Matching with Transfers
2015 Koopmans Lecture, Yale University
Part 2: Empirical applications

Pierre-André Chiappori

Columbia University

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Roadmap

1. Empirical implementation
2. The US education puzzle
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - Matching patterns and behavior: CCM 2015
3. Job matching by skills Lindenlaub (2014)

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- ... unless we can observe more than only matching patterns!

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- Alternative approach: use the stability inequalities

$$u_i + v_j \geq g_{ij}^{IJ} \quad \text{for any} \quad (i, j)$$

  $\rightarrow$ large number (one inequality *per potential couple*)

Empirical implementation

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Theorem

Under S, there exists U^{IJ} *and* V^{IJ} *such that* $U^{IJ} + V^{IJ} = Z^{IJ}$ *and for any match* $(i \in I, j \in J)$

$$\begin{aligned} u_i &= U^{IJ} + \alpha_i^{IJ} \\ v_j &= V^{IJ} + \beta_j^{IJ} \end{aligned}$$

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- Lastly, *parcimony!*

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A NSC for $i \in I$ being matched with a spouse in J is:

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  - and expected utility:

$$\bar{u}^I = E\left[\max_J \left(U^{IJ} + \alpha_i^{IJ}\right)\right] = \ln\left(\sum_J \exp U^{IJ} + 1\right) = -\ln\left(a^{I0}\right)$$

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  Then G_I^* is convex, and envelope theorem: $\partial G_I^* / \partial \gamma^J = U^{IJ}$
- $G^*\left(\gamma^I\right)$ is called the *generalized entropy* of the corresponding discrete choice problem

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    - ... for instance the 'supermodular core' ('preferences for assortativeness')

$$Z_t^{II} + Z_t^{JJ} - Z_t^{IJ} - Z_t^{JI} = K \Rightarrow Z_t^{IJ} = \zeta_t^I + \xi_t^J + Z_0^{IJ}$$

    - ... or at least some restrictions on its variations (e.g. linear trend): $Z_0^{IJ} + z^{IJ} \times t$
  - Alternatively, *more information is needed*

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  - or simulated moments ...
  - ... especially since simulating the model is easy (linear optimization)

Empirical implementation 3: matching patterns and transfers

- Basic reference: *hedonic models*
- Strong, non parametric identification results
- See f.i. Ekeland, Heckman and Nesheim (2004), Heckman, Matzkin and Nesheim (2010), Chernozhukov, Galichon and Henry (2014) and Nesheim (2013)

Roadmap

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   - Two-dimensional version: Low (2014)
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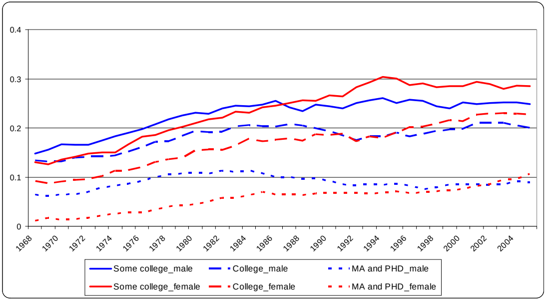
- Motivation: remarkable increase in female education, labor supply, incomes during the last decades.



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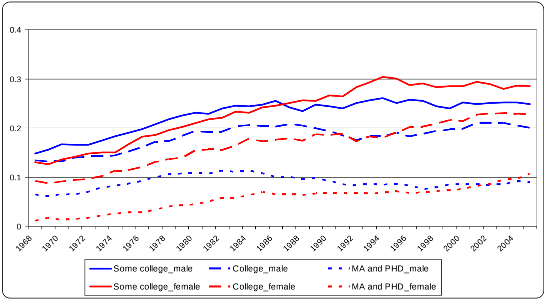

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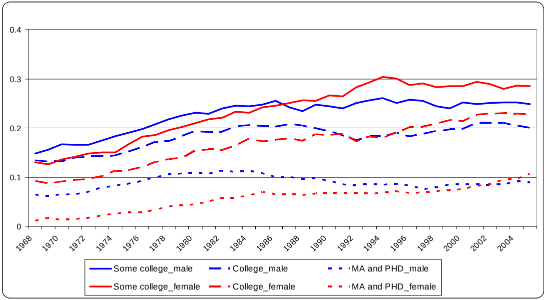

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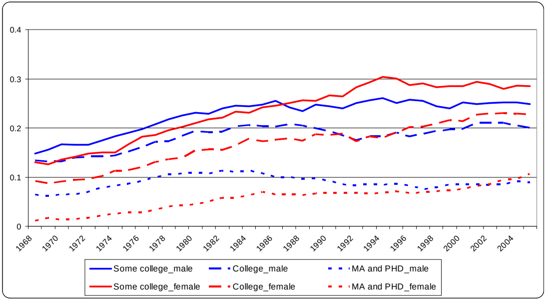

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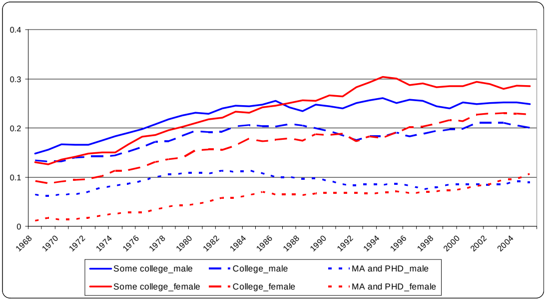

