Trade Integration, Global Value Chains, and Capital Accumulation*

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Abstract

Motivated by increasing trade and fragmentation of production across countries, accompanied by income convergence by many emerging economies, we build a dynamic two-country model featuring sequential, multi-stage production and capital accumulation. As trade costs decline over time, global-value-chain (GVC) trade expands across countries, particularly more in the faster growing country, consistent with the empirical pattern. Via Heckscher-Ohlin forces, GVC trade can generate back-and-forth feedback between comparative advantage and capital accumulation (growth). Moreover, GVC trade increases both steady-state and dynamic gains from trade.

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1 Introduction

A key feature of post-World War II globalization has been the increase in vertical specialization or global value chains (GVCs). Instead of exporting goods produced entirely within its borders, each country has increasingly specialized in particular stages of a good's production. Early evidence of this phenomenon was provided by Hummels, Ishii, and Yi (2001). Recently, Johnson and Noguera (2017) document that the domestic value-added embodied in gross exports (VAX) fell from 82 percent to 69 percent of gross exports from 1970 to 2007. Moreover, GVC trade grew especially rapidly during the growth miracle period of many countries, such as the four Asian "Tiger" economies, Ireland, China, and Malaysia. Johnson and Noguera (2017) show that countries that expanded GVCs more were those with higher GDP growth in their sample of more than 40 countries over the period 1970–2007.

In this paper, we build a dynamic two-country model featuring sequential, multi-stage production and capital accumulation to study the joint dynamics of investment, growth and GVC trade. Specifically, we introduce capital accumulation and growth into the GVC trade model of Antràs and de Gortari (2020). There are two main findings. First, via Heckscher-Ohlin forces, we show that GVCs can generate back-and-forth feedback between comparative advantage and capital accumulation (growth). Second, with our model, we find that GVCs increase both steady-state and dynamic gains from trade.

In the model, there is a large number of varieties, each of which requires multiple stages of production. In every country and for each variety, there is a lead firm that decides the optimal supply chain for the production of that variety (i.e., the location for each stage of production). The lead firm independently draws an efficiency level for any possible chain from a Fréchet distribution. There is a country-stage-specific productivity level that also affects the attractiveness of each chain. Moreover, stages differ in the factor intensity of capital and labor, meaning that capital abundant countries are more likely to engage in the capital intensive stage of production. Thus, our framework features both Ricardian and Heckscher-Ohlin (H-O) comparative advantage forces. Finally, trade costs are incurred at each stage when goods cross international borders, further shaping the pattern of trade. Capital accumulation results from a consumption-saving problem as in the neoclassical growth model and is influenced by comparative advantage forces across stages whereby the pattern of trade and aggregate growth evolve in an interdependent fashion.

To clearly elicit the implications of trade integration in such a model, we focus on a setting with two countries and two stages. Each country starts in the autarky steady state at period

0. Trade costs decline at a constant rate from period 1 to 50, and remain constant afterwards. Agents have perfect foresight. GVC trade patterns and capital adjust endogenously over time until each country reaches the new steady state. To highlight the role of GVC trade and capital accumulation, we also study two counterfactual cases: one without GVC trade, in which trade is allowed only for the last stage, i.e., finished goods, and one with fixed capital in which investment is exogenous, and in each period, equal to the value of the depreciated capital. These two counterfactual cases, together with the baseline, are solved numerically to study the entire transition dynamics.[1]

We start with symmetric countries. As trade integration deepens, import shares in each country and each stage increase. Fragmentation across borders occurs more in earlier stages of production, as pointed out by Antràs and de Gortari (2020). This is because trade costs apply to the gross value, and the later stages compound trade costs from all previous stages; hence, trade in earlier stages effectively incurs lower costs. Owing to the symmetry, the presence of endogenous capital accumulation does not affect the dynamics of GVC trade, but GVC trade does impact the dynamics of investment and capital accumulation quantitatively and qualitatively. Without GVC trade, investment declines initially in response to trade integration, lowering capital in the early periods, due to the incentive to postpone investment. With GVC trade, investment and capital rise throughout the transition. This is because production efficiencies are realized sooner and to a greater degree, which more than offsets the incentive to postpone investment.

We next explore asymmetric countries. In the initial steady state, country 1 has lower TFP and a lower capital-labor ratio than country 2. In addition to trade integration, this scenario has TFP growth in country 1 as a driving force. Country 1's TFP growth is set so that after 50 periods, the country has the same TFP as country 2. For simplicity, TFP is stage-neutral so that it is the same across the two stages in each country. Differences in the capital-labor ratio across countries give rise to H-O forces in which the poor country performs more of the stage-2 labor-intensive production, and the rich country performs more of the stage-1 capital-intensive production. Over time, as the poor country experiences faster productivity growth, it accumulates more capital and the H-O comparative advantage diminishes. This shifting comparative advantage alters the GVC trade patterns substantially, thus providing a channel from growth to trade patterns. Moreover, the pattern of GVC trade reinforces the growth patterns: as the fast growing country moves upstream, it accumulates

[1] We employ a similar method as in Ravikumar, Santacreu, and Sposi (2019) to compute the transitional dynamics in the open economy.

even more capital.

We use our model structure to derive an expression for the domestic value-added content of exports (DCE). During the trade integration process, DCE declines in both countries. This is because GVC trade becomes increasingly more important, especially for the smaller country, and international production fragmentation lowers the DCE. More importantly, the decline in DCE differs across countries according to their comparative advantage, which is determined by the endogenous capital accumulation process. The country with a lower capital-labor ratio takes on a greater share of stage 2 production, contributing to a lower overall DCE. Over time, as the small country accumulates capital at a faster pace, it takes on a greater share of stage 1 production, dampening the decline in its DCE. Indeed, in a model with fixed capital, the DCE for the fast growing country would decline even more, and for the large country, even less.

Our model's implications for the DCE dynamics are consistent with empirical findings. First, in our model the small, fast growing country has a comparative advantage in the labor-intensive final stage, implying a larger reduction in DCEs during the transition. This is consistent with the above-mentioned empirical pattern documented in Johnson and Noguera (2017): countries with higher output growth experience larger expansions in GVC trade, i.e., bigger reductions in the DCE. Second, our model implies that country 1's DCE rises once the trade integration process stops (constant trade costs) and productivity growth converges zero, because capital accumulation persists, further diminishing comparative advantage. This implication of the non-monotonic dynamics of the DCE is also consistent with the experience of fast-growing emerging economies. From 2010 to 2016, the DCE increased by about 6 percentage points in China and 8 percentage points in South Korea.

Lastly, we show that the transition dynamics are important for the gains from trade, and the presence of GVC trade and capital accumulation amplifies the gains from trade. The dynamic gains, computed over the entire transition period, are much smaller than the steady state gains. This is because consumption gradually rises to the new steady state level owing to consumption smoothing and capital accumulation. GVC trade amplifies the dynamic and steady state gains by about four-fold, relative to the no-GVC case, by promoting greater efficiency gains and capital accumulation. Capital accumulation amplifies the steady state gains substantially, but not as much for the dynamic gains. This is again because consumption is postponed to make room for investment during the transition, which implies much lower dynamic gains than the steady state gains. In addition, the dynamic gains from trade are greater for the small, fast-growing country: 4.8% compared to 3% for the larger country

Broadly, there have been two sets of frameworks embodying intermediate goods: "spiders" and "snakes". The former framework involves "roundabout" production, in which output of any firm is produced in one stage using a common bundle of all output as intermediates. The latter framework involves multiple, sequential, stages of production with each stage using specific inputs produced in the previous stage. The snake framework allows for explicit production fragmentation across countries, i.e., the location of production of each stage is endogenous. Hereafter, "GVC" refers to a snake framework. We compare the gains from trade in our snake model to those generated by a standard spider model and find that, under a comparable calibration, the gains in the snake model exceed those from the spider model. For example, with symmetric countries, when trade costs are reduced to generate an increase in the import share of GDP from 0 (autarky) to 0.16, the dynamic gains from trade are twice as large in the snake model relative to the spider model: 3.6% versus 1.8%.

The trade literature has made substantial progress in modeling GVC trade in recent years. Yi (2003) demonstrated that production fragmentation across two stages magnifies the effects of reductions in tariffs and other *ad valorem* trade costs, relative to standard trade models with just one stage of production. Most recently, Antràs and de Gortari (2020) have extended and generalized the snake framework to many stages and many countries in a tractable way. Both of the above papers have shown that GVC trade enlarges the gains from trade. However, existing analyses of the snake framework have abstracted from transitional dynamics and endogenous capital accumulation. Other recent papers include de Gortari (2019), Johnson and Moxnes (2019), Lee and Yi (2018), Fally and Hillberry (20018), Antràs and Chor (2013), Costinot, Vogel, and Wang (2013), Yi (2010), and Connolly and Yi (2015). The latter paper and Yi (2003) incorporate endogenous capital, but their analysis does not study transition dynamics, and focuses only on steady-states.

Our paper is also related to the quantitative Eaton-Kortum trade literature. Most of the studies in this literature model supply chains using a spider framework in a static setting. Several papers include capital accumulation in multi-country multi-sector trade models (i.e., Alvarez, 2017; Eaton, Neiman, and Kortum, 2016; Eaton et al., 2016; Levchenko and Zhang, 2016; Ravikumar, Santacreu, and Sposi, 2019). In addition to lower trade costs boosting investment and capital accumulation, our model has H-O forces, which give rise to cross-country differences in investment dynamics.

Finally, our paper is related to the literature on dynamic Heckscher-Ohlin trade models. This literature goes back to the 1960s. Recent contributions include Ventura (1997), Atkeson and Kehoe (1998), Cuñat and Maffezzoli (2004), Cuñat and Maffezzoli (2007), Bajona and Kehoe (2010), and Caliendo (2011). Of these, only Cuñat and Maffezzoli (2007) studies a scenario with declining trade costs over time. The other papers study free trade only, or compare free trade to autarky.

In the next section, we lay out the model. Then, in section 3, we numerically study the steady-state, transition dynamics, domestic content in exports, and welfare implications of the model. We pay particular attention to the interaction between GVCs and capital accumulation as a propagation mechanism from trade barriers to trade patterns and growth. In section 4, we further analyze the importance of H-O forces and the snake structure of production. The final section concludes.

2 Model

Our model extends a one-sector version of Antràs and de Gortari (2020) to a dynamic setting. Time t is discrete and infinite. The world economy consists of N countries, indexed by n. Each country is populated by a representative household and firms. Households decide on final consumption and investment under perfect foresight. A continuum of varieties, $v \in [0, 1]$, are produced over S stages, indexed by s. Products of *intermediate* stage $s < S$ are unfinished goods, and those of the final stage S are finished goods. Both unfinished and finished goods are traded across borders subject to trade costs, so the production of each variety v can come from different countries for different stages. Unfinished goods in stage $s < S$ are used directly and only as an input to produce goods in stage $s + 1$, capturing the "snake" structure of value added chains as in Yi (2003). All *finished* varieties are combined to produce a composite "retail" good that is used in final demand. The model provides a framework to analyze endogenous formation and dynamics of global value-added chains (GVC) over time.

2.1 Preferences

A representative household in each country owns primary factors of production: capital $K_{n,t}$ and labor $H_{n,t}$ at the beginning of period t. The household supplies capital and labor inelastically to domestic firms at competitive rates $r_{n,t}$ and $w_{n,t}$, respectively. She spends factor income on consumption $C_{n,t}$ and investment $X_{n,t}$ to maximize lifetime utility

$$\max_{\{C_t,X_t\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty} \beta^{t-1} H_{n,t} \ln\left(\frac{C_{n,t}}{H_{n,t}}\right), \tag{1}$$

subject to the budget constraint

$$P_{n,t}C_{n,t} + P_{n,t}X_{n,t} = r_{n,t}K_{n,t} + w_{n,t}H_{n,t}, \tag{2}$$

and the law of motion of capital

$$K_{n,t+1} = (1-\delta)K_{n,t} + X_{n,t}. \tag{3}$$

The parameter β denotes the discount factor and δ denotes the capital depreciation rate. The variable $P_{n,t}$ denotes composite, retail good price.

