

On Speculative Frenzies and Stabilization Policy*

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Abstract

This paper examines whether tasking central banks with leaning against asset booms can conflict with their existing mandates to stabilize goods prices and output. The paper embeds the Harrison and Kreps (1978) model of speculative booms in a monetary model based on Rocheteau, Weill, and Wong (2018). In the model, a speculation shock that generates an asset boom is associated with higher output but a lower price level, unlike aggregate demand shocks that raise both output and prices. Relying on contemporaneous monetary policy to respond to asset booms in the model creates a trilemma, since such a policy cannot simultaneously stabilize output, the price level, and real asset prices. Stabilizing all three requires other approaches beyond contemporaneous liquidity management.

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Introduction

The Global Financial Crisis has led to growing calls for policymakers to act against asset booms to avoid the potential consequences should these booms end in collapse as they did during the crisis. These interventions are often framed as part of a proposed financial stability mandate for central banks. Since central banks already face mandates to stabilize consumer prices and output, an important question before tasking central banks with responding to asset booms is whether leaning against asset prices is compatible with their existing mandates. Is financial stability inherently complementary to what central banks already do, or would central banks be forced to decide between mandates if asked to take on additional responsibilities?

In an influential paper, Bernanke and Gertler (1999) considered this question and argued that “central banks should view price stability and financial stability as highly complementary and mutually consistent objectives” (p78). They reached this conclusion by introducing an exogenous wedge between the price of capital and the present discounted value of capital income and showing that it leads to higher output and higher inflation in an otherwise standard macroeconomic model. This suggests there is no conflict between aligning asset prices with fundamentals and stabilizing prices and output. It also suggests little benefit from letting monetary policy respond directly to asset prices, since responding to inflation and output would already lead central banks to effectively respond to an asset boom should one arise.

This paper revisits the question of whether financial stability is consistent with the goals of stabilizing prices and output. Rather than assume an exogenous wedge between the price of an asset and its fundamental value, I allow for an endogenous asset boom as in the Harrison and Kreps (1978) model. In that model, time-varying beliefs combined with short-selling constraints give rise to a speculative boom in which the asset price can exceed what any agent expects the asset will pay out in dividends. To study the implications of such a speculative boom on output and prices, I embed a version of their model into a tractable monetary framework recently proposed by Rocheteau, Weill, and Wong (2018). I find that a shock that induces agents to speculate and generates a sufficiently large asset boom will be associated with higher output but a *lower* price level. A speculative boom driven by shocks to beliefs will not correspond to an aggregate demand shock of the type central banks are already tasked with offsetting.

The logic behind this result is due to the interaction between speculation and savings. Agents in the model buy assets intending to sell them to others later. These other agents must save to be able to buy assets in the future. To ensure future asset buyers don’t just save but also buy goods so the goods market clears, the return they expect to earn when they buy assets must not be too high. In some circumstances, this requires a low price level to ensure the *real* price for the asset is sufficiently high to keep the expected return on the asset in check. Basically, the high expected return to speculation during a boom induces agents to save, at least in part by hoarding money rather than using it to consume, until the real price of the asset rises enough to ensure an equilibrium. Caballero (2006) conjectured an asset boom might lead to lower inflation for similar reasons, although he did not formally model this phenomenon.

Since asset booms need not coincide with aggregate demand shocks, a financial stability mandate may conflict with stabilizing output and prices. Indeed, I show that my model implies a trilemma: It is impossible to use contemporaneous monetary policy to stabilize asset prices, goods prices, and output in the face of a speculation shock. Increasing liquidity can help offset the lower price level during the boom, but depending on who receives this liquidity, either the asset boom or the output boom will be amplified. Essentially, the asset boom is driven by optimism. Tighter monetary policy does not cure agents of their optimism; it only prevents them from acting on it to buy assets. As long as agents remain optimistic, their incentive to work and save may remain high even if they can't buy the asset, so tighter monetary policy may not stabilize output or the price level. Effective stabilization requires an intervention to offset optimism. For example, a financial transactions tax that makes speculation less profitable might be better suited for stabilization.

The tension between financial stability and stabilizing output and goods prices in the model seems relevant in practice. Okina, Shirakawa, and Shiratsuka (2001) describe the challenge for the Bank of Japan in the mid 1980s when it faced asset and output booms with little sign of inflation. The Bank of Japan reported a similar tension just before the Global Financial Crisis.¹ As I discuss in more detail below, Bordo and Wheelock (2007) and Christiano et al. (2010) identify historical stock market booms in various countries and find that they tend to be associated with lower rather than higher inflation. I also provide some evidence from the 1980s boom in Japan that large corporations, many of whom engaged in stock speculation, increased their cash holdings, in line with the key mechanism in my model.

Beyond the insights on asset booms, my model suggests new factors that can affect inflation. New Keynesian models emphasize the role of demand shocks in shaping inflation through their effect on marginal costs: When goods prices are rigid, lower demand for goods can lead to lower output, which decreases the marginal costs producers expect to face now and in the near future. By contrast, prices in my model are flexible, and what drives the price level are shocks in asset markets. In that sense, my model is closer in spirit to Brunnermeier and Sannikov (2016) and Piazzesi and Schneider (2018) who also study the effect of shocks originating in the asset market on the price level in flexible price models. These papers consider shocks that lower the value of risky assets and increase demand for safe money just when financial intermediaries create fewer money-like assets. That makes money more valuable, implying a fall in the price level. In my model, a speculation shock *increases* asset valuations but still leads to a fall in the price level. This is because future asset buyers save more, including by holding liquid assets they would have spent on goods.

The paper is organized as follows. The remainder of this section reviews the related literature. Section 1 lays out a purely monetary version of the Rocheteau, Weill, and Wong (2018) model in which money is the only asset. I then add a partially illiquid asset in Section 2 and allow for heterogeneous beliefs about its dividend payments in Section 3. In Section 4, I endogenize output and show that the asset boom and the lower price level will be associated with an output boom. Section 5 considers policy interventions and illustrates the trilemma for monetary policy. Section 6 reviews the evidence on asset booms, inflation, and liquidity hoarding. Section 7 concludes. Proofs of all propositions are in an Appendix.

¹See, for example, the Bank of Japan Outlook for Economic Activity and Prices, April 27, 2007.

Related Literature. As noted above, Bernanke and Gertler (1999) already examined whether financial stability is compatible with stabilizing output and prices. Using the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), they find that a wedge between the price of capital and the present discounted value of the income from capital acts like an aggregate demand shock. While they motivate the wedge as something that could reflect excessive optimism, they introduce the wedge exogenously. As such, their model ignores the possibility that during a boom agents might save to buy the asset in the future as in my model. Conversely, my model abstracts from important features in their model such as sticky goods prices and the role of assets as collateral against which agents can borrow.

Gilchrist and Leahy (2002), using the same Bernanke, Gertler, and Gilchrist (1999) model, study the role of asset price booms in the model in more detail. First, they consider a news shock about future productivity, which like the wedge in Bernanke and Gertler (1999) increases the price of capital without affecting current productivity. They find that a central bank which targets inflation but sets a higher real interest rate in response to higher future productivity comes close to replicating the first-best outcome. This suggests no conflict between price stability and responding to asset booms. Gilchrist and Leahy (2002) then consider a shock to the net worth of the producers who own capital. They confirm that this shock increases both inflation and output in the benchmark version of the model, and that aggressive inflation targeting can help lower inflation, output, and asset prices. At the same time, they observe that a net-worth shock is similar to a cost-push shock in that it reduces the cost of borrowing for firms, implying it will be impossible for a central bank to *fully* stabilize both inflation and output. Although my paper considers a different shock and a different model, my result that some shocks that lead to asset booms create a conflict between different stabilization mandates is similar in spirit to this finding.

Christiano et al. (2010) also consider the effect of a news shock about future productivity as in Gilchrist and Leahy (2002). However, they assume the central bank does not aim for a higher real interest rate in response to this shock. They find that when the central bank is aggressive in targeting inflation without aiming for a higher real rate, a news shock will be associated with an asset boom, lower inflation, and an output boom. This is similar to the effect of a speculation shock in my model, although the mechanism is different. In addition, the main insight from Christiano et al. (2010) is that a central bank should aim for a higher real rate in response to a news shock about future productivity, which would also help dampen asset prices. In my model, by contrast, contemporaneous monetary policy cannot simultaneously stabilize asset prices, output, and goods prices in the midst of a speculation shock.

A separate literature studies the tradeoff between financial stability and price and output stability not in models of asset booms but models with financial crises when agents borrow and then default. Examples include Svensson (2017), Gourio, Kashyap, and Sim (2018), and Boissay et al. (2021). In principle, an asset boom and bust could trigger default. Christiano et al. (2010) show that asset booms are associated with credit growth in both their model and the data. My model abstracts from credit, although I could add limited borrowing and lending without changing my key results. Geanakoplos (2010) and Simsek (2013) show that with heterogeneous beliefs, agents would naturally want to borrow and lend.

The model I use to study these questions draws on two literatures. The first concerns beliefs-driven asset booms. Miller (1977) first argued that the combination of heterogeneous beliefs and short-sales constraints can lead to high asset prices. I follow Harrison and Kreps (1978) in studying a model with *time-varying* beliefs that can give rise to speculation. Harris and Raviv (1993) and Scheinkman and Xiong (2003) show how time-varying beliefs can emerge endogenously when agents are overconfident in their own models or signals. Both emphasize that these models can generate asset booms that feature realistically high trading volumes. For this reason, such models has become a popular framework for studying asset booms. Simsek (2021) offers a recent survey of various macroeconomic applications of these models. Among these, perhaps the most closely related paper is Bigio and Zilberman (2020). Their model also implies that beliefs-driven asset booms will be associated with higher output. In their setup, this is because employers must hire workers before they see realized productivity, and so more optimism implies more hiring. This is similar to how agents in my model produce more when they expect to earn a higher return on the income they earn from producing. However, the motive to save that is central in my model does not play a key role in their setup, and Bigio and Zilberman (2020) abstract from both money and monetary policy.

A related strand of the literature on heterogeneous beliefs concerns consumption and savings decisions, e.g., Guzman and Stiglitz (2016) and Iachan, Nenov, and Simsek (2021). These papers show that whether savings rise when agents hold different beliefs depends on whether the intertemporal elasticity of substitution for agents exceeds one. I follow Iachan, Nenov, and Simsek (2021) in assuming preferences that imply that agents want to save more when they expect the return on the assets they hold will be higher.

The other literature my paper draws are monetary models that explore the interaction between money and asset prices. For a review of the broad literature, see Williamson and Wright (2010). The framework I use draws on Rocheteau, Weill, and Wong (2018) and Herrenbrueck (2019). One closely related paper in this literature is Lagos and Zhang (2019), which also develops a model with money and a dividend-bearing asset that agents value differently. However, they focus on the opposite direction of causality. That is, they examine how changes in the growth rate of money, and hence changes in inflation, affect asset prices. The key feature in their paper is that inflation discourages agents from holding money, which discourages asset trading and lowers the value of the asset given it may not be held by the agents who value it the most. By contrast, I examine how a shock to what agents believe about the asset affects goods prices.

Finally, an important feature of my model is that when agents are sufficiently optimistic, the price of the asset will be pinned down by the wealth agents have to spend on assets rather than their expectations of dividends. This corresponds to the notion of “cash-in-the-market” asset pricing first discussed in Allen and Gale (1994). In their model, such pricing occurs when agents first invest in liquid assets and then realized demand for liquid assets turns out to be high. Cash-in-the-market pricing arises because agents try to unload their illiquid assets to agents with limited liquidity, leading the asset price to fall. My model is closer in spirit to in Bolton, Santos, and Scheinkman (2021). In their model, some agents expect to profit more than others from buying assets, not because of optimism but because they are better informed. As in my model, the agents who stand to profit from buying assets are borrowing constrained, and so asset

prices depend on how much wealth these agents have rather than what they expect to earn. Closer still is the work of Caballero and Farhi (2017) on demand for safe assets. Their setup also assumes some agents have a stronger preference for an asset than others, although in their case it is because of different degrees of risk-aversion. My assumption that pessimists avoid the asset is equivalent to their assumption that risk-averse agents are infinitely risk-averse and refuse to hold the asset. The question then is whether natural buyers for the asset only hold the asset or hold both money and the asset. Cash-in-the-market pricing in my model corresponds to what they describe as the constrained regime equilibrium.

1 Preliminaries: A Purely Monetary Economy

As anticipated in the Introduction, this paper seeks to embed a model of speculative trade as in Harrison and Kreps (1978) into a monetary setting. I do this in steps. I begin with a purely monetary version of the Rocheteau, Weill, and Wong (2018) model that clarifies the role of money in my economy and how the price level is determined. In the next section, I introduce a dividend-bearing asset that agents can hold in addition to money. Finally, I allow agents to hold different and time-varying beliefs about the payoffs on the dividend-bearing asset in Section 3 and analyze how this affects goods and asset prices.

The economy is populated by a mass 1 of infinitely-lived agents. The model is set in continuous time. Each agent is endowed with a constant flow of y units of a non-storable consumption good per instant. For now, the supply of goods is exogenously determined by these endowments. Later on, I will consider a production economy where agents are endowed with inputs and must choose how much to produce.

Agents derive utility from consumption only at individual-specific random dates $\{t_n\}_{n=1}^{\infty}$, where the gap between urges is distributed exponentially with rate λ . In other words, agents occasionally experience idiosyncratic urges to consume. I assume the law of large numbers holds, meaning that a constant flow λ of agents chosen at random will have an urge to consume at each instant.

For ease of notation, I omit reference to a household's identity. Let c_t denote the amount an agent consumes at date t . Agents are risk-neutral and discount the future at rate ρ , so their utility is just the discounted sum of their consumption at the dates in which they have an urge to consume:

$$\sum_{n=1}^{\infty} e^{-\rho t_n} c_{t_n} \tag{1}$$

Strict risk-neutrality is not essential for my main results, although it greatly simplifies the analysis.² That said, linear preferences imply an infinite elasticity of intertemporal substitution. The fact that agents in

²Rocheteau, Weill, and Wong (2018) show how to solve the general case in which (1) is replaced by $\sum e^{-\rho t_n} u(c_{t_n})$ for some concave function $u(\cdot)$. Herrenbrueck (2019) similarly chooses to focus on the linear case to simplify the analysis.

my model want to save more when they expect a higher return on their savings requires that this elasticity exceed unity. See Iachan, Nenov, and Simsek (2021) for a discussion in a related context.

In this setup, consumption should only be allocated to those agents with an urge to consume. Following Rocheteau, Weill, and Wong (2018), I assume agents cannot rely on intertemporal trade to reallocate goods, e.g. because of limited tools to enforce contracts. However, agents are endowed with money that they can exchange for goods. The model is thus essentially a continuous-time version of Bewley (1980).³

The total supply of money is fixed at an amount \bar{M} at all dates. Allowing the money supply to grow over time would not change the results. Let $F_t(M)$ denote the fraction of households whose money holdings at date t is less than or equal to M for any value $M \in [0, \infty)$. The initial distribution $F_0(M)$ at $t = 0$ is assumed to contain no mass points to ensure no single household's money holding matters for aggregates.

Let P_t denote the price of goods at date t . An equilibrium is a path for the price of goods and the distribution of money holdings $\{P_t, F_t(M)\}_{t \geq 0}$ such that the market between goods and money clears at all dates when households choose their spending optimally. As usual in monetary models, the model admits many equilibria. Following Rocheteau, Weill, and Wong (2018), I restrict attention to *stationary monetary* equilibria. A monetary equilibrium is one in which $P_t < \infty$ for all t so money always has positive value. A monetary equilibrium is stationary if the the price level grows at the same rate as money. Given the money supply is constant, this amounts to equilibria in which $\frac{\dot{P}_t}{P_t} = 0$.

In any monetary equilibrium, agents without an urge to consume prefer to sell their entire endowment y : Holding on to it yields no utility, while selling it allows agents to consume more when they do have an urge to consume. In between urges to consume, then, individual money holdings satisfy

$$\dot{M}_t = P_t y \tag{2}$$

In the remaining dates $\{t_n\}_{n=1}^\infty$, a household must choose how much of its money holdings to spend given the urge to consume. In a stationary equilibrium, a household with linear preferences would choose to spend all of its money holdings. This is because when the price level P_t is constant, a unit of money will buy the same amount at any date. With discounting and a constant marginal utility from consumption, the household should consume immediately rather than wait and obtain the same utility flow later.⁴

Given these optimal decisions, solving for the price level P_t in a stationary monetary equilibrium is straightforward. At any instant, a flow of λ agents chosen at random will have an urge to consume and spend all of their money holdings. Under the law of large numbers, the average money holdings of these

³Bewley (1980) assumed agents have random endowments and a fixed concave utility. Here, agents have a fixed endowment and random utility. Both cases imply agents' marginal utilities would fluctuate over time in autarky, encouraging trade.

⁴If I let the stock of money grow at rate μ , a household would continue to prefer consuming immediately in a stationary monetary equilibrium where $\frac{\dot{P}_t}{P_t} = \mu$ as long as $\mu > -\rho$, i.e., as long as prices do not fall enough to overcome discounting.

agents will be the same as the average money holdings of all agents, which is given by

$$\int_0^\infty M dF_t(M) = \overline{M} \quad (3)$$

Total spending on goods at each instant will equal $\lambda \overline{M}$ regardless of how money holdings are distributed across agents. Since only a measure zero of agents will want to consume at any instant while all other agents sell their endowment, the total value of goods up for sale at each instant is $P_t y$. Equating the two yields a unique value for the price level P consistent with a stationary monetary equilibrium:

$$P_t = \frac{\lambda \overline{M}}{y} \quad (4)$$

The fact that spending on goods is always equal to $\lambda \overline{M}$ regardless of how money holdings are distributed is what makes the Rocheteau, Weill, and Wong (2018) framework convenient. Even though the money holdings of any individual household rise and fall over time, and the distribution of money holdings across households may converge to a steady state distribution only gradually, there is no need to track the distribution of money holdings across households to determine the price level P_t . For the same reason, nothing prevents P_t from jumping to the value in (4) immediately for any initial distribution of money holdings.

To recap, the stationary monetary equilibrium of the model features agents who save in between urges to consume. When money is the only asset, as I have assumed so far, all savings are in money. The next section introduces a dividend-bearing asset so that agents can choose how to save.

2 Assets with Homogeneous Beliefs

Suppose that in addition to a stock of money \overline{M} , the economy is now endowed with one unit of an asset that yields a constant flow of D consumption goods per instant. Let p_t denote the price of the asset and a_t denote a household's asset holdings at date t . The holdings of all households integrate up to 1. Rocheteau, Weill, and Wong (2018) study a similar extension of their model in which agents can hold either money or a dividend-bearing asset, although they consider a government bond that pays out money rather than goods.