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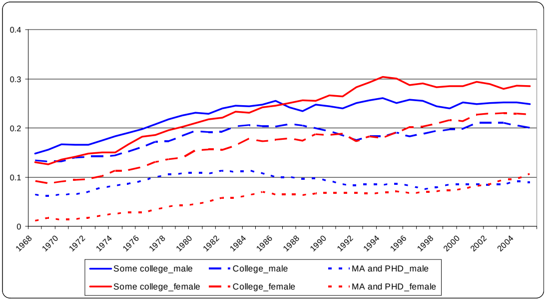

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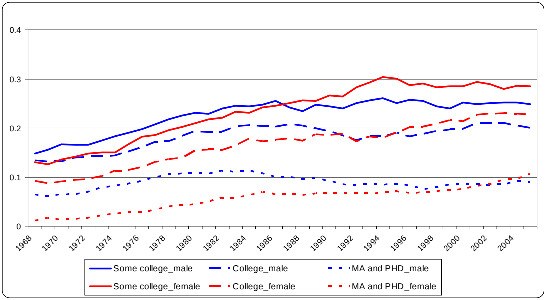

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  - But a structural model is needed!

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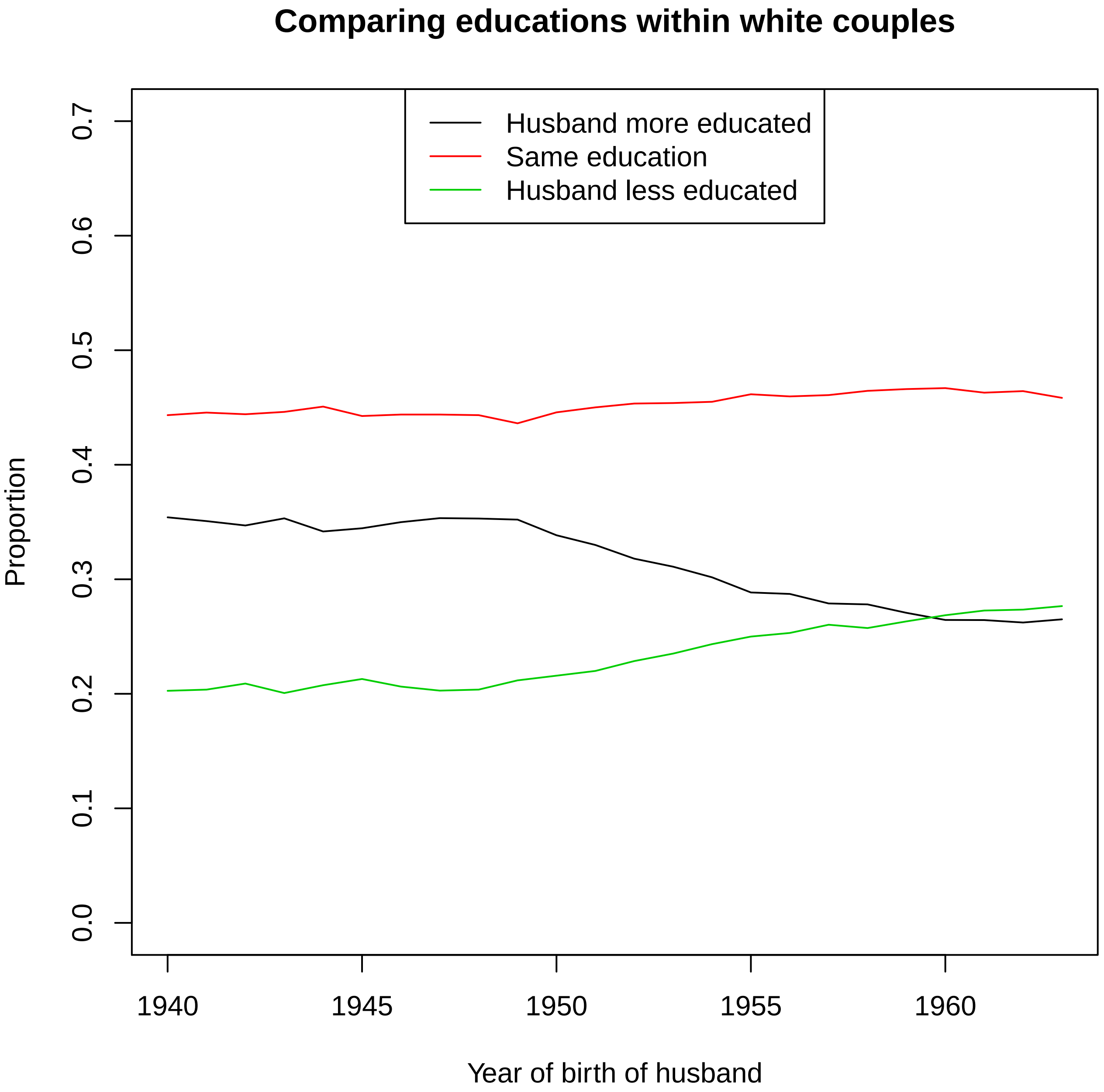
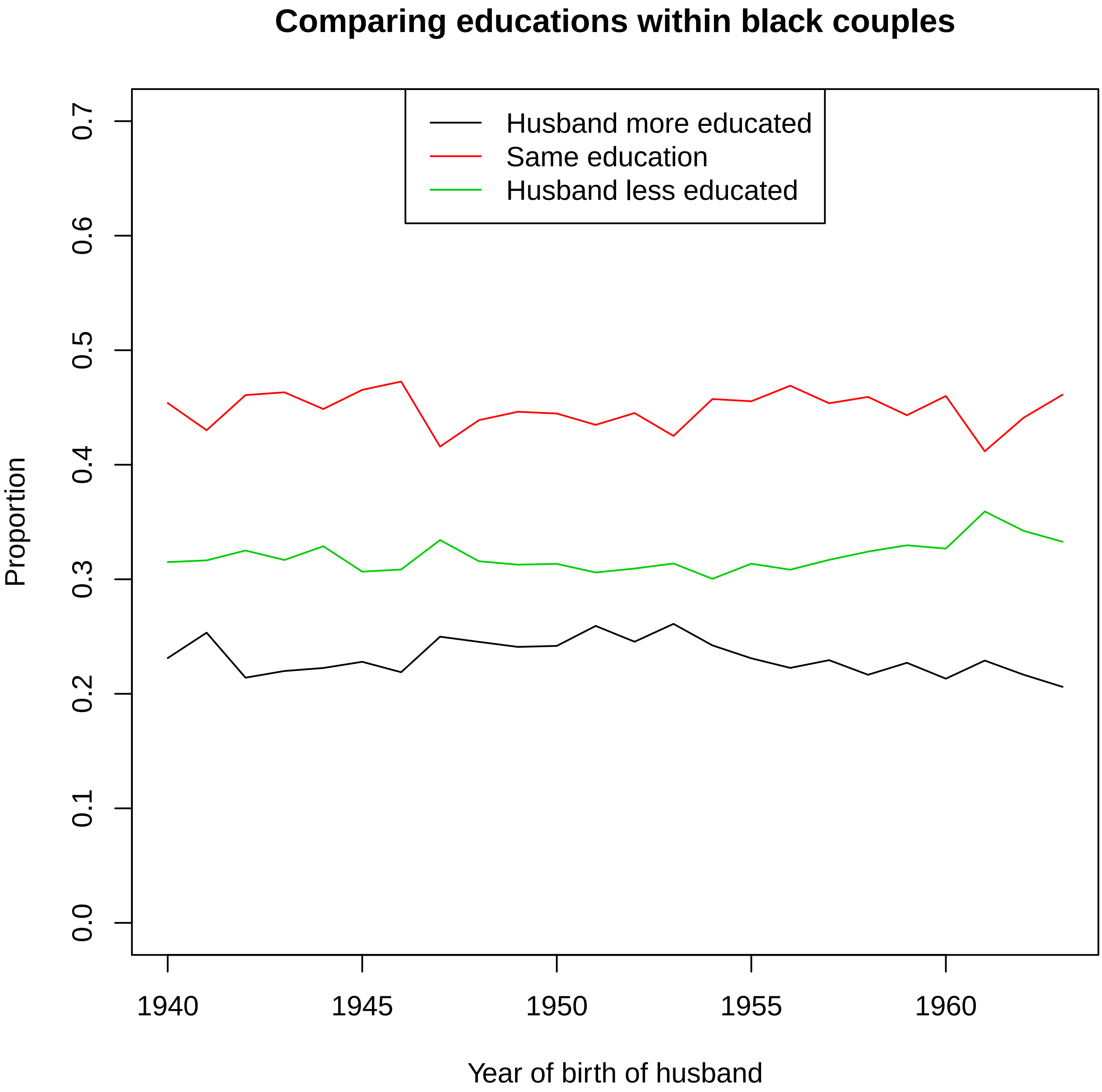
What do raw data say?



