB)\), at the MRSs of the candidate equilibrium (X)
Short-sales and intermediation

- Short-sales are not necessarily priced as equity (e.g., the last is guaranteed by an implicit collateral)
Short-sales and intermediation

- Short-sales are not necessarily priced as equity (e.g., the last is guaranteed by an implicit collateral)
- The self-financing constraint of the intermediary intermediating $m$ units of the derivative on the firm’s equity is:

\[ m \leq m(1 - \delta) + \gamma \]

where $\gamma$ is the amount of equity of the firm retained as collateral by the intermediary
Short-sales and intermediation

- Short-sales are not necessarily priced as equity (e.g., the last is guaranteed by an implicit collateral)
- The self-financing constraint of the intermediary intermediating $m$ units of the derivative on the firm’s equity is:

$$m \leq m(1 - \delta) + \gamma$$

where $\gamma$ is the amount of equity of the firm retained as collateral by the intermediary
- The intermediary chooses $m$, and $\gamma$, to maximize total revenue at date 0:

$$\max_{m,\gamma} (q^+ - q^-) m - q\gamma$$

subject to

$$m \leq m(1 - \delta) + \gamma$$
The consumers’ budget constraints are then in this set-up modified as follows:

\[ c^i_0 = w^i_0 + [-k + q + p B ] \theta^i_0 - q \theta^i - p b^i - q^+ \lambda^i_+ - q^- \lambda^i_- \]

\[ c^i(s_1) = w^i(s_1) + [f(k; s_1) - B ] (\theta^i + \lambda^i_+ - \lambda^i_-) + b^i, \quad \forall s_1, \]
The consumers’ budget constraints are then in this set-up modified as follows:

\[
c_0^i = w_0^i + [-k + q + pB] \theta_0^i - q \theta^i - p b^i - q^+ \lambda_+^i - q^- \lambda_-^i
\]

\[
c^i(s_1) = w^i(s_1) + [f(k; s_1) - B] (\theta^i + \lambda_+^i - \lambda_-^i) + b^i, \quad \forall s_1,
\]

The consistency condition for \( q(k, B) \) is:

\[
q(k, B) = \max \left\{ \max_i \mathbb{E}_{s_0} MRS^i(s_1) [f(k; s_1) - B], \right. \\
\left. \max_i \mathbb{E}_{s_0} MRS^i(s_1) [f(k; s_1) - B] - \min_i \mathbb{E}_{s_0} MRS^i(s_1) [f(k; s_1) - B] \right\}
\]
Market clearing of equity is:

\[ \gamma + \sum_{i \in I} \theta^i = 1, \]

\[ \sum_{i \in I} \lambda^i_+ = \sum_{i \in I} \lambda^i_- = m, \]
For the firm issuing equity, two possible situations can arise at equilibrium:

1. \( q = (q^+ - q^-)/\delta > q^+ \), which is in turn equivalent to \( q^+ > q^-/(1 - \delta) \): equity sells at a premium over the long positions on the derivative claim issued by the intermediary; all the amount of equity outstanding is purchased by the intermediary, who can bear the additional cost of equity thanks to the presence of a sufficiently high spread \( q^+ - q^- \) between the cost of long and short positions on the derivative.

2. \( q = q^+ \): there is a single price at which equity and long positions in the derivative can be traded; consumers are indifferent between buying long positions in equity and the derivative and some if not all the outstanding amount of equity is held by consumers; when consumers hold all the outstanding amount of equity, intermediaries are non active at equilibrium and the bid ask spread \( q^+ - q^- \) is sufficiently low (in particular, it is less or equal than \( \delta q \)).
For the firm issuing equity, two possible situations can arise at equilibrium:

1. \[ q = \frac{(q^+ - q^-)}{\delta} > q^+ \], which is in turn equivalent to \[ q^+ > \frac{q^-}{1 - \delta} \]: equity sells at a premium over the long positions on the derivative claim issued by the intermediary; all the amount of equity outstanding is purchased by the intermediary, who can bear the additional cost of equity thanks to the presence of a sufficiently high spread \( q^+ - q^- \) between the cost of long and short positions on the derivative.

