

Monetary Policy, Segmentation, and the Term Structure*

Rohan Kekre[†] Moritz Lenel[‡] Federico Mainardi[§]

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PRELIMINARY

Abstract

We develop a segmented markets model which rationalizes the effects of monetary policy on the term structure of interest rates. As in the preferred habitat tradition, habitat investors and arbitrageurs trade bonds of various maturities. As in the intermediary asset pricing tradition, the wealth of arbitrageurs is a state variable which affects equilibrium term premia. When arbitrageurs' portfolio features positive duration, an unexpected fall in the short rate revalues wealth in their favor and compresses term premia. A calibration to the U.S. economy accounts for the effects of monetary shocks along the yield curve while simultaneously rationalizing classic evidence on bond return predictability which does not condition on identified shocks.

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[†]Chicago Booth and NBER. Email: rohan.kekre@chicagobooth.edu.

[‡]Princeton. Email: lenel@princeton.edu.

[§]Chicago Booth. Email: fmainard@chicagobooth.edu.

1 Introduction

The effect of a change in short rates on long rates is central to the monetary transmission mechanism. It determines how monetary policy affects mortgage rates, corporate borrowing rates, and other determinants of aggregate demand. Long rates reflect the expected path of short rates plus term premia. There is accumulating empirical evidence that expansionary monetary policy lowers long rates by more than can be accounted for by the change in the expected path of short rates.¹ This implies that expansionary monetary policy operates in part by lowering term premia.

This evidence poses a challenge to existing models of monetary transmission and the term structure. Representative agent models typically imply that monetary policy shocks have negligible effects on the price and quantity of interest rate and inflation risks. Market segmentation opens the door for transitory shocks to have more substantial effects on term premia if they have relatively large effects on the subset of agents pricing long-term bonds. However, existing models of this kind, most notably those in the preferred habitat tradition, counterfactually imply that a monetary easing *raises* term premia, as the associated decline in long yields causes habitat investors to borrow more long-term and thus expose arbitrageurs to more risk.

In this paper, we propose a model which rationalizes the effects of monetary policy shocks on the term structure of interest rates. We build on the preferred habitat tradition by studying an environment in which habitat investors and arbitrageurs trade bonds of various maturities. We integrate this with the intermediary asset pricing tradition by studying an environment in which arbitrageur wealth is an endogenous state variable relevant for equilibrium term premia. When arbitrageurs' portfolio features positive duration, an unexpected fall in the short rate revalues wealth in their favor and lowers term premia. Quantitatively, this mechanism jointly rationalizes the financial sector's exposure to interest rate risk and the responses of the yield curve to monetary shocks in the data. While monetary shocks induce a negative relationship between the slope of the yield curve and expected excess returns on long-term bonds, shocks to the demand of habitat investors in the model induce a positive relationship. Because demand shocks are sufficiently volatile relative to monetary shocks, the model thus also matches the classic evidence of Fama and Bliss (1987) and Campbell and Shiller (1991) on bond return predictability which does not condition on identified shocks.

The model integrates elements of the preferred habitat and intermediary asset pricing

¹See, e.g., Cochrane and Piazzesi (2002), Gertler and Karadi (2015), and Hanson and Stein (2015).

ing traditions. As in existing preferred habitat models, time is continuous. A continuum of preferred habitat investors elastically demand bonds of each maturity. Overlapping generations of arbitrageurs trade with the central bank at the short rate and with habitat investors at each maturity. There are two risk factors: the short rate and the demand of habitat investors across maturities. Unlike existing preferred habitat models, arbitrageurs have log (rather than CARA) preferences, and are characterized by perpetual youth (rather than living only instantaneously). These two changes imply that the wealth of arbitrageurs is an endogenous state variable relevant for risk pricing, as in the intermediary asset pricing tradition.

We first study a simplified version of this environment which allows us to analytically characterize each of our main results. In the simplified environment, time is discrete and only one- and two-period bonds are traded. We first show that, when arbitrageurs die after one period and thus their endowment is exogenous, we recover the existing result from preferred habitat models that an unexpected fall in the short rate raises the term premium on two-period bonds: the associated decline in the two-period yield causes habitat investors to borrow more at this maturity and thus exposes arbitrageurs to more interest rate risk. We next allow arbitrageurs to live for more than one period, in which case the revaluation of arbitrageurs' wealth also determines the response of the term premium to a short rate shock. In particular, if arbitrageurs' portfolio features positive duration — in this simple setting, if they are long two-period bonds — an unexpected fall in the short rate raises their wealth. If this force is sufficiently strong relative to the demand elasticity of habitat investors, the term premium falls. We finally characterize the Fama and Bliss (1987) and Campbell and Shiller (1991) coefficients which would be obtained if the model was the true data-generating process. We show that these coefficients must be positive and negative, respectively, if the volatility of demand shocks is high enough relative to monetary shocks. This is because demand shocks imply that all changes in the slope of the yield curve are due to changes in term premia, unlike monetary shocks.

We then numerically study and quantify these mechanisms in the full, continuous-time model. When arbitrageur wealth is endogenous in the ways described above, bond prices no longer take an exponentially affine structure, and the model does not admit a closed form solution. We can nonetheless describe the equilibrium in terms of a system of four partial differential equations: equilibrium in the bond market implied by arbitrageurs' optimization and market clearing; the endogenous evolution of

arbitrageur wealth; and the exogenous evolutions of the short rate and habitat demand. We solve this system numerically using finite difference methods and collocation.

We confront our model with evidence on the effects of monetary policy shocks. We focus on the response of intermediary wealth and the yield curve around Federal Open Market Committee (FOMC) announcements. We isolate monetary shocks from information shocks by focusing on the subset of announcements during which bond yields and the S&P 500 move in opposite directions, building on Jarocinski and Karadi (2020). We further isolate monetary policy shocks from other shocks by studying tight intraday windows around FOMC announcements. Our baseline estimate is that a fall in the one-year real yield by $1pp$ on these announcement days raises the equity prices of primary dealers by $8.8pp$. This is consistent with arbitrageurs' portfolio having positive duration. Our baseline estimates further imply that a fall in the one-year real yield by $1pp$ lowers the 20-year real forward rate by $0.39pp$; more generally, the shock lowers long-dated real forward rates by statistically and economically significant amounts. This is consistent with a decrease in term premia, as any reasonable estimate of nominal rigidity implies that the expected real interest rate is essentially unchanged several years after a monetary shock.

Our core quantitative result is that the model can jointly match these responses of intermediary wealth and the yield curve to monetary shocks. We simulate a monetary easing as a negative innovation to the real short rate. Such a shock lowers forward rates at all maturities by more than the expectations hypothesis would imply, owing to the positive revaluation of arbitrageur wealth. The overreaction of the forward rate is reversed in a counterfactual economy with exogenous arbitrageur wealth, consistent with our analytical results. Quantitatively, a shock which lowers the one-year real yield by $1pp$ generates effects on arbitrageur wealth and real forward rates which lie within the confidence intervals estimated in the data. In the counterfactual economy with exogenous arbitrageur wealth, the same shock undershoots the observed response of long-dated forward rates.

At the same time, the model also rationalizes the bond return predictability evidence of Fama and Bliss (1987) and Campbell and Shiller (1991) which does not condition on identified shocks. Consistent with the analytical results, the effects of a shock to the short rate are in tension with this evidence because they imply that when the yield curve steepens, future excess returns on long-term bonds are low. An increase in the borrowing of habitat investors, however, implies that a steep yield curve

is followed by high excess returns on long-term bonds. Our final quantitative result is that demand shocks calibrated to match moments on the volatility in yields allow us to match empirical estimates of bond return predictability using model-generated data.

Related literature Our paper builds on preferred habitat models of the term structure of interest rates. The preferred habitat view was proposed by Culbertson (1957) and Modigliani and Sutch (1966) and formalized by the seminal work of Vayanos and Vila (2021). A growing theoretical literature has used this framework to study the implications for corporate finance (Greenwood, Hanson, and Stein (2010)), government debt policy (Guibaud, Nosbusch, and Vayanos (2013)), exchange rates (Gourinchas, Ray, and Vayanos (2021) and Greenwood, Hanson, Stein, and Sunderam (2020)), and the real economy (Ray (2021)). An enormous empirical literature has drawn on this framework to inform analyses of unconventional monetary policies. In the existing framework, the effects of the key driving force (the short rate) are counterfactual. We enrich this framework to match evidence on the response to such shocks by allowing the wealth of arbitrageurs to be an endogenous state variable relevant for risk pricing.

In doing so, our paper builds on the literature linking changes in intermediary net worth with asset prices. This is at the core of the intermediary asset pricing tradition in finance (He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014)) as well as the financial accelerator tradition in macroeconomics (Bernanke, Gertler, and Gilchrist (1999)). Our contribution is to embed this insight into a leading model of the term structure of interest rates.² Our focus on intermediaries' risk-bearing capacity contrasts with an alternative explanation for the term premium effects of monetary shocks focused on habitat investors' demand (Hanson and Stein (2015), Hanson, Lucca, and Wright (2021)). These papers propose models in which habitat investors save more (or borrow less) at long maturities after a monetary easing, perhaps due to "reaching for yield" behavior. Our paper instead maintains the standard assumption that investors borrow more when yields fall, and accounts for the term premium effects of monetary policy via a change in the endogenous price of risk.

In this respect, our paper is part of a broader agenda studying links between macroeconomic shocks and the price of risk in heterogeneous agent models. Alvarez, Atkeson,

²In their empirical analysis of government bond supply and excess returns, Greenwood and Vayanos (2014) anticipate that if arbitrageurs' coefficient of absolute risk aversion is a declining function of their wealth, changes in their wealth will have effects on term premia. Our paper formalizes this idea and traces out its theoretical and quantitative implications.

and Kehoe (2002, 2009) study monetary economies with segmented financial markets in which monetary shocks change the price of risk. Kekre and Lenel (2021a,b) build on these insights in conventional New Keynesian models enriched with agents having heterogeneous risk-bearing capacity. They find that a monetary easing lowers the risk premium on capital by redistributing wealth to agents who wish to invest more of their marginal wealth in capital. The present paper shows that a similar mechanism is at work in the bond market in a preferred habitat environment.³ While we do not extend the model to feature a New Keynesian production block, we expect that the effects of policy shocks on the term premium would imply that monetary policy is more potent in stimulating the real economy to the extent that aggregate demand is rising in the amount habitat investors borrow long-term.⁴

Outline In section 2 we outline the model environment. In section 3 we characterize our main results analytically in a simple version of this environment. In section 4 we estimate the effects of policy shocks on the yield curve and intermediary wealth in the data. In section 5 we calibrate the full model, study its impulse responses, and demonstrate its ability to rationalize the data. Finally, in section 6 we conclude.

2 Model

In this section we outline our model of the term structure of interest rates. The model integrates features of the preferred habitat and intermediary asset pricing traditions.

Timing and assets Time t is continuous. At time t there is a continuum of zero coupon bonds with maturities $\tau \in (0, \infty)$. A bond trading at t with maturity τ pays 1 unit of the numeraire at $t + \tau$ and its price is $P_t^{(\tau)}$. The instantaneous return on

³We conjecture that introducing heterogeneity in risk aversion into representative agent models in which aggregate comovements deliver a positive term premium, as in Piazzesi and Schneider (2007), Rudebusch and Swanson (2012), and Campbell, Pflueger, and Viceira (2020), would lead to similar results. With a positive price on term risk, relatively risk tolerant agents would endogenously be more exposed to it, implying a redistribution of wealth which affects the price of risk on impact of policy shocks. One important difference in the preferred habitat environment is that it does not rely on aggregate comovements generating a positive term premium, and thus implies that this mechanism remains operative even if, as in recent years, aggregate comovements may have flipped signs.