2.2 Technology

Much of the layout here follows the *lead-firm* approach of Antràs and de Gortari (2020). Production of each variety v involves multiple stages, and these stages can potentially be produced in different countries. A chain of production is denoted by the sequence $\ell = (\ell^1, \ldots, \ell^S)$, where ℓ^s indicates the location that production of stage s occurs. There are N^S such possible paths for each variety, and the set of possible chains is denoted by $\mathcal{C}$.[2] Input sourcing at all stages is determined by a *lead-firm* that handles production of the finished variety at stage $s = S$. The lead firm chooses a production chain ℓ_n to minimize its final costs, and the production at stage s has the form:

$$y^s_{\ell^s,t}(v) = a_\ell(v)\left(A^s_{\ell^s,t} k^s_{\ell^s,t}(v)^{\alpha^s} h^s_{\ell^s,t}(v)^{1-\alpha^s}\right)^{\gamma^s} m^s_{\ell^s,t}(v)^{1-\gamma^s}, \tag{4}$$

where $k^s_{\ell^s,t}(v)$ and $h^s_{\ell^s,t}(v)$ denote the amount of capital and labor, respectively, employed in stage s country ℓ^s for variety v. The term $m^s_{\ell^s,t}(v)$ is the input from stage $s-1$ used in production of stage s—the snake form. We abstract from the round-about form of input demands to highlight the role of vertical, or sequential, value-added chains.

The value added share of production is governed by γ^s. The snake form of the input chain

[2] For instance, with two countries and three stages, the set of 8 possible chains is given by $\mathcal{C} = \{(1,1,1);$ $(1,1,2);$ $(1,2,1);$ $(1,2,2)$ $(2,1,1);$ $(2,1,2);$ $(2,2,1);$ $(2,2,2)\}$. Another example is three countries and two stages, then the set consists of 9 chains: $\mathcal{C} = \{(1,1);$ $(1,2);$ $(1,3);$ $(2,1);$ $(2,2);$ $(2,3);$ $(3,1);$ $(3,2);$ $(3,3)\}$.

implies $\gamma^1 = 1$. The capital share in value-added is given by α^s. These share parameters are specific to the stage of production. Hence, owing to potentially varying capital shares, our framework embeds Heckscher-Ohlin forces, in addition to the primary Ricardian forces. For parsimony, we assume these parameters are identical across countries. We define a term $\widetilde{\gamma}^s = \prod_{s'=s+1}^{S}(1-\gamma^{s'})$, for $s < S$, which gives the share of stage-s gross output in the gross value of the finished good. By definition, $\widetilde{\gamma}^S = 1$. Furthermore, $\gamma^s\widetilde{\gamma}^s$ gives the share of stage-s value-added in the gross value of the finished good. This implies that $\sum_{s=1}^{S}\gamma^s\widetilde{\gamma}^s = 1$.

Production efficiency is described by two terms. One is a country-stage-specific term, $A^s_{\ell^s,t}$ which scales the production efficiency of all varieties in country ℓ^s, stage s, and period t. The other is a chain-variety-specific component, $a_\ell(v)$, which is randomly drawn from a Fréchet distribution captured by a unit location parameter and a shape parameter θ.[3]

Trade in both finished and unfinished goods is subject to iceberg costs that reflect many barriers including transportation costs owing to geography and technology, as well as policy-induced costs in the form of tariffs, quotas, and various regulations. The cost of shipping any stage s variety from origin country i to destination country n at time t is denoted by $d^s_{i,n,t}$. That is, to deliver one variety to country n, $d^s_{i,n,t}$ varieties must be shipped, and $d^s_{i,n,t} - 1$ units melt away in transit. We impose the usual restrictions that there is no internal trade friction: $d^s_{n,n,t} = 1$.

For each chain, the lead firm takes into account that trade costs must be paid whenever two adjacent links are located in different countries. That is, the input quantity in stage s must equal output from the previous stage with adjustments for trade costs:

$$m^s_{\ell^s,t}(v)d^s_{\ell^{s-1},\ell^s,t} = y^{s-1}_{\ell^{s-1},t}(v). \tag{5}$$

All finished varieties, which are potentially traded internationally, are assembled into a non-traded composite retail good, as in Eaton and Kortum (2002)

$$Q_{n,t} = \left(\int_0^1 q_{n,t}(v)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{6}$$

[3]The assumption of a unit location is without loss of generality, because country-stage-time efficiency differences that are common to all varieties are captured by $A^s_{\ell^s,t}$. In our framework, for a given country in a given time period, $\prod_{s=1}^{S}\left[a_\ell(v)\left(A^s_{\ell^s}\right)^{\gamma^s}\right]^{\widetilde{\gamma}^s}$ corresponds to $\prod_{n=1}^{N}(a^n_{l(n)}(z))^{-\alpha_n\beta_n}$ in Antràs and de Gortari (2020). Antràs and de Gortari (2020) show that the lead-firm approach is isomorphic to an alternative framework with stand-alone producers of different stages making cost-minimizing sourcing decisions for their input in a decentralized manner, with additional assumptions on information available to producers at each stage about the exact costs of producers at earlier stages. We describe our model using only the lead-firm approach.

where $q_{n,t}(v)$ indicates the quantity of the finished (stage S) variety purchased by country n. All varieties, regardless of the source, enter symmetrically. As in the tradition of Ricardian trade theory, varieties are differentiated only by cost from the faced by the purchaser. Retailers, which offer zero value added, sell the composite retail good domestically to households for consumption and investment.

This model setup allows for GVC trade, in which intermediate stages can be produced offshore and in different countries. If we set trade costs in intermediate stages to infinity, such trade will disappear, and we are back to the standard Eaton-Kortum trade model in which only finished goods are traded. We refer to this case as *the model without GVC trade*.

2.3 Equilibrium

All markets are perfectly competitive so prices are equal to marginal costs. Equilibrium is characterized by sequences of prices, quantities, and allocations across input chains such that (i) the quantities maximize household lifetime utility; (ii) input chains are optimal in minimizing production costs; and (iii) allocations are feasible. We first describe the household's solution, taking prices as given. Then, we characterize the solution to the prices that the household faces along with the underlying pattern of trade. Finally, we describe the market clearing conditions.

Final demand Given sequences of factor prices and retail prices, $\{r_{n,t}, w_{n,t}, P_{n,t}\}$, the household maximizes its lifetime utility by choosing consumption and investment in every period. The solution is governed by a standard Euler equation and budget constraint:

$$\frac{C_{n,t+1}/H_{n,t+1}}{C_{n,t}/H_{n,t}} = \beta\left(\frac{r_{n,t+1}}{P_{n,t+1}} + 1 - \delta\right), \tag{7}$$

$$P_{n,t}C_{n,t} + P_{n,t}K_{n,t+1} = \left(\frac{r_{n,t}}{P_{n,t}} + 1 - \delta\right)P_{n,t}K_{n,t} + w_{n,t}H_{n,t}. \tag{8}$$

Prices We first define the unit cost of factors in stage s country n as

$$u^s_{n,t} = B^s\left((r_{n,t})^{\alpha^s}(w_{n,t})^{1-\alpha^s}\right),$$

where $B^s = (\alpha^s)^{-\alpha^s}(1-\alpha^s)^{-(1-\alpha^s)}$ is a constant.

Next consider a chain ℓ demanded by country n. Each country involved in chain ℓ adds to the cost of the finished good. We denote the cost contributed in stage s as $\mathcal{U}^s_{\ell,n,t}$. Given the definition of $\widetilde{\gamma}^s$ (the share of stage-s gross output in the finished good), we can write

$$\mathcal{U}^s_{\ell,n,t} = \left[\left(\frac{u^s_{\ell^s,t}}{A^s_{\ell^s,t}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1},t} \right]^{\widetilde{\gamma}^s},$$

which incorporates the efficiency adjusted unit cost and the trade cost along chain ℓ from stage s to $s+1$. Note that since the chain is demanded by country n, $d^s_{\ell^S,\ell^{S+1},t} = d^s_{\ell^S,n,t}$. Under perfect competition, the price of chain ℓ in country n for finished variety v is given

$$p_{\ell,n,t}(v) = \frac{1}{a^\ell(v)} \prod_{s=1}^{S} \mathcal{U}^s_{\ell,n,t}.$$

The lead firm for any variety v sources inputs using the chain with the lowest price:

$$p_{n,t}(v) = \min_{\ell \in \mathcal{C}} \quad \{p_{\ell,n,t}(v)\}.$$

Aggregating all varieties under the relevant probability distribution of prices across varieties yields the composite retail price index:

$$P_{n,t} = \left(\int_0^1 p_{n,t}(v)^{1-\eta} dv \right)^{\frac{1}{1-\eta}} = \zeta \times \left(\sum_{\ell \in \mathcal{C}} \prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell^s,t}}{A^s_{\ell^s,t}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1},t} \right)^{-\theta \widetilde{\gamma}^s} \right)^{-\frac{1}{\theta}}, \tag{9}$$

where $\zeta = \Gamma\left(1 + (1-\eta)/\theta\right)^{1/(1-\eta)}$ is a constant, and $\Gamma(\cdot)$ is the Gamma function.[4]

Final demand shares across chains Usual properties the Fréchet distribution give rise to equilibrium sourcing shares, expressed as probabilities of a particular finished variety following a particular path $\ell = (\ell^1, \ldots, \ell^S) \in \mathcal{C}$. In other words, the fraction of finished varieties that country n sources through a particular chain, $\ell \in \mathcal{C}$, is given by

$$\lambda_{\ell,n,t} = \frac{\prod_{s=1}^{S} \left(\mathcal{U}^s_{\ell,n,t} \right)^{-\theta}}{\sum_{\ell' \in \mathcal{C}} \prod_{s=1}^{S} \left(\mathcal{U}^s_{\ell',n,t} \right)^{-\theta}} = \frac{\prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell^s,t}}{A^s_{\ell^s,t}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1},t} \right)^{-\theta \widetilde{\gamma}^s}}{\sum_{\ell' \in \mathcal{C}} \prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell'^s,t}}{A^s_{\ell'^s,t}} \right)^{\gamma^s} d^s_{\ell'^s,\ell'^{s+1},t} \right)^{-\theta \widetilde{\gamma}^s}}. \tag{10}$$

[4] The usual restriction requires that $(1-\eta)/\theta > -1$, beyond which, η plays no substantial role.

By construction, $\sum_{\ell \in \mathcal{C}} \lambda_{\ell,n,t} = 1$. Thus, a country on average utilizes a chain more heavily if countries along this chain have either lower unit costs of value added, or higher efficiency, or lower trade costs, at any stage in the chain.