2. \( q = q^+ \): there is a single price at which equity and long positions in the derivative can be traded; consumers are indifferent between buying long positions in equity and the derivative and some if not all the outstanding amount of equity is held by consumers; when consumers hold all the outstanding amount of equity, intermediaries are non active at equilibrium and the bid ask spread \( q^+ - q^- \) is sufficiently low (in particular, it is less or equal than \( \delta q \)).

**Proposition:** Competitive equilibria with short-sales are constrained Pareto efficient.
Unobservable risk composition: moral hazard

- Production takes place according to the function \( f(k, \phi; s) \), where \( \phi \) represents a technological choice, affecting the stochastic structure of the firm’s future output; e.g.,

\[
f(k, \phi; s) = [a(s) + \phi e(s)] k^\alpha,
\]

\( \phi \) is chosen by the firm’s manager - agent \( i \); the manager is hired by initial shareholders; \( \phi, k \) and \( B \) are chosen at time \( t = 0 \), before financial markets open; but, unlike the choice of \( B \) and \( k \), \( \phi \) is not observed by bond-holders nor by shareholders in financial market at time 0.

- The choice of \( \phi \) affects the manager’s disutility, \( v^i(\phi) \).

- The manager’s compensation package is observable and consists of a net payment \( x_0 \), in units of the consumption good at date 0, together with a portfolio of \( \theta^m \) units of equity and \( b^m \) units of bonds.
The cost to the firm of the manager $i$’s compensation package, $W^i(\phi, k, B; q, p)$ is

- the payment made to this agent at date 0, $x_0^i$,
- plus the value of the portfolio
  
  
  \[ q(k, B, \phi) \left( \theta^{i,m} - \theta_0^{i,m} \right) + p(k, B, \phi)b^{i,m} \]

  attributed to him,
- minus the amount of the dividends due to this agent on account of his initial endowment $\theta_0^{i,m}$ of equity, $\theta_0^{i,m} \left[ \begin{array}{c} -k + p(k, B, \phi)B \\ -W^i(\phi, k, B; q, p) \end{array} \right]$. 

The optimal choice problem of a firm who has a hired as manager a type \( i \) agent is then the following:

\[
V^i = \max_{k, B, \phi, x^i_0, \theta^i, m, b^i, m} -k + q(k, B, \phi) + p(k, B, \phi) B - W^i(\phi, k, B; q, p)
\]

s.t.

\[
\mathbb{E} u^i \left( w^i_0 + x^i_0, w^i(s) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^i, m \right] + \min \left\{ 1, \frac{f(k, \phi; s)}{B} \right\} b^i, m \right) - v^i(\phi) \geq \mathbb{E} u^i \left( w^i_0 + x^i_0, w^i(s) + \left[ \max\{0, f(k, \phi'; s) - B\} \theta^i, m \right] + \min \left\{ 1, \frac{f(k, \phi'; s)}{B} \right\} b^i, m \right) - v^i(\phi')
\]

for all \( \phi' \in \Phi \)  

\[
\mathbb{E} u^i \left( w^i_0 + x^i_0, w^i(s) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^i, m \right] + \min \left\{ 1, \frac{f(k, \phi; s)}{B} \right\} b^i, m \right) - v^i(\phi) \geq \bar{U}^i
\]
The optimal choice problem of a firm who has a hired as manager a type $i$ agent is then the following:

$$V^i = \max_{k,B,\phi,x^i,\theta^i,m,b^i,m} -k+q(k, B, \phi)+p(k, B, \phi)B-W^i(\phi, k, B; q, p)$$

s.t.

$$\mathbb{E}u^i\left( w_0^i + x_0^i, w^i(s) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^i,m \right. \\ + \min \left\{ 1, \frac{f(k,\phi; s)}{B} \right\} b^i,m) - v^i(\phi) \right) \geq \mathbb{E}u^i\left( w_0^i + x_0^i, w^i(s) + \left[ \max\{0, f(k, \phi'; s) - B\} \theta^i,m \right. \\ + \min \left\{ 1, \frac{f(k,\phi'; s)}{B} \right\} b^i,m) - v^i(\phi') \right)$$

for all $\phi' \in \Phi$ (9)

$$\mathbb{E}u^i\left( w_0^i + x_0^i, w^i(s) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^i,m \right. \\ + \min \left\{ 1, \frac{f(k,\phi; s)}{B} \right\} b^i,m) - v^i(\phi) \right) \geq \bar{U}^i$$

(10)

The reservation utility for a manager of type $i$, $\bar{U}^i$, is endogenously determined in equilibrium (see below).
The type $\bar{i} \in I$ of agent to be hired as manager is then chosen by selecting the type which maximizes the firm’s value:

$$\max_{i \in I} V^i, \quad (12)$$

for $V^i$ as determined in (9).