Comparing educations within white couples
Husband more educated
Same education
Husband less educated
Proportion
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1940
1945
1950
1955
1960
Year of birth of husband



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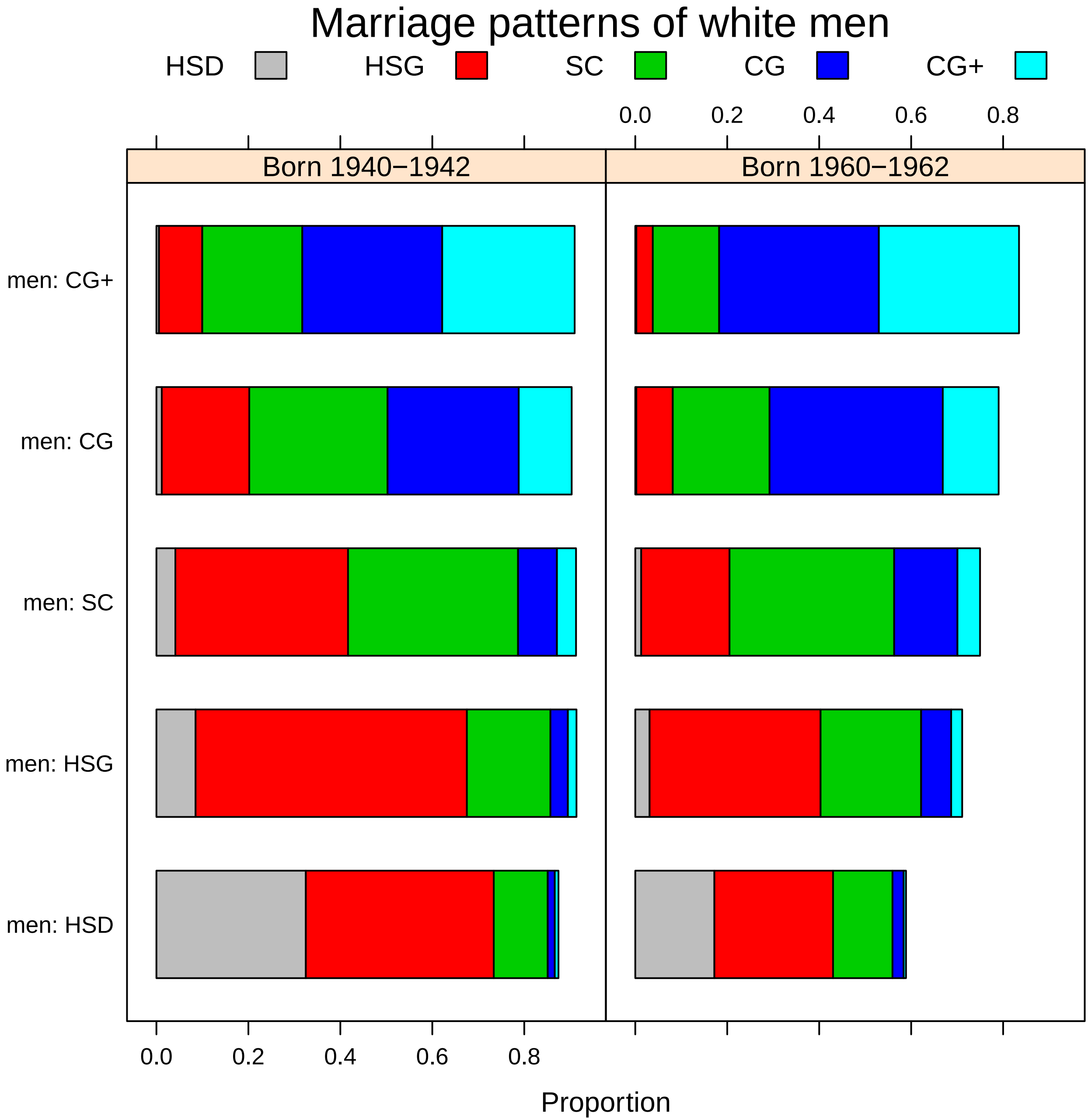
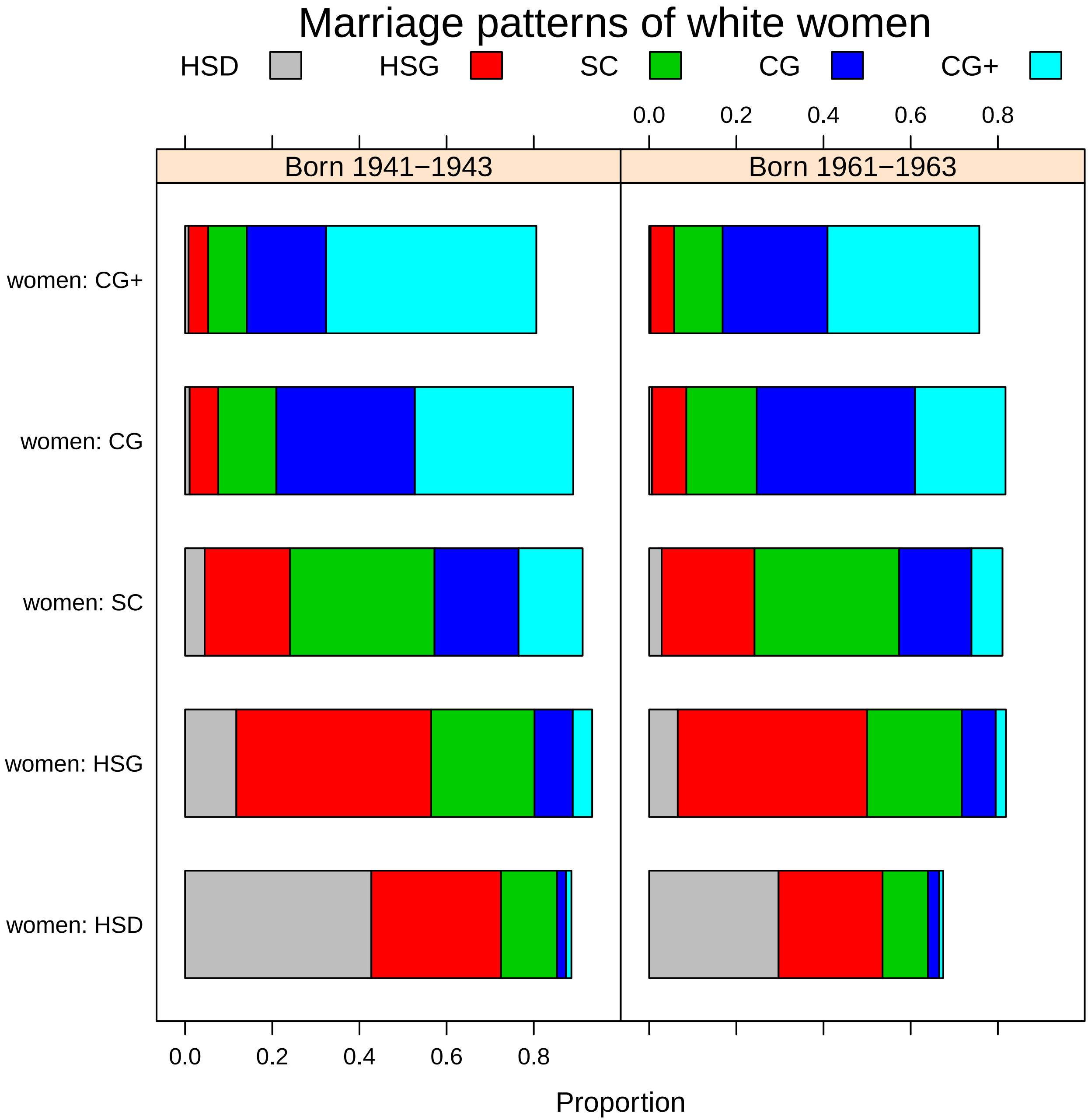