Input demand Firms demand inputs so that marginal products of all inputs are equated. In particular, this implies that revenues are exhausted on the cost of inputs. Define $\Lambda_n^s \equiv \{\ell \in \mathcal{C} : \ell^s = n\}$ as the set of chains that country n occupies position s. Then

$$r_{n,t} k_{n,t}^s = \alpha^s \gamma^s \widetilde{\gamma}^s \sum_{\ell \in \Lambda_n^s} \sum_{i=1}^{N} P_{i,t} Q_{i,t} \lambda_{\ell,i,t},$$

$$w_{n,t} h_{n,t}^s = (1-\alpha^s) \gamma^s \widetilde{\gamma}^s \sum_{\ell \in \Lambda_n^s} \sum_{i=1}^{N} P_{i,t} Q_{i,t} \lambda_{\ell,i,t}.$$

Trade flows Final demand $\{P_{i,t}Q_{i,t}\}_{i=1}^N$ and the corresponding shares along each chain $\{\lambda_{\ell,i,t}\}_{i=1}^N$ imply trade flows across countries. To calculate exports of a country along a chain, we consider two situations. First, foreign countries' demand on this chain generates exports, as long as this country occupies some stage, regardless whether it is intermediate or final. Second, a country's domestic demand might also generate exports for itself, as long as it occupies an intermediate stage and a foreign country occupies a subsequent stage on this chain. This second case is the novel feature of GVC trade.

To operationalize this accounting, we introduce some notation. For any stage $s < S$, define $\mathcal{E}_n^s = \{\ell : \ell^s = n \text{ and } \ell^{s+1} \neq n\}$. This set includes chains that generate exports of intermediate goods of stage s for country n from any country's demand on these chains, including its own. We also define $\mathcal{E}_n^S = \{\ell : \ell^S = n\} = \Lambda_n^S$, which is the set of chains that country n occupies the final stage. These chains generate exports of final goods for country n from foreign demand.

Country n's gross exports through chain ℓ are thus described by

$$E_{n,t}^{\ell} = \sum_{i \neq n} P_{i,t} Q_{i,t} \lambda_{\ell,i,t} \sum_{s=1}^{S} \widetilde{\gamma}^s \mathbf{1}_{\ell \in \mathcal{E}_n^s} + P_{n,t} Q_{n,t} \lambda_{\ell,n,t} \sum_{s=1}^{S-1} \widetilde{\gamma}^s \mathbf{1}_{\ell \in \mathcal{E}_n^s}.$$

Gross exports in each stage is the value of the final spending adjusted by $\widetilde{\gamma}^s$ (the value of flows passing through stage s). A country might export in more than one stage of a chain, implying a summation across all exporting stages.

Now we do a similar accounting for country n's imports. For any stage $s < S$, define the set of chains such that country n occupies stage $s+1$ and a different country $i \neq n$ occupies stage s as $\mathcal{M}_n^s = \{\ell : \ell^s \neq n \text{ and } \ell^{s+1} = n\}$. This is the set of chains that country n incurs imports at stage s to use in production in stage $s+1$. Also, define $\mathcal{M}_n^S = \{\ell : \ell^S \neq n\} = \cup_{i \neq n} \Lambda_i^S$ as the set of chains that country n imports final goods. Country n's gross imports through chain ℓ are thus given by

$$M_{n,t}^{\ell} = P_{n,t} Q_{n,t} \lambda_{\ell,n,t} \sum_{s=1}^{S} \widetilde{\gamma}^s \mathbf{1}_{\ell \in \mathcal{M}_n^s} + \sum_{i \neq n} P_{i,t} Q_{i,t} \lambda_{\ell,i,t} \sum_{s=1}^{S-1} \widetilde{\gamma}^s \mathbf{1}_{\ell \in \mathcal{M}_n^s}.$$

The first term on the right of the above equation is country n's imports generated by its own final demand on the chain. The second term gives its imports due to foreign final demand on the chain, which is the novel feature of GVC trade.

Total exports (imports) is the sum of trade flows along all chains:

$$E_{n,t} = \sum_{\ell \in \mathcal{C}} E_{n,t}^{\ell}, \quad M_{n,t} = \sum_{\ell \in \mathcal{C}} M_{n,t}^{\ell}.$$

Feasibility Factor markets must clear, which pins down equilibrium factor prices. Because capital and labor are immobile across countries, domestic supply must equal domestic demand for these inputs:

$$\sum_{s=1}^{S} k_{n,t}^s = K_{n,t}, \quad \sum_{s=1}^{S} h_{n,t}^s = H_{n,t}.$$

Since the composite retail good is not traded across countries, the supply of retail goods must equal domestic demand:

$$Q_{n,t} = C_{n,t} + X_{n,t}.$$

Finally, trade is assumed to balance period-by-period, i.e., for any country n

$$M_{n,t} = E_{n,t}.$$

3 Model Analysis

This section analyzes the implications of trade integration in the model with GVC trade across two stages and two countries. In period 0, countries start at the steady state of closed economies. In period 1, countries receive an unanticipated trade integration shock, whereby trade costs decline gradually for 50 periods, and remain constant thereafter. In the baseline model, agents have perfect foresight of this process as of period 1. In response to the trade integration shock, countries trade and invest, moving toward a new steady state. In this environment we illustrate how trade integration impacts endogenous formations of global value-added chains, trade, and growth.

We start by considering a case with symmetric countries to build understanding of the dynamics of GVC trade patterns, capital accumulation, and growth in response to trade integration. We then consider asymmetric countries that differ in aggregate productivity. In the asymmetric case, we explore how comparative advantage, both Heckscher-Ohlin and Ricardian, evolves endogenously and interacts with dynamics for capital accumulation and growth. We also illustrate the dynamics of domestic value added content in exports during trade integration. Finally, we present the welfare analysis of trade integration.

3.1 Symmetric Countries

We numerically evaluate the model in a two-country, two-stage setting: $N = 2$ and $S = 2$. In this environment there are four possible paths for global value chains: $\mathcal{C} = \{(1,1); (1,2); (2,1); (2,2)\}$. For simplicity, we assume that trade costs are identical across countries and stages in each period: $d^s_{1,2,t} = d^s_{2,1,t} = d_t$. Trade cost d_t declines at a constant rate over 50 periods from an initial steady state level $d^\star$ to a new steady state level $d^{\star\star}$ to capture the process of trade integration. We set $d^\star$ at 8 to render the world economy effectively in autarky initially, and $d^{\star\star}$ at 1.7 to generate the degree of global trade openness close to the one observed in 2010s. Specifically, we have

$$\frac{d_t}{d_{t-1}} - 1 = \left(\frac{d^{\star\star}}{d^\star}\right)^{1/50} - 1 = g_d, \tag{11}$$

where the growth rate of trade costs over time is about $g_d = -3\%$ per period.

We set all parameter values to be common across countries in the analysis of symmetric countries; see in Table 1. The household's period discount factor is set to $\beta = 0.96$ and the rate of depreciation is set to $\delta = 0.06$. The shape parameter of the Fréchet distribution, which governs the trade elasticity, is set to $\theta = 4$ as in Simonovska and Waugh (2014). The elasticity of substitution between varieties within the composite retail good is set to $\eta = 2$. For production shares, we set $\gamma^2 = 0.5$. Recall that $\gamma^1 = 1$ by design. This implies that the gross output of finished, stage 2 varieties consists equally of value added from both stage 1 and stage 2. We set capital's share in stage 1 to $\alpha^1 = 0.75$, and that for stage 2 to $\alpha^2 = 0.25$, implying that stage 1 production is more capital intensive than stage 2. This is consistent with the evidence presented in Antràs et al. (2012) that, in manufacturing, upstream stages are more capital-intensive. We set productivity $A^s_{n,t}$ at 1 for all countries, stages, and periods. Labor endowment H is constant at 1 over time in both countries. The initial capital stock $K_1 \equiv K^\star$ is the steady level under autarky.

Table 1: Symmetric Parameter Values

Discount factor	β	0.96
Depreciation rate	δ	0.06
Trade costs (equation 11)		
Initial:	d^*	8
Terminal:	d^{**}	1.7
Growth rate for periods 1–50:	g_d	−3%
Input share in gross output		
Stage 1	γ^1	1
Stage 2	γ^2	0.5
Capital's share in value added		
Stage 1	α^1	0.75
Stage 2	α^2	0.25
Value-added productivity	A	1
Labor endowment	H	1

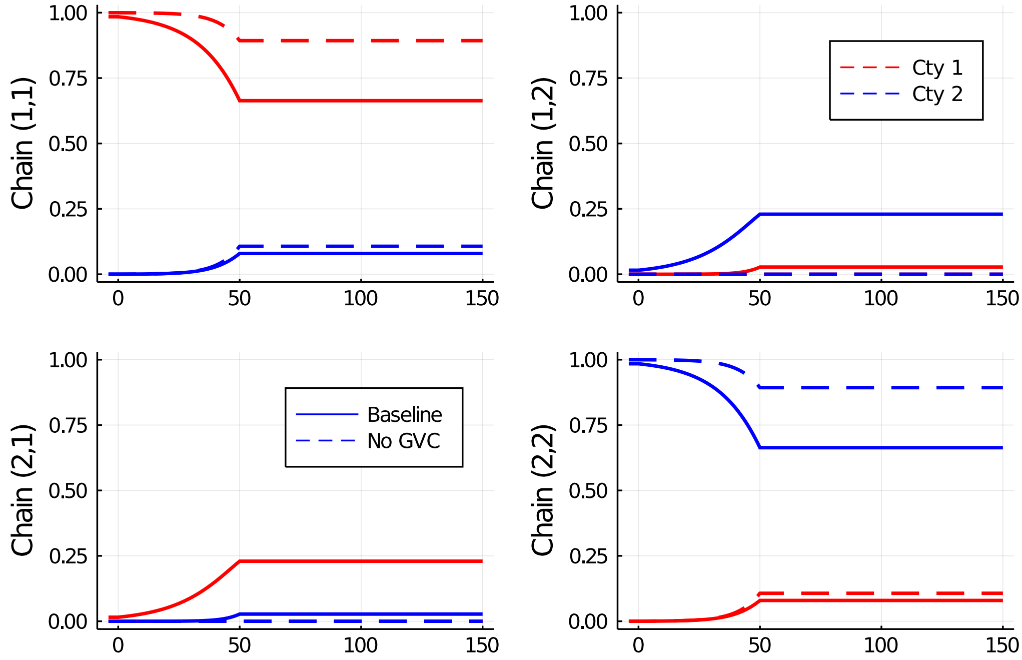
Note: Trade costs remain constant from period 50 onwards.

Given these parameter values and the process of trade costs, we solve the equilibrium of the model numerically to trace out the transition dynamics from the initial steady state to the new steady state. Our goal is to highlight novel results arising from GVC trade and capital accumulation. To highlight the implications of GVC trade, we conduct a counterfactual experiment where GVC trade is turned off. Specifically, we assume the trade costs in stage one are infinite so there is no possibility for trade in stage 1. To highlight the role of endogenous capital accumulation, we conduct a counterfactual experiment where the capital stock is fixed at the initial steady state. To be clear, the case with fixed capital is not the same as a purely static model with exogenous capital, or with no capital. In such purely static models, all of the output is consumed. In our model with fixed capital, the capital stock still depreciates each period and some of the output must be allocated to replace the depreciated stock. This distinction is important when evaluating the welfare implications.

Trade shares We start with the dynamic trade patterns. Each panel of Figure 1 compares trade shares for each chain $\ell \in \mathcal{C}$ in the model with GVC trade versus without

GVC trade. Red lines denote country 1 and blue lines denote country 2. Also, solid lines denote the baseline model with GVC trade, and dashed lines denote the case without GVC trade.

Figure 1: Trade shares with symmetric countries, with and without GVC trade



Note: Trade shares $\lambda_{\ell,n,t}$ are plotted for each chain $\ell \in \mathcal{C}$. The model without GVC trade imposes infinite trade costs in stage 1 and collapses to the Eaton-Kortum model. Periods up through 0 correspond to the initial steady state. Trade costs decline from period 1 to 50 and remain constant afterwards. Red lines are for country 1 and blue lines are for country 2. Solid lines are for the baseline model with GVC trade and dashed lines are for the model without GVC trade.