This determines $\bar{U}^i$.
The type $\bar{i} \in I$ of agent to be hired as manager is then chosen by selecting the type which maximizes the firm’s value:

$$\max_{i \in I} V^i, \quad (12)$$

for $V^i$ as determined in (9).

Each consumer of a given type $j$ not chosen as a manager chooses:

$$\max \mathbb{E} u^j(c^j_0, c^j(s)) \quad (13)$$

subject to

$$c^j_0 = w^j_0 + \{ -k + q + pB - W^i(\phi, k, B; q, p) \} \theta^j_0 - q \theta^j - p b^j \quad (14)$$

$$c^j(s) = w^j(s) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^j + \min\left\{1, \frac{f(k, \phi; s)}{B}\right\} b^j \right], \quad \forall s, \quad (15)$$

and

$$b^j \geq 0, \quad \theta^j \geq 0, \quad \forall j \quad (16)$$

This determines $\bar{U}^i$.
In equilibrium, the bond and equity price maps faced by the firm must satisfy the following consistency conditions for all $k, B, \phi$:

\[ p(k, B, \phi) = \max_i EMRS^i(s) \min \left\{ 1, \frac{f(k, \phi; s)}{B} \right\} \]

\[ q(k, B, \phi) = \max_i EMRS^i(s) \max \left\{ f(k, \phi; s) - B, 0 \right\} \]
In equilibrium, the bond and equity price maps faced by the firm must satisfy the following consistency conditions for all \( k, B, \phi \):

\[
[p] \quad p(k, B, \phi) = \max_i \mathbb{EMRS}^i(s) \min \left\{ 1, \frac{f(k, \phi; s)}{B} \right\}
\]

\[
[q] \quad q(k, B, \phi) = \max_i \mathbb{EMRS}^i(s) \max \left\{ f(k, \phi; s) - B, 0 \right\}
\]

These consistency conditions guarantee the following:

\( i \) Investors correctly anticipate the payoff distribution of the risky bond and equity, given the observed levels of \( k \) and \( B \) and the manager’s choice of the risk composition parameter \( \phi \) given \( k \), \( B \) and his compensation package. In particular, investors correctly anticipate that \( \phi \) satisfies (10).

\( ii \) The value of the bond and the equity price maps faced by each firm equal, for each \( k, B, \phi \), the highest marginal valuation across all consumers, evaluated at their equilibrium consumption choices, of the return on these assets.
Proposition: At a competitive equilibrium of the economy with moral hazard, shareholders unanimously support the production and financial decisions of firms as well as the choice of management, \( \bar{k}, \bar{B}, \bar{\phi}, \bar{i}, \bar{x}_0, \bar{\theta}^{i,m}, \bar{b}^{i,m} \); that is, every agent \( i \) holding a positive initial amount \( \theta^i_0 \) of equity of the representative firm will be made - weakly - worse off by any other admissible choice of a firm (that is, any \( k', B', \phi', i', x'_0, \theta'^{i',m}, b'^{i',m} \) which satisfies (10) and (11)).
A consumption allocation \((c^i)_{i=1}^I\) is admissible if:

1. It is feasible: there exists a production plan \(k\) and a risk composition choice \(\phi\) of firms such that

\[
\sum_i c^i_0 + k \leq \sum_i w^i_0
\]

\[
\sum_i c^i(s) \leq \sum_i w^i(s) + f(k, \phi; s) \quad \text{for all } s
\]

2. It is attainable with the existing asset structure: that is there exists \(B\) and, for each consumer’s type \(i\), a pair \(\theta^i, b^i\) such that

\[
c^i(s) = w^i(s) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^i \\
+ \min\left\{1, \frac{f(k, \phi; s)}{B}\right\} b^i \right], \quad \forall s
\]