Marriage patterns of white men
HSD
HSG
SC
CG
CG+
Born 1940−1942
Born 1960−1962
men: CG+
men: CG
men: SC
men: HSG
men: HSD
0.0
0.2
0.4
0.6
0.8
Proportion


Marriage patterns of white women
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HSG
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CG
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Born 1941−1943
Born 1961−1963
women: CG+
women: CG
women: SC
women: HSG
women: HSD
0.0
0.2
0.4
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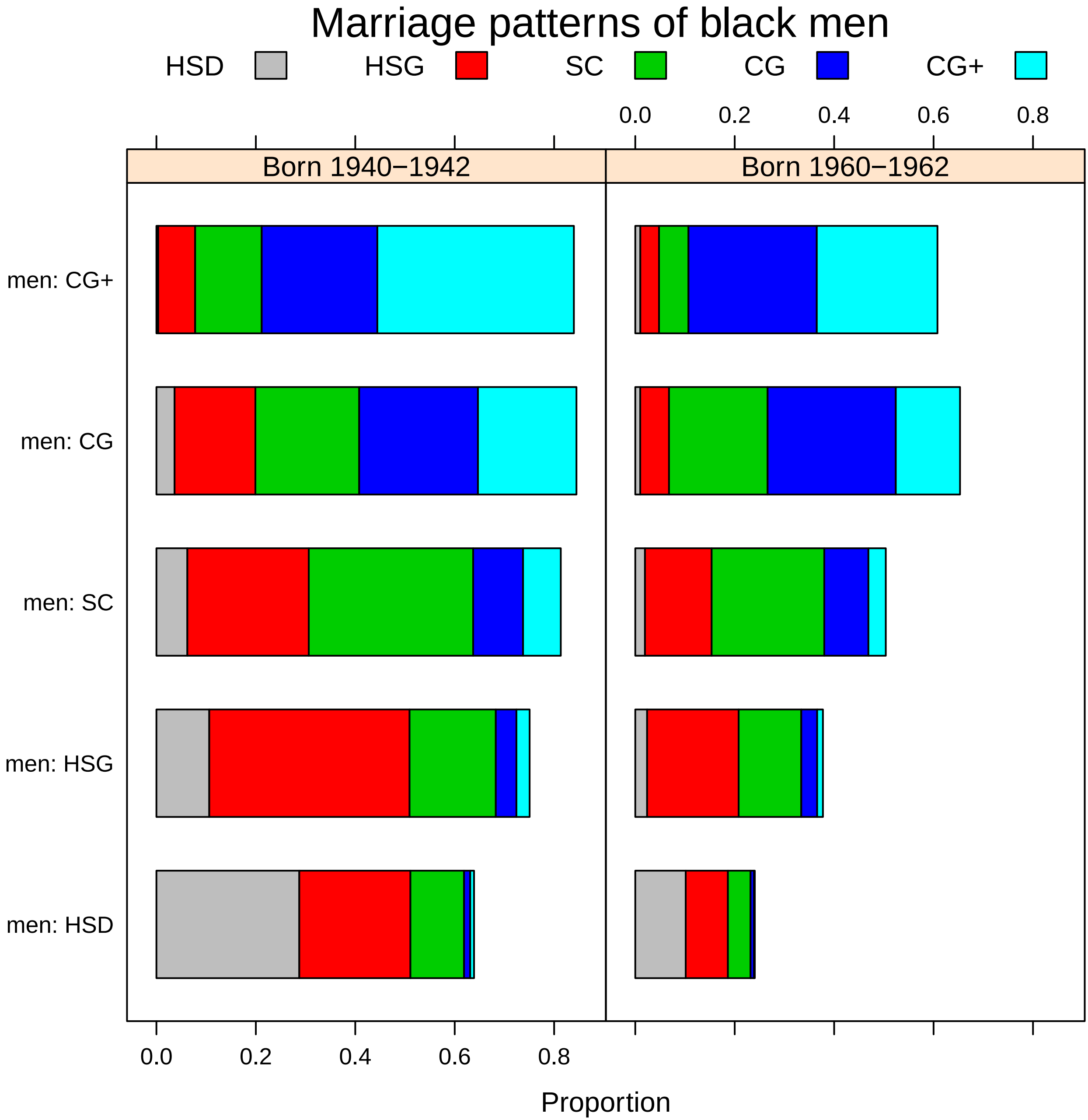
0.8
Proportion