As in Rocheteau, Weill, and Wong (2018), agents can exchange money for goods and money for assets, but not goods for assets. Technically, neither an agent's money nor her asset holdings at date t are measurable with respect to whether she has an urge to consume at date t . Agents must allocate their wealth before knowing if they have an urge to consume. Once they learn if they want to consume, they can only exchange their money for goods. Agents can exchange assets for money within an infinitesimally short period of time, but by that point the urge to consume would have lapsed. Agents thus face a tradeoff: Saving with money allows them to buy goods if hit with an urge to consume, but they must forgo dividends they could have earned holding the asset. Money has the advantage of being liquid, but it offers a zero nominal return.

Formally, let W_t denote an agent's nominal wealth at date t , where

$$W_t = M_t + p_t a_t \quad (5)$$

The fact that agents cannot trade goods for assets implies that their consumption must satisfy

$$P_t c_t \leq M_t \quad (6)$$

I continue to assume households cannot engage in intertemporal trade. This means that agents cannot short assets, i.e. they cannot borrow assets with a promise to deliver similar assets in the future. Household asset holdings must therefore be non-negative and cannot exceed their total wealth, i.e.,

$$0 \leq p_t a_t \leq W_t \quad (7)$$

An equilibrium is a path of prices $\{P_t, p_t\}_{t \geq 0}$ for goods and assets, respectively, together with a path for the distribution of nominal wealth holdings $F_t(W)$ such that the market between goods and money and the market between assets and money both clear at all dates. I again restrict attention to stationary monetary equilibria. Given the path of dividends is the same from any starting date, a stationary equilibrium now requires that both P_t and p_t grow at the same rate as money, i.e., $\frac{\dot{P}_t}{P_t} = \frac{\dot{p}_t}{p_t} = 0$.

As in the purely monetary economy, in any monetary equilibrium households prefer to sell any goods they have in between urges to consume. This now includes their endowment y and the Da_t goods they earn as dividends on their assets. In between urges to consume, then, W_t evolves according to

$$\dot{W}_t = P_t y + (P_t D + \dot{p}_t) a_t \quad (8)$$

When an agent is hit with an urge to consume, she will once again prefer to spend her entire money holdings M_t to buy goods in any stationary equilibrium. To see this, note that the instantaneous return to holding an asset in a stationary equilibrium is $\frac{D + \dot{p}_t}{p_t} = \frac{D}{p_t} > 0$. Since money offers a zero nominal return, forgoing a positive return can only be optimal if the agent intends to spend her money on goods whenever she has an urge to consume; holding money for the next instant would have been a waste otherwise. The total amount spent on goods at each instant thus remains $\lambda \bar{M}$, while the value of goods supplied at any instant is $P(y + D)$. The equilibrium price level in a stationary monetary equilibrium is then

$$P = \frac{\lambda \bar{M}}{y + D} \quad (9)$$

The presence of an asset does not change how the equilibrium price level P is determined.

I next turn to the equilibrium asset price p . Agents have linear utility, so the tradeoff between money and assets will not depend on wealth. Since money and the asset must both be held in equilibrium, agents

must be indifferent between the two. The price p must therefore ensure households value money and the asset equally. Let $V(W_t)$ denote the expected utility of an optimizing household with nominal wealth W_t , and let T denote the (random) time of their next urge to consume. Since agents will optimally spend all of their money holdings at date T , the optimal choice of asset holdings will satisfy the recursive equation

$$V(W_t) = \max_{\{a_T\}_{T=t}^{\infty}} \int_t^{\infty} \lambda e^{-(\rho+\lambda)(T-t)} [(W_T - pa_T) + V(pa_T)] dT \quad (10)$$

subject to the constraints (6), (7), and (8). The equilibrium price p must ensure that $V(W_t)$ in (10) is independent of the choice of $\{a_T\}_{T=0}^{\infty}$. This implies the gain from buying an asset for an instant dt and exchanging it back for money must equal the value of holding money throughout.⁵

In a stationary equilibrium where p is constant, holding the asset for an instant dt increases an agent's nominal wealth by $PD \cdot dt$. Let v_t denote the agent's value from a marginal unit of nominal wealth at date t . The gain from buying the asset is just $(PD \cdot dt) \times v_t$. To solve for v_t , note that since agents are always indifferent between money and the asset, they would be willing to hold their nominal wealth as money and spend it at the next urge to consume goods. This means the marginal value v_t of nominal wealth is

$$v_t = \int_t^{\infty} \lambda e^{-(\rho+\lambda)(T-t)} \frac{1}{P} dT = \frac{\lambda}{\lambda + \rho} \frac{1}{P} \quad (11)$$

The gain from buying an asset in a stationary equilibrium is $(PD \cdot dt) \times v_t = \frac{\lambda D}{\rho + \lambda} \cdot dt$. If the agent instead held the amount p needed to buy the asset as money, she would face a λdt chance of an urge to consume at date t . In that case, she could buy $\frac{p}{P}$ goods immediately rather than wait for the next urge to consume. Since the value of holding a unit of nominal wealth until the next urge to consume is $v_t = \frac{\lambda}{\rho + \lambda} \frac{1}{P}$, the expected gain from being able to consume immediately is equal to $\lambda dt \left[1 - \frac{\lambda}{\rho + \lambda} \right] \frac{p}{P} + (1 - \lambda dt) \cdot 0$. Equating the expected gain from holding money and the gain from holding the asset yields

$$\frac{p}{P} = \frac{D}{\rho} \quad (12)$$

Equilibrium prices are thus given by (9) and (12). At these prices, agents are indifferent and willing to save with either money or the asset in between urges to consume. They have no need to trade in assets.

In the next section, I allow agents to hold different beliefs about the asset's payoff, leading them to actively trade it. Starting with a framework in which there is no motive for trade helps isolate the role of beliefs. However, this does not mean trade requires agents to hold different beliefs. Rocheteau, Weill, and Wong (2018) consider the same setup but where agents have strictly concave utility over consumption. In that case, agents who go a long time without an urge to consume amass significant wealth and may not want to spend it all when they next have an urge to consume given diminishing returns. This would lead them to buy illiquid assets from agents who just experienced an urge to consume and now prefer liquid

⁵To be precise, I consider the payoff to holding an asset for a period of length Δ and take the limit as $\Delta \rightarrow 0$. The limit corresponds to the heuristic argument in the text.

assets. Herrenbrueck (2019) generates a motive for trade by assuming households are more likely to have an urge to consume at certain times than at others. Households who are more likely to have an urge to consume would sell their assets to those who are less likely to have an urge to consume.

3 Heterogeneous Time-Varying Beliefs: Steady State

I now introduce the last ingredient to generate an asset boom by letting agents disagree about the asset. Suppose each agent believes there is a set of random dates $\{\tau_n\}_{n=1}^\infty$ in which the asset might pay out lump-sum payments of Δ_{τ_n} per share beyond the flow dividend D . Payoff dates are independent across agents, which captures the idea that households fixate on different aspects of the asset and as a result have different expectations about its payoffs. For example, if the asset represents equity in a firm, one household might track the firm's R&D and news about its patents, while another household might track the firm's competitors and news about its market share. Agents can therefore have different perceptions of whether a payoff-relevant event just took place. Agents perceive the time between potential payoff events as distributed exponentially with rate α . Hence, only a flow α of households believe a payoff event occurs at any given instant, just as only a flow λ of households have an urge to consume at any instant.

For any given agent, define $\mathcal{T} \equiv \min\{\tau_n : \tau_n \geq t\}$ as the date of the next payoff event they anticipate and Δ as the lump-sum dividend payment they expect at date \mathcal{T} . While all agents assign the same likelihood that a payoff event occurs in the next instant, they do not have the same beliefs about the distribution of Δ if it occurs. Rather, agents are either optimistic or pessimistic. Optimists expect Δ will equal $\frac{\Delta^+}{q} > 0$ with probability q and 0 with probability $1 - q$, for an average payoff of Δ^+ . Pessimists expect to incur a lump-sum cost of $\frac{\Delta^-}{q}$ to maintain the asset with probability q and a cost of 0 with probability $1 - q$, for an average cost of Δ^- . Half the agents start as optimists at date 0 and half as pessimists. Following Harrison and Kreps (1978) and Scheinkman and Xiong (2003), beliefs can vary over time: Agents who expect the next payoff Δ to be positive on average expect the payoff after that to be negative on average, and vice versa. Thus, an agent who believes a payoff event just occurred will switch from optimism to pessimism and vice versa. By symmetry, half of all agents will be optimists at any point in time.

The true payoff Δ_t is assumed to equal 0 for all t , i.e., the asset never actually offers a lump-sum payment. This is consistent with what agents believe can happen: At any given instant, almost all agents believe no payoff occurred, while a flow α of agents believe a payoff occurred and that it could have been 0. Thus, the true realization of Δ_t does not falsify any beliefs. Abstracting from shocks to the true Δ_t isolates the effect of beliefs. Since Δ_t is always equal to 0, in my setup optimists will be disappointed at payoff events to see no windfall and turn pessimistic while pessimists will be relieved to see no loss and turn optimistic.⁶

⁶This pattern is reminiscent of aspects of diagnostic beliefs described in Bordalo, Gennaioli, and Shleifer (2018), whereby agents modify their beliefs in light of salient recent experiences, including surprises and disappointments.

As in the previous section, agents allocate their wealth between money and assets before learning whether they have an urge to consume. They also do so before learning whether a payoff event will occur. Optimists expect the return to holding the asset over the next instant to be

$$P_t (D + \alpha \Delta^+) + \dot{p}_t$$

Pessimists expect the return to be

$$P_t (D - \alpha \Delta^-) + \dot{p}_t$$

When $\alpha = 0$, both agents expect the same return on the asset, which is the same as the return on the asset in the previous section where agents had homogeneous beliefs. The fact that optimists expect the next payoff to be positive while pessimists expect it to be negative is irrelevant if neither thinks such an event will occur. My model of disagreement thus nests homogeneous beliefs as a special case.⁷

Suppose α starts at 0 and unexpectedly rises from 0 to some positive value while Δ^+ and Δ^- remain fixed. I will refer to this as a *disagreement shock*. I assume the positive value of α is large enough so that

$$\alpha \Delta^- > D \tag{13}$$

This restriction implies that when $\dot{p}_t = 0$, pessimists will expect a negative nominal return on the asset over the next instant and would prefer to hold money. The exact value of Δ^- does not matter in a stationary equilibrium, although it can matter along the transition path. What matters in the stationary equilibrium are the expected payoff Δ^+ for optimists and the rate α at which agents expect payoff events to occur.

While (13) implies only optimists would be willing to hold the asset in a stationary equilibrium, they need to have enough resources to buy the entire stock of the asset. Define W_t^+ as the total wealth of optimists and W_t^- as the wealth of pessimists at date t . These must add up to total available wealth, i.e.,

$$W_t^+ + W_t^- = \bar{M} + p_t \tag{14}$$

If the initial wealth of optimists W_0^+ is too low, they would not be able to immediately buy up all available assets at the price that clears the market. In that case, the economy will not immediately jump to the stationary equilibrium as in the previous two sections. Instead, the total wealth of optimists acts as a state variable that governs equilibrium prices, even as the distribution of wealth among optimists (or among pessimists) remains irrelevant for equilibrium prices. I therefore introduce the notion of an *asymptotically* stationary monetary equilibrium, or a monetary equilibrium that is stationary in the limit as $t \rightarrow \infty$. With a fixed supply of money, that means equilibria in which $\lim_{t \rightarrow \infty} \frac{\dot{P}_t}{P_t} = \lim_{t \rightarrow \infty} \frac{\dot{p}_t}{p_t} = 0$ but where both growth rates may differ from zero early on. In this section, I focus on the long-run steady states of these equilibria, i.e., when p_t , P_t , and W_t^+ have had enough time to converge to their long-run levels. I consider the transitional

⁷Setting $\Delta^+ = \Delta^- = 0$ also captures the case of homogeneous beliefs. However, setting the one parameter $\alpha = 0$ does so for any values of Δ^+ and Δ^- , while replicating full agreement for any α requires setting *both* Δ^+ and Δ^- to zero.

dynamics of the asymptotically stationary equilibrium in Section 5.

Up to now, stationary equilibria featured the following: (1) agents sell any goods they have in between urges to consume; (2) agents are willing to save the income they earn in between urges as money; and (3) when agents have an urge to consume, they spend all of their money holdings on goods. With time-varying beliefs, agents still sell any goods they have in between urges to consume. But the other two features may no longer hold. First, agents might not be willing to save in between urges to consume using money. If optimists expect a high return on the illiquid asset, they may prefer to use their income to speculate. By contrast, pessimists will still save their income as money. Second, when agents have an urge to consume, they might no longer spend all of their money on goods. Pessimists who expect a higher return to buying the asset in the future may prefer to hold on to money to be able to buy assets later rather than spend them on consumption now. By contrast, optimists see no such benefit to holding on to their money when faced with an urge to consume given the return on the asset will be the same or lower in the future.

The observations above allow me to write down a law of motion for W_t^+ when agents face a constant path for prices:

$$\dot{W}_t^+ = \frac{\alpha}{2} (W_t^- - W_t^+) + P \left(\frac{y}{2} + D \right) - \frac{\lambda}{2} (W_t^+ - p) \quad (15)$$

The first term denotes the change in the total wealth of optimists that is due to changes in beliefs. A flow $\frac{\alpha}{2}$ of optimists chosen at random turn pessimists in the next instant, and their wealth would be subtracted from the total wealth of optimists. At the same time, a flow $\frac{\alpha}{2}$ of pessimists chosen at random become optimists, and their wealth would be added to the total wealth of optimists. The second term denotes the income optimists earn from selling any goods they have. Since half of the agents are optimists, they account for half of the total endowment. But optimists own all of the asset in steady state, so they receive the dividend payment in full. Finally, a flow $\frac{\lambda}{2}$ of optimists chosen at random will have an urge to consume and will spend any money they hold. The money holdings of optimists is equal to the difference between their total wealth W_t^+ and the value p of the asset they own.

Setting $\dot{W}_t^+ = 0$ and substituting in for W_t^- from (14), we can solve for the steady-state level of wealth of optimists \bar{W}^+ at which $\dot{W}_t^+ = 0$:

$$\bar{W}^+ = \frac{P(y + 2D)}{2\alpha + \lambda} + \frac{\alpha}{2\alpha + \lambda} \bar{M} + \frac{\alpha + \lambda}{2\alpha + \lambda} p \quad (16)$$

The steady-state wealth of optimists depends on the price level P and the asset price p . While total wealth $\bar{M} + p$ increases one-for-one with p , the steady-state wealth of optimists \bar{W}^+ rises *less* than one-for-one with p . Intuitively, a fraction of the asset at any instant is held by agents who just turned pessimist, so a higher asset price benefits more than just optimists. This has an important implication: If optimists are more bullish on the asset, say because Δ^+ is high, and they bid up the asset price, their wealth will not rise by the same amount as the asset price. When optimists are very bullish, they might not be able to bid up asset prices in line with their beliefs. This leads to what Allen and Gale (1994) define as *cash-in-the-market pricing*, when the price of the asset is determined by the resources agents have to buy it rather than its

(perceived) fundamentals. In Allen and Gale (1994), such pricing occurs because of unexpectedly high demand for liquid assets. Here, such pricing occurs when borrowing constraints limit demand for illiquid assets, similarly to the cash-in-the-market pricing in Bolton, Santos, and Scheinkman (2021).

To fully characterize equilibrium asset prices, recall from Section 2 that in a stationary equilibrium agents are indifferent between money and the asset if and only if the expected nominal return on the asset is ρ . Optimists expect a return of $P(D + \alpha\Delta^+)/p$ on the asset, so the price that ensures this return is ρ is

$$p^* = \frac{P(D + \alpha\Delta^+)}{\rho} \quad (17)$$

If $\bar{W}^+ > p^*$, optimists have enough steady state wealth to buy the asset at the price p^* that ensures they expect a return of ρ from the asset. In that case, optimists will bid up the asset price to p^* and still have additional wealth they hold as money. Optimists will thus hold both money and the asset (and are indifferent between the two) while pessimists only hold money. If $\bar{W}^+ < p^*$, optimists will not be able to buy the asset if it were priced to ensure an expected return of ρ . The price of the asset will then be \bar{W}^+ , meaning optimists expect to earn a return above ρ and so will strictly prefer to hold the asset.

Since optimists would bid the asset price p up to p^* if they had enough wealth, and since p^* is increasing in the degree of optimism Δ^+ , the fact that \bar{W}^+ rises less than one-for-one with p suggests that for sufficiently high Δ^+ , optimists will be constrained and assets will be priced according to cash-in-the-market pricing. This is illustrated graphically in Figure 1. The blue lines show the equilibrium real asset price p/P and the return optimists expect $\frac{P(D+\alpha\Delta^+)}{p}$ when $\alpha = 0$. If agents expect payoff events to never happen, the stationary equilibrium coincides with the homogeneous beliefs case in Section 2 and is independent of Δ^+ . The black lines show the case where $\alpha > \frac{D}{\Delta^-} > 0$. The real asset price p/P and the return optimists expected to earn $\frac{P(D+\alpha\Delta^+)}{p}$ rise with Δ^+ . Intuitively, a larger degree of optimism makes the asset more valuable to the agents who hold it, and so they will bid its price up. When Δ^+ is small, optimists have sufficient wealth to buy the asset at the price p^* , and they expect a return ρ from the asset. When Δ^+ is large, the price p^* needed to ensure optimists expect a return of ρ exceeds \bar{W}^+ . Optimists can only spend \bar{W}^+ , which implies a return from the asset above ρ that leads optimists to invest all their wealth in the asset. A disagreement shock in which α rises from 0 to a positive value lifts the real asset price above $\frac{D}{\rho}$ and, when optimists are sufficiently bullish, increases the expected return of optimists above ρ .

As evident in Figure 1, when the degree of optimism Δ^+ exceeds some higher threshold Δ^{**} , the real price of the asset rises with Δ^+ once again. The reason for this has to do with what happens to the equilibrium price level P when Δ^+ is very large. This is illustrated in Figure 2. The blue line once again shows the case of $\alpha = 0$, where $P = \frac{\lambda M}{y+D}$ for all Δ^+ . The black line corresponds to the case where $\alpha > \frac{D}{\Delta^-} > 0$. For small degrees of optimism, the price level is the same as under homogeneous beliefs: Agents who hold money, be they optimists or pessimists, will spend it all if they have an urge to consume. However, this becomes unsustainable for very high degrees of optimism: If the price level P remained constant and the nominal asset price p were also constant and equal to \bar{W}^+ , the expected return to holding the asset $P(D + \alpha\Delta^+)/p$

would rise linearly Δ^+ . Eventually, the return to holding the asset would be high enough that pessimists would prefer to hold on to their money and buy assets when they turn optimists rather than consuming now. But if nobody wants to consume, the goods market will not clear. For very large degrees of optimism, then, the price level P must fall to ensure pessimists are indifferent between consuming and saving to buy assets in the future. If we let v_t^- denote the value of an additional unit of nominal wealth for pessimists, this means the price P must adjust in equilibrium so that v_t^- does not exceed $1/P$ and holding on to money becomes more valuable than consumption. This adjustment occurs when pessimists hoard cash rather than spend it all when they have an urge to consume. Such hoarding lowers the price of goods P , which increases the real price of the asset p/P and lowers the return $P(D + \alpha\Delta^+)/p$ that optimists expect. If pessimists hoard just the right amount, v_t^- will equal $1/P$ and pessimists would be willing to both hoard money and spend on consumption so the goods market clears. Essentially, when the disagreement shock leads to high returns, agents will want to save more, and that in turn will depress the price level P .