3. It is incentive compatible: given the production plan \(k\) and the financing plan \(B\), there exists \(\bar{i}\) such that:

\[
\mathbb{E}u^i(\bar{c}_0, \bar{w}(s)) + \left[ \max\{0, f(k, \phi; s) - B\} \theta^\bar{i} \\
+ \min\left\{1, \frac{f(k, \phi; s)}{B}\right\} b^\bar{i} \right] - v^\bar{i}(\phi) \geq
\]

\[
\mathbb{E}u^i(\bar{c}_0, \bar{w}(s)) + \left[ \max\{0, f(k, \phi'; s) - B\} \theta^\bar{i} \\
+ \min\left\{1, \frac{f(k, \phi'; s)}{B}\right\} b^\bar{i} \right] - v^\bar{i}(\phi')
\]

for all \(\phi' \in \Phi\).
Constrained Pareto optimality is now straightforwardly defined before with respect to the stronger notion of admissibility described above.
Constrained Pareto optimality is now straightforwardly defined before with respect to the stronger notion of admissibility described above.

And the First Welfare theorem readily applies. It can be established by an argument essentially analogous to the one used to establish the Pareto efficiency of competitive equilibria in Arrow Debreu economies.

**Proposition:** Competitive equilibria of the economy with moral hazard are constrained Pareto efficient.
The risk composition $\phi$ is not an unobservable choice of the manager of a firm, but rather is private information of the agent who is hired as manager of the firm at time $t = 0$, before the level of the firm’s capital $k$ and financial structure $B$ are chosen.
Unobservable risk composition: hidden information

- The risk composition $\phi$ is not an unobservable choice of the manager of a firm, but rather is private information of the agent who is hired as manager of the firm at time $t = 0$, before the level of the firm’s capital $k$ and financial structure $B$ are chosen.

- The cost of the compensation package for a manager of type $i$ is:

$$ W^i_\phi(k_\phi, B_\phi; q, p) = x^i_0 + \left[ \frac{q(k_\phi, B_\phi; \phi)(\theta^i,m - \theta^i,0) + p(k_\phi, B_\phi; \phi)b^i,m}{(1 - \theta^i,m)} \right] $$

$$ - \frac{\theta^i,m}{1 - \theta^i,0} \left[ p(k_\phi, B_\phi; \phi)B_\phi - k_\phi - x^i_0 \right] $$
The firm’s problem, given the type $i$ of the agent hired as manager, is then the following:

$$V^i = \max_{(k_{\phi}, B_{\phi}, x_{0\phi}, \phi, \theta_{\phi}^i, b_{\phi}^i, \phi) \in \Phi} \sum_{\phi \in \Phi} \Pr(\phi) \left[ -k_{\phi} + q(k_{\phi}, B_{\phi}; \phi) + p(k_{\phi}, B_{\phi}; \phi) - W_{\phi}^i(k_{\phi}, B_{\phi}; q, p) \right]$$

s.t.

$$\mathbb{E} u^i(0) + x_{0\phi}^i, w^i(s) + \left[ \max\{0, f(k_{\phi}, \phi; s) - B_{\phi}\} \theta_{\phi}^i, m + \min\{1, \frac{f(k_{\phi}, \phi; s)}{B_{\phi}}\} b_{\phi}^i, m \right] \geq \mathbb{E} u^i(0) + x_{0\phi}^i, w^i(s) + \left[ \max\{0, f(k_{\phi'}, \phi; s) - B_{\phi'}\} \theta_{\phi'}^i, m + \min\{1, \frac{f(k_{\phi'}, \phi; s)}{B_{\phi'}}\} b_{\phi'}^i, m \right]$$

for all $\phi$ and all $\phi' \neq \phi$

$$\sum_{\phi \in \Phi} \Pr(\phi) \mathbb{E} u^i(0) + x_{0\phi}^i, w^i(s) + \left[ \max\{0, f(k_{\phi}, \phi; s) - B_{\phi}\} \theta_{\phi}^i, m + \min\{1, \frac{f(k_{\phi}, \phi; s)}{B_{\phi}}\} b_{\phi}^i, m \right] \geq \bar{U}^i$$
It is tedious (very!) but not difficult to show that competitive equilibria for this economy also satisfy unanimity and constrained efficiency.
Unobservable leverage

- Debt-holders of each firm do not observe the total amount of debt issued by the firm in the market, i.e., its leverage.
Unobservable leverage