⁴See Caballero and Simsek (2020) for recent work linking risk premia, aggregate demand, and output in the New Keynesian environment. See Caramp and Silva (2020) for recent work linking term premia and aggregate demand in such an environment in particular.

holding such a bond is $dP_t^{(\tau)}/P_t^{(\tau)}$. The yield of the bond is given by

$$y_t^{(\tau)} = -\frac{\log\left(P_t^{(\tau)}\right)}{\tau}$$

and the short rate r_t is limit of the yield as τ goes to zero.

Decision problems There are two types of agents: habitat investors and arbitrageurs.

Habitat investors are indexed by τ and, at time t , are uniformly distributed over $\tau \in (t, \infty)$. An investor with habitat τ at time t holds a position

$$Z_t^{(\tau)} = -\alpha(\tau) \log\left(P_t^{(\tau)}\right) - \theta_t(\tau) \quad (1)$$

in bonds with maturity τ (and zero bonds of all other maturities), where a positive position implies that the investor is saving in this security. The parameter $\alpha(\tau) \geq 0$ controls the elasticity of demand to price. $\theta_t(\tau)$ controls the level of habitat demand and is given by

$$\theta_t(\tau) = \theta_0(\tau) + \theta_1(\tau)\beta_t, \quad (2)$$

where β_t is a demand factor, the parameter $\theta_1(\tau)$ controls the loading of demand on that factor, and the parameter $\theta_0(\tau)$ controls the time-invariant level of demand.

Arbitrageurs can trade at all maturities as well as at the short rate r_t with the central bank. Arbitrageurs are born and die at rate ξ and have separable log preferences over consumption upon death.⁵ Here (and only here) we depart from typical preferred habitat models which assume arbitrageurs are alive instantaneously and have CARA preferences over consumption. Using lower case to denote the endowment and choices of an individual arbitrageur with wealth w_t , this arbitrageur chooses its sequence of financial portfolios to maximize

$$v_t(w_t) = \max_{\{\{x_{t+s}^{(\tau)}\}_{\tau \in (0, \infty)}\}} E_t \int_0^\infty \exp(-\xi s) \log w_{t+s} ds \quad (3)$$

⁵Preferences with a constant coefficient of relative risk aversion are key for our results. A coefficient of one (log utility) is convenient because it implies myopic portfolio choice, but the precise value is not essential for our results. Finally, we assume that arbitrageurs consume only upon death for parsimony. Our results generalize easily to the case in which they intertemporally smooth consumption, since they will optimally consume at a constant rate out of wealth given their unitary elasticity of substitution.

subject to the budget constraint

$$dw_t = r_t w_t dt + \int_0^\infty x_t^{(\tau)} \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt \right) d\tau. \quad (4)$$

where $x_t^{(\tau)}$ denotes its position in bonds with maturity τ . Using upper case to denote aggregates across arbitrageurs, aggregate arbitrageur wealth thus follows

$$dW_t = W_t r_t dt + \int_0^\infty X_t^{(\tau)} \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt \right) d\tau + \xi (\bar{W} - W_t) dt, \quad (5)$$

where \bar{W} is the exogenous endowment of newborn arbitrageurs (perhaps zero). When $\xi \rightarrow \infty$, this converges to the constant endowment process in Vayanos and Vila (2021). For finite ξ , W_t will be an endogenous state variable of the model as in intermediary asset pricing models such as He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014).

Driving forces There are two driving forces in this economy: the short rate set by the central bank r_t , and the demand factor β_t . These follow the exogenous processes

$$dr_t = \kappa_r (\bar{r} - r_t) dt + \sigma_r dB_{r,t}, \quad (6)$$

$$d\beta_t = -\kappa_\beta \beta_t dt + \sigma_\beta dB_{\beta,t}, \quad (7)$$

where the Brownian motions $dB_{r,t}$ and $dB_{\beta,t}$ are independent and $\kappa_r, \kappa_\beta > 0$. We assume independent shocks and a single-factor demand structure since our calibration will, for simplicity, focus on this case. We expect our main insights would generalize to correlated shocks and multiple demand factors and leave this for future work.

Market clearing and equilibrium The bond market must clear for each maturity τ

$$Z_t^{(\tau)} + X_t^{(\tau)} = 0 \quad (8)$$

at each point in time t . The definition of an equilibrium is standard.

Interpretation We interpret the model in fully real terms. We do this for two reasons. First, focusing on real bonds allows us to study our mechanism focused on

interest rate risk in a more parsimonious setting which can abstract away from inflation risk. Second, focusing on the real term structure allows us to uncover the effects of monetary shocks on term premia purged from any effects on long-run inflation. In particular, monetary policy shocks may contain news about the long-run inflation target, which in turn will affect long-dated nominal forwards (Gurkaynak, Sack, and Swanson (2005b)). Long-dated real forwards are immune from this issue, and moreover monetary neutrality in the long run implies that expected real interest rates in the distant future should be unaffected by monetary shocks. In both model and data, this allows a tight analysis of the effects of a monetary shock on term premia by studying the response of real forwards on impact of the shock.

There is a related nuance in how we should understand monetary shocks in the model. Conventional macroeconomic models imply that the real short rate largely reflects the natural rate of interest, which in turn reflects productivity, demographics, and other driving forces which are independent of monetary policy (and can be expected to have a more persistent effect on the real interest rate than will monetary shocks). With this in mind, our approach to simulating a monetary shock is to study an unexpected, one-time shock to the real short rate with mean reversion κ_m which need not equal κ_r (in particular, most natural is that $\kappa_m > \kappa_r$). Since the mean reversion of the shock is only important for quantitative purposes, we will use the terms “monetary shock” and “short rate shock” interchangeably until we quantify the model.

3 Analytical insights

We now study a simplified version of the model which allows us to analytically characterize our main results. When arbitrageur wealth is endogenous and their portfolio features positive duration, an unexpected fall in the short rate revalues wealth in their favor and compresses term premia. While this induces a negative relationship between the slope of the yield curve and expected excess bond returns, habitat demand shocks induce a positive relationship and thus still allow the model to rationalize classic evidence on bond return predictability which does not condition on identified shocks.

3.1 Simplified environment

In this section we assume time is discrete and only two bonds are traded: maturities one and two periods. Together, this environment captures the essential forces at play

in our full model with much simpler mathematics.

We now spell out the details. Arbitrageurs trade in one-period bonds with the central bank at price $\exp(-r_t)$ and with habitat investors in two-period bonds at price P_t , where we now dispense with the notation for maturity τ since it is unambiguous. Habitat investors hold a position

$$Z_t = -\alpha \log P_t - \theta_t$$

in two-period bonds with $\alpha \geq 0$, as in (1). An arbitrageur with wealth w_t chooses its position in two-period bonds x_t to maximize

$$\max_{\{x_{t+s}\}} E_t \sum_{s=1}^{\infty} \exp(-\xi s) \log w_{t+s}$$

subject to the evolution of wealth

$$w_{t+1} = w_t \exp(r_t) + x_t \left(\frac{\exp(-r_{t+1})}{P_t} - \exp(r_t) \right),$$

the discrete time counterparts to (3)-(4). Aggregate arbitrageur wealth follows

$$W_{t+1} = \exp(-\xi) \left[W_t \exp(r_t) + X_t \left(\frac{\exp(-r_{t+1})}{P_t} - \exp(r_t) \right) \right] + (1 - \exp(-\xi)) \bar{W},$$

the discrete time counterpart to (5). The short rate and habitat demand follow the AR(1) processes

$$\begin{aligned} r_{t+1} - \bar{r} &= (1 - \kappa_r) (r_t - \bar{r}) + \sigma_r \epsilon_{r,t+1}, \\ \theta_{t+1} - \bar{\theta} &= (1 - \kappa_\theta) (\theta_t - \bar{\theta}) + \sigma_\theta \epsilon_{\theta,t+1}, \end{aligned}$$

where $\epsilon_{r,t+1}$ and $\epsilon_{\theta,t+1}$ are independent standard Normal innovations. $\kappa_r \in (0, 1)$ and $\kappa_\theta \in (0, 1)$ can be interpreted as the degree of mean reversion in these driving forces, as in (6) and (7). We dispense with β_t in this section because it is isomorphic to θ_t since there is only one long-term bond. Finally, bond market clearing requires

$$X_t + Z_t = 0,$$

as in (8).

3.2 Equilibrium

Following standard arguments, each arbitrageur's value function is characterized by

$$v_t(w_t) = \log w_t + v_t,$$

where v_t is common to arbitrageurs and invariant to their individual level of wealth. Arbitrageurs' optimality condition with respect to x_t implies

$$E_t \left(\exp(r_t) + \frac{x_t}{w_t} \left(\frac{\exp(-r_{t+1})}{P_t} - \exp(r_t) \right) \right)^{-1} \left[\frac{\exp(-r_{t+1})}{P_t} - \exp(r_t) \right] = 0, \quad (9)$$

clarifying that their portfolio share $\frac{x_t}{w_t}$ is also invariant to wealth. Defining the log one-period holding return on a two-period bond

$$r_{t+1}^{(2)} \equiv -r_{t+1} - \log P_t$$

and making use of

$$\frac{x_t}{w_t} = \frac{X_t}{W_t} \quad (10)$$

by aggregation, a second-order Taylor approximation of (9) around $r_{t+1}^{(2)} = r_t$ implies

$$E_t r_{t+1}^{(2)} - r_t + \frac{1}{2} \sigma_r^2 \approx \frac{X_t}{W_t} \sigma_r^2. \quad (11)$$

This has an intuitive interpretation. Arbitrageurs require non-zero expected excess returns to compensate them for bearing interest rate risk on two-period bonds. In particular, when $X_t > 0$, arbitrageurs are long two-period bonds and thus expected excess returns on two-period bonds must be positive; the opposite is true if $X_t < 0$. The higher is arbitrageur wealth W_t , the smaller (in absolute value) expected excess returns must be, because two-period bonds are a smaller share of their wealth and arbitrageurs have CRRA preferences. In the limit $W_t \rightarrow \infty$, arbitrageurs are effectively risk neutral and thus the (local) expectations hypothesis holds.⁶

The above condition is the only approximation we use in the rest of this section; all other conditions hold exactly. Combining the above condition with market clearing

⁶The standard Jensen's inequality term $\frac{1}{2}\sigma_r^2$ implies that the expectations hypothesis does not hold. See Piazzesi (2010) for further discussion of this point.

in two-periods bonds and habitat investors' demand yields

$$E_t r_{t+1}^{(2)} - r_t + \frac{1}{2} \sigma_r^2 = \frac{1}{W_t} (\alpha \log P_t + \theta_t) \sigma_r^2. \quad (12)$$

Recall that holding period returns are given by

$$r_{t+1}^{(2)} = -r_{t+1} - \log P_t. \quad (13)$$

and the evolution of the short rate and habitat demand are given by

$$r_{t+1} - \bar{r} = (1 - \kappa_r) (r_t - \bar{r}) + \sigma_r \epsilon_{r,t+1}, \quad (14)$$

$$\theta_{t+1} - \bar{\theta} = (1 - \kappa_\theta) (\theta_t - \bar{\theta}) + \sigma_\theta \epsilon_{\theta,t+1}. \quad (15)$$

Finally, combining the evolution of aggregate arbitrageur wealth with market clearing in two-period bonds and habitat investors' demand yields

$$W_{t+1} = \exp(-\xi) \left[W_t \exp(r_t) + (\alpha \log P_t + \theta_t) (\exp(r_{t+1}^{(2)}) - \exp(r_t)) \right] + (1 - \exp(-\xi)) \bar{W}. \quad (16)$$

The dynamical system (12)-(16) is thus five equations in five unknowns $r_{t+1}^{(2)}$, r_{t+1} , θ_{t+1} , P_t , and W_{t+1} , given r_t , θ_t , and W_t . The rest of this section proceeds through four main results characterizing the equilibrium.