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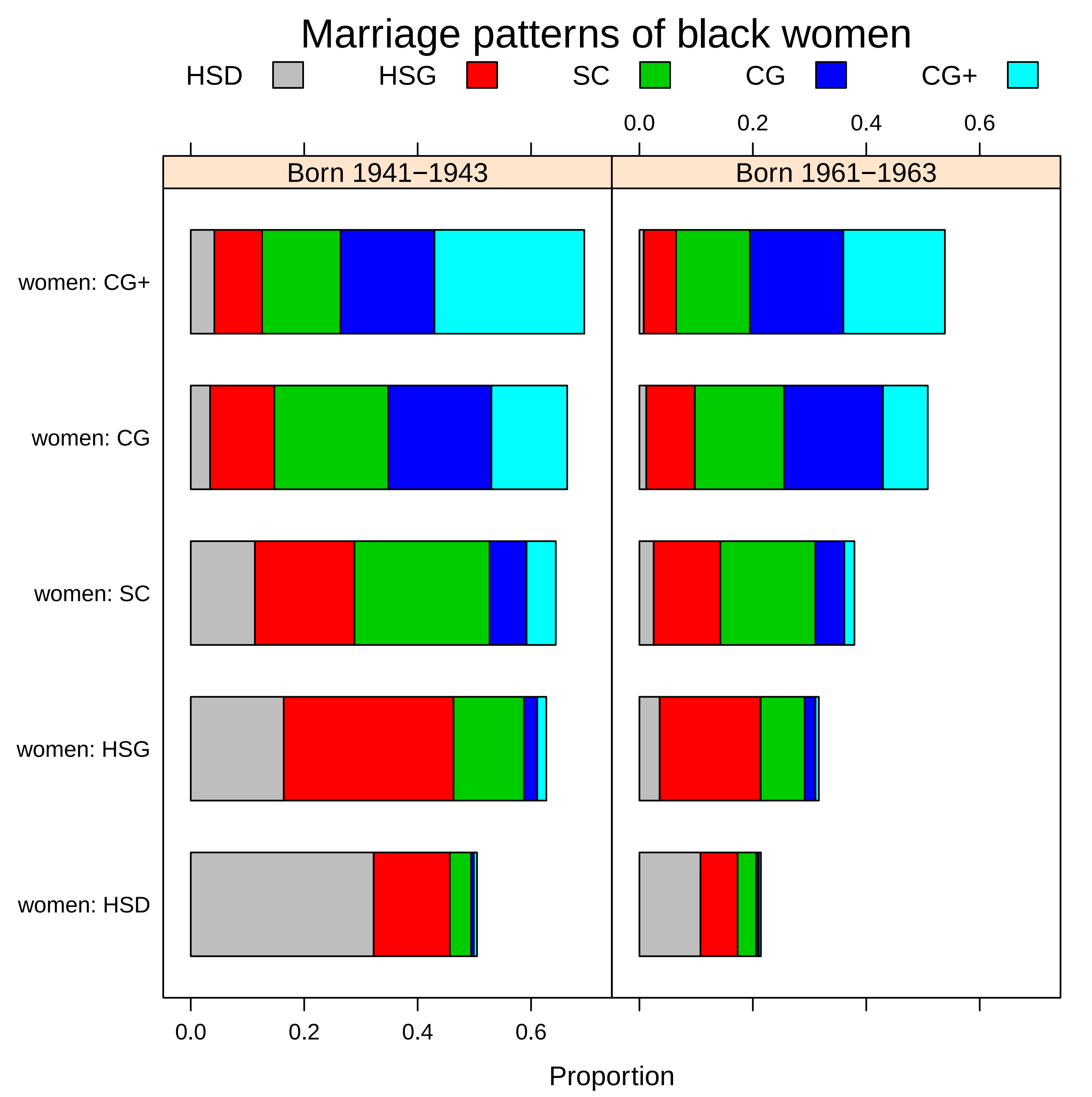
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Results: preferences for assortativeness

		Women				
		HSD	HSG	SC	CG	CG+
Men	HSD	0.0118***	0.0067***	0.0146***	-0.0023	-0.0366
		(0.0015)	(0.0012)	(0.0018)	(0.0017)	(0.0017
	HSG	-0.0237***	0.0024	0.011***	-0.0009	-0.01**
		(0.0011)	(0.0008)	(0.0008)	(0.0009)	(0.0014
	SC	-0.0198***	-0.001	0.0056***	0.004***	0.0001
		(0.0013)	(0.0006)	(0.0013)	(0.0015)	(0.0014
	CG	0.0187***	-0.0011	-0.0093***	0.0079***	0.015**
		(0.0012)	(0.0009)	(0.0013)	(0.0015)	(0.0018
	CG+	0.0436***	0.0055***	-0.0087***	-0.0059***	0.0149*
		(0.0004)	(0.0006)	(0.0008)	(0.001)	(0.0017

Table: Slopes - linear extension

Results: college premium

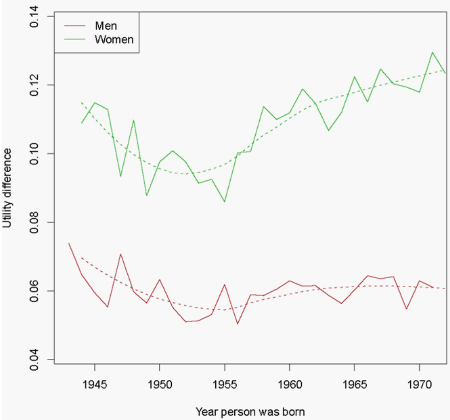

Figure 12: The marital college premium

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- Impact on marital prospects?