The next proposition formalizes how the steady-state equilibrium depends on Δ^+ . Ensuring that the cutoff Δ^* is positive as drawn in Figures 1 and 2 requires a parametric restriction. In particular, when $\Delta^+ = 0$ and the threshold $p^* = \frac{PD}{\rho}$ as in the homogeneous beliefs case, optimists may lack the wealth to afford the asset at p^* . This was not an issue when all agents had the same beliefs: In that case, all agents were willing to buy the asset, and their collective wealth is $W = \bar{M} + p > p$ which necessarily allows them to afford the asset. But when only optimists are willing to buy the asset, their wealth may not suffice. A sufficient condition that ensures optimists can afford to buy the asset when $\Delta^+ = 0$ is

$$\frac{y + D}{\lambda} > \frac{D}{\rho} \quad (18)$$

Intuitively, (18) stipulates that the real income agents earn in between urges to consume on average exceeds the real value of the asset, allowing agents to amass enough wealth to afford the asset.

Proposition 1: *When $\alpha > \frac{D}{\Delta^-} > 0$, there exists a unique steady state monetary equilibrium. Given (18), there exist cutoffs Δ^* and Δ^{**} where $0 < \Delta^* < \Delta^{**}$ such that*

1. *If $\Delta^+ < \Delta^*$, the steady state equilibrium is $\frac{p}{P} = \frac{D + \alpha\Delta^+}{\rho}$ and $P = \frac{\lambda\bar{M}}{y + D}$. In this equilibrium, optimists hold both assets and money, pessimists only hold money, and both optimists and pessimists spend down their money balances when they have an urge to consume.*
2. *If $\Delta^+ \in (\Delta^*, \Delta^{**})$, the steady state equilibrium is $\frac{p}{P} = \frac{y + D}{\lambda} + \frac{y + 2D}{\alpha}$ and $P = \frac{\lambda\bar{M}}{y + D}$. In this equilibrium, optimists only hold assets, pessimists only hold money, and pessimists spend all of their money balances when they have an urge to consume.*
3. *If $\Delta^+ \geq \Delta^{**}$, the steady state equilibrium is $\frac{p}{P} = \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)}$ and $P = g(\Delta^+)\bar{M}$ for some function g where $g'(\cdot) < 0$. In this equilibrium, optimists only hold assets, pessimists only hold money, and pessimists hoard some of their money to buy assets in the future.*

In equilibrium, agents buy assets when they are optimistic and sell them when they turn pessimistic. If the boom is small, they will consume throughout and will spend all of their cash holdings when they have an urge to consume as before. If the boom is large, they will consume only when they are pessimistic, and may not spend all of their cash holdings when they have an urge to consume because they want to save to buy assets later. A disagreement shock leads to a higher real asset price and, when the asset boom is sufficiently large, to a lower price level. The latter occurs as agents hoard liquid assets to speculate in the future. In Section 6, I describe some of the empirical work which shows that at least some notable asset booms in the past were indeed associated with lower rates of overall inflation.

A few remarks about this result are in order. First, the notion that asset booms feature an increased desire to save may seem surprising given asset booms in practice are often associated with consumption booms. However, higher desired savings are consistent with higher consumption. Since my model assumes an endowment economy with a fixed supply of nondurable goods, the increase in desired savings does not lead to lower consumption but to higher *real* savings. When I allow for production in the next section, I find that the disagreement shock leads to higher output, so the disagreement shock increases both consumption *and* real savings. Indeed, some of the empirical work on the recent U.S. housing boom has highlighted its connection to higher savings. Bernanke (2005) argued the housing boom was associated with a global savings glut. Caballero (2006) argued the boom was associated with a shortage of assets to satisfy demand, and points out that the demand for assets may have led to lower inflation. Bates, Kahle, and Stulz (2009) documented an increase in cash savings by corporations during the dot com and housing booms.

Second, an arguably peculiar feature of the steady-state equilibrium is that the agents who sell their assets turn around and start saving to buy assets again in the future. Equally odd is that it is pessimists who save to buy the asset. But these features are not essential. I could have equally assumed that optimists who sell their asset leave the market and are replaced by new agents who intend to buy assets in the future. In addition, the agents who save to buy assets in the future can be interpreted not as pessimists but as traders who expect high returns but are not yet ready to buy the asset. In the model, this is because they believe the current return on the asset is low. More realistically, agents may wait until they have saved enough to meet down-payment requirements to buy the asset, or because information frictions lead them to wait to identify buying opportunities among the assets they know. In both of these cases, a higher return on the asset would lead agents to save more, so the implications should be similar to my model.

Another possible concern is that there is no asset other than money or the speculative asset, so agents who want to speculate in the future must hold money. In reality, agents can save using various assets, and it is not obvious they will specifically hoard money and as a result lower the price level. Suppose I added a third asset into the model that is also illiquid but which agents agree on. As in Section 2, in equilibrium agents will be indifferent between money and illiquid assets they agree on. As long as there is a sufficiently large degree of optimism Δ^+ for the asset agents disagree on, optimists would only hold the asset about which agents disagree and pessimists would hold money and the illiquid asset that agents agree on. Agents who intend to buy assets in the future would be willing to save using illiquid assets rather than just money.

However, equilibrium still requires that the price level P adjusts to ensure pessimists don't just want to save. In other words, the price level adjusts not because agents are forced to hold money, but because they wish to save more and in equilibrium their wealth must invariably flow into money.

Finally, I turn to the question of whether the asset boom in the model when agents disagree corresponds to a bubble. As Harrison and Kreps (1978) originally pointed out, the equilibrium price of the asset may exceed what any agent believes the asset will pay out as dividends. In particular, as long as the price of the asset is not determined according to cash-in-the-market pricing, the asset will be priced as if the expected payoff at each payoff date is Δ^+ . But no agent believes all payoff events have this payoff. Scheinkman and Xiong (2003) and others have interpreted this pattern to mean the asset should be viewed as a bubble. Barlevy (2015) argues against this interpretation. Regardless, when the price of the asset is determined by cash-in-the-market pricing, there is no guarantee that the price will necessarily exceed what all agents believe the asset will pay out. Moreover, whether the asset price exceeds what agents believe about discounted dividends does not play an important role for the results. Instead, the key feature is that a disagreement shock associated with a large degree of optimism will lead agents to expect a higher return on the asset. Greenwood and Shleifer (2014) document that high realized asset price growth is often associated with higher expected returns.⁸ Higher expected returns is the reason agents in the model hoard money and drive down the price level, and, as we shall see in the next section, is why agents will want to produce more.

4 Endogenous Output

Up to now, I considered an endowment economy in which the amount of goods available for consumption was exogenously fixed. I now endogenize the amount of goods by letting agents decide how much to produce. This allows me to examine whether an asset boom will be associated with an output boom.

My model of production follows Rocheteau, Weill, and Wong (2018) and Herrenbrueck (2019). Rather than being endowed with a fixed amount of goods y , agents are endowed with a productive input, e.g. labor. At each instant, they choose the amount of input n_t to use to produce. Agents produce on their own rather than sell their services in a labor market. In other words, households choose how much to produce given the price P_t rather than how much to work at a given wage.

I assume a linear production technology for a household in which $y_t = n_t$. Each household incurs a cost of using its input that is given by a differentiable function $\phi(n)$ with $\phi'(n) \geq 0$, $\phi''(n) > 0$, and

⁸Realized returns on the asset will be high (technically infinite) on impact when the disagreement shock hits and the asset price jumps. Thereafter, the direction is ambiguous. Section 5 shows that if the initial wealth of optimists W_0^+ is below the steady state \bar{W}^+ , the asset price p_t will rise along the transition and the realized return on the asset can remain high. However, since the true $\Delta_t = 0$ for all t , realized returns in steady state are lower than before the disagreement shock.

$\lim_{n \rightarrow \infty} \phi'(n) = \infty$. Households choose production n_t , consumption c_t , and asset holdings a_t to maximize

$$E \left[\sum_{n=1}^{\infty} e^{-\rho t_n} c_{t_n} - \int_0^{\infty} e^{-\rho t} \phi(n_t) dt \right] \quad (19)$$

given their initial wealth W_0 and subject to the budget constraint

$$\dot{W}_t = P_t n_t + (P_t (D + \Delta_t) + \dot{p}_t) a_t \quad (20)$$

and the constraints that stem from the lack of intertemporal trade:

$$0 \leq p_t a_t \leq W_t \quad (21)$$

$$P_t c_t \leq W_t - p_t a_t \quad (22)$$

Although the true Δ_t in the budget constraint is equal to 0 for all t , optimists and pessimists have different expectations about Δ_t , and these will affect their production decisions.

Recall that I defined v_t as the marginal utility value of nominal wealth. Let v_t^+ and v_t^- denote this value for optimists and pessimists, respectively. Under the production technology, each unit of input produces one unit of a good. Hence, the value of the marginal unit of effort is just the value $P_t v_t$ the household earns from the additional good it sells. Agents will choose n_t to satisfy

$$\phi'(n_t) = P_t v_t$$

As in Section 3, I focus on the steady state of the asymptotically stationary monetary equilibrium. When $\alpha = 0$ and agents agree about dividend payments, I already showed in (11) that $v_t = \frac{\lambda}{\lambda + \rho} \frac{1}{P}$ where P is the equilibrium price level. Optimists and pessimists will thus produce the same, i.e.,

$$n_t^+ = n_t^- = \phi'^{-1} \left(\frac{\lambda}{\lambda + \rho} \right) \equiv n^*$$

Total output will be constant and equal to n^* when $\alpha = 0$. Figure 3 illustrates this graphically. The blue line denotes the quantity of goods all agents produce when they agree $\Delta_t = 0$ for all t .

I next turn to the case where $\alpha > D/\Delta^+ > 0$. Recall that in the endowment economy, I imposed a regularity condition (18) to ensure that when $\Delta^+ = 0$, optimists had enough wealth to buy the asset at the real price $\frac{D}{\rho}$. The analogous condition in the production economy is given by

$$\frac{n^* + D}{\lambda} > \frac{D}{\rho} \quad (23)$$

When Δ^+ is close to 0, condition (23) ensures that the steady state wealth of optimists \bar{W}^+ exceeds p .

Optimists will then hold both money and the asset, and so must be indifferent between the two. Just as in Section 3, this requires that the steady state return on the asset equal ρ . In that case, both optimists and pessimists would be willing hold their nominal wealth as money until their next urge to consume. This implies $v_t^+ = v_t^- = \frac{\lambda}{\lambda + \rho} \frac{1}{P}$. Hence, for small degrees of optimism, optimists and pessimists will continue to produce n^* just as when they agree. This is illustrated in Figure 3, where the black and gray lines depict the steady state production of optimists and pessimists, respectively, when $\alpha > \frac{D}{\Delta^-} > 0$.

Once the degree of optimism Δ^+ exceeds some cutoff Δ^* , the wealth of optimists who produce a fixed amount n^* will not suffice to buy the asset at the price p^* which ensures optimists expect to a return of ρ from holding the asset. Unlike the endowment economy, optimists can produce more, increase their wealth, and spend more on the asset. But they would only be willing to produce more if Pv_t^+ were higher. As Figure 3 illustrates, in equilibrium optimists do end up working more. However, they will only agree to work more if they are rewarded more for the additional unit they are produced. What happens in equilibrium is similar to what we saw in the endowment economy: For higher values of Δ^+ , the real asset price will not rise by the amount needed to keep the return to holding the asset at ρ . Optimists therefore expect a higher return from holding the asset, which implies their marginal value of nominal wealth v_t^+ increases with Δ^+ . Essentially, for higher Δ^+ , optimists view speculation as more profitable and are therefore willing to produce more in order to undertake more speculation. One might have expected pessimists to produce less given they believe the asset is overvalued. However, since they do not hold the asset, the low return does not discourage them from producing. Moreover, given they expect to speculate in the future, pessimists will be willing to produce more income to save and speculate in the future. This is why, as shown in Figure 3, pessimists produce more when Δ^+ is large, although not as much as optimists.

Finally, just as in the endowment economy, once Δ^+ exceeds a still higher cutoff Δ^{**} , the return that optimists would expect from holding the asset if the price level P remained constant would be sufficiently high that pessimists would prefer to hoard money than spend it on consumption. Once again, hoarding would lower the price level P until the return to holding the asset made pessimists indifferent about consuming. That is, the marginal value of nominal wealth for pessimists v_t^- must equal the marginal utility of using nominal wealth to consume when they have an urge. The latter is $1/P$, so in equilibrium $Pv^- = 1$, and pessimists produce $n^- = \phi'^{-1}(1)$. The formal analysis in the Appendix shows that if $Pv^- = 1$ then $Pv^+ = 1 + \frac{\rho}{\alpha}$, so optimists produce $n^+ = \phi'^{-1}\left(1 + \frac{\rho}{\alpha}\right)$. For $\Delta^+ > \Delta^{**}$, then, larger degrees of optimism Δ^+ are no longer associated with higher production. The next proposition formalizes these results.

Proposition 2: *When $\alpha > \frac{D}{\Delta^-} > 0$, there exists a unique steady state equilibrium. Given (23), there exist cutoffs Δ^* and Δ^{**} where $0 < \Delta^* < \Delta^{**}$ such that*

1. *If $\Delta^+ < \Delta^*$, optimists and pessimists produce a constant amount in equilibrium for all levels of Δ^+ . In particular, $n^+ = n^- = n^* \equiv \phi'^{-1}\left(\frac{\lambda}{\lambda + \rho}\right)$*
2. *If $\Delta^+ \in (\Delta^*, \Delta^{**})$, optimists produce more than pessimists in equilibrium, and the amount both groups produce increases in Δ^+ , i.e., $n^+ > n^- > n^*$.*

3. If $\Delta^+ > \Delta^{**}$, optimists produce more than pessimists in equilibrium, and the amount both produce is the same for all Δ^+ . In particular, $n^+ = \phi'^{-1}(1 + \frac{\rho}{\alpha}) > n^*$ and $n^- = \phi'^{-1}(1) > n^*$.

The proposition implies that a disagreement shock will have a limited effect on the macroeconomy when the degree of optimism Δ^+ is small: Optimists will bid up the price of the asset, but there will be no effect on the return agents who hold the asset expect to earn, the level of output y , or the price level P . But when Δ^+ is large, a disagreement shock will have a broad effect on the macroeconomy: Optimists still bid up the price of the asset, but the agents who hold the asset expect to earn a higher return, output will rise, and the price level will fall. In the latter case, the disagreement shock will not correspond to a standard aggregate demand shock in which output and prices both rise.

For intermediate values of Δ^+ between the cutoffs Δ^* and Δ^{**} , a disagreement shock will induce agents to produce more but will not induce pessimists to hoard money. The price level in this case is given by

$$P = \frac{\lambda \bar{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}$$

If production rises while the money supply \bar{M} remains fixed, the price level will fall. Intuitively, a fixed money supply must buy more goods, so the price level must fall. The central bank could simply increase \bar{M} in line with output to ensure the price level remained fixed. By contrast, when Δ^+ exceeds Δ^{**} , pessimists hoard money. In that case, the money supply \bar{M} would have to rise by more than output to keep the price level stable. This highlights the distinct forces acting on the price level in my model: The decision by agents to produce more following a disagreement shock acts like an aggregate supply shock, while the decision by pessimists to save using liquid assets leads the price level to change even when output is fixed. These forces matter not only for how a disagreement shock affects the price level, but for what the central bank must do if it wants to stabilize output and prices. The next section turns to these issues.

5 Trilemma and Transitional Dynamics

In this section, I study the effects of monetary policy. I focus not on what a central bank *should* do but what it *can* do, i.e., on what is feasible rather than on what is optimal. This analysis requires going beyond steady state equilibria, since a monetary intervention can have different effects in the short and long run.

To fix ideas, suppose that when the disagreement shock hits at date 0, the economy immediately transitions to the steady-state equilibrium when $\alpha > 0$ that is described in Propositions 1 and 2. Since agents are indifferent between assets and money when $\alpha = 0$, any distribution of asset holdings is compatible with equilibrium before the shock hits. That includes the one in which optimists hold the same mix of assets and money they would in the steady state with disagreement. In that case, the wealth of optimists immediately after the shock will equal \bar{W}^+ . So I am effectively choosing a particular equilibrium before the disagreement shock. I denote the steady-state prices after the shock hits by \hat{p} and \hat{P} , respectively.

Consider what would happen if the central bank intervened contemporaneously by increasing the money supply from \bar{M} to $(1 + \mu)\bar{M}$ at date 0, just when the disagreement shock hits. To characterize the effect of this intervention requires solving for the equilibrium along the transition to a new steady state. While I frame the discussion in terms of the equilibrium path following a change in the money supply, the same analysis can be used to characterize the equilibrium path after a disagreement shock hit if the initial wealth of optimists W_0^+ after the shock was different from \bar{W}^+ while the money supply stayed fixed.

The steady-state equilibrium prices p and P in Propositions 1 and 2 are proportional to \bar{M} . After increasing the money supply from \bar{M} to $(1 + \mu)\bar{M}$, then, the new steady state equilibrium prices would equal $(1 + \mu)\hat{p}$ and $(1 + \mu)\hat{P}$, respectively. The real price of the asset in the new steady state will equal \hat{p}/\hat{P} , just as without intervention. Likewise, the return that the agents who hold the asset expect to earn in the new steady state will equal $\frac{\hat{P}(D + \alpha\Delta^+)}{\hat{p}}$, just as without intervention. In other words, money is neutral in the long run, just as in most standard monetary models. However, a one-time injection can still have real effects in the short run, depending on how it is distributed between optimists and pessimists.⁹

I begin with a short-run neutrality result. Suppose the additional $\mu\bar{M}$ of liquidity at date 0 was distributed between optimists and pessimists in proportion to their initial money holdings. That is, if we denote the original money holdings of optimists and pessimists by M_0^+ and M_0^- , respectively, then the amounts of additional liquidity the two groups receive would be μM_0^+ and μM_0^- , respectively. In this case, the intervention can leave the asset market unaffected at all dates.¹⁰

Proposition 3: *Suppose the central bank injects $\mu\bar{M}$ worth of liquidity to optimists and pessimists in proportion to their money holdings at date 0 when the economy is at its original steady state. Then there exists an asymptotically stationary equilibrium in which $p_t = (1 + \mu)\hat{p}$ and $P_t = (1 + \mu)\hat{P}$ for all t .*

Proposition 3 establishes that there is a way to inject liquidity following a disagreement shock that has no impact on the real economy. Such an injection can still be used to offset the effect of the disagreement shock on the price level P by choosing μ to match the original price level before the disagreement shock. But it would not be useful for stabilizing the asset price or output, even temporarily. However, the fact that neutrality rests on a particular allocation between optimists and pessimists suggests there may be scope for using liquidity policy to temporarily stabilize more than just the price level by directing the liquidity to favor some agents over others. To restore both asset prices and output to their levels before the disagreement shock, the central bank would need to direct liquidity in a way that dampens the price of

⁹One motivation for studying the short-run effects of liquidity injections is that we can modify the model to allow a temporary disagreement shock, i.e., if α is positive for an exponentially distributed period of time and then reverts back to 0. The asset price would reflect the expected capital loss on the asset when disagreement ends, but the steady-state equilibrium in the disagreement regime would be qualitatively similar to the one with permanent disagreement. The long-run effects of an injection are irrelevant given the economy will almost surely revert to full agreement, but short-run effects still matter.