- Debt-holders of each firm do not observe the total amount of debt issued by the firm in the market, i.e., its leverage.
- The production function is given by $y = f(k; s_1)$. 
Unobservable leverage

- Debt-holders of each firm do not observe the total amount of debt issued by the firm in the market, i.e., its leverage.
- The production function is given by \( y = f(k; s_1) \).
- The firm’s problem becomes:

\[
V = \max_{k, B} -k + q(k, B) + p(k, B)B \\
\text{s.t.} \\
q(k, B) + p(k, B)B \geq q(k, B) + p(k, B)B' \text{ for all } B' \neq B
\]
The firm’s shareholders have an incentive to expand the amount of debt issued to increase the revenue from its sale.
The firm’s shareholders have an incentive to expand the amount of debt issued to increase the revenue from its sale.

This in turn reduces the equilibrium price of debt, as bondholders correctly anticipate the dilution of the payoff of the bonds issued: the only value of $B$ which satisfies the above incentive constraint is $B = \infty$ (and $p = 0$) and the firm defaults in every state on its debt which becomes then equivalent to equity.
The firm’s shareholders have an incentive to expand the amount of debt issued to increase the revenue from its sale.

This in turn reduces the equilibrium price of debt, as bondholders correctly anticipate the dilution of the payoff of the bonds issued: the only value of $B$ which satisfies the above incentive constraint is $B = \infty$ (and $p = 0$) and the firm defaults in every state on its debt which becomes then equivalent to equity.

Competitive equilibria for this economy continue to satisfy unanimity and constrained efficiency properties.
The technology of the firm is $f(k, \phi; s)$, but $\phi$ represents now its manager’s quality, which affects the stochastic structure of the firm’s future output.
Unobservable manager’s quality - adverse selection

- The technology of the firm is $f(k, \phi; s)$, but $\phi$ represents now its manager’s quality, which affects the stochastic structure of the firm’s future output.
- Managers receive benefits from control $\varsigma_\phi$, in units of the consumption good, diverted from the firm’s output at time 1.
The technology of the firm is $f(k, \phi; s)$, but $\phi$ represents now its manager’s quality, which affects the stochastic structure of the firm’s future output.

Managers receive benefits from control $\varsigma_\phi$, in units of the consumption good, diverted from the firm’s output at time 1.

The problem of the (shareholders) of the firm is choosing the production plan $k$ and the financial structure $B$, as well as the type of agent serving as manager, were the type is now given by an observable component $i$ and a second, unobservable component, $\phi$, together with the associated compensation package.
Unobservable manager’s quality - adverse selection

- The technology of the firm is $f(k, \phi; s)$, but $\phi$ represents now its manager’s quality, which affects the stochastic structure of the firm’s future output.
- Managers receive benefits from control $\varsigma_\phi$, in units of the consumption good, diverted from the firm’s output at time 1.
- The problem of the (shareholders) of the firm is choosing the production plan $k$ and the financial structure $B$, as well as the type of agent serving as manager, were the type is now given by an observable component $i$ and a second, unobservable component, $\phi$, together with the associated compensation package.
- The manager’s compensation package consists of an amount $x_0$ of the consumption good at date 0, $\theta^m$ units of equity and $b^m$ of bonds.
Redefine the size of the mass of firms and assume that it is less than $\chi^i_\phi$, the mass of agents of type $i$ and quality $\phi$. 
Redefine the size of the mass of firms and assume that it is less than \( \chi_\phi^i \), the mass of agents of type \( i \) and quality \( \phi \).

Assume that the firms’ technology is such that some production and financing levels and a compensation package can always be found so as to separate managers of different unobservable types (\textit{single crossing property}): The firms’ technology is such that, for any tuple \( v = (x_0, b, \theta, B, k) \in \mathbb{R}^5_+ \) the vectors

\[
D_v \mathbb{E} u^i (w^i_0 + x_0, w^i(s)) + \left[ \varsigma_\phi + \max\{0, f(k, \phi; s) - \varsigma_\phi - B\} \theta \\
+ \min\{1, \frac{f(k, \phi; s) - \varsigma_\phi}{B} \} b \right],
\]