3.3 Short rate shock with constant arbitrageur wealth

Our first result characterizes the effects of a short rate shock $\epsilon_{r,t}$ assuming $\xi \rightarrow \infty$, and thus $W_t = \bar{W}$ for all t . We focus on the impact response of the one-period ahead forward rate

$$f_t \equiv -\log P_t - r_t \quad (17)$$

since this contains the essential economics, though it is straightforward to characterize the full impulse response of the forward rate or transformations such as bond yields. We obtain:⁷

Proposition 1. *If $\xi \rightarrow \infty$, then the response of the one-period ahead forward rate to*

⁷The proof of this result, together with the proof of all results in this section, is in appendix A.

a conventional monetary shock is

$$df_t = \frac{1 - \kappa_r - \frac{1}{W} \alpha \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} \sigma_r d\epsilon_{r,t}.$$

Thus, there is underreaction of the forward rate relative to the expected short rate

$$df_t < (1 - \kappa_r) \sigma_r d\epsilon_{r,t} = dE_t r_{t+1}$$

if $\alpha \sigma_r^2 > 0$.

Thus, when $\xi \rightarrow \infty$ we recover the effects of short rate shocks in existing preferred habitat models.⁸ Intuitively, consider an unexpected fall in the short rate. Holding fixed habitat investor borrowing, this lowers the two-period bond yield. If habitat investors are price elastic ($\alpha > 0$), this causes them to borrow more in two-period bonds. If arbitrageurs face price risk in these bonds ($\sigma_r > 0$), this raises the term premium, reflected in underreaction of the forward rate. To summarize: a fall in the short rate raises the term premium because arbitrageurs must bear more risk.

3.4 Short rate shock with endogenous arbitrageur wealth

In the remainder of this section, we focus on the case with finite ξ and thus allow arbitrageurs' endowment to evolve endogenously over time. We study impulse responses and comovements around the model's stochastic steady-state, which we denote without time subscripts.

In this setting, our first result characterizes the impact effect of a short rate shock $\epsilon_{r,t}$ on arbitrageur wealth W_t :

Proposition 2. *In response to a conventional monetary shock starting from the stochastic steady-state,*

$$d \log W_t = - \exp(-\xi) \omega \sigma_r d\epsilon_{r,t},$$

where ω is the duration of arbitrageurs' wealth and satisfies

$$\omega \propto \frac{X}{W}.$$

⁸See for instance Proposition 2 in Vayanos and Vila (2021).

Intuitively, consider an unexpected fall in the short rate. When arbitrageurs' endowment is endogenous, their wealth will be revalued upwards if and only if their portfolio has positive duration at the stochastic steady-state, which amounts in this environment to a positive position in two-period bonds X .

Given the revaluation of arbitrageur wealth, our next result revisits the impact effect of a short rate shock on the forward rate:

Proposition 3. *The response of the one-period ahead forward rate to a conventional monetary shock is*

$$df_t = \left[\frac{1 - \kappa_r - \frac{1}{W}\alpha\sigma_r^2}{1 + \frac{1}{W}\alpha\sigma_r^2} + \frac{\frac{1}{W}X\sigma_r^2}{1 + \frac{1}{W}\alpha\sigma_r^2} \exp(-\xi)\omega \right] \sigma_r d\epsilon_{r,t}.$$

Thus, assuming $\sigma_r^2 > 0$, there is overreaction of the forward rate relative to the expected short rate

$$df_t > (1 - \kappa_r)\sigma_r d\epsilon_{r,t} = dE_t r_{t+1}$$

if $\exp(-\xi)|\omega|$ is sufficiently high relative to α .

Thus, when arbitrageurs' wealth is a relevant state variable for risk pricing, we can reverse the effects of a short rate shock on the term premium. In particular, if arbitrageurs have positive duration $\omega \propto \frac{X}{W}$, we know from (12) that the steady-state term premium is positive. A rise in their wealth lowers their price of bearing interest rate risk. If this force is sufficiently strong relative to the increase in the quantity of risk they bear — characterized in Proposition 1, and controlled by α — the term premium will fall. This is reflected in overreaction of the forward rate.

3.5 Slope of yield curve and bond return predictability

We finally characterize the model-implied relationship between the slope of the yield curve and bond return predictability.

In the data, a steep yield curve predicts high excess returns on long-term bonds (Fama and Bliss (1987), Campbell and Shiller (1991)). The overreaction of the forward rate on impact of a short rate shock in Proposition 3 is in tension with this evidence: it implies that a fall in the short rate which steepens the yield curve should predict subsequently *low* excess bond returns because the term premium has fallen.

Our final result is that the model can nonetheless be consistent with the classic bond return predictability evidence in the literature if shocks to the demand of habitat

investors $\epsilon_{\theta,t}$ are sufficiently large relative to shocks to the short rate $\epsilon_{r,t}$. Intuitively, the *only* effect of demand shocks on the yield curve are via their induced effects on the term premium. Thus, a demand shock which steepens the yield curve must necessarily be raising the term premium, and vice-versa.⁹

We summarize these results by characterizing the coefficients on Fama and Bliss (1987) and Campbell and Shiller (1991) regressions estimated on model-generated data. In the present simplified environment, the Fama and Bliss (1987) regression would estimate

$$r_{t+1}^{(2)} - r_t = \alpha_{FB} + \beta_{FB} (f_t - r_t) + \epsilon_{FB,t+1},$$

while the Campbell and Shiller (1991) regression would estimate

$$r_{t+1} - y_t = \alpha_{CS} + \beta_{CS} (y_t - r_t) + \epsilon_{CS,t+1},$$

given the yield on the two-period bond

$$y_t \equiv -\frac{\log P_t}{2}. \tag{18}$$

While the expectations hypothesis predicts $\beta_{FB} = 0$ and $\beta_{CS} = 1$, the empirical evidence suggests these coefficients are positive and less than one (in fact negative). Treating our model as the data-generating process, we obtain:

Proposition 4. *If ξ is finite, the Fama-Bliss coefficient β_{FB} can be above or below zero, and the Campbell-Shiller coefficient β_{CS} can be above or below one. However, as $\sigma_\theta \rightarrow \infty$, $\beta_{FB} \rightarrow 1$ and $\beta_{CS} \rightarrow -1$.*

Thus, even if a fall in the short rate lowers the term premium, our model will still be consistent with the classic evidence on bond return predictability which does not condition on identified shocks when demand shocks are sufficiently important.

3.6 Taking stock

In this simplified environment, we have obtained four main results. First, when arbitrageurs' endowment is constant as in existing preferred habitat models, an unexpected fall in the short rate raises the term premium because habitat investors seek to borrow more long-term. Second, when arbitrageurs' endowment is endogenous, the same

⁹The importance of demand shocks in rationalizing evidence on bond return predictability echoes previous results in the literature. See for instance Proposition 6 in Vayanos and Vila (2021).

shock revalues arbitrageur wealth upwards if their portfolio features positive duration. Third, the latter mechanism implies that the term premium will fall if the duration of arbitrageurs’ portfolio is sufficiently high relative to the price elasticity of habitat investors. Finally, habitat demand shocks always induce a positive relationship between the slope of the yield curve and expected excess bond returns, unlike short rate shocks. Thus, when the volatility of demand shocks is sufficiently high, the Fama and Bliss (1987) and Campbell and Shiller (1991) regressions yield coefficients above and below zero, respectively.

4 Identified effects of monetary shocks

Before quantitatively studying these mechanisms in the full model, we provide two key pieces of empirical evidence used to discipline and validate it: the effects of monetary policy shocks on the yield curve and intermediary wealth.

4.1 Empirical strategy

We study the response of the yield curve and intermediary equity prices around announcements of the Federal Open Market Committee (FOMC). Given an outcome variable x_t (a forward rate or measure of intermediary wealth) and one-year yield $y_t^{(1)}$ measured at the end of day t , we estimate the effect of a change in $y_t^{(1)}$ on the change in x_t , instrumenting the former with the high-frequency change in Fed funds futures in a 30 minute window around the FOMC announcement. By focusing on variation induced by the high-frequency change in Fed funds futures, we address the point made by Nakamura and Steinsson (2018) that even on days with FOMC announcements, there are many other sources of news orthogonal to monetary policy which simultaneously affect yields and other outcome variables. By nonetheless summarizing our results in terms of the effect of a daily change in the one-year yield on outcome variables, we provide estimates which are easy to interpret and compare to the model.

A long-standing challenge in the identification of monetary policy shocks is that, even using intraday data, it is difficult to decouple them from “information shocks”: information about the state of the economy revealed at the time of FOMC announcements which is distinct from a shock to the Federal Reserve’s monetary policy rule. In the spirit of Jarocinski and Karadi (2020), we seek to identify the effects of monetary shocks alone by focusing only on FOMC announcement days in which the high

frequency change in the S&P 500 and one-year bond yield have opposite signs. Intuitively, if an increase in the one-year bond yield is due to good news about the state of the economy, it should be reflected in an increase in the S&P 500. Instead, if an increase in the one-year bond yield is due to a monetary policy shock, it should be reflected in a fall in the S&P 500 (due to the higher discount rate and, consistent with Kekre and Lenel (2021a,b) as well as the present paper, a higher price of risk). We discuss the robustness to using all FOMC announcement days, as well as a number of other alternative robustness exercises, later in this section.

4.2 Data

For high-frequency measures of monetary policy surprises and changes in the S&P 500, we use the data constructed by Jarocinski and Karadi (2020). They measure the monetary surprise in particular using the three-month ahead Fed funds futures contract. As they argue, this horizon combines information about near term policy shocks and forward guidance, useful during times when the zero lower bound was binding.

For data on the yield curve, we use Gurkaynak, Sack, and Wright (2008)'s interpolation interpolated yield curve on each day to compute yields and forwards at all maturities and horizons at a daily frequency. We use in particular the updated data maintained by the Federal Reserve. As previously noted, we focus on the real yield curve since our model is silent about inflation. For completeness, we present empirical estimates using the nominal yield curve in appendix B.

For data on intermediary wealth, we construct value-weighted indices of stock returns for publicly traded primary dealers. We use the list of primary dealers provided by the Federal Reserve and obtain daily closing prices and market capitalizations from CRSP, and intraday quotes using TAQ. Our focus on primary dealers follows He, Kelly, and Manela (2017), who more broadly study the relevance of their balance sheet health for prices in many different asset classes.

We use the January 2004 through December 2016 period for our analysis. While TIPS have been traded since the late 1990s, two- and three-year maturities were only included in Gurkaynak, Sack, and Wright (2006)'s interpolated real yield curve since 2004. We thus begin our sample at this point since we will express the effects on all outcome variables relative to a $1pp$ change in the one-year yield.¹⁰ We end our sample

¹⁰None of our findings would meaningfully change if we expressed effects relative to a $1pp$ change in the two-year yield instead. We prefer using the one-year yield (which requires extrapolating from

in 2016 as this is the last year in Jarocinski and Karadi (2020)’s sample.

In robustness exercises described further below, we also use the classification of FOMC announcements of Cieslak and Schrimpf (2019) and the measure of monetary policy surprises constructed by Nakamura and Steinsson (2018).

4.3 Response of yield curve to monetary shocks

We first characterize the response of the yield curve to monetary shocks. Our outcome variables of interest are the changes in one-year real forward rates paying between 2 and 20 years from each date t .

Figure 1 plots the regression coefficients and associated 90% confidence intervals. There is a striking U -shaped pattern in these effects, consistent with two effects which move in opposite directions as the maturity rises. First, there is the standard effect of monetary policy on the real interest rate arising from nominal rigidity. Given nominal rigidity, a persistent rise in the nominal interest rate will induce an immediate rise in the real interest rate which dissipates over time. At long horizons, the real interest rate should be unchanged because nominal prices will have adjusted to the shock. This mechanism is consistent with the fall in the estimated coefficients through ten years maturity. Second, there can be an effect of monetary policy on term premia. To the extent a monetary tightening raises term premia, this will be reflected in a rise in forward rates (*overreaction* of the forward rate, following section 3). This mechanism is consistent with the rise in the estimated coefficients from 10 to 20 years maturity, since longer maturity bonds are exposed to more risk.