Model

- Two commodities, private consumption and child expenditures; utility:

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- Transferable utility: any efficient allocation maximizes $u_h + u_w$; therefore surplus with a child

$$s\left(y_h, y_w\right) = \frac{\left(y_h + y_w + 1\right)^2}{4}$$

and without a child ($Q = 0$)

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therefore, if π probability of a child:

$$s\left(y_h, y_w\right) = \pi \frac{\left(y_h + y_w + 1\right)^2}{4} + \left(1-\pi\right)\left(y_h + y_w\right)$$

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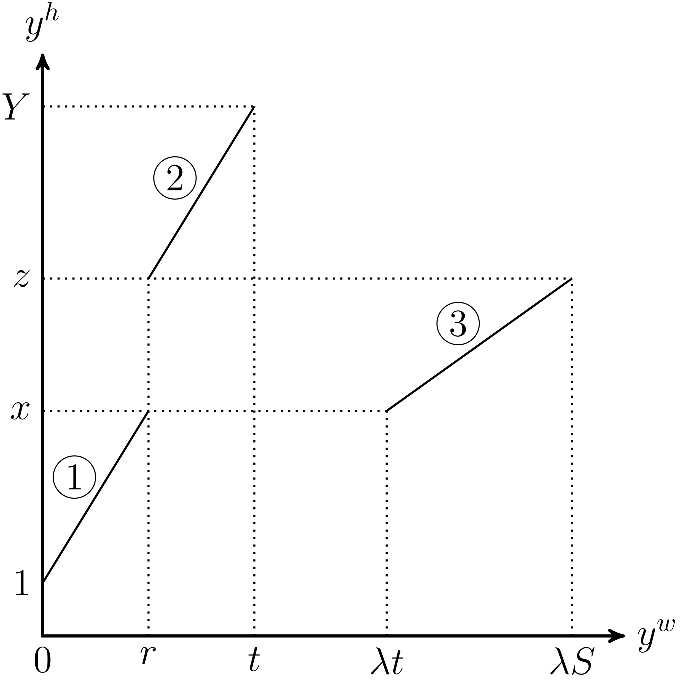

y^h
Y
z
x
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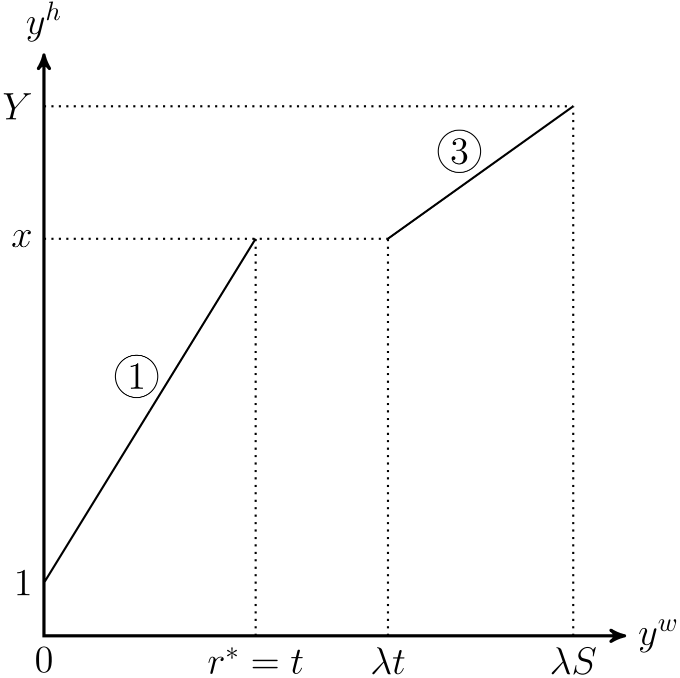