¹⁰The qualifier that intervention *can* rather than *must* be neutral reflects the fact that I have not been able to prove this transition path is the unique continuous asymptotically stationary equilibrium, although I conjecture it is. The price paths are governed by a system of differential equation, and if these equations are well behaved, they should admit a unique solution.

the asset and, to discourage agents from producing more, also dampens the expected return on the asset. I now argue that this will not be possible; a central bank that tries to use injections to stabilize output, asset prices, and the price level will face a trilemma. Intuitively, temporarily depressing the price of the asset increases the expected return to holding the asset, since a lower price implies a higher dividend yield and, to the extent the fall in price is temporary, higher future capital gains. Directing liquidity in a way that temporarily increases the relative wealth of either optimists or pessimists will dampen asset prices or the expected return on the asset, but not both.

For analytical tractability, I establish the impossibility of stabilizing both the asset price and the expected return of optimists in the endowment economy. The same logic would carry over to the production economy, but in that case I can only confirm the results numerically.

Consider first an injection that favors optimists, i.e., in which optimists receive more than μM_0^+ and pessimists receive less than μM_0^- . When the degree of optimism $\Delta^+ \leq \Delta^*$, the original wealth of optimists exceeds the value of the asset. In that case, given optimists more resources should not matter for asset prices; if optimists wanted to spend more on the asset, they could have done so already. For a liquidity injection to have real effects, optimists would have had to be wealth-constrained in the original steady state, which is only true when $\Delta^+ > \Delta^*$. In that case, giving optimists more resources allows them to spend more on the asset and temporarily bid up its price. I confirm this in Proposition 4 below, but it is easier to convey the result graphically, as in Figure 4. Following a liquidity injection that favors optimists, the real asset price temporarily rises, although by less than the real wealth of optimists. Optimists are thus indifferent between money and the asset during the transition, so the expected return on the asset during the transition must be ρ . In the new steady state, however, the expected return when $\Delta^+ > \Delta^*$ exceeds ρ . Directing liquidity to optimists thus temporarily drives up the asset price and temporarily depresses the expected return on the asset. Figure 4 also shows that the price level can overshoot $(1 + \mu) \hat{P}$. Essentially, a continuous path for the price level requires that when $\Delta^+ > \Delta^{**}$, pessimists hoard money not just at the new steady state but earlier as well. Deflation makes holding money more attractive, encouraging pessimists to hoard it during the transition.

Proposition 4: *Suppose the central bank injects $\mu \bar{M}$ worth of liquidity at date 0 in the endowment economy in a way that gives optimists more than their original share of the money supply. Then*

1. *If $\Delta^+ \leq \Delta^*$, there exists an asymptotically stationary equilibrium in which $p_t = (1 + \mu) \hat{p}$ and $P_t = (1 + \mu) \hat{P}$ for all t .*
2. *If $\Delta^+ > \Delta^*$, there exists an asymptotically stationary equilibrium in which there is some date $T < \infty$ such that $W_t^+ = p_t = (1 + \mu) \hat{p}$ for all $t \geq T$. Before T , the real asset price $p_t/P_t > \hat{p}/\hat{P}$, optimists expect a return of $\rho < \frac{\hat{P}(D + \alpha \Delta^+)}{\hat{p}}$, and, if $\Delta^+ > \Delta^{**}$, the price level $P_t > (1 + \mu) \hat{P}$.*

When the degree of optimism Δ^+ is small, a liquidity injection that favors optimists has no effect on the asset market. Recall that in this case, the disagreement shock itself has limited macroeconomic effects. But

when the degree of optimism Δ^+ is large and the asset is priced according to cash-in-the-market pricing, such an injection will temporarily raise asset prices and depress expected returns. In a production economy, a lower expected return discourages production, so in principle the central bank could restore output to its level before the disagreement shock. But it would drive the real asset price away from its value before the disagreement shock. This is intuitive: Giving optimists more resources will not help stabilize asset prices.

What about a liquidity injection that increases favors pessimists, i.e., in which pessimists receive more than μM_0^- and optimists receive less than μM_0^+ ? That depends on whether the injection leaves optimists with enough resources to still afford the asset in date 0. When $\Delta^+ < \Delta^*$, optimists would have held both money and the asset in the original steady state, and giving more liquidity to pessimists would reduce the relative wealth of optimists but could still leave them with enough wealth to afford that asset. In that case, the intervention would be neutral. When $\Delta^+ > \Delta^*$, optimists would have only held the asset in the original steady state. A liquidity injection that favors pessimists would require a negative injection to optimists that would force them to sell some of their asset holdings. Following such an intervention, the asset price would temporarily fall until optimists amassed enough wealth to afford to buy up the asset again. Figure 5 illustrates the effects of such an intervention. A liquidity injection that favors pessimists temporarily depresses the real asset price. Pessimists will retain a fraction of the assets and sell them to optimists gradually over time. Since pessimists are worried about a negative payoff event, they would only agree to hold the asset if they anticipated a large capital gain. Optimists would also benefit from this capital gain, so the expected return on the asset will be higher during the transition. The price level will once again overshoot $(1 + \mu) \hat{P}$ if $\Delta^+ > \Delta^{**}$, for the same reason as with an injection that favored optimists.

Proposition 5: *Suppose the central bank injects $\mu \bar{M}$ worth of liquidity at date 0 in the endowment economy in a way that gives pessimists more than their original share of the money supply. Then*

1. *If $W_0^+ \geq (1 + \mu) \hat{p}$ there exists an asymptotically stationary equilibrium in which $p_t = (1 + \mu) \hat{p}$ and $P_t = (1 + \mu) \hat{P}$ for all t .*
2. *If $W_0^+ < (1 + \mu) \hat{p}$, there exists an asymptotically stationary equilibrium in which there is some date $T < \infty$ such that $W_t^+ = p_t = (1 + \mu) \hat{p}$ for all $t \geq T$. Before T , the real asset price $p_t/P_t < \hat{p}/\hat{P}$, optimists expect a return that exceeds $\frac{\hat{P}(D + \alpha \Delta^+)}{\hat{p}}$, and, if $\Delta^+ > \Delta^{**}$, the price level $P_t > (1 + \mu) \hat{P}$.*

An injection that favors pessimists in the face of a disagreement shock will either have no real effects or dampen the asset boom. Such an intervention thus helps stabilize asset prices and the price level, at least temporarily. However, in dampening the price of the asset, this intervention increases the expected return to the agents who hold the asset. This is inherent to the intervention: By shifting wealth away from optimists, it prevents them from being able to afford all of the asset and forces pessimists to hold on to some of the asset for a time. But pessimists would only be willing to hold the asset if the expected return on the asset was higher than what they expect to earn in steady state. Specifically, they must anticipate the price of the asset will grow enough to offset the potential loss in case of a payoff event. But then optimists

would also expect to earn a higher return from holding the asset. This implication would remain true in a production economy. An intervention that directed liquidity to pessimists would make production more attractive rather than less, and would amplify the output boom after a disagreement shock. The central bank could temporarily rein in asset prices, but only by stimulating output even more. The central bank thus faces a trilemma with regards to liquidity injections: To stabilize the price level and asset prices, it needs to inject liquidity and direct it to pessimists. But that must lead to a higher expected return on the asset in order to induce pessimists to temporarily hold the asset, which will in turn stimulate output.

I conclude this section with a few remarks. First, note that if the central bank increased the wealth of pessimists, the asset price p_t would fall immediately and then rise during the transition back to steady state. As p_t rises, the expected dividend yield on the asset falls. Hence, pessimists must expect p_t to grow even faster to remain willing to hold the asset. The asset price will thus grow at an accelerating rate during the transition, up until optimists buy all of the asset and the asset price growth stops abruptly. While directing liquidity toward pessimists helps depress the real price of the asset, it also fuels explosive price growth. This is reminiscent of a result in Galí (2014) that a monetary intervention designed to dampen an asset boom may lead to faster asset price growth. In Galí's model, the central bank raises the real interest rate rather than shifts wealth towards pessimists. But in his model, just like here, the agents who hold the asset demand a higher return as a result, which requires that the asset price grows more rapidly.

Second, the trilemma for monetary policy arises because the central bank is tasked with three targets to stabilize but only two tools – the amount of liquidity and how to direct that liquidity. This suggests it might be possible to avoid the trilemma by using both current and future monetary policy or by relying on additional tools. A full analysis along these lines is beyond the scope of the paper, but my setup does offer some insights. First, liquidity injections shift wealth between optimists and pessimists, which moves the asset price and the expected return on the asset in opposite directions. So a different type of intervention would be needed to avoid the trilemma. One possibility is if monetary policy affects what agents expect to earn in the future. For example, if the central bank promised to intervene in case of a windfall in a way that would hurt asset owners, that should make the asset less attractive. This would lower both the price of the asset and the expected return on it. Allen, Barlevy, and Gale (2021) obtain a related result in which promises by the monetary authority to intervene in the future may be more effective against an asset boom than direct intervention. But the current setup is not well suited for exploring this.¹¹

Another possibility is to use tools other than monetary policy. One natural candidate is macroprudential policy that affects credit rather than liquidity. Although my setup rules out all intertemporal trade, including credit, optimists would have a natural incentive to borrow from pessimists to buy more assets. One could allow for some borrowing in the model, similarly to the way Geanakoplos (2010) and Simsek (2013) combine credit and disagreement. However, regulating credit in such a setup wouldn't necessarily

¹¹To discourage optimists, the monetary authority would need to promise to transfer resources to pessimists after a windfall event to depress the price of the asset and offset the windfall dividend. But the model is unclear on what pessimists would do if they observed an event they believe occurs with zero probability.

resolve the trilemma. Credit restrictions effectively limit the amount of resources optimists have to buy the asset, and so should resemble liquidity injections to pessimists in depressing asset prices while increasing the return that optimists would expect to earn from the asset. A more promising intervention would be financial transactions tax or a windfall tax that makes the asset less attractive to optimists in the same way that contingent monetary policy would. But that seems more suited to a tax authority than a central bank.

Finally, my results only concern the feasibility of stabilizing multiple targets, not its desirability. As specified, the model is not well suited for welfare analysis. There is nothing in the model that makes price stability desirable, in contrast to models with price rigidity in which price stability can mitigate distortions to production that arise when some prices are rigid and some are not. It is also not obvious that stabilizing asset prices is desirable. Brunnermeier, Simsek, and Xiong (2014) discuss the difficulty of doing welfare analysis when agents hold different beliefs. They offer a welfare criterion which suggests that trade rooted in disagreement should be discouraged. But their argument is to discourage trade, not to stabilize asset prices. Moreover, one could make the case that policymakers should encourage trade rooted in disagreement just as they would trade rooted in differences in preferences or productivity. To see this, note that we can reinterpret the model as one in which the asset yields some private payoff that some people enjoy and others find a burden. In that case, efficiency dictates the assets should be held by those who enjoy the private payoff. If agents hold different beliefs despite knowing that others disagree with them, it is not obvious that their actions should be limited. In any event, in practice it would be difficult for a policymaker to identify why agents trade. Caballero and Simsek (2020) and Farhi and Werning (2020) argue that there may scope for policy when agents disagree about an asset and optimists borrow to buy the asset. In this case, a negative shock might force leveraged optimists to sell their asset holdings, and the asset ends up in the hands of pessimists. That on its own is ex-post inefficient, but if prices are rigid, these papers show that a transfer of ownership can be associated with aggregate demand externalities that harm all agents. Allen, Barlevy, and Gale (2021) consider externalities that arise even without rigid prices when agents face information frictions and default is costly. Adding price rigidity, credit, and other frictions may provide insights on the tradeoff when stabilization goals are in conflict.

6 Empirical Evidence

I now turn to empirical evidence regarding the key implications of the model. I first review the evidence on inflation and asset booms, which shows that quite a few significant historical asset booms do appear to be associated with lower inflation rather than higher inflation. The discrepancy between these episodes and the predictions of the Bernanke and Gertler (1999) model suggests not all asset booms manifest as aggregate demand shocks. My model offers one potential explanation for why an asset boom may be associated with lower rather than higher price pressures. Since this explanation relies on the notion that agents hoard liquid assets during asset booms, I then turn to evidence on liquidity hoarding during asset booms and show there is some evidence consistent with this implication.

I begin with the evidence on inflation during asset booms. Recall that in the model, disagreement shocks associated with small degrees of optimism have little effect on the macroeconomy: asset prices will be bid up, but the price level will remain unchanged. Only shocks associated with large degrees of optimism will create downward pressure on prices. The predictions of the model thus concern inflation during significant asset booms rather than the overall correlation between inflation and asset prices.¹² Fortunately, existing work has already attempted to identify large asset booms and characterize the macroeconomic conditions associated with them. For example, Bordo and Wheelock (2007) use statistical methods to identify large asset booms as the run-up before isolated peaks in real stock prices. Their analysis covers 10 developed countries starting in 1900: Australia, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, United Kingdom, and the United States. They summarize their results as follows:

Stock market booms typically arose when output growth exceeded its long-run average and when inflation was below its long-run average... We find less variation in the association of booms with low inflation than we do in the association of booms with rapid output or productivity growth. (p115)

In subsequent work, Christiano et al. (2010) identify stock market booms in the United States going back to the 1800s. They follow a narrative approach. First, they look for historical episodes from 1800 until just before World War I (but excluding the Civil War) that were commonly described as financial panics. In each case, they define the boom as the run-up in stock prices leading to the peak just before the respective panic occurred. In these episodes, they find that

In virtually every stock market boom, the price level actually declined. Moreover, in no case did the price level rise more than its average in the non-boom, non-Civil War periods. (p93)

The fact that the price level declined during these episodes lines up with the implication of the model of what would happen in the case of a disagreement shock and a fixed supply of money. All of the stock market booms that Christiano et al. (2010) identify during this period occurred before the creation of the Federal Reserve system, and so before there would have been any coordinated policy response that would have changed money supply during these episodes. Christiano et al. (2010) then use statistical methods to identify stock market booms after World War I but excluding the World War II period. For these, they report the following:

As in the earlier data set, each boom episode is a time of non-accelerating inflation. In several cases, inflation actually slowed noticeably from the earlier period. (p95)

¹²Historically, inflation has tended to be negatively correlated with stock returns, as prominently documented in Fama and Schwert (1977) and Modigliani and Cohn (1979). That is, inflation tends to be low when stock prices rise, just as the model predicts for large asset booms. However, Gourio and Ngo (2020) find that this correlation turned positive around 2008.

Finally, Christiano et al. (2010) look separately at Japan between 1960 and 2010, and find the same pattern for the stock market boom in the mid 1980s:

CPI inflation is significantly positive before the start of the 1980s stock market boom, and it then slows significantly as the boom proceeds. Inflation even falls below zero a few times in the second half of the 1980s. (p95)

The evidence above concerns stock market booms. However, the model does not specifically concern equity booms, and its logic should in principle apply equally to any dividend-bearing asset that agents can potentially disagree about. A natural candidate to look at are housing booms, since the model suggests downward price pressures only for sufficiently large booms, and housing is another asset which has exhibited dramatic price growth. Here, the evidence suggests that some but not all housing booms were associated with low inflation. For example, Laidler (2003) argues that price stability does not appear to have fostered financial stability, citing examples related to both housing and equity:

Even at the end of the 1980s, real estate bubbles occurred in some economies without being accompanied by any obvious general inflationary pressures. The Nordic countries provide a notable example here. Furthermore, the high-tech bubble that shocked North American and European markets in the late 1990s occurred in markets where monetary policy was aimed at domestic goals and inflation remained low. (p1)

Piazzesi and Schneider (2008) discuss the fact that the housing boom in the United States that started around 2003 occurred during a period of low inflation, and which Laidler presciently identifies in his article as well. At the same time, they argue that this is not a universal pattern, citing the (considerably smaller) housing boom that occurred in the U.S. in the 1970s during a period of high inflation. Brunnermeier and Julliard (2008) look at housing data in the United Kingdom between 1966 and 2004 and find that inflation tends to predict a lower price-rent ratio. This suggests low inflation is associated with higher house prices, although their approach does not try to make a distinction between large and small housing booms. It is therefore not obvious that this evidence relates to disagreement shocks as in my model. Indeed, Brunnermeier and Julliard (2008) interpret their evidence to mean that inflation affects house prices rather than responds to a shock that affects house prices. That said, the housing boom in the US between 2003 and 2007 occurred in a period widely acknowledged to be relatively low inflation, when Fed officials were publicly expressing concern about the prospect of price deflation.¹³

Given the difficulty in identifying or even defining significant asset booms, it is hard to describe the pattern of lower inflation during asset booms that previous work has documented as a stylized fact. However, the evidence does suggest that at least some asset booms do not conform to the prediction of the Bernanke and Gertler (1999) model that asset booms correspond to aggregate demand shocks. Some other framework is

¹³See, for example, Bernanke (2003).

needed to explain why quite a few asset booms are not associated with higher inflationary pressures. My model offers one explanation based on the idea that asset booms encourage agents to save in order to buy assets in the future, including through liquid assets that would have otherwise been spent on consumption. I now examine whether there is any evidence liquidity hoarding during asset booms.

Documenting the presence of liquidity hoarding during asset booms turns out to be somewhat subtle. In the model, a disagreement shock induces agents to want to hold more liquid assets. However, the total supply of liquid assets \bar{M} is exogenously fixed by assumption. The fact that a disagreement shock can lead to both an asset boom and more hoarding of liquid assets thus need not imply that the total stock of liquid assets held by all agents must rise during booms. In the model, the higher demand for liquid assets results in a lower price level, which in turn increases *real* money balances \bar{M}/P . But looking at the behavior of real balances would use data on the price level and so would not provide independent evidence of the channel through which the price level adjusts. An alternative approach would go beyond the model as it is specified and look at the money holdings of individuals. To see why this might be informative, suppose that in addition to optimists and pessimists, I introduced a third type of agent into the model who believed that $\Delta^+ = 0$ for all t . Such agents would not hold the asset in the steady state following a disagreement shock: Optimists would bid up the asset price enough that these agents would expect to earn a return below ρ from the asset and would prefer to hold money. At the same time, these agents would see no reason to hoard money given they do not expect to earn high returns from the asset in the future. Their share of money holdings should therefore fall relative to the agents who alternate between optimism and pessimism.