This evidence rationalizes competing findings in the literature. Hanson and Stein (2015) estimate that in two-day windows around FOMC announcements, a $1pp$ increase in the two-year nominal yield is associated with a $0.42pp$ increase in the 10-year instantaneous real forward and $0.30pp$ increase in the 20-year instantaneous real forward, both of which are statistically significantly different from zero at all conventional levels (their Table 1). Since estimates of nominal rigidity cannot account for changes in real interest rates this far in the future, they conclude that a monetary tightening raises term premia. Nakamura and Steinsson (2018) argue that using two- or even one-day changes in yields as a measure of monetary policy surprises is misleading, because even on FOMC announcement days most of the variation in yields is induced by

the interpolated yield curve) simply because this is also what we use in our bond return predictability regressions, as is standard.

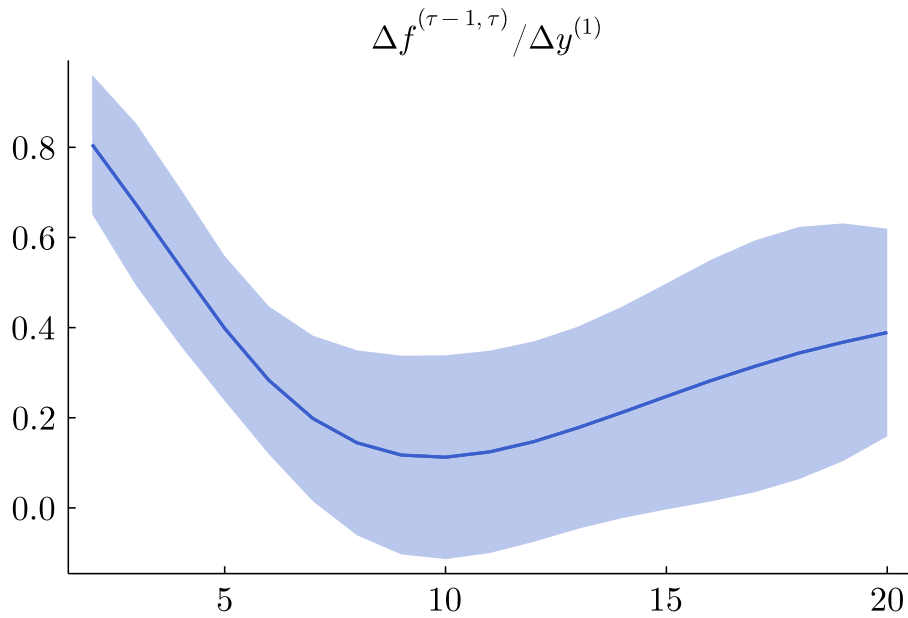


Figure 1: $\Delta f_t^{(\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise

Notes: at each integer between 2 and 20 on the x-axis, we plot coefficients and 90% confidence interval using $\Delta f_t^{(\tau)}$ as the outcome variable. Confidence interval based on robust standard errors.

non-monetary shocks. Using high-frequency intraday measures of monetary surprises, they estimate that a surprise associated with a $1pp$ increase in the one-year nominal yield induces only a $0.12pp$ increase in the 10-year instantaneous real forward, and this is statistically indistinguishable from zero (their Table 1). We follow Nakamura and Steinsson (2018) in using intraday measures of monetary policy surprises. Our results verify their finding that at the 10-year maturity, a monetary tightening has only a small effect on forward rates. We then demonstrate that when considering *longer maturity* forward rates, which were not studied in Nakamura and Steinsson (2018), a monetary tightening economically and statistically significantly raises forward rates.¹¹ This is consistent with the findings of Hanson and Stein (2015).

Figure 2 visually depicts the second-stage in our IV estimation for the 20-year forward rate. The scatterplots make evident that the positive slope is not driven by any one observation. At the same time, these figures make clear that the distribution of monetary policy surprises is leptokurtic, with large observations in absolute value

¹¹We note that the response of the yield curve at maturities as high as 20 years is not based on any extrapolation. Gurkaynak et al. (2008) demonstrate that TIPS with time to maturity exceeding 20 years have been outstanding since 1998. For instance, there were four such issues outstanding in 2004 at the beginning of our sample period.

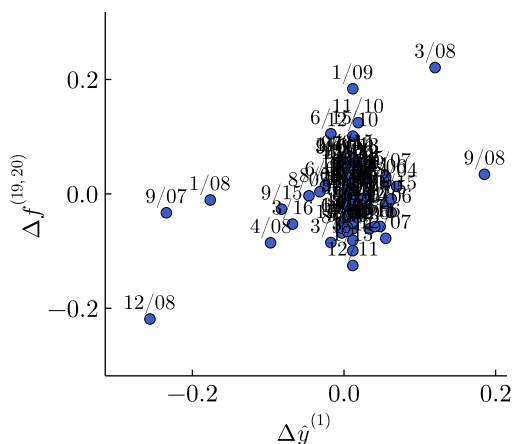


Figure 2: $\Delta f_t^{(19,20)}$ on $\widehat{\Delta y}_t^{(1)}$

Notes: $\widehat{\Delta y}_t^{(1)}$ is estimated based on first-stage projection on high-frequency monetary surprise estimated by Jarocinski and Karadi (2020).

particularly concentrated around the global financial crisis. One may be worried, then, that our results are entirely driven by anomalies during the most acute phase of this crisis, or reflect news other than conventional monetary policy such as to QE.

Table 1 demonstrates that this is in fact not the case, and that our results are robust to a wide variety of alternative specifications. Proceeding from top to bottom, the first panel summarizes the baseline estimates of a monetary tightening on the 5-, 10-, 15-, and 20-year forwards (the same as the relevant points in Figure 1). The next three panels consider alternative samples: we first consider all FOMC announcements rather than only on those in which the one-year yield and S&P 500 move in opposite directions; we drop all announcements between July 2008 and June 2009 to eliminate the most acute phase of the financial crisis; and we finally drop all announcements involving any news about asset purchases or non-standard credit operations, as classified by Cieslak and Schrimpf (2019). The next panel considers an alternative measure of monetary surprises as an instrument: we use the first principal component of five futures contracts in a 30 minute window around policy announcements as estimated by Nakamura and Steinsson (2018). The final panel uses the latter instrument and drops all announcements between July 2008 and June 2009, corresponding most closely to the benchmark specification in Nakamura and Steinsson (2018). In all five alternative specifications, we estimate a *U*-shape as in our baseline. In all cases, the response of the 20-year forward is economically significant, ranging between 0.27pp and 0.50pp for

Specification	$\Delta f_t^{(5)}$	$\Delta f_t^{(10)}$	$\Delta f_t^{(15)}$	$\Delta f_t^{(20)}$
Baseline	0.40 (0.10)	0.11 (0.14)	0.25 (0.15)	0.39 (0.14)
All FOMC announcements	0.38 (0.10)	0.11 (0.11)	0.13 (0.15)	0.27 (0.13)
Excluding 7/08-6/09	0.46 (0.22)	-0.26 (0.30)	0.21 (0.21)	0.50 (0.29)
Excluding announcements with LSAP news	0.28 (0.12)	-0.12 (0.17)	0.07 (0.14)	0.30 (0.19)
Nakamura and Steinsson (2018) IV	0.72 (0.32)	-0.07 (0.26)	0.13 (0.19)	0.29 (0.26)
Nakamura and Steinsson (2018) IV, ex. 7/08-6/09	0.64 (0.15)	0.27 (0.13)	0.35 (0.11)	0.40 (0.13)

Table 1: $\Delta f_t^{(\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise

Notes: robust standard errors provided in parenthesis.

a *1pp* increase in the one-year yield. And in three of these five specifications, the effect is statistically significantly different from zero at a 90% level.

4.4 Response of arbitrageur wealth to monetary shocks

We next characterize the response of intermediary wealth to monetary shocks. We focus on the change in the equity prices of primary dealers around FOMC announcements. By definition, primary dealers are the trading counterparties of the New York Fed in its implementation of monetary policy, and thus market-makers in Treasury securities. They thus correspond most closely to arbitrageurs in our model. The balance sheet capacity of primary dealers has been more broadly central in the literature on intermediary asset pricing (He et al. (2017)), so that our results may also be of interest for asset classes beyond government bonds.

We measure the response of primary dealer equity prices in 30 minute windows around FOMC announcements. For each publicly traded and active dealer around an FOMC announcement, we measure the closest prices of transactions 10 minutes prior to the FOMC announcement and 20 minutes after the FOMC announcement.¹² We then aggregate the change in dealer prices in this 30 minute window, weighting by

¹²For FOMC announcements occurring outside NYSE trading hours, we use the preceding closing price and following opening price.

yield curve, which detected statistically significant effects even on one day changes in forward rates. This makes sense because equity prices are much more volatile than forward rates and thus the signal to noise ratio is lower. The literature has generally found it difficult to detect any meaningful effect of monetary shocks on bank equity prices (Drechsler, Savov, and Schabl (2021), Haddad and Sraer (2020)), even though bank portfolios appear to exhibit positive duration (Begenau, Piazzesi, and Schneider (2015)) and bank exposures forecast bond returns (Haddad and Sraer (2020)). We focus on a subset of intermediaries particularly relevant for risk pricing — primary dealers — and demonstrate that using tight windows around FOMC announcements, the effects of monetary shocks on their equity prices are evident.¹⁵

Table 2 demonstrates that our estimated effects on dealer equity prices are again robust to alternative specifications. We again consider all FOMC announcements rather than the subset in which the one-year yield and S&P 500 move in opposite directions; drop all announcements between July 2008 and June 2009; drop all announcements involving any news about asset purchases or non-standard credit operations; use the Nakamura and Steinsson (2018) measure of monetary surprises; and use the latter measure and drop all announcements between July 2008 and June 2009. Across specifications, we find that dealer equity prices fall by $3.1pp - 24.1pp$ in response to a $1pp$ rise in the one-year yield. In all but the first case, the response is statistically significantly different from zero at a 90% level. In contrast, the results using one-day changes in dealer equity prices are sometimes negative and sometimes positive, and in all but one case not statistically significantly different from zero. This reinforces the importance of using tight event windows to have enough power to detect these effects.

While our results make clear that primary dealers are in fact exposed to interest rate risk, they do not speak to the precise mechanism by which they are. It could be that dealers are exposed because their portfolios have positive duration. But it also could be that dealers are exposed because, say, they are exposed to credit risk, and credit risk rises when monetary policy tightens because it affects the ability of borrowers to repay. In ongoing work, we are studying whether heterogeneity in the responses to monetary shocks *among* primary dealers can shed light on the mechanisms by which a monetary tightening lowers their wealth. This can be used to further discipline the mechanism operating through portfolio duration which is at the core of our model.

¹⁵English, den Heuvel, and Zakrajsek (2018) also use tight intraday windows to study the response of commercial bank equity prices to monetary shocks. Our analysis complements theirs but focuses on primary dealers.

Specification	30 minute change	One day change
Baseline	-8.8 (2.7)	-8.2 (10.0)
All FOMC announcements	-3.1 (4.1)	-2.8 (8.2)
Excluding 7/08-6/09	-15.5 (8.7)	-3.4 (20.2)
Excluding announcements with LSAP news	-10.5 (4.7)	3.7 (10.3)
Nakamura and Steinsson (2018) IV	-24.1 (10.0)	-21.3 (17.7)
Nakamura and Steinsson (2018) IV, ex. 7/08-6/09	-12.3 (4.0)	-21.7 (6.6)

Table 2: change in dealer prices on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise

Notes: robust standard errors provided in parenthesis.