Without GVC trade, countries can trade only along chains $(1,1)$ and $(2,2)$. By construction, there is no trade along chains $(1,2)$ and $(2,1)$. As trade costs decline, Figure 1 shows that country 1's home trade share $(1,1)$ decreases, and its import share $(2,2)$ increases. When the trade costs become constant in period 50, trade shares reach the new steady state at the same time.

When GVC trade is allowed, all four chains can be traded, i.e., production stages can be fragmented across countries. As trade costs decline, country 1's home trade share $(1,1)$ still decreases over time, but by much more than in the case without GVC trade. Also, country 1 imports more finished goods with both stages produced in country 2—chain (2,2)—but not by as much as in the case without GVCs. Instead, country 1's shares of the fragmented chains (1,2) and (2,1) increase, particularly chain (2,1), as trade costs decline. That is, country 1 increases its imports of stage-1 unfinished goods to use as inputs for its stage-2 production at home. In contrast, the chain (1,2), where stage one is produced home, then shipped abroad to finish stage-2 production abroad, and finally shipped home for final demand, experiences a much smaller increase over time because trade costs are incurred twice with this "back-and-forth" trade.

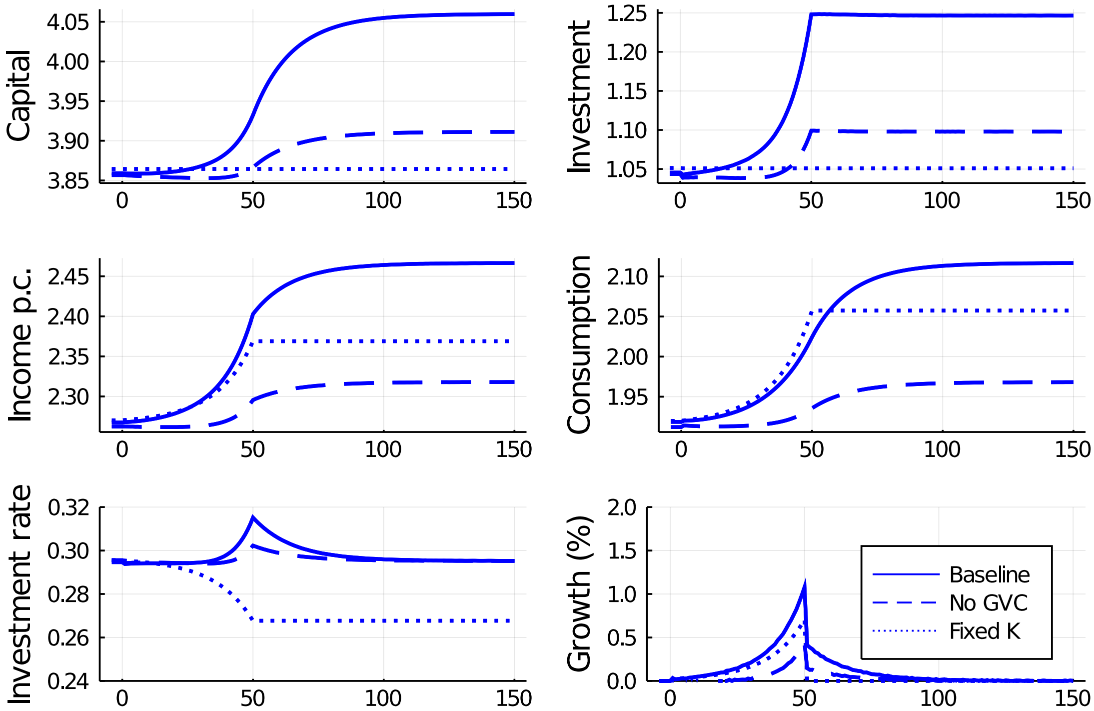
In general, fragmentation of stages across borders occurs more frequently in earlier stages of production. This pattern is also pointed out by Antràs and de Gortari (2020). This is because trade costs apply to the gross value, and the second stage compounds trade costs from every stage, so trade in earlier stages effectively incurs lower costs.

Interestingly, in this simple symmetric model, there is no impact of endogenous capital accumulation on the dynamics of trade shares. Trade shares for each chain in the baseline model with endogenous capital accumulation are identical to those in the model with fixed capital. This is the artifact of the symmetric country framework. Both countries accumulate capital over the transition path at the same rate, which leaves no impact on relative factor prices across countries and no impact on trade shares. In other words, all of the increase in trade shares is a result of lower trade costs leading to increased specialization via Ricardian forces. Heckscher-Ohlin (H-O) forces play no role. Later, we will study an asymmetric model, in which capital accumulation will impact trade shares and GVC patterns.

Macroeconomic growth Now we turn to macroeconomic growth. The dynamics of capital, investment, income per capita, consumption, the investment rate, and the growth rate are plotted for the baseline case, the case without GVC trade, and the case with fixed capital in Figure 2. Let's first look at the long-run impact across the three cases. Start with the baseline case plotted by solid lines. It is well known that trade integration leads to a reduction in home trade shares and a rise in production efficiency induced by trade selection. With endogenous capital, countries accumulate a higher level of capital in the new steady state. Accordingly, levels of income per capita, consumption and investment are also higher in the new steady state. On the other hand, the investment rate in the long run is the same as in autarky, and the long-run growth rate is zero.

Consider the case without GVC trade plotted as dashed lines. Since the degree of trade integration is substantially limited, gains in effective efficiency from trade are also limited. As a result, the long-run capital, investment, consumption and output are much lower than those in the case with GVC trade. The investment rate and the growth rate in the long run are the same as the baseline case. Consider the case with fixed capital plotted as dotted

Figure 2: Macroeconomic dynamics with symmetric countries



Note: Consumption is $C_{n,t}$; Investment is $X_{n,t}$; Capital stock is $K_{n,t}$; Income p.c. is $Y_{n,t} = (r_{n,t}K_{n,t} + w_{n,t}H_{n,t})/P_{n,t}$. Each of these variables are in log units. Investment rate is defined as the share of income spent on new investment: $X_{n,t}/Y_{n,t}$. The model with fixed capital restricts investment to equal depreciated capital. Periods up through 0 correspond to the initial steady state. Trade costs decline from period 1 to 50 and remain constant afterwards. Solid lines are for the baseline model with endogenous investment, dashed lines are for the model without GVC trade, and dotted lines are for the model with fixed capital.

lines. Although capital and investment is fixed, output and consumption rise substantially in the new steady state due to the presence of GVC trade. These increases are smaller than those in the baseline case, but quantitatively larger than those in the case without GVC trade. The investment rate in the long run is lower than the autarky by construction, since investment is held fixed and output is higher.

We now focus on transitional dynamics. Given the incentives to smooth consumption over time, the transition process of capital takes over 100 periods in the baseline model. Capital continues to rise to the new steady state level for over 50 periods after trade costs become constant. This pattern of capital accumulation drives the dynamics of consumption and income per capita. Specifically, before period 50, both rising TFP due to declining trade costs and capital accumulation contribute to higher output, and post period 50, capital accumulation drives all the growth in output. Consequently, the investment rate and the growth rate first rise and then decline over time. Particularly, once trade costs remain constant, TFP growth shuts down to zero and all growth is through capital accumulation so there is a discrete decline in aggregate growth. These patterns are similar to the impact of a series of transitory TFP shocks in a closed economy neoclassical growth model.

We next look at the dynamics without GVC trade. We observe a similar qualitative pattern for all variables except investment and capital. Here investment and capital initially decline as the trade integration process begins. The reason for this is that households postpone investment knowing that trade costs and prices of investment will decline in the future, which outweighs the incentives to invest in response to rising efficiencies of production. Indeed, it takes a while for the trade costs to become low enough for the latter force to dominate in this case. In the model with GVC trade, the efficiency gains are fast enough in early periods, so that the return to investment is large enough to increase investment in these periods. Moreover, the magnitude of the dynamics is much smaller without GVC trade than with GVC trade. This is because GVC trade leads to large increases in trade shares and production efficiencies.

Now let's look at the transition in the case with fixed capital. By construction, capital and investment are constant over time. Output rises in response to trade integration and trade-induced efficiency improvement. Given fixed capital, the investment rate declines over time and all of the additional output goes to consumption. There are no further dynamics post period 50: output growth drops to zero and consumption remains constant. With endogenous capital accumulation, consumption is back-loaded to make room for more investment. This can be seen in the panel for consumption, where the dotted line rises slightly above the solid line before period 50. After period 50, consumption continues to grow to a higher long run level in the baseline model. The area between the two curves underpins the additional gains from trade integration brought through by capital accumulation. Indeed, the dynamic gains including transition are clearly lower than the steady-state measure of the gains.

To summarize, with symmetric countries, the presence of endogenous capital accumulation does not affect the dynamics of GVC trade, but GVC trade does impact the dynamics of investment and capital accumulation, both quantitatively and qualitatively. Without GVC trade, investment declines initially in response to trade integration, lowering capital in early periods, due to the incentive to postpone investment. With GVC trade, investment and capital rise throughout the transition because production efficiencies are realized sooner and to a greater degree, which more than offsets the incentive to postpone investment.

3.2 Asymmetric Countries

We now consider an environment in which trade integration occurs between two asymmetric countries. This scenario aims to capture a broad picture: a poor country (e.g. China or South Korea) with low productivity and a low capital-labor ratio initially grows at a faster rate than the rest of the world (country 2) in a prolonged period of increasing trade integration. Specifically, two countries start with different initial productivity levels so that country 1 is half as productive as country 2: $(A_1^\star, A_2^\star) = (0.5, 1)$. Over time country 1's productivity grows for 50 periods, while country 2's does not:

$$\frac{A_{n,t}}{A_{n,t-1}} - 1 = \left(\frac{A_n^{\star\star}}{A_n^\star}\right)^{1/50} - 1 = g_{A_n}. \tag{12}$$

Country 1's productivity grows at a constant rate of 1.4%, while country 2's grows at a constant rate of zero. Starting from period 50, productivity in both countries levels up, just as the trade costs. This implies that from period 50 onward, country 1 is equally as productive as country 2: $(A_1^{\star\star}, A_2^{\star\star}) = (1, 1)$. Table 2 summarizes the specification of the model with asymmetric productivity across countries. All other parameters and the process of the trade costs remain identical to the symmetric case. To highlight the role of endogenous capital and GVC trade, we compute the outcomes for the fixed capital case and the no GVC case under this asymmetric calibration.

Table 2: Asymmetric Parameter Values

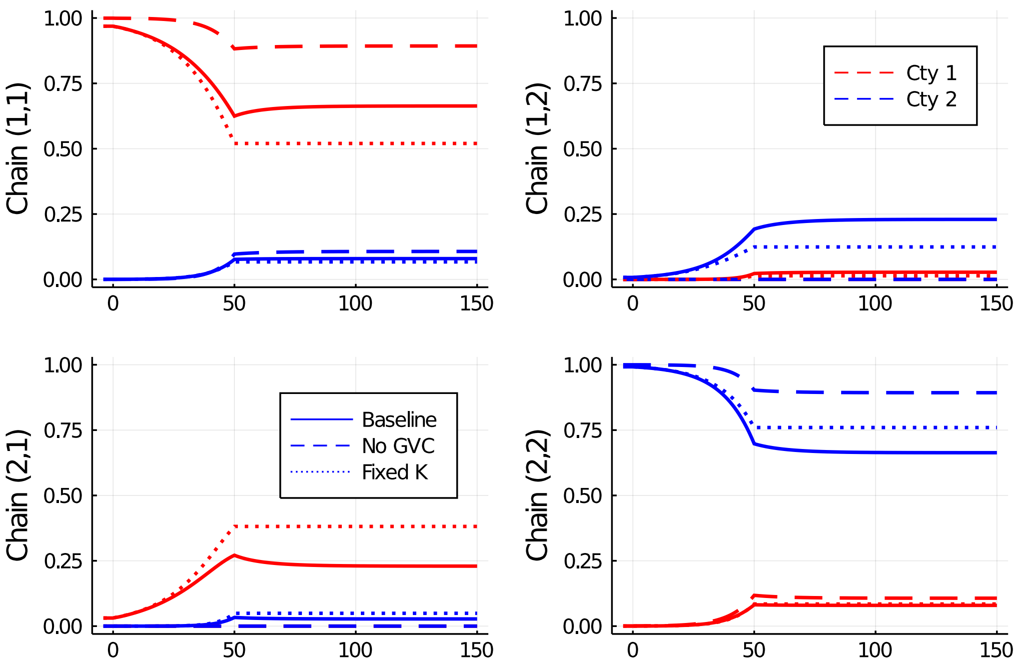
Value-added productivity (equation 12)		
Initial:	$(A_1^\star, A_2^\star)$	$(0.5, 1)$
Terminal:	$(A_1^{\star\star}, A_2^{\star\star})$	$(1, 1)$
Growth rate for periods 1–50:	(g_{A_1}, g_{A_2})	$(1.4\%, 0\%)$

Note: Productivity remains constant from period 50 onwards. The remaining parameters are identical to those in Table 1.