This suggests looking for a group of agents that ordinarily does not value liquid assets prior but do engage in speculation during asset booms and so would have reason to hoard liquidity. One candidate group is corporate entities. Ordinarily, corporations have an incentive to distribute their cash flow to shareholders or use it to repay debt rather than hold on to it. But in some countries and in some periods, firms do appear to hold in large amounts of cash. A substantial literature has emerged that tries to answer why this might be the case.¹⁴ Among the various reasons for why firms might hold on to cash is a precautionary motive in which firms want to ensure that they can meet future liquidity needs if they are concerned that their access to capital and debt markets will be costly or slow. Such needs include saving to meet sudden liquidity shortfalls, but it can also include investment or speculative opportunities.¹⁵ To the extent that firms participate in some of the speculation that occurs during asset booms, we can look at whether firms also hoard liquidity in line with the view that speculation should be associated with liquidity hoarding.

Earlier I cited work by Bates, Kahle, and Stulz (2009) which shows that corporate cash holdings in the United States increased in the period of the dot com and housing booms, specifically that “the average cash ratio of S&P 500 firms roughly doubled from 1998 to 2006.” (p1992) While they argue that the precautionary motive for holding cash played an important role in this increase, they do not provide evidence

¹⁴For a comprehensive survey of the work on corporate cash holdings, see Ferreira da Cruz, Kimura, and Sobreiro (2019).

¹⁵For an example of a model where firms hold liquidity for speculative reasons, see Gale and Yorulmazer (2013).

that specifically relates the increase in cash holding of U.S. corporations to a desire to speculate on assets. By contrast, there is more direct evidence that firms engaged in speculation during the Japanese asset boom starting in the mid 1980s. For example, Kester (1991) described the phenomenon in which firms began to engage in *zaitoku*, or financial engineering, as follows:

With financial emancipation has come the deployment of cash in ways that are of dubious value. Much corporate free cash flow in Japan is being used in *zaitoku* operations – essentially speculation on the stock market and other types of financial risk taking. (p65)

To see if these corporations held more cash, I look at data from the Financial Statements Statistics of Corporations reported by the Japanese Ministry of Finance. This data is reported in Figure 6. The data is reported in the aggregate and for groups of firms of different size. I specifically focus on data for the largest corporations, or firms with capital holdings of at least 1 billion yen. This is because Pinkowitz and Williamson (2001) argue that cash holdings at smaller firms are in part driven by the demands of monopolistic Japanese banks, who pressured borrowers to hold a large amount of deposits with their lender banks. Pinkowitz and Williamson (2001) argue that this practice can explain why Japanese firms hold considerably more cash than comparable firms in other countries. Since the market power of Japanese banks began to wane in the late 1980s, the cash holdings of smaller companies may have changed for unrelated reasons around the time of the stock market boom. By contrast, large corporations were less reliant on banks and thus less subject to such pressures.¹⁶ The red line in Figure 6 reports the ratio of cash and deposits to total assets among large Japanese corporations, while the blue reports the ratio of stocks to total assets for these same corporations. The black line corresponds to the Nikkei 225 index. The data suggests that from the mid 1960s to the mid 1980s, large corporations in Japan held about 10% of their assets in cash and deposits. They increased their cash holdings to 13% in the early 1970s, during an earlier smaller stock boom.¹⁷ They then increased their cash holdings again, to 15%, by the late 1980s. In the latter case, the data on stock holdings confirms that these firms as a whole were buying publicly traded stock. There is therefore some evidence that the same Japanese corporations who purchased stock during the 1980s stock market booms were also holding significantly more cash than usual at the same time. They also seemed to hold more cash back in the stock market boom of the early 1970s, although there is no data to confirm these firms increased their stock holdings at the same time.

To be sure, the evidence in Figure 6 is only suggestive. A more definitive analysis would attempt to control for other motives to hold cash among Japanese firms that varied at the time. It is also not clear whether the additional cash holdings were significant enough to have much impact on the overall price level.

¹⁶Interestingly, there is another (smaller) stock market boom in the early 1970s that occurred when the concentration of Japanese banks was relatively stable. In that episode, corporations in all three size bins reported by the Japanese Ministry of Finance increased their cash holdings. By contrast, the cash holdings of smaller firms in the late 1980s were flat or declining.

¹⁷French and Poterba (1991) show that stock boom in the early 1970s corresponded to a high real stock price as well as a high stock price relative to earnings as compared with the rest of the 1970s and early 1980s.

Still, it is noteworthy that at the height of the Japanese stock market boom in the 1980s, agents that were known to have actively engaged in speculation appear to have increased their cash holdings.

7 Conclusion

This paper explored the implications of the Harrison and Kreps (1978) model, which has become a common framework for studying asset booms, in a monetary setting. The analysis revealed that an asset boom fueled by disagreement is likely to be associated with an output boom, higher expected returns, and a lower price level for goods and services. This result offers a contrast to previous work, most notably Bernanke and Gertler (1999), which argued that an asset boom should operate similarly to an aggregate demand shock. The implications of my results are noteworthy for two reasons. First, it offers a potential explanation for the empirical pattern described in Section 6 that quite a few asset booms in practice occurred during a period of lower inflation. The explanation this paper offers is that a disagreement shock that leads to an asset boom also encourages agents to hoard liquidity, which other things equal would drive the price level down. This is a contrast to models such as Lagos and Zhang (2019) which emphasize the opposite, but mutually compatible, direction of causality in which an exogenous fall in inflation encourages agents to trade the asset and drives up asset prices as the asset is increasingly held by those who value it most.

The fact that an exogenously driven asset boom may create disinflationary pressures provides some insight on why central banks have at times expressed a reluctance to act against asset and output booms because of a lack of evidence of inflationary pressures they would have expected to see if the economy was overheating. This reaction by central banks suggests that the low inflation prevalent during some of these episodes was not something engineered by the central bank, but was instead something that central banks were reacting to. The model can also explain why central banks in these situations have expressed frustration about implementing monetary policy. In the model, central banks will find it impossible to use liquidity injections (or contractions) to stabilize asset prices, the price level, and output in the face of disagreement shocks. The central bank could in principle withdraw liquidity from those who are actively trading the asset to dampen its price, but this would only further lower the price level at a time when inflation is already low. It could instead inject liquidity to boost inflation but make sure to direct it away from those inclined to buy the asset. This could dampen the real asset price, but if so it would only increase what agents expect to earn from speculation and would only encourage an even bigger output boom. In the paper I suggest that there may be additional policy interventions that can help to avoid this trilemma. But solely tasking central banks to carry out monetary policy to stabilize multiple targets may be asking the impossible.

Finally, to the extent that restrictions on policymakers mean that current monetary policy is the only available tool, the analysis in this paper suggests a tradeoff between financial stability and stabilizing the price level and output. However, the simple framework I use to clarify the mechanism and potential conflicts between financial stability and price and output stability is not well suited for exploring this tradeoff. In particular, my model features none of the elements that make price stability or financial stability desirable.

The typical argument for why price stability is desirable is that in a world where some prices are rigid, changes in the price level can lead to misallocation. The argument for financial stability is that an asset boom that is financed by debt may lead to large-scale defaults that can be detrimental to the economy and lead to a severe recession that society would prefer to avoid even if it meant preventing optimists from buying the assets that they expect to offer a high return. An important direction for future research is exploring how the intuition from the model here would extend to environments with both price rigidity and credit. Both of these elements also figure prominently in the Bernanke and Gertler (1999) analysis, and so incorporating these elements should also help to assess how the incentive for speculators to save compares with the considerations highlighted in their framework.

Appendix

Proof of Proposition 1: Let p and P denote the constant price of the asset and of goods in a steady state equilibrium. Since $D < \alpha\Delta^-$, pessimists strictly prefer money in a steady state where prices are constant since the return on money 0 while the return on the asset is negative. Hence, in any stationary equilibrium, optimists own all assets and pessimists only hold money.

Given optimists hold the entire stock of assets in a stationary equilibrium, they will earn all of the dividends D . As described in the text, when p and P are constant, this means the wealth of optimists W^+ evolves as follows:

$$\dot{W}_t^+ = \frac{\alpha}{2} (W_t^- - W_t^+) + P \left(\frac{y}{2} + D \right) - \frac{\lambda}{2} (W_t^+ - p) \quad (\text{A.1})$$

Using the fact that $W_t^+ + W_t^- = \bar{M} + p$, we can rewrite this law of motion in terms of W_t^+ alone:

$$\dot{W}_t^+ = \frac{\alpha}{2} \bar{M} + \left(\frac{\alpha + \lambda}{2} \right) p + P \left(\frac{y}{2} + D \right) - \left(\alpha + \frac{\lambda}{2} \right) W_t^+$$

Setting $\dot{W}_t^+ = 0$ allows us to solve for the steady state value \bar{W}^+ :

$$\bar{W}^+ = \frac{\alpha \bar{M}}{2\alpha + \lambda} + \frac{P(y + 2D)}{2\alpha + \lambda} + \frac{\alpha + \lambda}{2\alpha + \lambda} p \quad (\text{A.2})$$

Given the steady state level for \bar{W}^+ in (A.2), we have that $\bar{W}^+ > p$ iff

$$\bar{M} + \frac{P(y + 2D)}{\alpha} > p$$

Optimists would be willing to hold the asset in steady state only if $p \leq \frac{P(D + \alpha\Delta^+)}{\rho}$, i.e., when the instantaneous return on the asset is at least ρ . Since optimists must hold the asset in equilibrium, a necessary condition for the existence of an equilibrium in which the wealth of optimists \bar{W}^+ strictly exceeds the asset price p is

$$\frac{\bar{M}}{P} + \frac{y + 2D}{\alpha} > \frac{D + \alpha\Delta^+}{\rho} \quad (\text{A.3})$$

If (A.3) holds for some value P , it will also hold for any lower P . Since $P \leq \frac{\lambda \bar{M}}{y + D}$ in equilibrium, inequality (A.3) holds in *any* stationary equilibrium if it holds at $P = \frac{\lambda \bar{M}}{y + D}$, i.e., if

$$\frac{y + D}{\lambda} + \frac{y + 2D}{\alpha} > \frac{D + \alpha\Delta^+}{\rho} \quad (\text{A.4})$$

Condition (A.4) ensures that \bar{W}^+ exceeds p in any steady state equilibrium. If (A.4) holds, optimists must hold both money and the asset and so must be indifferent between the two. Optimists would never hold money they don't intend to spend if they have an urge to consume given they can earn a positive return on the asset. Pessimists have no reason to hoard money to buy assets in the future, since they anticipate they will be indifferent between money for assets as optimists. Hence, all agents spend all of their money balances when they have an urge to consume. To ensure optimists hold both the asset and money, they must expect the return on the asset $\frac{P(D + \alpha\Delta^+)}{p}$ to equal ρ .

The equilibrium prices are thus given by

$$\begin{aligned} P &= \frac{\lambda \bar{M}}{y + D} \\ p &= \frac{\lambda \bar{M}}{y + D} \frac{D + \alpha \Delta^+}{\rho} \end{aligned} \quad (\text{A.5})$$

In sum, under condition (A.4), the steady state equilibrium prices P and p are given by (A.5). Let Δ^* denote the highest value of Δ^+ for which (A.4) holds, i.e.

$$\Delta^* \equiv \frac{\rho}{\alpha} \left[\frac{y + D}{\lambda} + \frac{y + 2D}{\alpha} - \frac{D}{\rho} \right]$$

Condition (18) ensures $\Delta^* > 0$ for any $\alpha \geq 0$. Hence, for $\Delta^+ < \Delta^*$, the unique steady state equilibrium is given by (A.5). This establishes the first part of proposition 1.

Next, suppose $\Delta^+ > \Delta^*$, i.e.,

$$\frac{y + D}{\lambda} + \frac{y + 2D}{\alpha} \leq \frac{D + \alpha \Delta^+}{\rho} \quad (\text{A.6})$$

In this case, there can be no stationary equilibrium in which $\bar{W}^+ > p$. For suppose there were such an equilibrium. Then p would have to equal $\frac{P(D + \alpha \Delta^+)}{\rho}$ to ensure optimists are willing to hold both money and assets. The fact that optimists are indifferent between money and the asset would imply that both optimists and pessimists spend all of their money holdings when they have an urge to consume. This implies $P = \frac{\lambda \bar{M}}{y + D}$, which contradicts our assumption that $\bar{W}^+ > p$. Instead, in any steady state equilibrium optimists only hold the asset, i.e.,

$$p = \bar{W}^+$$

Using (A.2), the equilibrium price p is given by

$$p = \bar{M} + \frac{P(y + 2D)}{\alpha} \quad (\text{A.7})$$

I next solve for the price level P . Since $W^+ = p$, all money is held by pessimists, and the price level P depends on their spending. If pessimists spend all of their money balances when they have an urge to consume, the equilibrium price would equal $P = \frac{\lambda \bar{M}}{y + D}$. Otherwise, the equilibrium price P will be below $\frac{\lambda \bar{M}}{y + D}$. To determine what pessimists do with their money holdings, let v^- denote the value of a unit of nominal wealth for an agent who is currently a pessimist, and v^+ the value of a unit of nominal wealth for an optimist. Whether pessimists spend all of their money holdings when they have an urge to consume thus depends on how v^- compares with the marginal utility from spending a unit of nominal wealth, which is $1/P$.

In equilibrium, v^- cannot exceed $1/P$. Otherwise, agents would never consume and the goods market cannot clear. This means an agent would always be willing to spend her money holdings when faced with an urge to consume as a pessimist, either out of indifference if $v^- = 1/P$ or because she strictly prefers to consume if $v^- > 1/P$. We can therefore characterize v^- using a Bellman equation that assumes the agent consumes the next time they have an urge to consume if she is still a pessimist:

$$\rho v^- = \lambda \left(\frac{1}{P} - v^- \right) + \alpha (v^+ - v^-) \quad (\text{A.8})$$

The value of a unit of nominal wealth v^- changes to $1/P$ if she has an urge to consume while still a pessimist, and changes to v^+ if an event payoff occurred. Next, consider the value of nominal wealth v^+ for an optimist. Since $\Delta > \Delta^*$, the optimist invests any nominal wealth in assets. The wealth of the agent then grows due to dividend payments from the asset, i.e., $\dot{v}^+ = \frac{PD}{p}v^+$. If a payoff event occurred, the optimist would also realize a windfall dividend of Δ^+ . Hence, v^+ satisfies the Bellman equation

$$\rho v^+ = \left(\frac{PD}{p}\right)v^+ + \alpha \left(\left(1 + \frac{P\Delta^+}{p}\right)v^- - v^+ \right) \quad (\text{A.9})$$

Solving the system of equations given by (A.8) and (A.9) yields

$$v^+ = \frac{1}{P} \frac{\alpha \lambda \left(1 + \frac{P\Delta^+}{p}\right)}{(\rho + \alpha + \lambda) \left(\rho + \alpha - \frac{PD}{p}\right) - \alpha^2 \left(1 + \frac{P\Delta^+}{p}\right)} \quad (\text{A.10})$$

$$v^- = \frac{1}{P} \frac{\lambda \left(\rho + \alpha - \frac{PD}{p}\right)}{(\rho + \alpha + \lambda) \left(\rho + \alpha - \frac{PD}{p}\right) - \alpha^2 \left(1 + \frac{P\Delta^+}{p}\right)} \quad (\text{A.11})$$

Pessimists will prefer to spend their money balances when $\frac{1}{P} > v^-$. Using the value of v^- , this condition implies

$$\frac{p}{P} > \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \quad (\text{A.12})$$

Recall that under (A.6), the equilibrium asset price is given by $p = \bar{M} + \frac{P}{\alpha}(y + 2D)$. Using the fact that in equilibrium $P \leq \frac{\lambda\bar{M}}{y+D}$, we have

$$\frac{p}{P} = \frac{\bar{M}}{P} + \frac{y + 2D}{\alpha} \geq \frac{y + D}{\lambda} + \frac{y + 2D}{\alpha}$$

Hence, if

$$\frac{D + \alpha\Delta^+}{\rho} > \frac{y + D}{\lambda} + \frac{y + 2D}{2\alpha} > \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \quad (\text{A.13})$$

the unique equilibrium is one where pessimists spend all of their money holdings when they have an urge to consume, and equilibrium prices are given by

$$\begin{aligned} P &= \frac{\lambda\bar{M}}{y + D} \\ p &= \left[1 + \frac{1}{\alpha} \frac{y + 2D}{y + D}\right] \lambda\bar{M} \end{aligned} \quad (\text{A.14})$$

Let Δ^{**} denote the highest value of Δ^+ for which the second inequality in (A.13) holds, i.e.,

$$\Delta^{**} = \left(2 + \frac{\rho}{\alpha}\right) \frac{\rho}{\alpha} \left[\frac{y + D}{\lambda} + \frac{y + 2D}{\alpha} - \frac{(\alpha + \rho)D}{(2\alpha + \rho)\rho} \right]$$

The unique equilibrium when $\Delta^+ \in (\Delta^*, \Delta^{**})$ is given by (A.14), establishing the second part of Proposition 1.