5 Quantitative analysis

We now assess the ability of our full model to rationalize the effects of monetary policy on the yield curve. The impulse responses to monetary policy shocks are quantitatively consistent with the yield curve and intermediary wealth responses estimated in the data. At the same time, the model still rationalizes the classic bond return predictability evidence of Fama and Bliss (1987) and Campbell and Shiller (1991) which does not condition on identified shocks.

5.1 Equilibrium and solution

We first summarize the equilibrium conditions of the full model environment described in section 2 and the computational algorithm we use to solve it.

Equilibrium As derived formally in appendix C, arbitrageurs' first-order conditions for the problem (3)-(4) imply that

$$\frac{x_t^{(\tau)}}{w_t} = \frac{X_t^{(\tau)}}{W_t}$$

and

$$E_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right) - r_t dt = \frac{1}{W_t} \int_0^\infty X_t^{(s)} Cov_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds,$$

natural generalizations of (10) and (11) in the simple model, respectively. As in that case, the latter optimality condition has an intuitive interpretation. Arbitrageurs require non-zero expected excess returns to compensate them for bearing interest rate and demand risk. Their exposure to a bond with maturity τ depends on the covariance of returns on that bond with all other bonds of maturity $s \in (0, \infty)$ and the arbitrageurs' position in those bonds $\{X_t^{(s)}\}_{s=0}^\infty$. As arbitrageurs' wealth rises to infinity, a given position in these bonds accounts for a smaller share of their wealth and thus their required risk compensation falls to zero.

Substituting habitat demand (1) and market clearing (8) into the above condition, we obtain

$$E_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right) - r_t dt = \frac{1}{W_t} \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) Cov_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds. \quad (19)$$

Substituting arbitrageurs' habitat demand and market clearing in arbitrageurs' aggregate evolution of wealth (5), we obtain

$$dW_t = W_t r_t dt + \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt \right] d\tau + \xi(\bar{W} - W_t) dt. \quad (20)$$

Together with the driving forces (6)-(7), this characterizes the equilibrium. These equilibrium conditions parallel (12)-(16) in the simple model.

Solution In a large class of term structure models, including existing models in the preferred habitat tradition, bond prices are exponentially affine function in the model's state variables. The dependence of the price of risk on arbitrageurs' wealth in our setting implies that bond prices are no longer exponentially affine in this way.

We therefore characterize bond prices as a general function of the three state vari-

able r_t , β_t and W_t

$$P_t^{(\tau)} \equiv P^{(\tau)}(r_t, \beta_t, W_t). \quad (21)$$

Writing the evolution of wealth as

$$dW_t = \omega(r_t, \beta_t, W_t)dt + \eta_r(r_t, \beta_t, W_t)dB_{r,t} + \eta_\beta(r_t, \beta_t, W_t)dB_{\beta,t} \quad (22)$$

for some functions ω , η_r , and η_β , we can use (6), (7), (19), and (22) together with Ito's Lemma to obtain a system of partial differential equations (PDEs) relating partial derivatives of $\{P^{(\tau)}\}_{\tau=0}^\infty$ and the state variables r_t , β_t , and W_t . Given the boundary condition

$$P^{(0)}(r_t, \beta_t, W_t) = 1,$$

we use (forward) finite differences in maturity and collocation in other state variables to numerically solve this system of PDEs, given conjectures for the functions ω , η_r , and η_β . We then repeat this process and iterate over our guesses for the functions ω , η_r , and η_β until (22) is consistent with (20). Our code is written in Julia and solves the model in less than a second on a standard desktop computer. Further details on the algorithm are in appendix C.

As previously noted, we then simulate a monetary shock as an unexpected, one-time shock to the short rate with persistence $\kappa_m \neq \kappa_r$. The algorithm to solve for an impulse response to such an unexpected shock is also provided in appendix C

5.2 Calibration

We assume an exponential form for the slope and intercept of habitat demand by maturity, as in Vayanos and Vila (2021) and facilitating comparison with the literature. In particular, we assume

$$\begin{aligned} \alpha(\tau) &= \alpha \exp^{-\delta_\alpha \tau}, \\ \theta_0(\tau) &= \theta_0 (\exp^{-\delta_\alpha \tau} - \exp^{-\delta_\theta \tau}), \\ \theta_1(\tau) &= \theta_1 (\exp^{-\delta_\alpha \tau} - \exp^{-\delta_\theta \tau}), \end{aligned}$$

for $\tau \leq 20$, and $\alpha(\tau) = \theta_0(\tau) = \theta_1(\tau) = 0$ for $\tau > 20$. We focus on trade in bonds out to 20 years maturity because these are the maximum maturity of TIPS outstanding.

The model admits two normalizations. First, only the product $\theta_1 \sigma_\beta$ rather than θ_1

Description		Value	Moment	Target	Model
<i>Unconditional moments of yields and volumes</i>					
\bar{r}	mean short rate	0	$y_t^{(1)}$	0.06%	0.21%
κ_r	mean rev. short rate	0.5	$\sigma(y_t^{(1)})$	1.66%	1.69%
σ_r	std. dev. short rate	0.022	$\sigma(\Delta y_{t+1}^{(1)})$	1.75%	1.62%
ξ	persistence arb. wealth	0.15	$y_t^{(20)} - y_t^{(1)}$	1.54%	2.15%
κ_β	mean rev. demand	0.1	$\frac{1}{20} \sum_{\tau=1}^{20} \sigma(y_t^{(\tau)})$	1.01%	1.87%
σ_β	std. dev. demand	0.65	$\frac{1}{20} \sum_{\tau=1}^{20} \sigma(\Delta y_{t+1}^{(\tau)})$	0.78%	1.41%
α	level price elast.	0.87	$\frac{1}{20} \sum_{\tau=1}^{20} \rho(\Delta y_{t+1}^{(1)}, \Delta y_{t+1}^{(\tau)})$	0.57	0.66
δ_α	sens. price elast. to τ	0.28	$\sum_{\tau=1}^2 \Delta X_t^\tau / \sum_{\tau=1}^{20} \Delta X_t^\tau $	0.20	0.18
δ_θ	sens. demand to τ	0.50	$\sum_{\tau=11}^{20} \Delta X_t^\tau / \sum_{\tau=1}^{20} \Delta X_t^\tau $	0.09	0.11
<i>Impact effects of monetary shock</i>					
κ_m	mean rev. monetary	0.5	$df_t^{(1,2)} / dy_t^{(1)}$	0.81	0.64
\bar{W}	arb. endowment	0.002	$dW_t / dy_t^{(1)}$	-8.8	-6.6

Table 3: baseline calibration

Notes: Δ denotes annual change in this table. σ denotes monthly standard deviation, ρ denotes monthly correlation, and moments without these symbols are simple time-series averages.

and σ_β matter for the equilibrium dynamics; we thus set $\theta_1 = 1$. Second, $\{\bar{W}, \theta_0, \sigma_\beta, \alpha\}$ can each be scaled without changing the state-contingent path of prices or returns. We thus set $\theta_0 = 1$.

The calibration of remaining moments is summarized in Table 3. We calibrate the model to match two sets of moments: unconditional moments of yields and transaction volumes, and key (conditional) moments summarizing the effects of monetary shocks. Unless otherwise specified, all moments in the data are computed over the same January 2004 through December 2016 period studied in section 4. We reiterate that our calibration focuses on the real yield curve, since our model is silent about inflation. Finally, we note that the current parameterization is preliminary as it does not tightly match the desired moments; in ongoing work we are refining the parameterization to more tightly match the targeted moments.

We first set a subset of parameters to match unconditional moments of yields and transaction volumes. In this respect we follow the strategy of Vayanos and Vila (2021) to ease comparison with the literature. We set the average level of the short rate \bar{r} to match the average one-year yield of 0.06%. We set the mean reversion of the short

rate process κ_r to match the monthly volatility of the one-year yield of 1.66%, and the volatility of shocks σ_r to match the monthly volatility of annual changes in the one-year yield of 1.75%. Analogously, we set the mean reversion of the demand process κ_β to match the average monthly volatility of yields at maturities $\{1, \dots, 20\}$ of 1.01%, and the volatility of shocks σ_β to match the average monthly volatility of annual changes in these same yields of 0.78%. We set ξ , which controls the persistence of arbitrageur wealth and thus the volatility of bond prices, to match the 20-year/1-year yield spread of 1.54%. We set the level of the demand elasticity α to match the average correlation between annual changes in the one-year yield and annual changes in yields at maturities $\{1, \dots, 20\}$ of 0.57. We set δ_α to match the share of transactions by primary dealers for bonds with maturities less than two years, and δ_θ to match the share of transactions for bonds with maturities greater than ten years.¹⁶

The remaining parameters are set to match our baseline estimates of the effects of monetary shocks in section 4. We set the mean reversion of a monetary shock κ_m to match a 0.81*pp* increase in the 2-year real forward rate on impact of a monetary shock which raises the one-year yield by 1*pp* (Figure 1). For now, we set $\kappa_m = \kappa_r$ for simplicity. The parameter \bar{W} controls the level of arbitrageur wealth and is calibrated to match the 8.8% by which intermediary wealth falls on impact of the same monetary shock (Table 2).

5.3 Forward curve and excess returns on carry trades

We now compare an important set of untargeted moments between model and data: the forward curve and average excess returns on carry trades. Generating realistic patterns in both dimensions has important implications for the effects of changes in arbitrageur wealth on the yield curve, as we show in the next subsection.

The first panel of Figure 4 compares the average one-year forward rates in the model versus those estimated over our January 2004 through December 2016 sample period. By construction, the mean forward rate across maturities 2 through 20 in the model and data will be comparable, given that we calibrate the model to target the 1- and 20-year yields. However, nothing in the calibration explicitly targets the shape of the

¹⁶Note that in the data, we compute these ratios for the nominal Treasuries rather than TIPS. This is because the revaluation of wealth on impact of a change in the short rate would affect both assets, and the quantities of Treasuries traded is an order of magnitude larger than TIPS. With that said, the ratios are comparable for TIPS: the fraction of TIPS transacted with maturity less than two years is 17%, and with maturity greater than ten years is 13%.

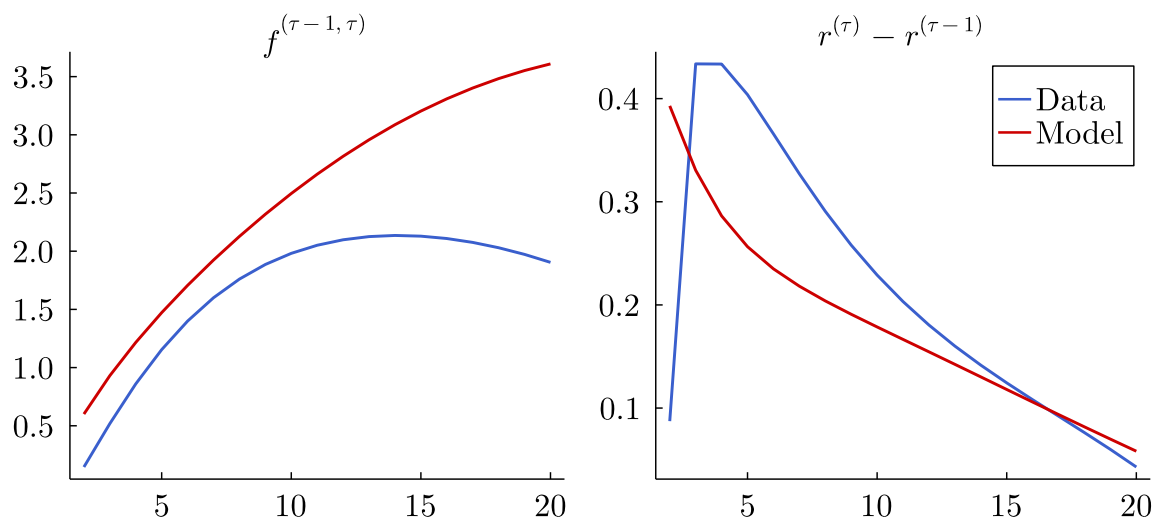


Figure 4: forward curve and average excess returns on carry trades: model vs. data

Notes: moments are time-series averages in both model and data.

forward curve. As the figure makes clear, the shape of the model-generated forward curve is comparable to the data, though it overshoots at long horizons.