In the initial steady state, the economies are almost in autarky due to the high trade costs. Due to its lower productivity, country 1 has lower levels of capital, output and consumption per capita than country 2. At the new steady state, countries are symmetric in all dimensions, and the outcomes are identical to those in the new steady state of the symmetric-country case. We will focus on the transitional dynamics of trade patterns and macroeconomic growth.

Trade shares Figure 3 plots trade shares in three cases under the asymmetric calibration: the baseline case, the no-GVC-trade case, and the fixed capital case. In contrast to the symmetric calibration, under which capital accumulation has no impact on the patterns of GVC trade, capital accumulation does influence these patterns under the asymmetric calibration. We first look at the results in the fixed capital case illustrated by dotted lines. With fixed capital, country 1 is smaller than country 2, even in the new steady state when they have the same productivity. As a result, country 1 trades more than country 2: country 1's usage of the purely domestic chain (1,1) is about 0.5 and country 2's usage of chain (2,2) is about 0.75 in the new steady state. In addition, with factors fixed at the initial steady state levels in both countries, country 1 has a lower capital-labor ratio than country 2 in all periods. This difference in relative factor abundance gives rise to the H-O motive of trade: country 1 specializes in stage 2 – the labor-intensive stage, while country 2 specializes in stage 1 – the capital-intensive stage.

Figure 3: Trade shares with asymmetric countries



Note: Trade shares $\lambda_{\ell,n,t}$ are plotted for each chain $\ell \in \mathcal{C}$. The baseline model begins with $(A_1^\star, A_2^\star) = (0.5, 1)$, then imposes $A_{1,t} \approx (1.04)A_{1,t-1}$ for $t = 2, \ldots, 50$, with $A_{1,t} = 1$ thereafter ($A_{2,t} = 1$ in every period). The model without GVC trade imposes infinite trade costs in stage 1 and collapses to the Eaton-Kortum model. The model with fixed capital restricts investment to equal depreciated capital. Periods up through 0 correspond to the initial steady state. Trade costs decline from period 1 to 50 and remain constant afterwards. Red lines are for country 1 and blue lines are for country 2. Solid lines are for the baseline model with endogenous investment, dashed lines are for the model without GVC trade, and dotted lines are for the model with fixed capital.

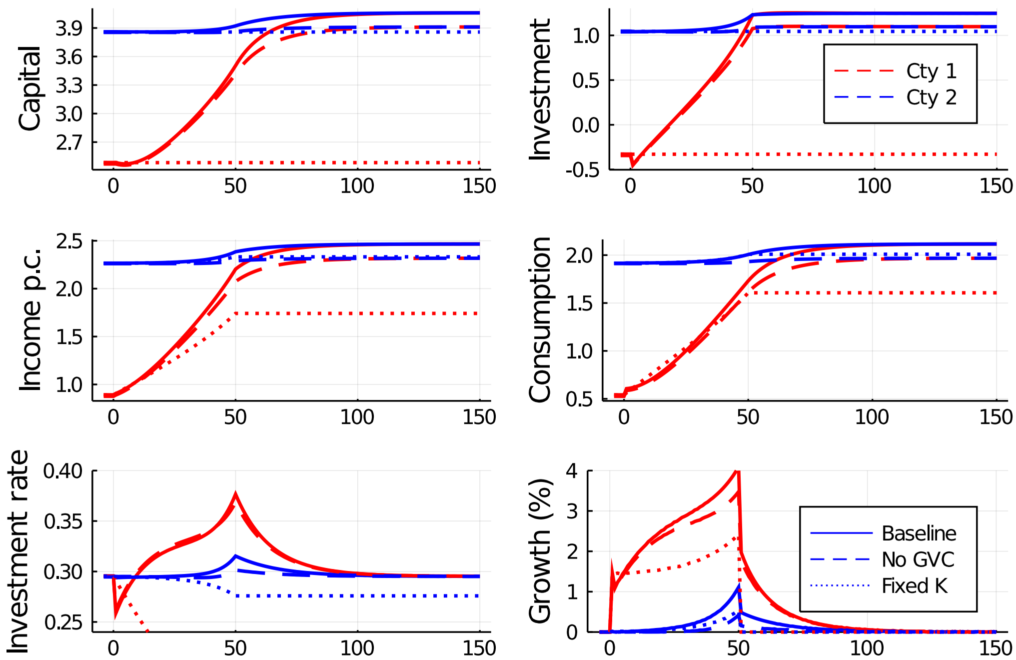
Now let's compare trade patterns in the baseline case with endogenous capital accumulation (solid lines) with the fixed capital case (dotted lines). With capital accumulation in response to rising productivity, country 1 becomes larger over time and ends up with the same size and the same capital-labor ratio as country 2 in the new steady state. This implies that the H-O motive for trade is dwindling over time. As a result, country 1 has larger domestic chain share, chain (1,1), in the baseline case than in the fixed capital case. Moreover, its GVC trade along chain (2,1) is also greatly reduced in the baseline case as a result of vanishing H-O forces. The same mechanisms work through country 2. As it becomes small in relative size, country 2 utilizes chain (2,2) more frequently in the baseline compared to the case with fixed capital. In addition, as the H-O comparative advantage vanishes, country 2 uses chain (1,2) more frequently by in the baseline case than in the fixed capital case.

Lastly, we turn to the case with no-GVC trade, plotted as dashed lines. In this case, as trade costs decline and trade flows grow, the shares of domestic chains – chain (1,1) for country 1 and chain (2,2) for country 2 – decline over time. Meanwhile, the declines in domestic chains are offset by the increases in foreign chains: country 1 increases chain (2,2) and country 2 increases chain (1,1); Neither country uses fragmented chains, by assumption. These results are similar to those in the no-GVC case under the symmetric calibration. Compared to the baseline case, trade openness is much lower with no GVC trade, because there is no scope for countries to specialize across stages.

Interestingly, the trade shares in the baseline case continues to adjust even after trade costs and productivities become constant in period 50. As in the symmetric calibration, countries continue to accumulate capital even when trade costs and productivities become constant at period 50 due to consumption smoothing. What is different in the asymmetric calibration is that capital accumulates at different rates across the two countries. Time-varying relative size shifts trade patterns by boosting openness in country 2 and reducing openness in country 1. In addition, as the capital-labor ratio converges across countries over time, the H-O comparative advantage disappears. As country 1's comparative advantage in stage 2 dissipates, it increases chain (1,1) and decreases chain (2,1). Similarly, country 2 decreases chain (2,2) and increases chain (1,2). These mechanisms are absent in the case with fixed capital, in which the trade shares become constant right at period 50.

Macroeconomic growth Now consider the dynamics of macroeconomic outcomes. Figure 4 plots the results for the baseline case, the no-GVC case, and the fixed capital case. For each individual country, the qualitative patterns are similar to those under the symmetric calibration. Economic growth is rooted in exogenous productivity growth and declining trade costs, and amplified by endogenous trade selection and capital accumulation. As shown in Figure 4, the presence of GVC trade and capital accumulation boost investment, capital, output and consumption for each country. Quantitatively, capital accumulation is important for country 1's growth dynamics, while GVC-trade induced efficiency gains are important for country 2's growth.

Figure 4: Macroeconomic dynamics with asymmetric countries



Note: Consumption is $C_{n,t}$; Investment is $X_{n,t}$; Capital stock is $K_{n,t}$; Income p.c. is $Y_{n,t} = (r_{n,t}K_{n,t} + w_{n,t}H_{n,t})/P_{n,t}$. Each of these variables are in log units. Investment rate is defined as the share of income spent on new investment: $X_{n,t}/Y_{n,t}$. The baseline model begins with $(A_1^\star, A_2^\star) = (0.5, 1)$, then imposes $A_{1,t} \approx (1.04)A_{1,t-1}$ for $t = 2, \ldots, 50$, with $A_{1,t} = 1$ thereafter ($A_{2,t} = 1$ in every period). The model without GVC trade imposes infinite trade costs in stage 1 and collapses to the Eaton-Kortum model. The model with fixed capital restricts investment to equal depreciated capital. Periods up through 0 correspond to the initial steady state. Trade costs decline from period 1 to 50 and remain constant afterwards. Red lines are for country 1 and blue lines are for country 2. Solid lines are for the baseline model with endogenous investment, dashed lines are for the model without GVC trade, and dotted lines are for the model with fixed capital.

What we want to highlight in this asymmetric-country scenario is how the heterogeneity of macroeconomic outcomes is impacted by capital accumulation and global GVC trade. Although country 1's productivity converges to that of country 2, it still has a permanent lower income level in the case with fixed capital. Capital accumulation reduces the income gap entirely in the long run. This is the outcome of persistently higher investment rates and growth rates in country 1 than in country 2, as illustrated in the bottom panels of figure 4.

Countries with faster productivity growth have higher investment rates, as in the standard neoclassical growth model. Trade integration induces further capital deepening as in the trade literature (see Ravikumar, Santacreu, and Sposi, 2019). What is new here is that countries with increasing comparative advantage in capital-intensive stages have yet even higher investment rates[5]

We now illustrate the dynamic spillover effects across countries through international trade by focusing on country 2, which experiences no productivity growth. Country 2, however, has persistent and positive output growth over time. There are four channels of spillover. First, lower trade costs boost its output over the transition through efficiency gains from trade. Second, country 1's productivity growth benefits country 2 exclusively through international trade. Third, the GVC trade amplifies the benefits of trade through comparative advantage across stages. Lastly, country 2 benefits also from country 1's capital accumulation because the gains from trading with a larger partner are bigger. In conclusion, countries with stagnant productivity growth are able to enjoy positive income growth by maintaining open trade with faster growing countries. Moreover, fragmentation of production based on H-O comparative advantage further amplifies the growth effects.

3.3 Value-Added Content of Trade

Earlier, we showed how the prevalence of each production chain ℓ evolves during trade integration and growth. To connect our model more closely to the empirical literature that measures the extent of GVCs, we now derive a model-based metric for the domestic content of exports. We then compare our metric to some of the leading empirical measures. Finally, we show how our metric evolves during trade integration and growth.

Domestic Content of Exports By domestic content of exports, we mean the domestic value-added embodied in the gross exports of a country. Consider the following simple example of the production of a smartphone. Suppose South Korea produces parts like the screen, memory, and processor, and exports these parts to China for \$100 (stage 1). China assembles all the parts, and, in so doing, adds value of \$100 (stage 2). China then exports the assembled smartphone to the U.S. for \$200. In gross terms, China exports \$200 to the U.S. In value added terms, however, China has only \$100 worth exports, while the remaining

[5] Cuñat and Maffezzoli (2007) study trade liberalization in which countries start out with permanent differences in TFP and initially different capital/labor ratios. In this scenario, countries diverge in their investment path, as the country with the higher initial capital/labor ratio accumulates capital, while the other country decumulates capital.