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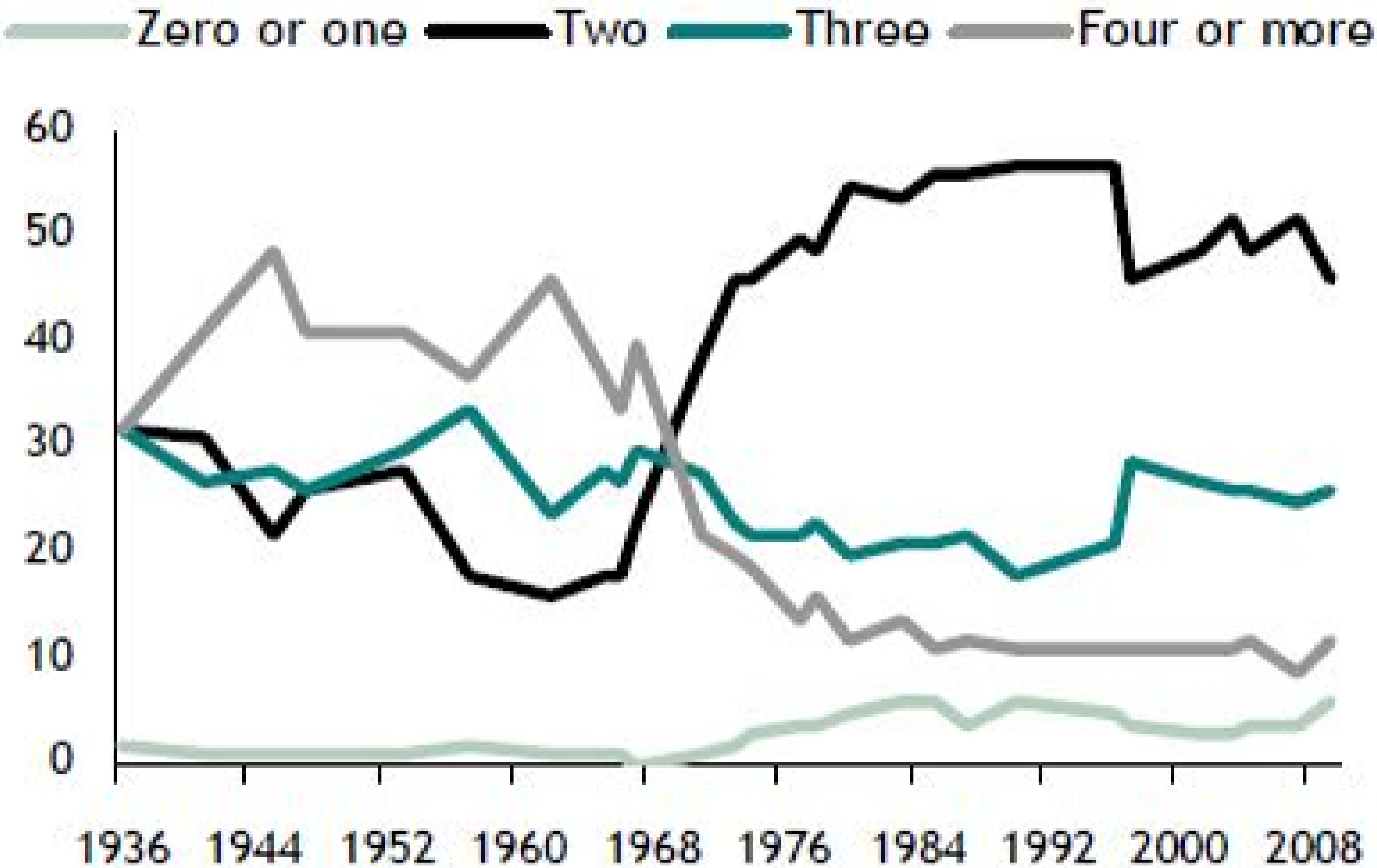

Notes: "Don't know/refused" responses not shown. Respondents were asked: "What is the ideal number of children for a family to have?"

Source: Gallup, 1936-2007; Pew Research Center, 2009

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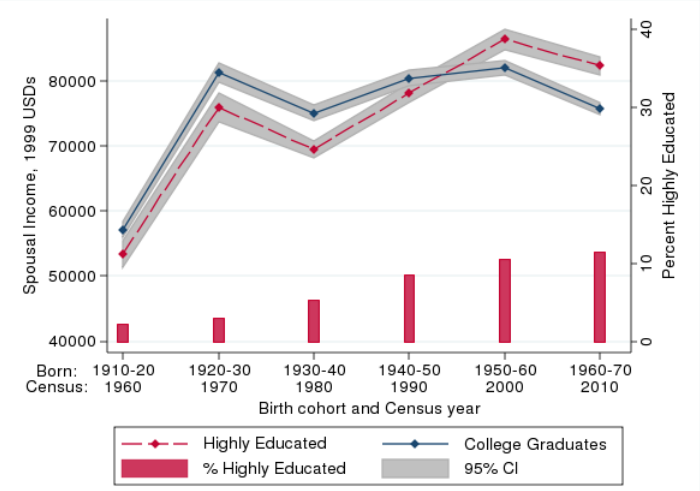
Spousal income by wife's education level, white women 41-50



Roadmap

1. Empirical implementation
2. *The US education puzzle*
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - *Matching patterns and behavior: CCM 2015*
3. Job matching by skills Lindenlaub (2014)

Matching patterns and behavior

Chiappori, Costa Dias, Meghir 2015

- The basic motivation for this project is to understand how policy affects individual life-cycle decisions
- Long term effects will change education choices and the marriage market
- In turn this will have effects on labor supply and will have intergenerational impacts
- Two fundamental, Beckerian insights: Notion of Human Capital and Matching as an equilibrium phenomenon

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    - ... including initial productivity (or HC) shock

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- Finally period 1: education decisions

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  - The fictitious game is much easier to simulate (matching $\longrightarrow$ linear programming)

Roadmap

1. Empirical implementation
2. The US education puzzle
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - Matching patterns and behavior: CCM 2015
3. *Job matching by skills Lindenlaub (2014)*

Job matching by skills (Lindenlaub 2014)

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- $\rightarrow$ Wages more convex in cognitive but less convex in manual skills
- $\rightarrow$ Increased wage inequality along the cognitive dimension, compressed inequality in the manual dimension.

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Conclusion

1. Frictionless matching: a powerful and tractable tool for theoretical analysis, especially when not interested in frictions
2. Crucial property: intramatch allocation of surplus derived from equilibrium conditions
3. Applied theory: many applications (abortion, female education, divorce laws, children, ...)
4. Can be taken to data; structural econometric model, over identified
5. Multidimensional versions: index (COQD 2010), general (CMcCP 2015)
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