Finally, we turn to the case where $\Delta^+ > \Delta^{**}$, we have

$$\frac{y + D}{\lambda} + \frac{y + 2D}{\alpha} < \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \quad (\text{A.15})$$

In this case, there cannot be an equilibrium in which pessimists spend all of their money balances when they have an urge to consume, since they would strictly prefer to hold on to money. Instead, we need pessimists to be indifferent between holding money and spending it, i.e., $v^- = 1/P$. This requires

$$\frac{p}{P} = \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \quad (\text{A.16})$$

Since

$$\frac{y + D}{\lambda} + \frac{y + 2D}{\alpha} < \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} < \frac{D + \alpha\Delta^+}{\rho}$$

it follows that p is equal to \bar{W}^+ , and so

$$p = \bar{M} + \frac{P}{\alpha}(y + 2D) \quad (\text{A.17})$$

Solving the system of equations given by (A.16) and (A.17) yields

$$\begin{aligned} P &= \left(\frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \left[1 - \frac{y + 2D}{\alpha} \right] \right)^{-1} \bar{M} \\ p &= \bar{M} + \frac{P}{\alpha}(y + 2D) \end{aligned} \quad (\text{A.18})$$

This establishes the third part of Proposition 1. Essentially, the stationary equilibrium depends on how the expression $\frac{y+D}{\lambda} + \frac{y+2D}{\alpha}$ falls between two cutoffs, $\frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$ and $\frac{D+\alpha\Delta^+}{\rho}$. This can be reinterpreted as how the degree of optimism Δ^+ compares to two cutoffs, Δ^* and Δ^{**} . ■

Proof of Proposition 2: Part of the proof mirrors the proof of Proposition 1. In any stationary equilibrium, the assumption that $D < \alpha\Delta^-$ implies pessimists will not hold the asset. The steady state wealth of optimists must therefore be enough to buy the asset, i.e., $\bar{W}^+ \geq p$. I first look for a steady state equilibrium in which $W^+ > p$. In any such equilibrium, optimists hold both assets and money. As in the endowment economy, we can solve for the steady state wealth of optimists as

$$\bar{W}^+ = \frac{\alpha\bar{M}}{2\alpha + \lambda} + \frac{P(n^+ + 2D)}{2\alpha + \lambda} + \frac{\alpha + \lambda}{2\alpha + \lambda}p \quad (\text{A.19})$$

This expression for \bar{W}^+ exceeds p iff

$$\frac{\bar{M}}{P} + \frac{n^+ + 2D}{\alpha} > \frac{p}{P}$$

However, in a stationary equilibrium, optimists will only hold the asset if $\frac{p}{P} < \frac{D+\alpha\Delta^+}{\rho}$ so that the return to holding the asset is at least ρ . Hence, a sufficient condition for a stationary equilibrium in which $\bar{W}^+ > p$ is if

$$\frac{\bar{M}}{P} + \frac{n^+ + 2D}{\alpha} > \frac{D + \alpha\Delta^+}{\rho}$$

Optimists will be indifferent between money and assets if the expected return on the asset equals ρ , i.e., $\frac{p}{P} = \frac{D+\alpha\Delta^+}{\rho}$. We can use this to solve for how optimists and pessimists value a marginal unit of nominal wealth. The equations for v^+ and v^- are the same as in the endowment economy, i.e., (A.10) and (A.11). When $\frac{p}{P} = \frac{D+\alpha\Delta^+}{\rho}$, these expressions imply $v^+ = v^- = \frac{1}{P} \frac{\lambda}{\lambda + \rho}$. Since agents choose n to solve $\phi'(n) = Pv$, optimists and pessimists produce the same amount, i.e., $n^+ = n^- = \phi'^{-1}\left(\frac{\lambda}{\rho + \lambda}\right) \equiv n^*$. If $\bar{W}^+ > p$, then, output per instant is $n^* + D$, and the price

of goods will equal $P^* = \frac{\lambda \bar{M}}{n^* + D}$. Thus, as long as

$$\frac{n^* + D}{\lambda} + \frac{n^* + 2D}{\alpha} > \frac{D + \alpha \Delta^+}{\rho} \quad (\text{A.20})$$

the equilibrium must satisfy

$$\begin{aligned} n^+ &= n^- = n^* \\ P^* &= \frac{\lambda \bar{M}}{n^* + D} \\ p^* &= \frac{\lambda \bar{M}}{n^* + D} \frac{D + \alpha \Delta^+}{\rho} \end{aligned} \quad (\text{A.21})$$

Condition (A.20) defines a cutoff Δ^* such that (A.21) is an equilibrium, and (23) ensures $\Delta^* > 0$. To show there is no equilibrium in which $\bar{W}^+ = p$ when $\Delta^+ < \Delta^*$, suppose there was such an equilibrium. Then we would have

$$\frac{p}{P} = \frac{\bar{M}}{P} + \frac{n^+ + 2D}{\alpha}$$

Since the most agents spend on goods is $\lambda \bar{M}$, we know that $P \leq \frac{\lambda \bar{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}$. Substituting in this inequality implies

$$\frac{p}{P} \geq \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{n^+ + 2D}{\alpha}$$

In any equilibrium, $\frac{p}{P} \leq \frac{D + \alpha \Delta^+}{\rho}$ to ensure optimists are willing to hold the asset. Suppose $\frac{p}{P} = \frac{D + \alpha \Delta^+}{\rho}$. Then $v^+ = v^- = \frac{\lambda}{\rho + \lambda} \frac{1}{P}$, which implies $n^+ = n^- = n^*$ and so

$$\begin{aligned} \frac{p}{P} &= \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{n^+ + 2D}{\alpha} \\ &= \frac{n^* + D}{\lambda} + \frac{n^* + 2D}{\alpha} \\ &> \frac{D + \alpha \Delta^+}{\rho} \end{aligned}$$

where the last inequality comes from the fact that $\Delta^+ < \Delta^*$. But this contradicts our original supposition that $\frac{p}{P} = \frac{D + \alpha \Delta^+}{\rho}$. Suppose instead that $\frac{p}{P} < \frac{D + \alpha \Delta^+}{\rho}$. From the solutions above, we know Pv^+ and Pv^- are both increasing in P/p . Hence, we would have $n^+ > n^*$ and $n^- > n^*$, in which case

$$\begin{aligned} \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{n^+ + 2D}{\alpha} &> \frac{n^* + D}{\lambda} + \frac{n^* + 2D}{\alpha} \\ &> \frac{D + \alpha \Delta^+}{\rho} \end{aligned}$$

where again the second inequality follows from the fact that $\Delta^+ < \Delta^*$. But this implies $\bar{W}^+ > p$, which is a contradiction. So when $\Delta^+ < \Delta^*$, there cannot be a stationary equilibrium other than (A.21).

I next turn to the case where $\Delta^+ > \Delta^*$. I first argue that the real price of the asset $\frac{p}{P} \geq \frac{(\alpha + \rho)D + \alpha^2 \Delta^+}{\rho(2\alpha + \rho)}$. For suppose this inequality was violated. Since $\frac{(\alpha + \rho)D + \alpha^2 \Delta^+}{\rho(2\alpha + \rho)} < \frac{D + \alpha \Delta^+}{\rho}$, optimists would prefer assets over money. At the same time, the equation for v^- would imply a value below $\frac{1}{P}$, and so pessimists would refuse to spend money when they have in urge to consume. But then the market for goods would not clear.

Consider the case where $\frac{p}{P} = \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. Substituting in for this value in the expressions for v^+ and v^- yields

$$Pv^+ = 1 + \frac{\rho}{\alpha} \quad (\text{A.22})$$

$$Pv^- = 1 \quad (\text{A.23})$$

In this case, we have

$$\begin{aligned} n^+ &= \phi'^{-1}\left(1 + \frac{\rho}{\alpha}\right) \equiv (n^{**})^+ \\ n^- &= \phi'^{-1}(1) \equiv (n^{**})^- \end{aligned}$$

The steady state equilibrium in this case is as follows. The real price of the asset is given by

$$\frac{p}{P} = \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \quad (\text{A.24})$$

Optimists and pessimists choose labor optimally, i.e.

$$n^+ = (n^{**})^+ \quad (\text{A.25})$$

$$n^- = (n^{**})^- \quad (\text{A.26})$$

Finally, since $\Delta^+ > \Delta^*$, the price $p = \overline{W}^+$, i.e.,

$$\frac{p}{P} = \frac{\overline{M}}{P} + \frac{\frac{1}{2}(n^{**})^+ + D}{\alpha} \quad (\text{A.27})$$

I can solve P using (A.24) and (A.27). Since $P \leq \frac{\lambda\overline{M}}{\frac{1}{2}(n^{**})^+ + \frac{1}{2}(n^{**})^- + D}$, this can only be an equilibrium if

$$\frac{\frac{1}{2}(n^{**})^+ + \frac{1}{2}(n^{**})^- + D}{\lambda} + \frac{\frac{1}{2}(n^{**})^+ + D}{\alpha} \leq \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \quad (\text{A.28})$$

Hence, there exists a cutoff Δ^{**} such that (A.28) holds iff $\Delta^+ > \Delta^{**}$. Moreover, since $(n^{**})^+$ and $(n^{**})^-$ both exceed n^* , it follows that $\Delta^{**} > \Delta^*$. Hence, (A.25)-(A.27) constitute an equilibrium whenever $\Delta^+ > \Delta^{**}$.

To show that (A.24) - (A.27) is the unique equilibrium when $\Delta^+ > \Delta^{**}$, recall that when $\Delta^+ > \Delta^{**} > \Delta^*$, we must have $\overline{W}^+ = p^*$. Equilibrium requires that $\frac{p}{P} \geq \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$ to ensure pessimists are willing to spend some money when hit with an urge to consume. If $\frac{p}{P} = \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$, the equilibrium will correspond to (A.24) - (A.27). So, for the equilibrium to be distinct requires that $\frac{p}{P} > \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. Since Pv^+ and Pv^- are both increasing in P/p and $\phi'(\cdot) > 0$, it follows that $n^+ < (n^{**})^+$ and $n^- < (n^{**})^-$. Moreover, since v^- is decreasing in p/P , pessimists will prefer to spend all of their money balances when they have an urge to consume. This implies $P = \frac{\lambda\overline{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}$. At the same time, since $\Delta^+ > \Delta^*$, we must have $W^+ = p$, which implies $\frac{p}{P} = \frac{\overline{M}}{P} + \frac{\frac{1}{2}n^+ + D}{\alpha}$. It follows that

$$\begin{aligned} \frac{p}{P} &= \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{\frac{1}{2}n^+ + D}{\alpha} \\ &< \frac{\frac{1}{2}(n^{**})^+ + \frac{1}{2}(n^{**})^- + D}{\lambda} + \frac{\frac{1}{2}(n^{**})^+ + D}{\alpha} = \frac{(\alpha + \rho)D + \alpha^2\Delta^+}{\rho(2\alpha + \rho)} \end{aligned}$$

But this contradicts our presumption that $\frac{p}{P} > \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. So there can be no other stationary equilibrium when $\Delta^+ > \Delta^{**}$.

Finally, I turn to the case where $\Delta^+ \in (\Delta^*, \Delta^{**})$. Since $\Delta^+ > \Delta^*$, optimists only hold assets and so $\bar{W}^+ = p$. At the same time, since $\Delta^+ < \Delta^{**}$, pessimists prefer to spend all of their available money holdings when they have an urge to spend. These two imply

$$\begin{aligned}\frac{p}{P} &= \frac{\bar{M}}{P} + \frac{\frac{1}{2}n^+ + D}{\alpha} \\ P &= \frac{\lambda\bar{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}\end{aligned}$$

Substituting in for P in the equation for p yields

$$\frac{p}{P} = \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{\frac{1}{2}n^+ + D}{\alpha} \quad (\text{A.29})$$

which is increasing in n^+ and n^- . The labor supplies n^+ and n^- in turn satisfy

$$\begin{aligned}\phi'(n^+) &= Pv^+\left(\frac{p}{P}, \Delta^+\right) \\ \phi'(n^-) &= Pv^-\left(\frac{p}{P}, \Delta^+\right)\end{aligned}$$

A higher value of $\frac{p}{P}$ holding all other terms fixed decreases Pv^+ and Pv^- . It follows that for any fixed Δ^+ , there can be at most one value of (n^+, n^-) that solves the above system, and this solution is increasing in Δ^+ . ■

Proof of Proposition 3: The proof proceeds in steps. I first show that the new steady state prices are given by $p = (1 + \mu)\hat{p}$ and $P = (1 + \mu)\hat{P}$. This is immediate for the endowment economy given the steady state p and P are proportional to \bar{M} per Proposition 1. But the argument for the production economy is a bit more subtle since agents choose how much to produce. I then argue that if \widehat{W}^+ denotes the steady state wealth of optimists before the liquidity injection, then steady state wealth of optimists after the liquidity injection is given by $\bar{W}^+ = (1 + \mu)\widehat{W}^+$. Finally, I show that a liquidity injection that is proportional to the original money holdings leaves optimists with a wealth equal to \bar{W}^+ at date 0, which implies that $p_t = (1 + \mu)\hat{p}$ and $P_t = (1 + \mu)\hat{P}$ for all t is an equilibrium.

I begin with the equations for the steady state values of v^+ and v^- of the value of marginal wealth for optimists and pessimists, respectively. These are given by (A.10) and (A.11). Multiplying these by P reveals that the steady state expressions Pv^+ and Pv^- can be expressed solely as a function of the steady state real asset price p/P . From the first order condition, we have

$$\begin{aligned}n^+ &= \phi'^{-1}(Pv^+) \\ n^- &= \phi'^{-1}(Pv^-)\end{aligned}$$

Hence, n^+ and n^- can also be expressed solely as a function of the real asset price p/P . Consider the prices $(1 + \mu)\hat{p}$ and $(1 + \mu)\hat{P}$. The ratio of these prices is equal to \hat{p}/\hat{P} , which is the same as the original steady state before the liquidity injection. If we let \hat{n}^+ and \hat{n}^- denote the production levels in the original steady state, then if $p_t = (1 + \mu)\hat{p}$

and $P_t = (1 + \mu) \widehat{P}$ for all t , optimists and pessimists would optimally choose to produce the same quantities as in the original steady state before the liquidity injection, i.e., $n^+ = \widehat{n}^+$ and $n^- = \widehat{n}^-$.

Next, from the proof of Proposition 2, we know that holding n^+ and n^- fixed, the steady state prices P and p will be proportional the \overline{M} . That is, holding production fixed, increasing liquidity by a factor of $1 + \mu$ would increase the steady state equilibrium prices to $p = (1 + \mu) \widehat{p}$ and $P = (1 + \mu) \widehat{P}$. Together, these results imply that $p = (1 + \mu) \widehat{p}$, $P = (1 + \mu) \widehat{P}$, $n^+ = \widehat{n}^+$ and $n^- = \widehat{n}^-$ is a steady state equilibrium when the money supply is $(1 + \mu) \overline{M}$ given that \widehat{p} , \widehat{P} , \widehat{n}^+ , and \widehat{n}^- is a steady state when the money supply is \overline{M} . From the proof of Proposition 2, we know this must be the unique steady state.

From the proof of Proposition 2, the steady-state wealth of optimists is given by

$$\overline{W}^+ = \frac{\alpha \overline{M}}{2\alpha + \lambda} + \frac{P(n^+ + 2D)}{2\alpha + \lambda} + \frac{\alpha + \lambda}{2\alpha + \lambda} p$$

It follows that

$$\overline{W}^+ = (1 + \mu) \widehat{W}^+ \tag{A.30}$$

Finally, let \widehat{M}^+ and \widehat{M}^- denote the money holdings of optimists and pessimists in the original steady state before the liquidity injection. Optimists hold all of the asset in steady state, so their original steady state wealth is given by $\widehat{W}^+ = \widehat{M}^+ + \widehat{p}$. If the price of the asset increased to $(1 + \mu) \widehat{p}$ and the money holdings of optimists is increased to $(1 + \mu) \widehat{M}^+$, then the wealth of optimists would immediately jump to $\overline{W}^+ (1 + \mu) \widehat{W}^+$. As such, jumping to the new steady state immediately is an equilibrium. ■

Proof of Proposition 4: The proof of Proposition 3 establishes that in the new steady state after a liquidity injection features $p = (1 + \mu) \widehat{p}$, $P = (1 + \mu) \widehat{P}$ and $\overline{W}^+ = (1 + \mu) \widehat{W}^+$. A liquidity injection that favors optimists would give them more than $\mu \widehat{M}^+$, and so their initial wealth at date 0 amounts to $W_0^+ > (1 + \mu) \widehat{M}^+ + p_t$. I now consider three cases in turn, depending on the value of Δ^+ .

Case (a): $\Delta^+ \leq \Delta^*$

I show there exists an equilibrium in which $p_t = (1 + \mu) \widehat{p}$ and $P_t = (1 + \mu) \widehat{P}$ for all t . Given prices p_t and P_t , the wealth of optimists W_t^+ evolves as

$$\begin{aligned} \dot{W}_t^+ &= \alpha \left((1 + \mu) \overline{M} + (1 + \mu) \widehat{p} - 2W_t^+ \right) + (1 + \mu) \widehat{P} \left(\frac{y}{2} + D \right) - \lambda (W_t^+ - (1 + \mu) \widehat{p}) \\ &= (2\alpha + \lambda) \left(\overline{W}^+ - W_t^+ \right) \end{aligned}$$

where $\overline{W}^+ = (1 + \mu) \left[\frac{\alpha \overline{M}}{2\alpha + \lambda} + \frac{P(n^+ + 2D)}{2\alpha + \lambda} + \frac{\alpha + \lambda}{2\alpha + \lambda} p \right] = (1 + \mu) \widehat{W}^+$. This implies

$$W_t^+ = \overline{W}^+ + \left[W_0^+ - \overline{W}^+ \right] e^{-(2\alpha + \lambda)t} \tag{A.31}$$

Since $p_t = (1 + \mu)\widehat{p}$, an initial injection that favors optimists would leave optimists with an initial wealth that exceeds the new steady state, since

$$\begin{aligned} W_0^+ &> (1 + \mu)M_0^+ + p_t \\ &= (1 + \mu)M_0^+ + (1 + \mu)\widehat{p} = \overline{W}_0^+ \end{aligned} \quad (\text{A.32})$$

It follows that $W_t^+ > \overline{W}^+ \geq (1 + \mu)\widehat{p} = p_t$ for all t , so at these prices, optimists always have sufficient wealth to buy the asset. Given it was optimal for optimists to hold the asset in the original steady state, it will be optimal for them to hold the asset when the prices $p_t = (1 + \mu)\widehat{p}$ and $P_t = (1 + \mu)\widehat{P}$ for all t when the expected return on the asset is unchanged. It will also be optimal for agents to spend their money holdings when they have an urge to consume given these prices, and so $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ is a market clearing price at each date t .

Case (b.1): $\Delta^+ \in (\Delta^*, \Delta^{})$**

I show there exists an equilibrium in which there is some finite date $T < \infty$ where $W_t^+ > p_t > (1 + \mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1 + \mu)\widehat{p}$ for $t \geq T$, while the price level $P_t = (1 + \mu)\widehat{P} = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for all t .

First, if the wealth of optimists $W_t^+ > p_t$ for all $t < T$, optimists must be willing to hold both money and assets prior to date T . Let v_t^+ denote the value of marginal wealth for an agent at date t . The instantaneous utility return to holding money is ρv_t^+ while the instantaneous return to holding the asset over the next instant is $\frac{P_t(D + \alpha\Delta^+) + \dot{p}_t}{p_t} v_t^+$. Optimists are indifferent only if the latter return is equal ρ . That is, for $t < T$, for the asset price p_t at $t < T$ to be an equilibrium as specified, it must satisfy the differential equation

$$P_t (D + \alpha\Delta^+) + \dot{p}_t = \rho p_t \quad (\text{A.33})$$

Given a value for T and a path for the price level P_t , we can solve this differential equation forward to obtain

$$p_t = \int_t^\infty e^{-\rho(s-t)} P_s (D + \alpha\Delta^+) ds + K e^{\rho t} \quad (\text{A.34})$$

where the constant K is determined by the boundary condition that the price p_t at $t = T$ must equal $(1 + \mu)\widehat{p}$, i.e.,

$$K = e^{-\rho T} (1 + \mu)\widehat{p} - \int_T^\infty e^{-\rho s} P_s (D + \alpha\Delta^+) ds \quad (\text{A.35})$$

If the price level is $P_s = (1 + \mu)\widehat{P}$ for all s as conjectured, the constant is given by

$$K = \int_T^\infty e^{-\rho s} P_s (D + \alpha\Delta^+) ds = (1 + \mu) e^{-\rho T} \left[\widehat{p} - \frac{\widehat{P}(D + \alpha\Delta^+)}{\rho} \right] < 0$$

where the last inequality uses Proposition 1 and the fact that $\Delta^+ > \Delta^*$.