The consistency between model and data in this dimension can be made more precise by comparing the pattern of carry trade returns between model and data. Following Cochrane and Piazzesi (2008), standard identities imply that

$$f_t^{(\tau-1, \tau)} - y_{t+\tau-1}^{(1)} = \left[r_{t+1}^{(\tau)} - r_{t+1}^{(\tau-1)} \right] + \left[r_{t+2}^{(\tau-1)} - r_{t+2}^{(\tau-2)} \right] + \dots + \left[r_{t+\tau-1}^{(2)} - y_{t+\tau-2}^{(1)} \right], \quad (23)$$

where $r_{t+1}^{(\tau)}$ denotes the log return to purchasing a τ -period bond at t and holding it for one year:

$$r_{t+1}^{(\tau)} \equiv \log P_{t+1}^{(\tau-1)} - \log P_t^{(\tau)}.$$

The left-hand side of (23) is the forward-spot spread. The right-hand side of (23) reflects the cumulative return to a sequence of carry strategies: purchasing a (τ) -year bond at t financed by a $(\tau - 1)$ -year bond, then purchasing a $(\tau - 1)$ -year bond at $t + 1$ financed by a $(\tau - 2)$ -year bond, and so on.

The right panel of Figure 4 computes the average excess return to each carry trade strategy $r_t^{(\tau)} - r_t^{(\tau-1)}$ on model-generated data and over the January 2004 through December 2016 period. In the data these excess returns have been on average positive, generally declining in maturity τ but non-trivial even at maturities above 10 years. The model-generated excess returns exhibit similar patterns. These patterns play an

important role in accounting for the model’s predictions for monetary shocks, to which we now turn.

5.4 Effects of monetary shock

Figure 5 depicts the impulse responses to an expansionary monetary shock. The shock is scaled to generate a $100bp$ fall in the one-year real yield on impact, facilitating comparison with the empirical results in section 4. Besides the short rate and one-year real yield shown in the first row, the figure depicts arbitrageur wealth. In the second row, the figure depicts the 20-year real forward rate; the spread between the 20-year real forward rate and one-year yield; and expected excess returns on the 20-year bond financed by the one-year bond over a one year holding period. The impulse responses are contrasted against those in a counterfactual economy in which $\xi \rightarrow \infty$ and thus arbitrageurs’ endowment is constant. All impulse responses are computed beginning from the equilibrium with $r = \bar{r}$, $\beta = 0$, and arbitrageur wealth W equal to the mean of the ergodic distribution.

The 20-year real forward rate falls in response to the shock, in contrast to the counterfactual model in which arbitrageurs’ endowment is constant. The difference in these responses is driven by the upward revaluation of arbitrageurs’ wealth in the baseline model, which lowers their price of bearing risk and compresses term premia. Notably, since the fall in the short rate is not permanent, the forward spread rises as the yield curve steepens. Since term premia have fallen, future excess returns on the 20-year bond are high — persistently so, reflecting the pattern of arbitrageurs’ wealth. It follows that a monetary shock (and more generally any shock to the short rate) induces a negative relationship between the slope of the yield curve and subsequent excess returns on long-term bonds. The opposite is true in the counterfactual model. All of these results are consistent with the analytical results in section 3.

Figure 6 depicts the impact effect of the monetary shock on the forward rate across maturities and compares it to the estimates from Figure 1. The model generates a U -shaped response of the forward rate as in the data, and the model-generated responses lie within the empirical confidence intervals at most maturities. The counterfactual model featuring underreaction of the forward rate instead undershoots the data at most maturities. We emphasize that the response of long-dated forward rates was not targeted in the calibration. We conclude that the model can successfully account for the response of the yield curve to monetary shocks in the data, and that accounting for

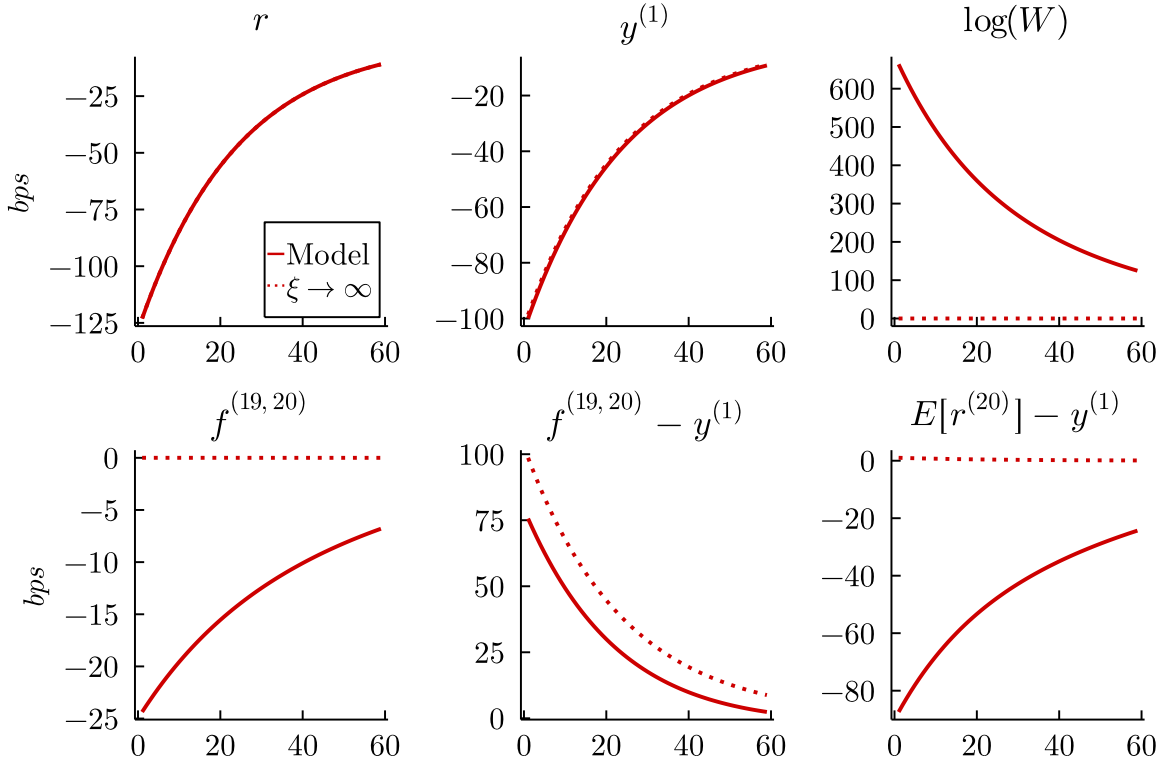


Figure 5: impulse responses to monetary shock

Notes: monetary shock is a one-time innovation to short rate with mean reversion $\kappa_m = 0.5$ as described in main text. Figure depicts responses to infinitesimal shock, scaled to generate 100bp fall in one-year yield on impact. Impulse responses computed from the equilibrium with $r = \bar{r}$, $\beta = 0$, and arbitrageur wealth W equal to the mean of the ergodic distribution.

an endogenous price of risk through the revaluation of arbitrageur wealth is essential to this result.

We can use our model to provide a deeper decomposition of why the forward curve in the baseline model responds in this way. Evaluating the identity (23) ex-ante instead of ex-post and taking expectations at t , we have that

$$f_t^{(\tau-1,\tau)} - E_t y_{t+\tau-1}^{(1)} = E_t \left[r_{t+1}^{(\tau)} - r_{t+1}^{(\tau-1)} \right] + E_t \left[r_{t+2}^{(\tau-1)} - r_{t+2}^{(\tau-2)} \right] + \dots + E_t \left[r_{t+\tau-1}^{(2)} - y_{t+\tau-2}^{(1)} \right]. \quad (24)$$

It follows that the response of the forward rate relative to the expected spot rate — the difference between the red line and black line in Figure 6 — encodes the response of expected excess returns on a sequence of carry trades at future dates.

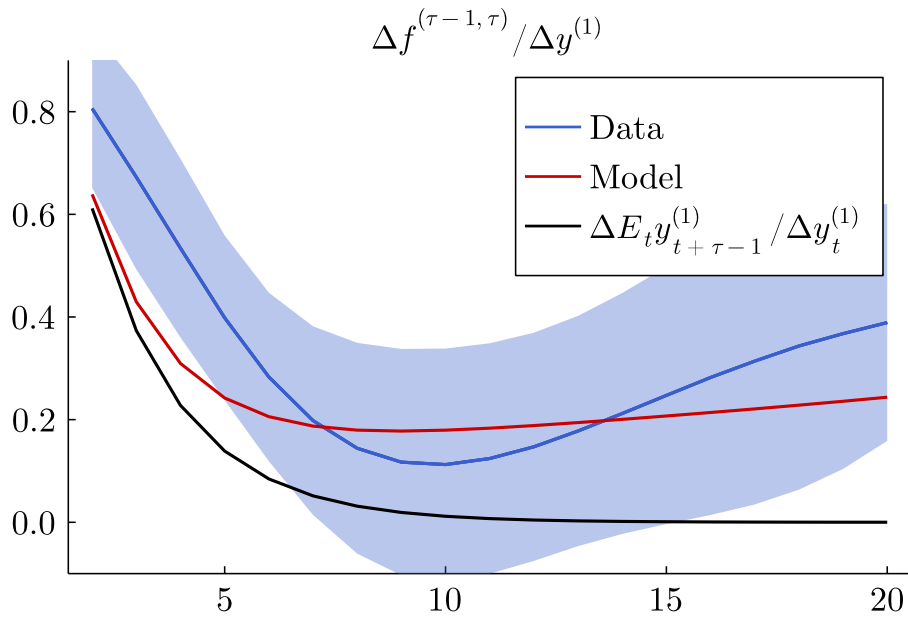


Figure 6: $f_t^{(\tau)}$ on $y_t^{(1)}$ given monetary shock: model vs. data

Notes: empirical estimates correspond to those in Figure 1.

Figure 7 depicts the response of each of these expected excess returns for $\tau = \{5, 10, 15, 20\}$ -year bonds. It contains two main insights. First, the response of even long-dated forward rates to a monetary shock largely reflects changes in expected excess returns in the first few years, since wealth eventually returns to steady-state (though it is still quite persistent in this calibration, with a half-life of roughly 5 years). Second, the response of long-dated forward rates is larger than short-dated forward rates (relative to expected spot rates at each horizon) because the former nonetheless cumulates over a longer period of expected excess returns.

