Irving Fisher, in a Ph.D. thesis submitted to Yale University in 1891, defined a fundamental market model. A remarkable convex program, given by Eisenberg and Gale in 1959, captures, as its optimal solutions, equilibrium allocations for the linear utilities case of this market. In recent joint work with Devanur, Papadimitriou and Saberi, we gave a combinatorial, polynomial time algorithm for computing equilibrium allocations, and hence optimal solutions to the Eisenberg-Gale program, for this case.

Our algorithm uses the classical primal-dual paradigm -- not in its usual setting of LP-duality theory but in the enhanced setting of KKT conditions and nonlinear convex programs. In this talk, I will describe the difficulties raised by this new setting and show how our algorithm circumvents them. I will also present a generalization to spending constraint utilities and show how they can be useful in Google's AdWords market. Finally, I will allude to several basic algorithmic questions that arise from these two works.