Turning to the wealth of optimists, given a path of prices p_t and P_t , the law of motion for W_t^+ for $t < T$ when agents act optimally is given by

$$\dot{W}_t^+ = \alpha(1 + \mu)\overline{M} + (\alpha + \lambda)p_t + P_t \left(\frac{y}{2} + D \right) - (2\alpha + \lambda)W_t^+ + \dot{p}_t \quad (\text{A.36})$$

Given a value for T and paths for P_t and p_t , we can solve the differential equation backward to obtain

$$W_t^+ = \int_0^t e^{-(2\alpha+\lambda)s} \left[\alpha(1+\mu)\overline{M} + P_s \left(\frac{y}{2} + D \right) + (\alpha + \lambda)p_s + \dot{p}_s \right] ds + K e^{-(2\alpha+\lambda)t} \quad (\text{A.37})$$

where now $K = W_0^+$ in the initial wealth that is determined by p_0 and the liquidity injection.

Turning to the price level P_t , we know that from date T on, optimists and pessimists would want to spend all of the money holdings given the steady state asset price if the price level was also constant. This implies $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for all $t \geq T$ is an equilibrium. Before date T , optimists must hold money if $W_t^+ > p_t$. But since they can earn a positive return of ρ from holding the asset, they would only agree to hold money at a given instant if they intended to spend it. So optimists would spend all of their money holdings before T as well. Pessimists expect the return on the asset to be highest at $t \geq T$. But at this return, they prefer to spend all of their cash when they have an urge to consume. It follows that they would also prefer to spend all of their cash when they have an urge to consume at earlier dates, when the return to holding the asset is lower and waiting will give them at best the return at date T . If all agents spend their available cash, the market clearing price would be given by $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$.

Finally, I need to confirm that $W_t^+ > p_t$ for $t < T$ and show how to solve for T . I first argue that the date T in which $p_t > W_t^+ > (1+\mu)\widehat{p}$ is finite. For suppose $T = \infty$. In that case, we know from (A.35) that $K = 0$. Given the price level P_t , the price of the asset in (A.34) would asymptotically converge to $(1+\mu)\frac{\widehat{P}(D+\alpha\Delta^+)}{\rho} > (1+\mu)\widehat{p}$. But this contradicts Proposition 1 which establishes that there is a unique steady state equilibrium in which the asset price is $(1+\mu)\widehat{p}$. Hence, there exists a finite date T at which $p_T = W_T^+$. By contrast, at date 0, the wealth of optimists exceeds the price of the asset by the amount of the liquidity injection optimists receive at that date. Let T' denote the first date at which the money holdings of optimists are equal to 0, i.e., $T' = \inf \{t : M_t^+ = 0\}$. Since the liquidity injection at date 0 favors optimists, and since the money holdings of optimists in the original steady state is 0, it follows that $M_0^+ > 0$ after the injection. Hence, $T' > 0$. By construction, $T' \leq T < \infty$. Since the wealth of optimists is given by $W_t^+ = M_t^+ + p_t$, then $W_t^+ - p_t = M_t^+ > 0$ for all $t \in [0, T')$. Hence, there exists a finite date T' such that $W_t^+ > p_t$ for $t < T'$ as desired. That is, $T = T'$, so T is the first date at which money holdings of optimists are equal to 0. To solve for T , we choose the value of T that ensures W_t^+ at $t = T$ is equal to $(1+\mu)\widehat{p}$. In particular, p_t in (A.34) depends on T through the coefficient K , so we can write p_t^T to indicate that it is a function of T . We then solve for T from the equation

$$\int_0^T e^{-(2\alpha+\lambda)s} \left[\alpha(1+\mu)\overline{M} + (1+\mu)\widehat{P} \left(\frac{y}{2} + D \right) + (\alpha + \lambda)p_s^T + \dot{p}_s^T \right] ds + \left(M_0^+ + p_0^T \right) e^{-(2\alpha+\lambda)T} = (1+\mu)\widehat{p} \quad (\text{A.38})$$

The existence of a solution follows from the existence of T' . We can thus find a date $T < \infty$ where $W_t^+ > p_t > (1+\mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1+\mu)\widehat{p}$ for $t \geq T$ that is consistent with optimization and market clearing.

Case (b.2): $\Delta^+ > \Delta^{**}$

Once again, I construct an equilibrium in which there is some finite date $T < \infty$ where $W_t^+ > p_t > (1+\mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1+\mu)\widehat{p}$ for $t \geq T$ while the price level $P_t = (1+\mu)\widehat{P}$ for all t .

The path for the asset price p_t is once again given by (A.34), and the path for the total wealth of optimists W_t^+

is given by (A.37). Both are governed by the path for the price level, P_t . I now look for a path for the price level in which $P_t > (1 + \mu) \widehat{P}$ for $t < T$ and $P_t = (1 + \mu) \widehat{P}$ for $t \geq T$. That is, the price level attains its steady state level at the same date as when the wealth of optimists is equal to the price of the asset.

For $\Delta^+ > \Delta^{**}$, we know from Proposition 1 that $\widehat{P} < \frac{\lambda \overline{M}}{y+D}$. Since the path for the price level is continuous, this means there exists an interval $(T_0, T]$ where $0 \leq T_0 < T$ in which the price level is close to $(1 + \mu) \widehat{P}$ and thus below $\frac{\lambda(1+\mu)\overline{M}}{y+D}$. This can only be an equilibrium if not all agents want to spend their money holdings for $t \in (T_0, T]$. Since optimists hold both money and the asset before date T , and the nominal return on the asset is positive, they will only hold money if they intend to spend it if they have an urge to consume. This suggests pessimists are either indifferent between holding money and spending it, or else strictly prefer to hold their money.

I begin with the case in which pessimists are indifferent between holding and spending money. Once again, let v_t^+ denote the value of a unit of money at date t for an optimist and v_t^- denote the analogous value for a pessimist. The proof of Proposition 1 implies that the steady state values of v_t^+ and v_t^- from date T on are given by

$$v^+ = \frac{1 + \rho/\alpha}{(1 + \mu) \widehat{P}} \quad (\text{A.39})$$

$$v^- = \frac{1}{(1 + \mu) \widehat{P}} \quad (\text{A.40})$$

Before date T , if pessimists are indifferent between holding money and spending it, it would be optimal for them to spend their money at the first urge to consume that occurred before date T regardless of their type at the time. I can therefore compute v_t^- assuming they spend their money holdings if they have an urge to consume before date T , and if they don't have an urge to consume by date T their payoffs will be given by (A.39) and (A.40). That is, v_t^- will satisfy the integral equation

$$v_t^- = \int_t^T \frac{\lambda e^{-(\lambda+\rho)(s-t)}}{P_s} ds + \frac{e^{-(\rho+\lambda)(T-t)}}{P_T} \left[\Pr(v_T = v^- | v_t = v^-) + \Pr(v_T = v^+ | v_t = v^-) \frac{\alpha + \rho}{\alpha} \right] \quad (\text{A.41})$$

For a two-state switching model, we know that

$$\begin{aligned} \Pr(v_T = v^- | v_t = v^-) &= \frac{1 + e^{-2\alpha(T-t)}}{2} \\ \Pr(v_T = v^+ | v_t = v^-) &= \frac{1 - e^{-2\alpha(T-t)}}{2} \end{aligned}$$

Substituting in for these probabilities yields

$$v_t^- = \int_t^T \frac{\lambda e^{-(\lambda+\rho)(s-t)}}{P_s} ds + \frac{e^{-(\lambda+\rho)(T-t)}}{P_T} \left(1 + \frac{\rho}{2\alpha} \left[1 - e^{-2\alpha(T-t)} \right] \right) \quad (\text{A.42})$$

If pessimists are indifferent between spending and holding money from date T_0 on, we must have $v_t^- = \frac{1}{P_t}$ for all $t > T_0$. Equating v_t^- above with P_t thus translates into the integral equation for the price P_t :

$$\int_t^T \frac{\lambda e^{-(\lambda+\rho)(s-t)}}{P_s} ds = \frac{1}{P_t} - \frac{e^{-(\lambda+\rho)(T-t)}}{P_T} \left(1 + \frac{\rho}{2\alpha} \left[1 - e^{-2\alpha(T-t)} \right] \right)$$

This is a linear Volterra integral equation of the second kind, i.e. it has the form

$$y(t) - \int_t^T y(s) k(s-t) ds = f(t)$$

where $y(t) = \frac{P}{P_t}$, $k(s-t) = \lambda e^{-(\lambda+\rho)(s-t)}$, and $f(t) = \frac{e^{-(\lambda+\rho)(T-t)}}{P^*} \left(1 + \frac{\rho}{2\alpha} [1 - e^{-2\alpha(T-t)}]\right)$. When $k(s-t)$ is an exponential, as is the case here, the Volterra equation has a closed form solution, i.e.,

$$\frac{1}{P_t} = \frac{1}{P_T} \left[\frac{\rho + \lambda + 2\alpha}{\lambda + 2\alpha} e^{-\rho(T-t)} - \frac{\rho}{\lambda + 2\alpha} e^{-(\rho+\lambda+2\alpha)(T-t)} \right]$$

Rearranging yields

$$P_t = \frac{(\lambda + 2\alpha) e^{-\rho(t-T)} P_T}{\lambda + 2\alpha + \rho [1 - e^{-(\lambda+2\alpha)(t-T)}]} \quad (\text{A.43})$$

This path exceeds $P_T = (1 + \mu) \widehat{P}$ for $t < T$, and reaches $(1 + \mu) \widehat{P}$ when $t = T$. Since the price level cannot exceed $\frac{\lambda(1+\mu)\overline{M}}{y+D}$, this path is only feasible for $t \geq T_0$ where T_0 solves

$$\frac{(\lambda + 2\alpha) e^{-\rho(T_0-T)} P_T}{\lambda + 2\alpha + \rho [1 - e^{-(\lambda+2\alpha)(T_0-T)}]} = \frac{\lambda \overline{M}}{y + D}$$

If $T_0 > 0$, then for $0 \leq t \leq T_0$, the price level $P_t = \frac{\lambda \overline{M}}{y+D}$ for $t < T_0$. At these dates, we have $v_t^- < 1/P$ and pessimists would strictly prefer to spend all of their money holdings for $t \in [0, T_0)$.

Although the path in (A.43) ensures pessimists are indifferent between holding money and spending it when they have an urge to consume, the fact that optimists always want to spend any money they have implies that the price level is bounded below given their spending, i.e.,

$$P_t > \frac{\lambda M_t^+}{y + D} \quad (\text{A.44})$$

Although we know that the money holdings $M_t^+ \rightarrow 0$ as $t \rightarrow T$, so that this condition will hold close to the terminal date, we cannot be sure that it also holds earlier. But I now argue that

$$P_t = \max \left\{ \frac{\lambda M_t^+}{y + D}, \frac{(\lambda + 2\alpha) e^{-\rho(t-T)} P_T}{\lambda + 2\alpha + \rho [1 - e^{-(\lambda+2\alpha)(t-T)}]} \right\} \text{ for } t > T_0 \quad (\text{A.45})$$

is an equilibrium. In particular, at any date in which $\frac{\lambda M_t^+}{y+D} > \frac{(\lambda+2\alpha)e^{-\rho(T_0-T)}P_T}{\lambda+2\alpha+\rho[1-e^{-(\lambda+2\alpha)(T_0-T)}]}$ and the continuation path weakly exceeds the path that leaves pessimists indifferent, it will be optimal for pessimists to hold on to their cash, since the return to holding the asset is higher while the utility value from spending is lower. But in that case, only optimists spend, and the equilibrium price will be given by $\frac{\lambda M_t^+}{y+D}$. In short, the price level in this case is given by

$$P_t = \begin{cases} \frac{\lambda(1+\mu)\overline{M}}{y+D} & \text{if } t \leq T_0 \\ \max \left\{ \frac{\lambda M_t^+}{y+D}, \frac{(\lambda+2\alpha)e^{-\rho(t-T)}(1+\mu)\widehat{P}}{\lambda+2\alpha+\rho[1-e^{-(\lambda+2\alpha)(t-T)}]} \right\} & \text{if } t \in (T_0, T) \\ (1 + \mu) \widehat{P} & \text{if } t \geq T \end{cases}$$

The final step is to confirm that $W_t^+ > p_t$ for $t < T$ and show how to solve for T . The argument here is analogous to case (b.1) above. First, we know that T is finite. This means the money holdings of optimists reaches 0 in finite

time. But then the path between date 0 and when optimists first run down their cash satisfies all of these conditions. This completes the proof. ■

Proof of Proposition 5: From the proof of Proposition 3, the new steady state after a liquidity injection is $p = (1 + \mu)\widehat{p}$, $P = (1 + \mu)\widehat{P}$ and $\overline{W}^+ = (1 + \mu)\widehat{\overline{W}}^+$. A liquidity injection that favors pessimists at date 0 leaves optimists with an initial wealth of $W_0^+ < (1 + \mu)\widehat{M}^+ + p_t$. I consider three cases in turn, depending on how the initial wealth of optimists W_0^+ compares with the new steady state asset price $(1 + \mu)p$ and on Δ^+ .

Case (a): $W_0^+ \geq (1 + \mu)\widehat{p}$

I show there exists an equilibrium in which $p_t = (1 + \mu)\widehat{p}$ and $P_t = (1 + \mu)\widehat{P}$ for all t . If $W_0^+ \geq (1 + \mu)\widehat{p}$, the same logic as in case (a) in the proof of Proposition 4 applies. The only difference is that now we have

$$\begin{aligned} W_0^+ &< (1 + \mu)M_0^+ + p_t \\ &= (1 + \mu)M_0^+ + (1 + \mu)\widehat{p} = \overline{W}_0^+ \end{aligned} \tag{A.46}$$

The wealth of optimists increases over time rather than decreases. The time it takes the wealth of optimists W_t^+ to reach the new steady state price $(1 + \mu)\widehat{p}$ is infinite.

Case (b.1): $W_0^+ < (1 + \mu)\widehat{p}$ and $\Delta^+ \leq \Delta^{**}$

I show there exists an equilibrium in which there is a finite date $T < \infty$ where $W_t^+ < p_t < (1 + \mu)\widehat{p}$ when $t < T$ and $W_t^+ = p_t = (1 + \mu)\widehat{p}$ for $t \geq T$. The price level $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for $t \in [0, T]$. If $\Delta^+ \in (\Delta^*, \Delta^{**}]$, the price level $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D} (1 + \mu)\widehat{P}$ for $t > T$ as well.

Since $p_t > W_t^+$ for all $t < T$, pessimists must own at least some of the asset given optimists can't afford to buy all of it. Since optimists expect a higher return than pessimists, if pessimists are willing to hold the asset, optimists would as well. Hence, before date T , pessimists must be indifferent between the asset and money and optimists must prefer the asset. Otherwise, nobody would hold money. Using the same argument as in the proof of Proposition 4, pessimists are indifferent between the asset and money only if they expect the return on the asset to equal ρ . This implies that for $t \in [0, T]$, the asset price must satisfy the condition

$$P_t (D - \alpha\Delta^-) + \dot{p}_t = \rho p_t \tag{A.47}$$

Solving this differential equation forward as in the proof of Proposition 4, we have

$$p_t = \int_t^\infty e^{-\rho(s-t)} P_s (D - \alpha\Delta^-) ds + K e^{\rho t} \tag{A.48}$$

where K is determined by the boundary condition that the price p_t at $t = T$ must equal $(1 + \mu)\widehat{p}$, i.e.,

$$K = e^{-\rho T} (1 + \mu)\widehat{p} - \int_T^\infty e^{-\rho s} P_s (D - \alpha\Delta^+) ds \tag{A.49}$$

If the price level is $P_s = (1 + \mu) \widehat{P}$ for all s as conjectured, the constant is given by

$$K = \int_T^\infty e^{-\rho s} P_s (D - \alpha \Delta^+) ds = (1 + \mu) e^{-\rho T} \left[\widehat{p} - \frac{\widehat{P} (D - \alpha \Delta^+)}{\rho} \right] < 0$$

Since pessimists expect to earn ρ , optimists expect to earn a return $\frac{P_t(D + \alpha \Delta^+) + \dot{p}_t}{p_t}$ that exceeds ρ .

The law of motion for the wealth of optimists is given by

$$\dot{W}_t^+ = P_t \left(\frac{y}{2} + D \frac{W_t^+}{p_t} \right) + \alpha (\overline{M} + p_t - 2W_t^+) + \frac{W_t^+}{p_t} \dot{p}_t \quad (\text{A.50})$$

The coefficient on W_t^+ is now a function of p_t^+ , in contrast to the proof of Proposition 4 for an injection that favors optimists. Given a path for p_t and P_t , we can solve for this differential equation.

Turning to the price level P_t , we know that from date T on, only pessimists hold money. From Proposition 1, we know that as long as $\Delta^+ < \Delta^{**}$, pessimists would want to spend all of their money holdings when they have an urge to consume. This implies $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for all $t \geq T$. Before date T , the fact that $W_t^+ < p_t$ implies pessimists must be indifferent between the money and the asset while optimists, who expect a higher return, strictly prefer the asset. Hence, pessimists hold all money. Since they are indifferent between money and the asset, they would want to spend all of their money holdings if they have an urge to consume, since they could have earned a positive return by holding the asset. If the agents who hold cash spend it all when they have an urge to consume, the market clearing price would be given by $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ at these dates as well.

Finally, I need to confirm that $W_t^+ < p_t$ for $t < T$ and show how to solve for T . The argument is analogous to the proof in Proposition 4. First, I argue that the date T in which $p_t < W_t^+$ is finite. For suppose $T = \infty$. In that case, we know from (A.35) that $K = 0$. Given the price level P_t , the price of the asset in (A.34) would asymptotically converge to $(1 + \mu) \frac{\widehat{P}(D - \alpha \Delta^+)}{\rho} < (1 + \mu) \widehat{p}$. But this contradicts Proposition 1 which establishes that there is a unique steady state equilibrium in which the asset price is $(1 + \mu) \widehat{p}$. Hence, there exists a finite date T at which $p_T = W_T^+$. Let T' denote the first date at which have enough wealth to buy the entire asset, i.e., $T' = \inf \{t : W_t^+ - p_t = 0\}$. By construction, $T' \leq T < \infty$. Hence, there exists a finite date T' such that $W_t^+ < p_t$ for $t < T'$ as desired. That is, $T = T'$, so T is the first date at which money holdings of optimists are equal to 0. To solve for T , we choose the value of T that ensures W_t^+ at $t = T$ is equal to $(1 + \mu) \widehat{p}$, analogously to the proof of Proposition 4.

Case (b.2): $W_0^+ < (1 + \mu) \widehat{p}$ and $\Delta^+ > \Delta^{**}$

Once again, I construct an equilibrium in which there is some finite date $T < \infty$ where $W_t^+ > p_t > (1 + \mu) \widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1 + \mu) \widehat{p}$ for $t \geq T$. The difference from case (b.1) above is the price level P_t .

The path for the asset price p_t is once again given by (A.48), and the path for the total wealth of optimists W_t^+ is governed by the differential equation in (A.50).