5.5 Slope of yield curve and bond return predictability

An implication of the previous section is that monetary shocks, and shocks to the short rate more generally, induce a negative relationship between the slope of the yield curve and future excess returns on long-term bonds via their effect on term premia. In this subsection we demonstrate that shocks to the demand of habitat investors instead imply that a steep yield curve predicts high excess returns on long-term bonds. These shocks are sufficiently important in our calibration that the model can rationalize estimates in classic Fama and Bliss (1987) and Campbell and Shiller (1991) regressions which do

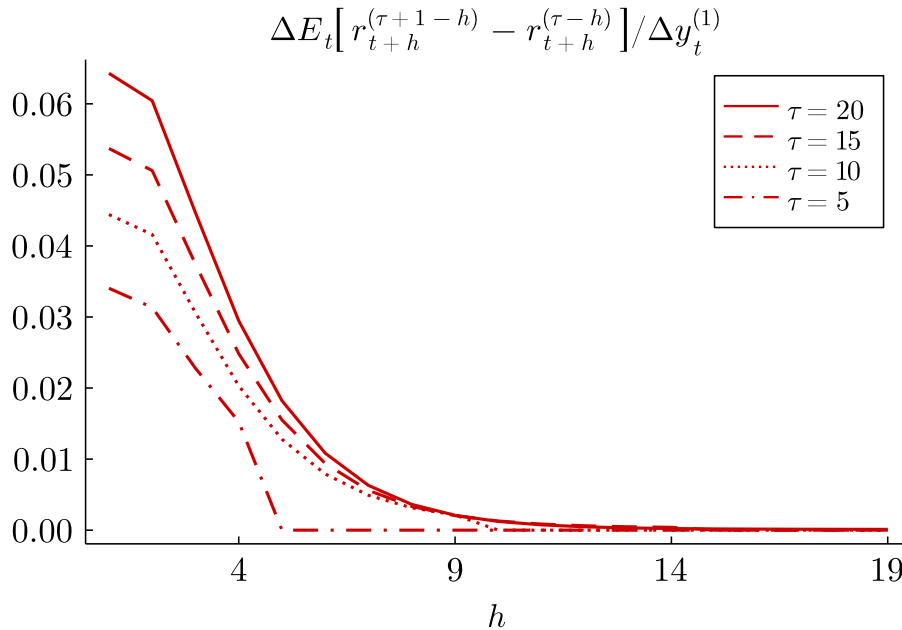


Figure 7: decomposing $d[f_t^{(\tau)} - E_t y_{t+\tau-1}^{(1)}] / dy_t^{(1)}$ on impact of monetary shock

Notes: as derived in (24), $\sum_{h=1}^{\tau-1} E_t [r_{t+h}^{(\tau+1-h)} - r_{t+h}^{(\tau-h)}] = f_t^{(\tau-1, \tau)} - E_t y_{t+\tau-1}^{(1)}$.

not condition on identified shocks.

We first consider the impulse responses to habitat demand shocks. Figure 8 presents the response to a one standard deviation demand shock $\epsilon_{\beta,0}$. The other panels further depict the same variables as in Figure 5. Figure 8 demonstrates that habitat demand shocks are effectively term premium shocks. Whether arbitrageurs' endowment is endogenous or exogenous, forward rates fall when habitat investors seek to borrow less in long-term bonds (a negative innovation to $\epsilon_{\beta,0}$) and thus arbitrageurs must bear less risk. Since expected short rates are unchanged, this fully reflects a decline in term premia. The decline in term premia manifests as a fall in the slope of the yield curve, as well as low subsequent excess returns on long-term bonds. Notably, these comovements are amplified on impact in the model with endogenous wealth because the decline in long yields revalues wealth in arbitrageurs' favor, further compressing term premia.¹⁷

The comovements induced by habitat demand shocks allow us to rationalize classic Fama and Bliss (1987) and Campbell and Shiller (1991) regressions which do not condition on identified shocks. Using the same sample period studied in section 4, we

¹⁷Because the volume of arbitrageurs' carry trade is smaller and they earn smaller excess returns in doing so, eventually arbitrageur wealth falls below its initial value and term premia reverse sign.

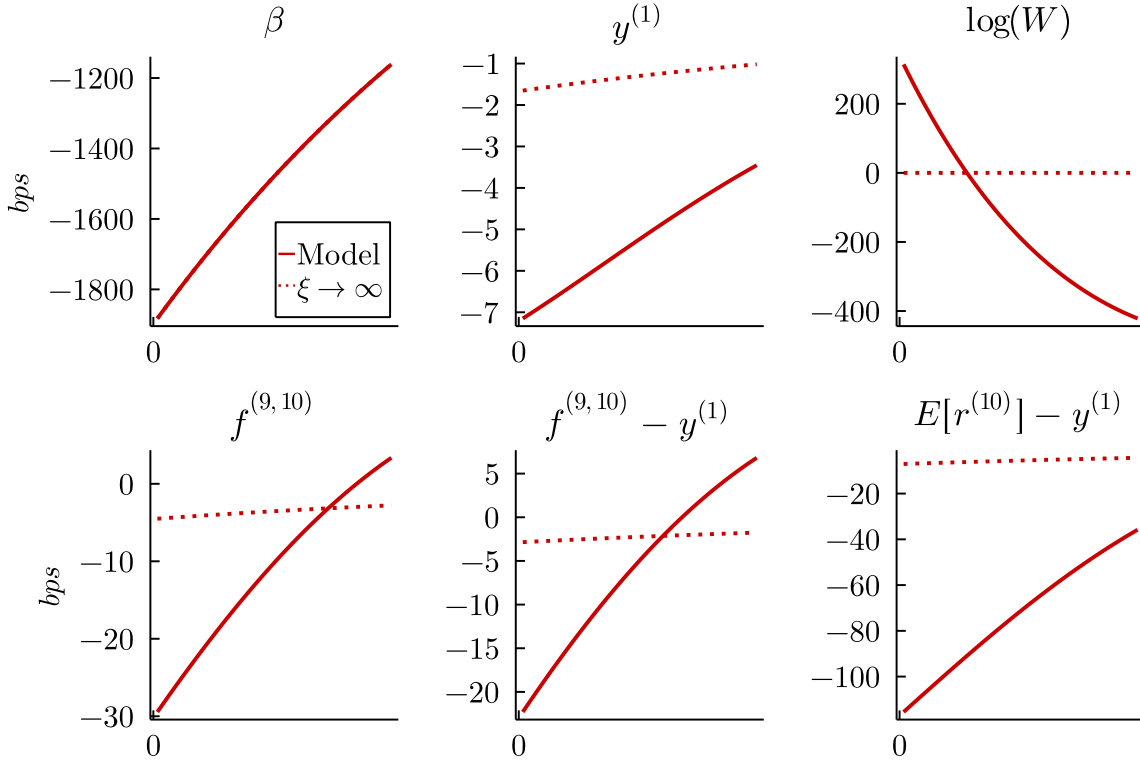


Figure 8: impulse responses to habitat demand shock

Notes: figure depicts one standard deviation shock. Impulse responses computed from the equilibrium with $r = \bar{r}$, $\beta = 0$, and arbitrageur wealth W equal to the mean of the ergodic distribution.

first re-estimate these regressions in the data. The shaded region in Figures 9 and 10 report the 90% confidence intervals for the coefficients $\beta_{FB}^{(\tau)}$ and $\beta_{CS}^{(\tau)}$ in the regressions

$$r_{t+1}^{(\tau)} - y_t^{(1)} = \alpha_{FB}^{(\tau)} + \beta_{FB}^{(\tau)} \left(f_t^{(\tau-1, \tau)} - y_t^{(1)} \right) + \epsilon_{FB, t+1}^{(\tau)} \quad (25)$$

and

$$y_{t+1}^{(\tau-1)} - y_t^{(\tau)} = \alpha_{CS}^{(\tau)} + \beta_{CS}^{(\tau)} \frac{1}{\tau-1} \left(y_t^{(\tau)} - y_t^{(1)} \right) + \epsilon_{CS, t+1}^{(\tau)}, \quad (26)$$

again using the real yield curve as we do throughout the main text. Consistent with Fama and Bliss (1987) and inconsistent with the expectations hypothesis, these estimates suggest $\beta_{FB}^{(\tau)}$ is positive and rising in τ . Consistent with Campbell and Shiller (1991) and inconsistent with the expectations hypothesis, these estimates suggest $\beta_{CS}^{(\tau)}$ is less than one, in fact negative for most maturities, and falling in τ . Both results imply that a steep yield curve predicts high excess long-term bond returns.

Figures 9 and 10 further plot the same regression coefficients estimated on long

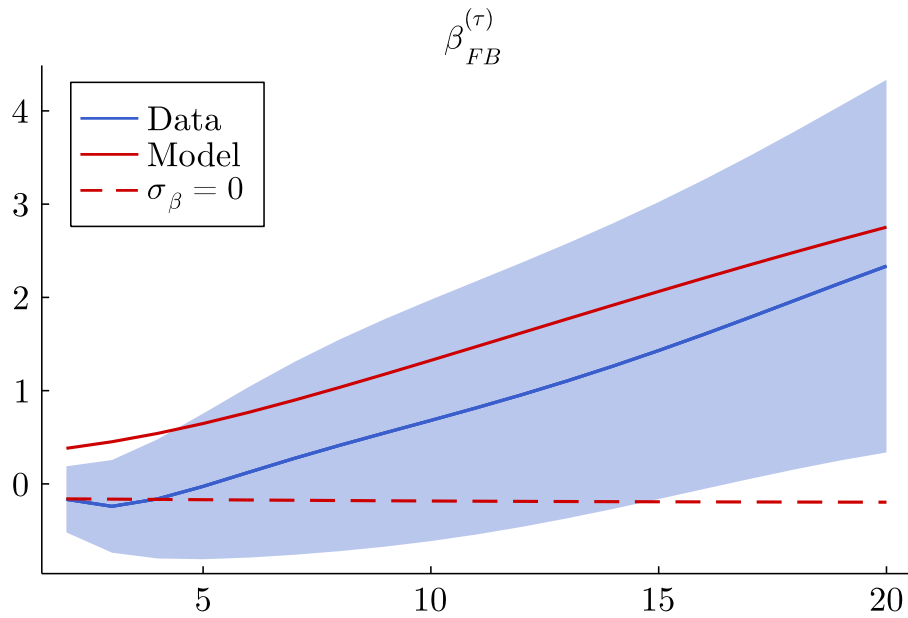


Figure 9: Fama-Bliss regression coefficients: model vs. data

Notes: figure depicts $\beta_{FB}^{(\tau)}$ estimated from (25) and, in the case of the data, the 90% confidence interval obtained using Hansen-Hodrick standard errors with 12 lags.

simulations of model-generated data. The model is largely consistent with the empirical patterns in $\beta_{FB}^{(\tau)}$ and $\beta_{CS}^{(\tau)}$. When habitat demand shocks are shut down ($\sigma_\beta = 0$), the figure finally demonstrates that the model-generated regression coefficients no longer can rationalize the data. Consistent with the analytical results in section 3, we conclude that habitat demand shocks are essential to match this classic evidence on bond return predictability.

6 Conclusion

In this paper, we propose a model which rationalizes the effects of monetary policy shocks on the term structure of interest rates. As in the preferred habitat tradition, habitat investors and arbitrageurs trade bonds of various maturities; as in the intermediary asset pricing tradition, arbitrageur wealth is an endogenous state variable relevant for equilibrium risk pricing. When arbitrageurs' portfolio features positive duration, an unexpected fall in the short rate revalues wealth in their favor and lowers term premia. A calibration matching the portfolio exposure of the U.S. financial sector rationalizes the identified effects of policy shocks along the yield curve, while

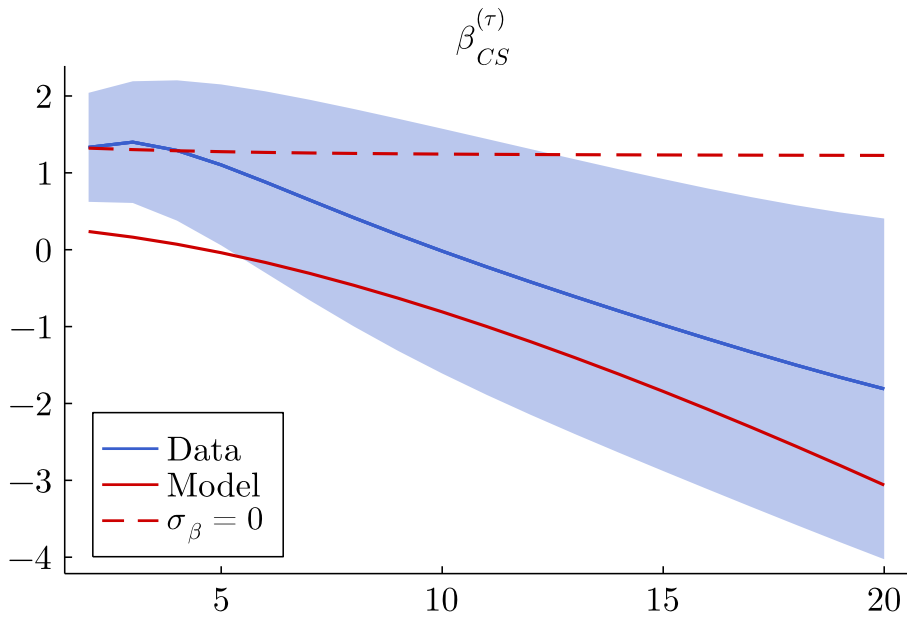


Figure 10: Campbell-Shiller regression coefficients: model vs. data

Notes: figure depicts $\beta_{CS}^{(\tau)}$ estimated from (26) and, in the case of the data, the 90% confidence interval obtained using Hansen-Hodrick standard errors with 12 lags.

simultaneously matching the classic evidence on bond return predictability over time.