\$100 is U.S. content. In this simple example, the domestic content of China's exports is one half—the ratio of domestic value added in exports to gross exports.

We derive separate expressions for domestic valued added embodied in exports ultimately absorbed in foreign countries and for domestic value added embodied in exports ultimately absorbed in the domestic country, where the ultimate absorption is from final demand. In the former case, country n's domestic value added along a particular production chain is the product of foreign demand and its value-added along that chain. In the latter case, we consider the domestic value added contribution to exports generated by domestic final demand along chains that generate exports in some intermediate stage. Define the set of such chains as $\mathcal{E}_n^\star = \{\ell : \ell \in (\cup_{j=1}^{S-1} \mathcal{E}_n^j)\}$. Combining domestic value added contributions to exports generated by both foreign and domestic final demand gives rise to country n's value-added exports through chain ℓ:

$$V_{n,t}^{\ell} = \sum_{i \neq n} P_{i,t} Q_{i,t} \lambda_{\ell,i,t} \sum_{s=1}^{S} \widetilde{\gamma}^s \gamma^s \mathbf{1}_{\ell(s)=n} + P_{n,t} Q_{n,t} \lambda_{\ell,n,t} \mathbf{1}_{\ell \in \mathcal{E}_n^\star} \sum_{s=1}^{S-1} \widetilde{\gamma}^s \gamma^s \mathbf{1}_{\ell(s)=n}, \qquad (13)$$

and its total value-added in exports are

$$V_n = \sum_{\ell \in \mathcal{C}} V_n^{\ell}.$$

We define the domestic value-added contribution by country n in our model as total value-added in exports expressed as a share of n's gross exports:

$$\text{DCE}_{n,t} = \frac{V_{n,t}}{E_{n,t}} \qquad (14)$$

where the denominator is gross exports of country n, and the numerator is domestic value-added in its exports. If there is no GVC trade, gross exports are produced fully with domestic value added, $\text{DCE}_{n,t} = 1$. Thus, $1 - \text{DCE}_{n,t}$ gives the foreign value added contribution to gross exports. The lower is the domestic content in exports, $\text{DCE}_{n,t}$, the more integrated are the global supply chains.

Given the complex trade patterns in the model with GVC trade, Table 3 shows the formulas for country 1's gross exports and value-added exports for each production chain (in our two-country, two-stage case).

Start with chains $(1,1)$ and $(2,2)$. Gross and value-added exports for these chains are identical, because production is not fragmented across borders. For chain (1,1), country 1's

Table 3: Example: country 1 gross and value-added exports

Chain	Gross exports	Value-added exports
(1,1)	$Y_{2,t}\lambda_{(1,1),2,t}\widetilde{\gamma}^2$	$Y_{2,t}\lambda_{(1,1),2,t}\widetilde{\gamma}^2$
(2,2)	0	0
(2,1)	$Y_{2,t}\lambda_{(2,1),2,t}\widetilde{\gamma}^2$	$Y_{2,t}\lambda_{(2,1),2,t}\widetilde{\gamma}^2\gamma^2$
(1,2)	$(Y_{2,t}\lambda_{(1,2),2,t} + Y_{1,t}\lambda_{(1,2),1,t})\widetilde{\gamma}^1$	$(Y_{2,t}\lambda_{(1,2),2,t} + Y_{1,t}\lambda_{(1,2),1,t})\widetilde{\gamma}^1\gamma^1$

Note: $Y_{n,t} = P_{n,t}\,(C_{n,t} + X_{n,t})$ is final spending in country n.

exports are simply country 2's final spending on that chain, given by $Y_{2,t}\lambda_{(1,1),2,t}$. Chain (2,2) does not involve flows from country 1 to country 2, by definition, so exports are zero for both measures. Next, look at chain (2,1), in which gross and value-added exports are different for country 1 owing to production fragmentation across countries. The gross value of exports of country 1 is recorded as $Y_{2,t}\lambda_{(2,1),2,t}$. However, country 1's value added exports are only a fraction $\gamma^2\widetilde{\gamma}^2$ of the gross value (because we only have two stages and $\widetilde{\gamma}^2$, this fraction is γ^2). The remaining value is from country 2 at stage 1 (used by country 1 as intermediates). Lastly, consider chain (1,2), in which country 1 exports the stage 1 good to country 2, then country 2 sells the finished good to both country 1 and to itself. Thus, final demand in both countries generates exports for country 1 in stage 1. Moreover, country 1's gross and value added exports coincide because all exports of stage 1 are value added.

Comparison to Empirical Measures Since Hummels, Ishii, and Yi (2001) developed a measure of the import content of exports, "VS", there have been a number of contributions that have extended and generalized that measure. Some of the major contributions include those of Johnson and Noguera (2012) and Koopman, Wang, and Wei (2014).[6] We now compare the measures in these two papers to our measure, $\text{DCE}_{n,t}$.[7] The primary measure in Johnson and Noguera (2012) is the "VAX" ratio, which measures the domestic value-added embodied in exports ultimately absorbed abroad. So, conceptually, the VAX ratio corresponds to the first term in equation (13) divided by total gross exports. Koopman, Wang, and Wei (2014) develop several measures to capture domestic and foreign value-added in exports. Their measure "domestic value added in exports" captures the value-added embodied in exports ultimately absorbed abroad and domestically, and is conceptually identical to $\text{DCE}_{n,t}$.

The empirical measures were all developed to make use of available input-output tables,

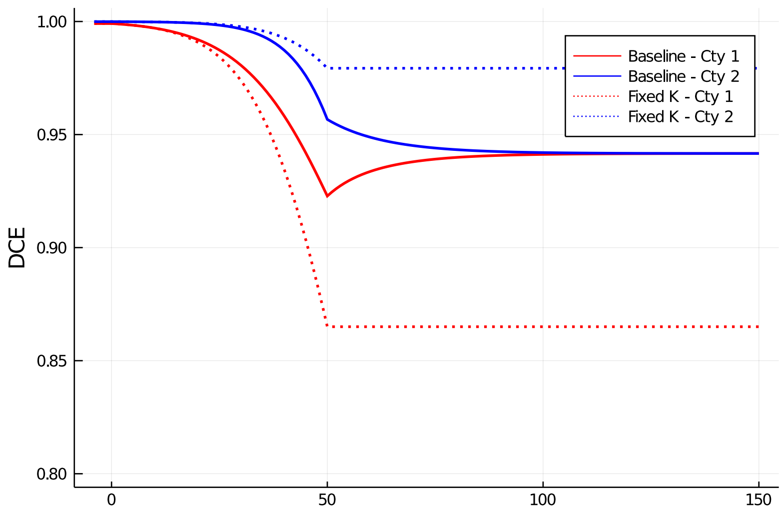
[6] Also, see Daudin, Rifflart, and Schweisguth (2011), Los, Timmer, and de Vries (2016), Johnson and Noguera (2017), Wang et al. (2017), and Timmer et al. (2021).

[7] $1 - \text{DCE}_{n,t}$ is a generalization of the "VS" measure from Hummels, Ishii, and Yi (2001).

especially those from WIOD and the OECD. However, our measure cannot be directly applied to input-output tables, because those data are presented in roundabout form with no explicit account of vertical linkages.[8]

Evolution of DCE in Model Figure 5 shows the dynamics of the DCE over time with asymmetric countries for both the baseline case (solid lines) and the fixed capital case (dotted lines).[9] During the first 50 periods with declining trade costs, countries trade more with each other, GVC trade becomes increasingly more important, and the DCEs fall over time in both cases and both countries. The patterns of the DCE ratios are strikingly different post period 50 across the two countries and across the two cases. In the baseline case, the DCE ratio post period 50 rises in country 1 but declines in country 2. In contrast, these ratios post period 50 are constant in the fixed-capital case.

Figure 5: The domestic content of exports with asymmetric countries



Note: DCE is computed using equation (14). The baseline model begins with $(A_1^\star, A_2^\star) = (0.5, 1)$, then imposes $A_{1,t} \approx (1.04)A_{1,t-1}$ for $t = 2, \ldots, 50$, with $A_{1,t} = 1$ thereafter ($A_{2,t} = 1$ in every period). The model with fixed capital restricts investment to equal depreciated capital. Periods up through 0 correspond to the initial steady state. Trade costs decline from period 1 to 50 and remain constant afterwards. Red lines are for country 1 and blue lines are for country 2. Solid lines are for the baseline model and dotted lines are for the model with fixed capital.

To understand this, first consider the case with fixed capital. There are permanent H-O effects since country 2 is relatively more capital abundant. As trade costs decline in the first

[8]More precisely, there exists a mapping from a multi-sector version of our model to an input-output table. However, as discussed and proved in de Gortari (2019), there is no unique mapping from an input-output table to a GVC model with more than two stages of production.

[9]For the no-GVC case, the DCE is of course equal to one, and thus omitted from the figure.

50 periods, the DCE ratio declines by less in country 2 than in country 1 due to two effects. The first effect is through comparative advantage across stages. Country 1 has a comparative advantage in stage 2, which involves both domestic and foreign imports in its production and exports. Country 2, on the other hand, has comparative advantage in stage 1, which has zero foreign content. Therefore, country 2's DCE ratio is higher. The second effect is due to relative size. Country 1 is permanently smaller so its import share is higher contributing to an even lower DCE ratio. After period 50, since factors of production and productivity and trade costs are all constant, the trade shares and DCE ratios are also constant. The DCE ratio is 0.86 in country 1 and 0.98 in country 2.

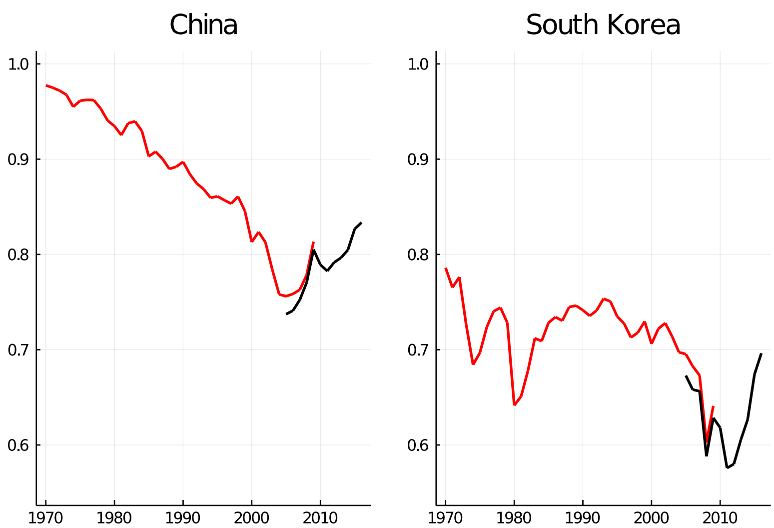
Now consider what happens in the presence of endogenous capital. In the new steady state the two countries are symmetric, so their GVC shares and DCEs are identical and equal to 0.94. The interesting differences arise along the transition. First consider periods 1–50. Relative to the case with fixed capital, the DCE for country 1 is considerably larger, while it is smaller for country 2. The reason is because H-O comparative advantage that drives the difference in the DCEs across countries in the fixed capital case diminishes endogenously over time with capital accumulation. Specifically, comparative advantage shrinks in stage 1 for country 2, and in stage 2 for country 1. Thus, the gap between the DCEs across countries is smaller in the baseline than in the fixed capital case. After period 50, trade costs and productivity are constant while capital accumulation persists in both countries, albeit at a higher rate in country 1. As a result, H-O comparative advantage continues to diminish over time to zero in the new steady state, which leads to the convergence of the DCEs across the two countries to the same steady state level.