I now turn to the equilibrium price level P_t . Before date T , the fact that $W_t^+ < p_t$ implies pessimists must be indifferent between the money and the asset while optimists, who expect a higher return, strictly prefer the asset. Hence, pessimists hold all money. Since they are indifferent between money and the asset, they would want to spend all of their money holdings if they have an urge to consume, since they could have earned a positive return by holding the asset. If the agents who hold cash spend it all when they have an urge to consume, the market clearing price would be given by $P_t = \frac{\lambda(1+\mu)\bar{M}}{y+D}$. From Proposition 1, we know that when $\Delta^+ > \Delta^{**}$, the original steady-state equilibrium price level $\hat{P} < \frac{\lambda\bar{M}}{y+D}$. Hence, $P_t > (1+\mu)\hat{P}$ when $t < T$.

After date T , the price level must converge towards its new steady state level $(1+\mu)\hat{P}$. Throughout this transition, when the price $P_t < \frac{\lambda(1+\mu)\bar{M}}{y+D}$, pessimists must be indifferent between spending and holding on their cash. This is because optimists do not hold cash at all. For pessimists to be indifferent, the value of marginal wealth for them, v_t^- , must equal $1/P_t$. Define $\eta_t^+ = P_t v_t^+$ and $\eta_t^- = P_t v_t^-$. Then equilibrium requires that $\eta_t^- = 1$ for all $t > T$. Using the laws of motion for v_t , we have

$$\begin{aligned}\rho\eta_t^+ &= \frac{P_t(D + \alpha\Delta^+)}{(1+\mu)\hat{p}}\eta_t^+ + \alpha(\eta_t^- - \eta_t^+) + \dot{\eta}_t^+ \\ \rho\eta_t^- &= \lambda(1 - \eta_t^-) + \alpha(\eta_t^+ - \eta_t^-)\end{aligned}$$

The second equation implies

$$\eta_t^- = \frac{\lambda + \alpha\eta_t^+}{\rho + \alpha + \lambda}$$

We can substitute this into the first equation to get a single differential equation for η_t^+ , i.e.,

$$\rho\eta_t^+ = \frac{P_t(D + \alpha\Delta^+)}{(1+\mu)\hat{p}}\eta_t^+ + \alpha\left(\frac{\lambda + \alpha\eta_t^+}{\rho + \alpha + \lambda} - \eta_t^+\right) + \dot{\eta}_t^+ \quad (\text{A.51})$$

The boundary condition for η_t^+ is that there exists some date T_1 such that $\eta_t^+ = 1 + \rho/\alpha$ at $t = T_1$. The equation for η_t^+ depends on the path of P_t . At the same time, we have the equilibrium condition $\eta_t^- = 1$, or

$$\frac{\lambda + \alpha\eta_t^+}{\rho + \alpha + \lambda} = 1 \quad (\text{A.52})$$

Combining (A.51) and (A.52) yields an integral equation for P_t that is analogous to the one in the proof of Proposition 4. To pin down T_1 , we have the boundary condition that $P_T = \frac{\lambda(1+\mu)\bar{M}}{y+D}$. This equation characterizes the path of the price level from date T on. Along this path, pessimists are indifferent and would be willing to spend the amount necessary needed to ensure the price level P_t that corresponds to η_t^+ . The equilibrium is consistent with all the characterizations in the statement of the proposition. ■

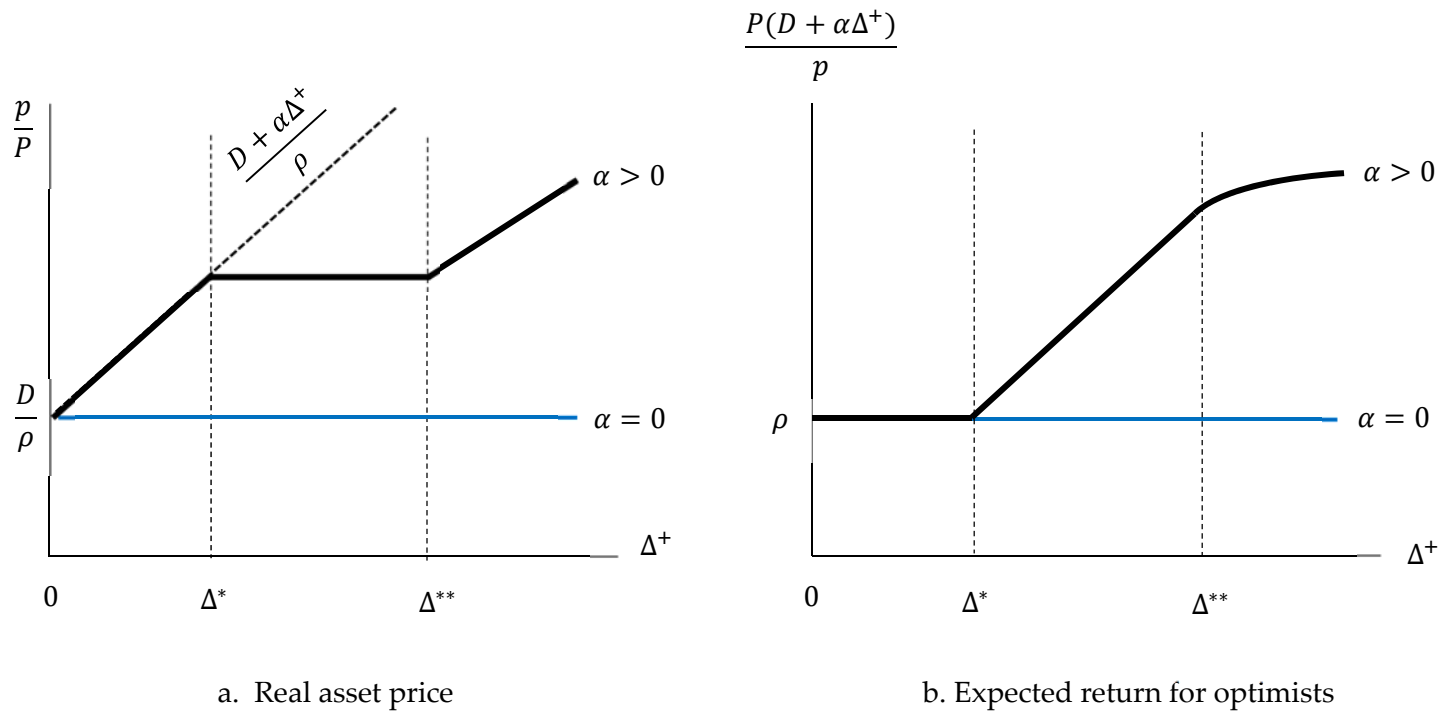


Figure 1: Equilibrium real asset price and expected return on asset as a function of the degree of optimism Δ^+

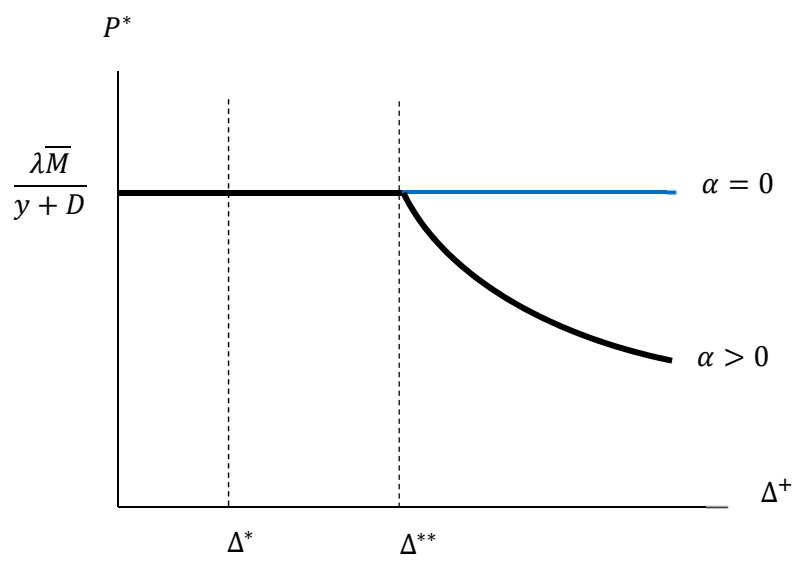


Figure 2: Steady state price level P as a function of the degree of optimism Δ^+

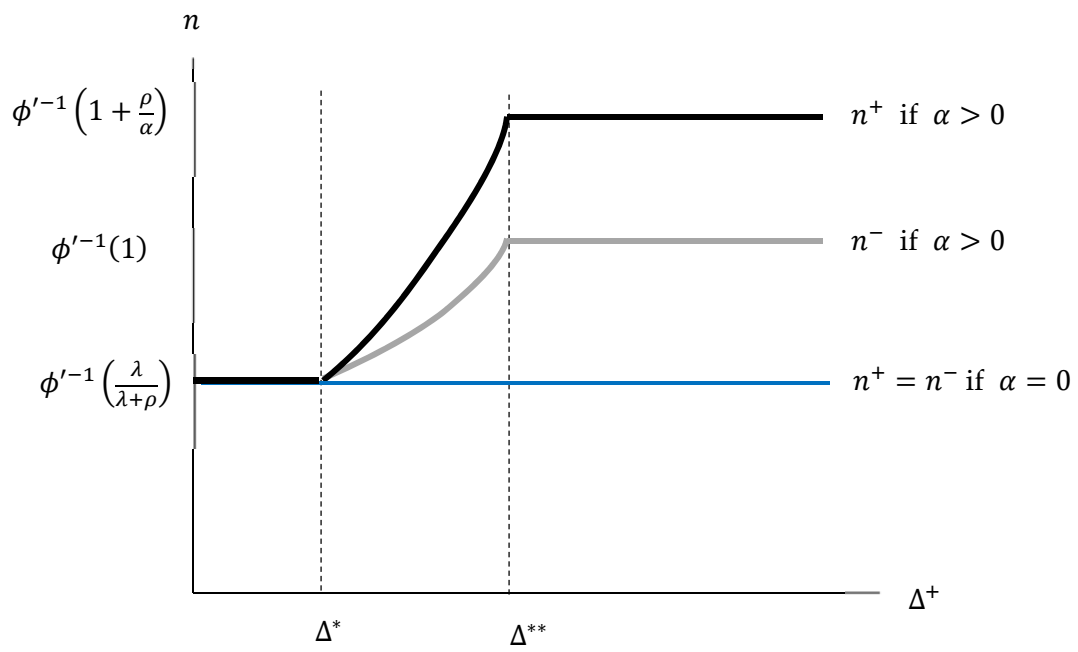


Figure 3: Production choices of optimists and pessimists as a function of the degree of optimism Δ^+

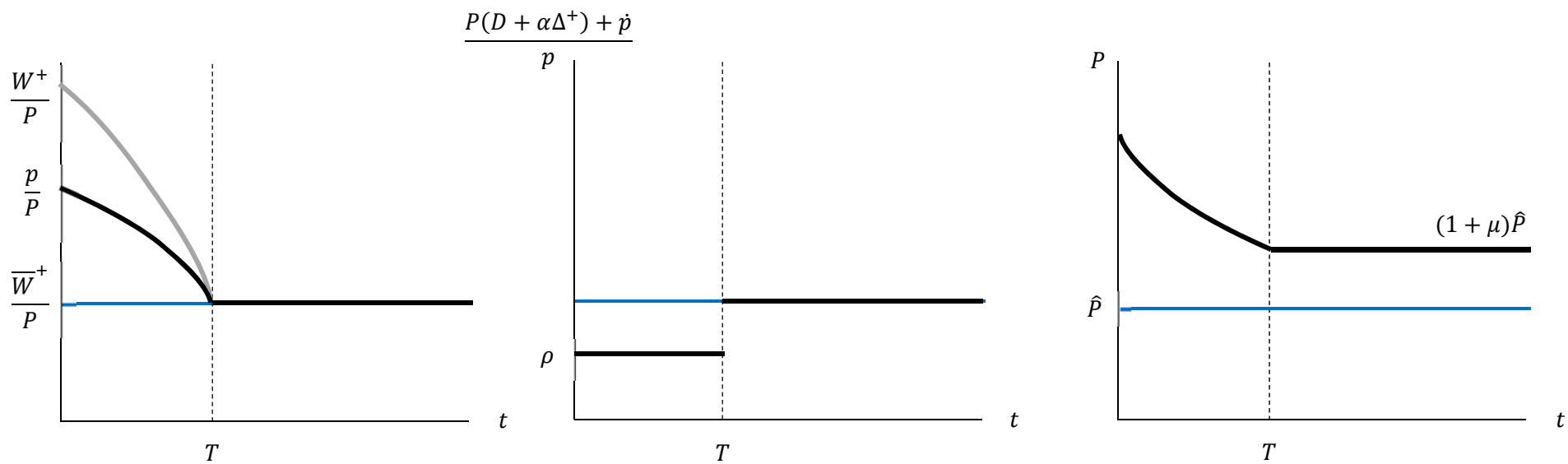


Figure 4: The effect of a liquidity injection that favors optimists when $\Delta^+ > \Delta^{**}$ (blue line = original steady state)

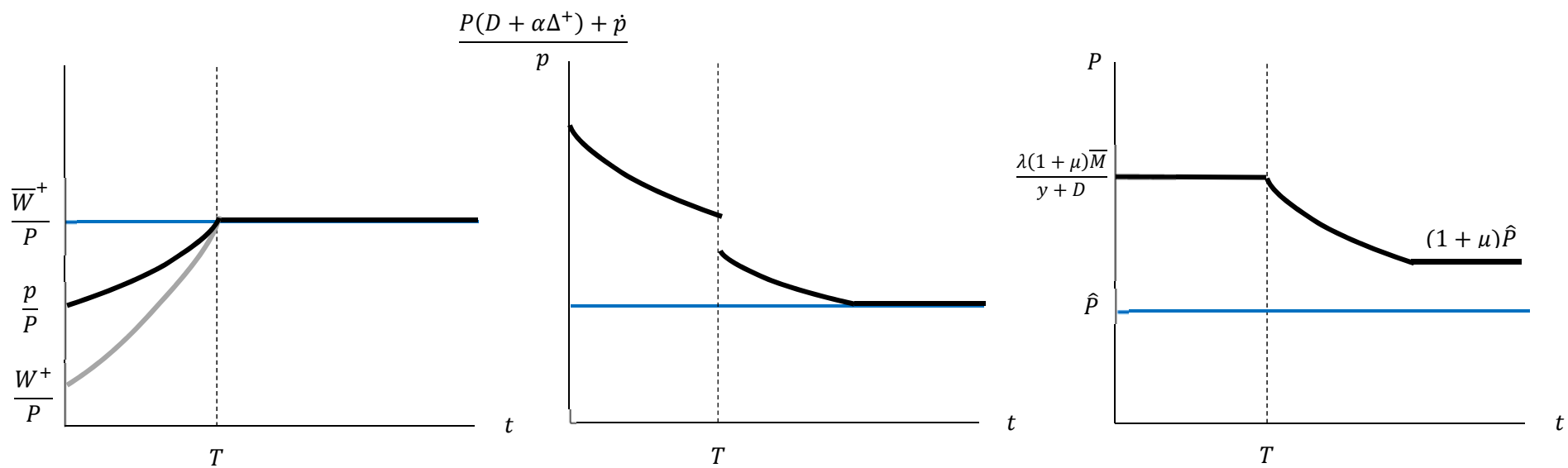


Figure 5: The effect of a liquidity injection that favors pessimists when $\Delta^+ > \Delta^{**}$ (blue line = original steady state)

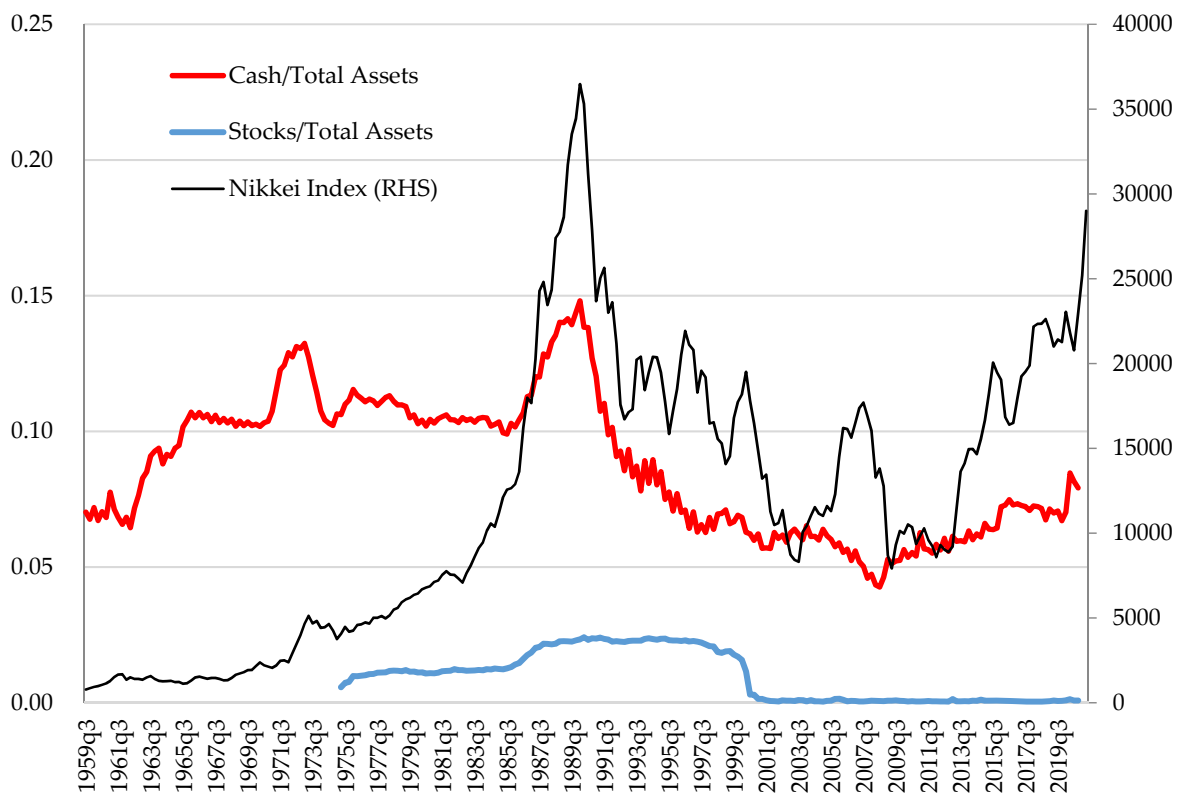


Figure 6: Asset Holdings of Large Non-Financial Corporations in Japan

The red and blue line denote the ratio of cash and deposits to total assets and the ratio of stocks to total assets of large Japanese corporations (with at least 1 billion yen in capital). The black line denotes the quarterly average of the Nikkei 225 Index. Source: Financial Statements Statistics of Corporations by Industry, Japanese Ministry of Finance for ratios and Federal Reserve Economic Data for the Nikkei 225 Index.

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