\( \phi \in \Phi \)

are linearly independent.
The cost of the compensation package for a manager of type $i$ and quality $\phi$ is

$$W^i(\phi, k, B; q, p, ) = \left[ x_0^i + \frac{q(k, B, \phi)(\theta^{i,m}_0 - \theta^{i,m}_0) + p(k, B, \phi)b^{i,m}}{(1-\theta^{i,m}_0)} - \frac{\theta^{i,m}_0}{1-\theta^{i,m}_0} [p(k, B, \phi)B - k - x_0^i] \right]$$
The value maximization problem of a firm who is hiring as manager an agent of type $i$ and unobservable quality $\phi$ takes the following form:

$$V^i(\phi) = \max_{k, B, x^i_0, \theta^i, m, b^i, m} -k + q(k, B, \phi) + p(k, B, \phi) B - W^i(\phi, k, B; q, p)$$

subject to

$$\bar{U}^i \geq \mathbb{E} u^i(w^i_0 + x^i_0, w^i(s)) + \left[ \varsigma_{\phi'} + \max\{0, f(k, \phi'; s) - \varsigma_{\phi'} - B\} \theta^i, m \right]$$

$$+ \min\left\{ 1, \left( \frac{f(k, \phi'; s) - \varsigma_{\phi'}}{B} \right) b^i, m \right\}$$

for all $\phi' \neq \phi$  \hspace{1cm} (22)

and

$$\bar{U}^i \leq \mathbb{E} u^i(w^i_0 + x^i_0, w^i(s)) + \left[ \varsigma_{\phi} + \max\{0, f(k, \phi; s) - \varsigma_{\phi} - B\} \theta^i, m \right]$$

$$+ \min\left\{ 1, \left( \frac{f(k, \phi; s) - \varsigma_{\phi}}{B} \right) b^i, m \right\}$$

(23)
If at equilibrium the optimal choice of the firm is to hire a single quality type $\bar{\phi}$ as manager, we call the equilibrium \textit{separating}, following Rothschild-Stiglitz (1979). On the other hand, if the optimal choice is to hire a nonsingleton set $\Phi' \subseteq \Phi$ of quality types, we say the equilibrium is (partially) \textit{pooling}, where agents of different quality become managers.
If at equilibrium the optimal choice of the firm is to hire a single quality type $\phi$ as manager, we call the equilibrium *separating*, following Rothschild-Stiglitz (1979). On the other hand, if the optimal choice is to hire a nonsingleton set $\Phi' \subset \Phi$ of quality types, we say the equilibrium is (partially) *pooling*, where agents of different quality become managers.

By a similar argument as in Bisin and Gottardi (2006), we can show that competitive equilibrium are necessarily separating and moreover that, differently from the economy with moral hazard, equilibrium allocations are not in general constrained Pareto efficient, in the sense of Diamond (1967) and Prescott-Townsend (1984). On the other hand, unanimity still holds in this environment.
Conclusions

- In the presence of financial frictions, such as incomplete markets and/or borrowing restrictions and informational asymmetries between managers and shareholders or bondholders, production decisions are not necessarily separated from financing decisions.
Conclusions

- In the presence of financial frictions, such as incomplete markets and/or borrowing restrictions and informational asymmetries between managers and shareholders or bondholders, production decisions are not necessarily separated from financing decisions.

- Corporate financing decisions, in these economies, are therefore interesting and one can investigate their interaction with the properties of the equilibrium allocation and prices.
Conclusions

- In the presence of financial frictions, such as incomplete markets and/or borrowing restrictions and informational asymmetries between managers and shareholders or bondholders, production decisions are not necessarily separated from financing decisions.

- Corporate financing decisions, in these economies, are therefore interesting and one can investigate their interaction with the properties of the equilibrium allocation and prices.

- The conceptual problems usually associated with modeling firm decisions when markets are incomplete or with asymmetric information can be overcome with appropriate, and natural modeling choices.
Conclusions

- In the presence of financial frictions, such as incomplete markets and/or borrowing restrictions and informational asymmetries between managers and shareholders or bondholders, production decisions are not necessarily separated from financing decisions.

- Corporate financing decisions, in these economies, are therefore interesting and one can investigate their interaction with the properties of the equilibrium allocation and prices.

- The conceptual problems usually associated with modeling firm decisions when markets are incomplete or with asymmetric information can be overcome with appropriate, and natural modeling choices.

- Corporate finance is thus ready to be passed on to macroeconomists.