The non-monotonic dynamics of country 1's DCE is particularly interesting, and results from a very simple process of productivity convergence coupled with trade integration. Next we discuss the empirical relevance of these implications.

Empirical Evidence The numerical exercise underlying Figure 5 is quite stylized. However, it is consistent with empirical estimates of the DCE in two ways. First, as the figure shows, the country that is initially smaller, with lower productivity, has a rapid decline in its DCE, followed by a subsequent partial reversal of that decline. This pattern is consistent with the existing empirical evidence. Figure 6 plots the VAX ratio for China and South Korea, using estimates from Johnson and Noguera (2012) and OECD (2019). Both countries experienced a decline in their VAX of about 20 percentage points before reversing. Between 2010 and 2016, the DCE rose by about 6 percentage point in China and about 8 percentage points in South Korea. The reversal pattern of the DCE ratio is also consistent with Kee and Tang (2016), who directly measure the domestic content in Chinese exports using detailed firm-level and transaction-level data. They find that the DCE in fact rises from 65% to 70% in the period of 2000–2007.

Second, in Figure 5, country 1 is the more rapidly growing country for 50 periods, and during this time it has a sharper fall in the DCE. Country 2 grows less rapidly, and has a smaller fall in the DCE. This pattern is consistent with the evidence in Figure 3b in Johnson and Noguera (2017). The figure shows, as mentioned previously, a statistically significant negative cross-country correlation of average annual growth of GDP and average annual change in VAX.[10] To summarize, our numerical exercise is consistent with the empirical evidence on both the time series and cross-section dimensions.

Figure 6: Domestic Content of Exports



Note: DCE is measured as VAX from Johnson and Noguera (2012) (red) and trade in value-added from OECD (2019) (black). VAX is constructed slightly differently from trade in value-added, and from DCE, but, in practice, all these measures yield similar numbers at the aggregate level.

3.4 Welfare Gains from Trade

After illustrating how capital accumulation and GVC trade interact to impact the dynamics of trade patterns and economic growth, we now show how this interaction influences the

[10] As mentioned above, the Johnson and Noguera (2012) measure of VAX is similar to our measure of DCE.

dynamic and steady state gains from trade. We measure the dynamic gains from trade by calculating the consumption equivalence between remaining in autarky and opening up to trade. In particular, let $\{C_{n,t}^{\text{aut}}\}$ and $\{C_{n,t}^{\text{trd}}\}$ denote equilibrium consumption paths under autarky and with trade integration, respectively. Following Lucas (2003), the measure of dynamic gains ξ_n^{dyn} is the per-period proportionate increase in consumption required to make the autarky consumption path yield the same lifetime utility as the stream with trade integration. In other words, it is the value ξ_n^{dyn} that satisfies

$$\sum_{t=1}^{\infty} \beta^{t-1} H_{n,t} \ln \left(\frac{\left(1 + \frac{\xi_n^{dyn}}{100}\right) C_{n,t}^{\text{aut}}}{H_{n,t}} \right) = \sum_{t=1}^{\infty} \beta^{t-1} H_{n,t} \ln \left(\frac{C_{n,t}^{\text{trd}}}{H_{n,t}} \right). \tag{15}$$

The steady state gains from trade is defined as the percentage difference between the long-run steady state consumption with trade and that under autarky:

$$1 + \frac{\xi_n^{ss}}{100} = \frac{\lim_{t\to\infty} C_{n,t}^{\text{trd}}}{\lim_{t\to\infty} C_{n,t}^{\text{aut}}}. \tag{16}$$

GVC trade and capital accumulation impact both the dynamic and steady state gains from trade, because they affect the consumption path. The results are reported in Table 4 for both the symmetric and asymmetric cases.

Table 4: Welfare gains from trade

	Baseline	No GVC trade	Fixed capital
Symmetric countries			
Steady state gains	21.9%	5.8%	14.7%
Dynamic gains	3.6%	0.8%	3.6%
Asymmetric countries			
Steady state gains			
Country 1	21.8%	5.8%	20.2%
Country 2	21.8%	5.8%	9.8%
Dynamic gains			
Country 1	4.8%	0.9%	5.2%
Country 2	3.0%	0.7%	2.3%

Note: Dynamic welfare gains are computed using equation (15). Steady state welfare gains are computed using equation (16).

Let's first look at the steady state gains in the case with symmetric countries. The gains in the baseline model with the GVC trade are substantially larger than those in the case without GVC trade and in the fixed capital case: 22% versus 5.8% and 15%. Clearly, GVC trade and capital accumulation amply the steady state gains from trade. To illustrate these mechanisms transparently, we derive an explicit formula for the steady state gains from trade when the capital intensity is constant across stages of production: $\alpha^s = \alpha$. In this case the steady-state gains from trade are captured entirely by changes in the purely domestic GVC share, $\lambda_{\ell_n,n}$, where $\ell_n = (n, n, n, ..., n)$, as

$$1 + \frac{\xi_n^{ss}}{100} \equiv \frac{C_n^{\text{trd}}}{C_n^{\text{aut}}} = \left(\frac{\lambda_{\ell_n,n}^{\text{trd}}}{\lambda_{\ell_n,n}^{\text{aut}}}\right)^{-\frac{1}{\theta(1-\alpha)}} = \underbrace{\left(\frac{\lambda_{\ell_n,n}^{\text{trd}}}{\lambda_{\ell_n,n}^{\text{aut}}}\right)^{-\frac{1}{\theta}}}_{\text{Trade contribution}} \quad \underbrace{\left(\frac{\lambda_{\ell_n,n}^{\text{trd}}}{\lambda_{\ell_n,n}^{\text{aut}}}\right)^{-\frac{\alpha}{\theta(1-\alpha)}}}_{\text{Capital contribution}}.$$

We drop the time subscript t for the steady state analysis. The first term on the right-hand side captures the standard Ricardian gains through measured productivity as in Arkolakis, Costinot, and Rodríguez-Clare (2012). The second term captures the gains through changes in capital stocks between the steady states. Because capital is a reproducible factor of production, in steady-state, it is effectively like an intermediate input, and thus amplifies the gains from trade in a similar way.

To drive the point home, consider the gains from trade in our model with symmetric countries and free trade. In this case, all N^S chains are symmetric, implying $\lambda_{\ell_n,n}^{\text{GVC}} = \frac{1}{N^S}$. Without GVC trade, only the final stage is traded, and N chains are available. This is equivalent to a standard Eaton-Kortum model with one stage of production. Free trade and symmetric countries imply that $\lambda_{\ell_n,n}^{\text{EK}} = \frac{1}{N}$. Clearly, $\lambda_{\ell_n,n}^{\text{GVC}} < \lambda_{\ell_n,n}^{\text{EK}}$. Since under autarky, the purely domestic GVC share is $\lambda_{\ell_n,n}^{\text{aut}} = 1$, the presence of GVC trade amplifies the gains from moving from autarky to free trade by a factor of N^{S-1}, for the same value of θ. This also shows clearly the complementarity between the number of countries and stages.[11] Introducing endogenous capital further amplifies the gains from trade. Relative to a model without reproducible capital, the presence of capital amplifies the gains by a factor of $\frac{1}{1-\alpha}$. In this simple example, the amplification from either GVCs, or from capital, is not affected by the presence, or absence, of the other. This result is not true in general, and the effects of the two channels are indeed interdependent in more general settings.

We next turn back to the dynamic gains from trade in Table 4. The dynamic gains are much smaller than the steady state gains in all three cases. This is because consumption gradually rises to the new steady state level due to consumption smoothing and capital accumulation, if allowed. Moreover, GVC trade amplifies the dynamic gains from trade substantially by more than four folds, but capital accumulation does not alter much the dynamic

[11] Antràs and de Gortari (2020) derive a similar result; this is demonstrated quantitatively by Yi (2003).

gains. Consumption paths in Figure 2 are the key to understanding this result. The levels of consumption in the baseline case are higher than those in the no-GVC case throughout the transition and in the steady state. However, consumption levels in the baseline are lower than those in the fixed capital case during the transition, because consumption is postponed to accumulate capital. With discounting, the dynamic gains from trade in the baseline turn out to be similar to those in the case with fixed capital.

Now let's turn to gains from trade under the asymmetric calibration. We first look at the steady state gains. In both the baseline case and the no-GVC case, the steady state gains are identical across the two countries. This is because in the new steady state, both countries are identical in productivities, capital stocks and trade costs. The gains are substantially larger for both countries with the GVC trade than without GVC trade: 22% versus 5.8%. In the fixed capital case, the steady state gains are much greater for country 1 than country 2: 20% versus 10%. This is the outcome of fixed differences in the capital stock, whereby country 1 permanently smaller in size.

We now turn to the dynamic gains. In the baseline case, the dynamic gains are larger for country 1 than for country 2: 4.8% versus 3%. This is because during the transition country 1 is smaller so it realizes a greater increase in openness early on, boosting consumption, investment, and welfare. Similarly, these benefits accrue to country 2 more slowly over time given its large initial size. As a result of discounting, the dynamic gains are smaller for country 2.

With no GVC trade, the dynamic gains are much smaller for both countries, following similar intuition in the symmetric calibration. What is interesting is that relative to country 2, country 1's dynamic gains are much larger with GVC trade than without GVC trade. The key to understand this result is the transitional consumption cost of capital accumulation for country 1 without the access to the GVC trade. With the GVC trade, country 1 can leverage its comparative advantage and concentrate more on labor intensive stages during the transition, which allows slower capital accumulation and less sacrifice in consumption. Moreover, the GVC trade allows country 1 to import stage 1 products which induces higher efficiency gains, facilitating capital accumulation. This example illustrates that the fragmentation of supply chains benefits countries with higher needs for capital accumulation relatively more.

With fixed capital the dynamic gains are larger for country 1, the smaller country. It is worth noting that the gains with fixed capital in country 1 exceed the gains in the baseline with reproducible capital. This is purely mechanical. With fixed capital, the share of income allocated to investment declines sharply since the level of investment is fixed. Thus, all of the additional income generated from trade goes directly toward consumption in the case with fixed capital, so boosts the gains from trade. Of course, the steady state level of consumption is lower because of the lower productive capacity, but along the transition, the boost in consumption in early periods generates huge welfare gains.

4 Importance of Key Model Mechanisms

This section investigates two key features of our theoretical framework. The first key feature is the presence of both Ricardian and H-O motives for trade. We evaluate the relative contribution of the H-O forces versus the Ricardian forces on the dynamic gains from trade. We further highlight the H-O forces using a case with asymmetric population growth across countries. The second key feature of our model is the snake-form supply chains. We compare the model implications under the snake-form supply chains with those under the spider-form supply chains, commonly used in the trade literature.

4.1 Ricardian and H-O Motives for Trade

Our framework captures both Ricardian and H-O motives for trade. To tease out the role of H-O forces, we study a case where H-O forces are shut off under the asymmetric calibration. Specifically, we turn off H-O channels by setting the factor intensity α identical across stages. Consequently, even with time-varying relative input abundance, there are no H-O forces because the two stages are identical in factor intensity. The remaining incentive for trade is Ricardian. There are two types of Ricardian comparative advantages in the Eaton-Kortum trade model. The first type is micro-Ricardian: the productivity differences across countries for each variety lead to intra-stage trade. The other type is macro-Ricardian: the differences in average productivity across stages and countries lead to inter-stage trade. One feature of our asymmetric experiments is that there are no macro-Ricardian forces built in, because productivity is country-specific, but not stage-specific. Instead, it is the micro-Ricardian forces that deliver gains from trade in the no H-O case.

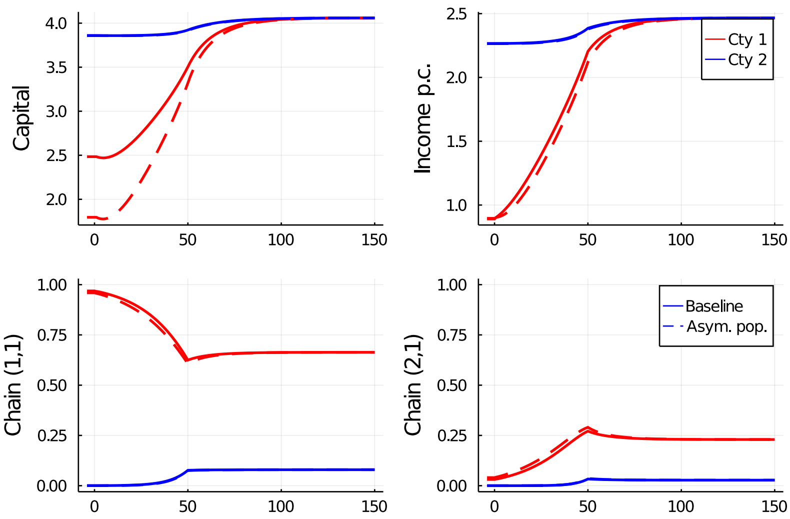
The dynamic gains from trade in the no H-O case are 4.4% and 3.1% for countries 1 and 2 respectively. Recall that these gains in the baseline are 4.8% and 3.0%, respectively. Thus, Ricardian forces are more important than H-O forces in accounting for the dynamic gains in this example of asymmetric countries. This result is not surprising because the two countries become similar and the H-O effects diminishes over time. The dynamic gains for country 1 are lower in the no H-O case than in the baseline case: 4.4% versus 4.8%. The intuition for this result is that country 1 benefits by importing the capital-intensive (stage 1) varieties from country 2 to fuel its investment demand in the baseline model. Without H-O forces, the fast growing country satisfies its investment demand by producing stage-1 goods itself, which is more costly as it requires foregoing more consumption along the transition. In contrast, the dynamic gains for country 2 are slightly higher in the no H-O case than in the baseline case: 3.1% versus 3%. This is because without the H-O forces, country 2 concentrates relatively less in stage 1, and therefore require less investment, allowing higher consumption along the transition.

Asymmetric population growth Given the presence of H-O forces, it is natural to examine how an exogenous change in relative population size plays out. Empirically, emerging economies have exhibited faster population growth than advanced economies. To capture this pattern in our model, we add to our asymmetric benchmark case the following: we start country 1 at half of the of population of country 2 at date 0, and let it grow gradually to the size of country 2 over 50 years. That is, population in country 1 grows from 1/2 by about 1.4 percent a year, then remains constant at 1 after period 50.

Figure 7 presents the results for this experiment, compared with those in the asymmetric benchmark. Consider the macroeconomic variables of country 1 in the top six panels. During the transition period, as labor increases, despite rapid investment growth, it takes many periods for capital to catch up owing to the incentives of consumption smoothing. Because of the lower capital-labor ratio during the transition, GDP per capita in this case is lower than in the asymmetric baseline. However, because both productivity and population are growing (in the small country), aggregate GDP growth is higher than in the asymmetric baseline. In terms of GVC implications, because country 1 is smaller in this case, its home chain share is lower, but its gains from trade are greater. Also, it is less capital intensive, implying more specialization in downstream production and a greater sahre in chain (2,1). Over time, these gaps vanish, and eventually country 1 reaches the same steady state as country 2

The dynamic gains from trade of country 1 are greater in this case than in the baseline: 7.0% versus 4.8%. The opposite is true for country 2: its dynamic gains are 2.8% in this case and 3.0% in the baseline. This is mainly due to the fact that in the asymmetric-population growth case, the gap in size is even greater than in the baseline model.

Figure 7: Effects of asymmetric population growth



Note: The model with asymmetric population growth begins with $(H_1^\star, H_2^\star) = (0.5, 1)$, then imposes $H_{1,t} \approx (1.04)H_{1,t-1}$ for $t = 2, \ldots, 50$, with $H_{1,t} = 1$ thereafter ($H_{2,t} = 1$ in every period). Both models have $(A_1^\star, A_2^\star) = (0.5, 1)$, then impose $A_{1,t} \approx (1.04)A_{1,t-1}$ for $t = 2, \ldots, 50$, with $A_{1,t} = 1$ thereafter ($A_{2,t} = 1$ in every period). Trade costs decline from period 1 to 50 and remain constant afterwards. Red lines are for country 1 and blue lines are for country 2. Solid lines are for the baseline model and dotted lines are for the model with asymmetric population. The top two panels depict the capital stock $K_{n,t}$ and income per capita $(r_{n,t}K_{n,t} + w_{n,t}H_{n,t})/(P_{n,t}H_{n,t})$. The bottom two panels depict the shares of pending on two select chains, $\lambda_{(1,1),n,t}$ and $\lambda_{(2,1),n,t}$.

4.2 Snake versus Spider Networks

We now compare the implications for trade flows and the gains from trade in our model with snake-form supply chains with those in a model with spider-form supply chains. The latter form is commonly used in the trade literature. In the typical spider model, there is one stage of production that uses capital, labor, and a composite intermediate input in a roundabout way (as in Caliendo and Parro, 2015; Levchenko and Zhang, 2016). The composite good of all varieties serves as intermediate inputs in each variety's production and final demand for consumption and investment. With only one stage of production, each variety is potentially traded and then enters directly into the basket of finished goods.

For the ease of comparison, we focus on the symmetric-country case. To discipline the comparison between the spider model and the snake model, we first calibrate the share of value added in gross production in the spider model, ν, to the aggregate value arising from the snake model: $\frac{\sum_s VA^s}{\sum_s GO^s} = 2/3$. Second, in the spider model there is only one stage of production so we set capital's share in value added equal, α, equal to the aggregate value arising from the snake model: $\frac{\sum_s \alpha^s VA^s}{\sum_s VA^s} = 1/2$. We then conduct two scenarios in the spider model. In the first scenario, the spider model is given the same exogenous processes of trade costs, TFPs, and labor endowments as in the snake model. Comparing this scenario with the snake model, we highlight the impact of the snake input structure on trade flows and gains from trade. In the second scenario, we re-calibrate the new steady state level of trade costs in the spider model to generate the same ratio of imports to absorption in the snake model. In particular, this requires a lower trade cost in the new steady state, about 1.5 instead of about 1.7. Comparing this scenario with the snake model highlights the difference in gains from trade when both models are disciplined by the same key observable.

Table 5 reports the summary statistics from these two spider scenarios, together with the baseline snake model. In our snake model, the steady state gains from trade are 21.8%, and the dynamic gains are 3.6%. In contrast, when countries experience the same exogenous processes, the spider model implies the steady state gains of 8.8% and the dynamic gains of 1.16%. When we look at the ratio of imports to absorption in the new steady state, it is 0.16 in the snake model, but only 0.11 in the spider model. Thus, the snake model generates larger trade flows and offers higher gains from trade than the spider model. Once the trade costs are lowered in the spider model to deliver the same ratio of imports and absorption in the snake model, we still find substantially higher gains in the snake model than the spider 2 model: 21.8% versus 13.5% for steady state gains, and 3.6% versus 1.8% for dynamic gains.

Table 5: Snake versus Spider

	Snake	Spider 1	Spider 2
Steady state gains from trade	21.8%	8.8%	13.5%
Dynamic gains from trade	3.6%	1.2%	1.8%
Steady state imports-to-absorption	0.16	0.11	0.16

Note: We consider the symmetric countries in each model specification. The Snake and Spider 1 cases have the same exogenous processes of trade costs and TFPs. In the Spider 2 case, we re-calibrate the new steady state trade cost to generate the ratio of imports-to-absorption in steady state equal to that in the snake model. In both spider cases, the parameter for the ratio of value added to gross output is set equal to the aggregate value arising from the snake case.

5 Conclusion

As the world's economies globalized over the past 75 years, there have been increasing opportunities for these economies to reap the gains from specializing in goods and sectors of their comparative advantage. Continued reductions in barriers to trade over decades have pushed specialization to even finer levels than goods – increasingly to particular stages of a good's production sequence. The fragmentation of production chains across multiple countries has become an important source of gains and gives rise to global value chains (GVCs). These gains are the primary reason why economies have chosen to globalize in the first place.

To understand the dynamics of GVCs over the long horizon during which trade barriers have fallen, we have studied the implications of trade integration in a two-country model with Ricardian and Heckscher-Ohlin comparative advantage forces, GVCs, and capital accumulation. From our model analysis, we identify a two-way interplay from comparative advantage and GVCs to capital accumulation and from capital accumulation to comparative advantage and GVCs. The additional aggregate productivity gains from specializing in GVCs induces a sufficiently large boost to capital accumulation to offset the incentive to delay investment until trade barriers are even lower. In addition, when the countries are asymmetric, the dynamics of capital accumulation endogenously affects comparative advantage so that the country with more rapid capital accumulation increasingly specializes in the capital-intensive stage. In turn, this provides even more incentive for that country to accumulate capital.

Eventually, as the capital-labor ratios of the two economies converge, the motive for Heckscher-Ohlin specialization in particular stages diminishes. As a result, the domestic content of exports (DCE), which initially declines as GVCs expand, eventually rises again for the fast growing country. This implication is supported by empirical patterns on the DCE. We also find that the gains from trade in a framework with GVCs are several times larger than in a similar dynamic framework without GVCs. Finally, we compare the gains from our snake GVC framework to the more commonly used "spider" framework with roundabout production and also find that the gains are two to three times larger.

In our perfect competition framework, the only frictions preventing the formation of GVCs are iceberg trade costs. We have abstracted from any frictions associated with market power and asymmetric information. To the extent capital and labor in emerging economies are not paid their marginal products, this could hinder the growth-enhancing role of GVCs and dynamic gains from trade. In addition, as Antràs and Chor (2013) has shown, bargaining and information frictions can affect whether upstream or downstream stages of production are outsourced. To the extent outsourced stages tend to occur abroad, there would be implications for GVC formation.

Our framework assumes balanced trade to focus on the main mechanisms of comparative advantage, GVCs, and capital accumulation. Allowing for current account imbalances will clearly have implications for each of these. Specifically, introducing international borrowing and lending will speed up the convergence of the capital-labor ratios, which in turn accelerates the evolution of H-O comparative advantage and GVCs. On the other hand, financial autarky has been shown to be a reasonable approximation to the world, because the observed trade imbalances are small. For example, Caselli et al. (2020) show that income and consumption are highly positively correlated for most countries, consistent with the implications of financial autarky.[12]

Our analysis has also focused on trade integration under perfect foresight. It would be useful to study the effects of trade integration when it embodies an unexpected shock element. In addition, the effects of shocks, such as pandemics, on GVCs could be studied. In particular, to the extent tail risks may have increased, this could have implications for GVC formation.[13] Finally, our framework could be thought of as a multi-sector framework with each stage representing a sector. It would be useful to explicitly introduce into our framework the standard notion of sectors, such as manufacturing and services, as in Lee and Yi (2018). We leave these and additional analyses for future research.

[12]This is related to the "Feldstein-Horioka" puzzle. See Bai and Zhang (2010) for an explanation for the puzzle based on financial frictions. Also, Heathcote and Perri (2002) show that financial autarky is a modeling benchmark closest to international business cycle statistics.

[13]Caselli et al. (2020) study the effects of increased openness and exposure to global shocks, and find that international trade, through its diversification channel, can lead to lower income volatility.

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