Our analysis has stopped short of tracing out the consequences for the real economy so as to focus on the novel mechanisms in financial markets relative to existing term structure models. Embedding our model in a New Keynesian production economy, we expect that the effects of policy on the price of risk will amplify the real effects of monetary policy, to the extent that aggregate demand is rising in the amount habitat investors borrow long-term. This seems natural if we interpret long-term borrowers as mortgagors or non-financial corporates whose marginal propensity to consume or invest is higher than the owners of financial firms. We view this as among the most interesting applications of our framework in future work.

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Appendix for Online Publication

A Proofs of analytical results

We first provide proofs of all analytical results in the main text.

A.1 Proposition 1

Proof. When $\xi \rightarrow \infty$, (16) implies $W = \bar{W}$. (13) and (14) imply

$$\begin{aligned} E_t r_{t+1}^{(2)} &= -E_t r_{t+1} - \log P_t, \\ &= -\kappa_r \bar{r} - (1 - \kappa_r) r_t - \log P_t \end{aligned}$$

Substituting these into (12) yields

$$-\kappa_r \bar{r} - (2 - \kappa_r) r_t - \log P_t + \frac{1}{2} \sigma_r^2 = \frac{\alpha \log P_t + \theta_t}{\bar{W}} \sigma_r^2.$$

Re-arranging yields

$$\log P_t = \frac{1}{1 + \frac{1}{\bar{W}} \alpha \sigma_r^2} \left[-\kappa_r \bar{r} - (2 - \kappa_r) r_t + \frac{1}{2} \sigma_r^2 - \frac{1}{\bar{W}} \theta_t \sigma_r^2 \right].$$

It follows from (17) that

$$f_t = -\frac{1}{1 + \frac{1}{\bar{W}} \alpha \sigma_r^2} \left[-\kappa_r \bar{r} - (2 - \kappa_r) r_t + \frac{1}{2} \sigma_r^2 - \frac{1}{\bar{W}} \theta_t \sigma_r^2 \right] - r_t.$$

The response of the forward rate to a short rate shock follows. □

A.2 Proposition 2

Proof. Combining (13) and (16), wealth evolves according to

$$\begin{aligned} W_t &= (1 - \exp(-\xi)) \bar{W} + \\ &\quad \exp(-\xi) \exp(r_{t-1}) [W_{t-1} + (\alpha \log P_{t-1} + \theta_{t-1}) (\exp(-r_t - r_{t-1} - \log P_{t-1}) - 1)]. \end{aligned}$$

Around the stochastic steady-state, this implies

$$\begin{aligned}
d \log W_t = & \frac{\exp(-\xi) \exp(\bar{r}) [W - X]}{W} dr_{t-1} + \exp(-\xi) \exp(\bar{r}) d \log W_{t-1} + \\
& \frac{\exp(-\xi) \exp(\bar{r}) (\exp(-2\bar{r} - \log P) - 1)}{W} d\theta_{t-1} - \frac{\exp(-\xi) X \exp(-\bar{r} - \log P)}{W} dr_t + \\
& \frac{\exp(-\xi) \exp(\bar{r}) [\alpha (\exp(-2\bar{r} - \log P) - 1) - X \exp(-2\bar{r} - \log P)]}{W} d \log P_{t-1}. \quad (27)
\end{aligned}$$

The impact response of wealth to a short rate shock follows, with

$$\omega \equiv \frac{X}{W} \exp(-\bar{r} - \log P)$$

summarizing the duration of arbitrageurs' wealth. \square

A.3 Proposition 3

Proof. The same steps as in the proof of Proposition 1 imply that around the stochastic steady-state

$$d \log P_t = -\frac{2 - \kappa_r}{1 + \frac{1}{W} \alpha \sigma_r^2} dr_t - \frac{\frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t + \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t. \quad (28)$$

It follows from (17) that

$$df_t = \frac{1 - \kappa_r - \frac{1}{W} \alpha \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} dr_t + \frac{\frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t. \quad (29)$$

The response of the forward rate to a short rate shock follows from Proposition 2. \square

A.4 Proposition 4

Proof. A standard Taylor approximation up to first order in the model's driving forces around the stochastic steady-state implies that

$$\begin{aligned}
\beta_{FB} & \equiv \frac{\text{Cov}(r_{t+1}^{(2)} - r_t, f_t - r_t)}{\text{Var}(f_t - r_t)} \approx \frac{\text{Cov}(dr_{t+1}^{(2)} - dr_t, df_t - dr_t)}{\text{Var}(df_t - dr_t)}, \\
\beta_{CS} & \equiv \frac{\text{Cov}(r_{t+1} - y_t, y_t - r_t)}{\text{Var}(y_t - r_t)} \approx \frac{\text{Cov}(dr_{t+1} - dy_t, dy_t - dr_t)}{\text{Var}(dy_t - dr_t)}.
\end{aligned}$$

Around the stochastic steady-state, (13), (14), (18), (28), and (29) imply

$$\begin{aligned}
dr_{2,t+1} - dr_t &= -(2 - \kappa_r) \frac{\frac{1}{W} \alpha \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} dr_t + \frac{\frac{1}{W_t} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t - d\sigma_r \epsilon_{r,t+1}, \\
df_t - dr_t &= -\frac{\kappa_r + 2 \frac{1}{W} \alpha \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} dr_t + \frac{\frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t, \\
dr_{t+1} - dy_t &= -\frac{1}{2} \frac{\kappa_r - 2 \frac{1}{W} \alpha \sigma_r^2 (1 - \kappa_r)}{1 + \frac{1}{W} \alpha \sigma_r^2} dr_t - \frac{1}{2} \frac{\frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t + \frac{1}{2} \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t + d\sigma_r \epsilon_{r,t+1}, \\
dy_t - dr_t &= -\frac{1}{2} \frac{\kappa_r + 2 \frac{1}{W} \alpha \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} dr_t + \frac{1}{2} \frac{\frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{1}{2} \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t.
\end{aligned}$$

Substituting (28) into (27) and collecting terms, we further have

$$d \log W_t = \mu_{r,-1} dr_{t-1} + \mu_{\theta,-1} d\theta_{t-1} + \mu_{W,-1} d \log W_{t-1} + \mu_r dr_t,$$

where

$$\begin{aligned}
\mu_{P,-1} &\equiv \frac{\exp(-\xi) \exp(\bar{r}) [\alpha (\exp(-2\bar{r} - \log P) - 1) - X \exp(-2\bar{r} - \log P)]}{W}, \\
\mu_{r,-1} &\equiv \frac{\exp(-\xi) \exp(\bar{r}) [W - X]}{W} - \mu_{P,-1} \frac{2 - \kappa_r}{1 + \frac{1}{W} \alpha \sigma_r^2}, \\
\mu_{\theta,-1} &\equiv \frac{\exp(-\xi) \exp(\bar{r}) (\exp(-2\bar{r} - \log P) - 1)}{W} - \mu_{P,-1} \frac{\frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2}, \\
\mu_{W,-1} &\equiv \exp(-\xi) \exp(\bar{r}) + \mu_{P,-1} \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2}, \\
\mu_r &\equiv -\frac{\exp(-\xi) X \exp(-\bar{r} - \log P)}{W}.
\end{aligned}$$

By (14) and (15), we thus have

$$\begin{aligned}
dr_t &= \sum_{\tau=0}^{\infty} (1 - \kappa_r)^\tau \sigma_r d\epsilon_{r,-\tau}, \\
d\theta_t &= \sum_{\tau=0}^{\infty} (1 - \kappa_\theta)^\tau \sigma_\theta d\epsilon_{\theta,-\tau}, \\
d \log W_t &= \sum_{\tau=0}^{\infty} \left[\mu_r \mu_{W,-1}^\tau + [\mu_r (1 - \kappa_r) + \mu_{r,-1}] \frac{(1 - \kappa_r)^\tau - \mu_{W,-1}^\tau}{1 - \kappa_r - \mu_{W,-1}} \right] d\sigma_r \epsilon_{r,-\tau} +
\end{aligned}$$

$$\sum_{\tau=1}^{\infty} \left[\mu_{\theta,-1} \frac{(1 - \kappa_{\theta})^{\tau} - \mu_{W,-1}^{\tau}}{1 - \kappa_{\theta} - \mu_{W,-1}} \right] \sigma_{\theta} d\epsilon_{\theta,-\tau}.$$

We complete the proof in two steps.

First, we prove that β_{FB} can be above or below zero, and β_{CS} can be above or below one. Focusing on the useful benchmark when $\alpha = 0$ and $\sigma_{\theta} = 0$, straightforward algebra implies

$$\beta_{FB} \propto 1 - \beta_{CS} \propto \sum_{\tau=0}^{\infty} \left[-\frac{1}{W} \theta \sigma_r^2 \left(\mu_r \mu_{W,-1}^{\tau} + [\mu_r(1 - \kappa_r) + \mu_{r,-1}] \frac{(1 - \kappa_r)^{\tau} - \mu_{W,-1}^{\tau}}{1 - \kappa_r - \mu_{W,-1}} \right) \right] \times \\ \left[-\kappa_r(1 - \kappa_r)^{\tau} - \frac{1}{W} \theta \sigma_r^2 \left(\mu_r \mu_{W,-1}^{\tau} + [\mu_r(1 - \kappa_r) + \mu_{r,-1}] \frac{(1 - \kappa_r)^{\tau} - \mu_{W,-1}^{\tau}}{1 - \kappa_r - \mu_{W,-1}} \right) \right].$$

Now assume $\exp(-\xi)$ is finite but sufficiently small that we can ignore terms of order two or higher in $\exp(-\xi)$. When $\kappa_r \rightarrow 1$ and $\theta > 0$, the above expression is negative; when $\kappa_r \rightarrow 0$, the above expression is positive regardless of θ . The same argument implies β_{CS} can be above or below one.

Second, we characterize the limits as $\sigma_{\theta} \rightarrow \infty$. In this case it is clear from the above results that

$$\beta_{FB} \rightarrow \frac{\text{Var} \left(\frac{\frac{1}{W_t} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t \right)}{\text{Var} \left(\frac{\frac{1}{W_t} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t \right)} = 1, \\ \beta_{CS} \rightarrow -\frac{\text{Var} \left(\frac{\frac{1}{2} \frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{1}{2} \frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t \right)}{\text{Var} \left(\frac{\frac{1}{2} \frac{1}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d\theta_t - \frac{\frac{1}{2} \frac{X}{W} \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} d \log W_t \right)} = -1,$$

where we have abused notation (in writing infinity divided by infinity) to clarify the mechanics of this result. \square

B Effects of monetary shocks on nominal yield curve

In the empirical analysis in the main text we focus on real yields and forwards given that our model is silent about inflation. Here we replicate our analysis using the nominal yield curve. In particular, we regress the change in the one-year nominal yield on our high-frequency monetary policy surprise measure in the first stage, and then

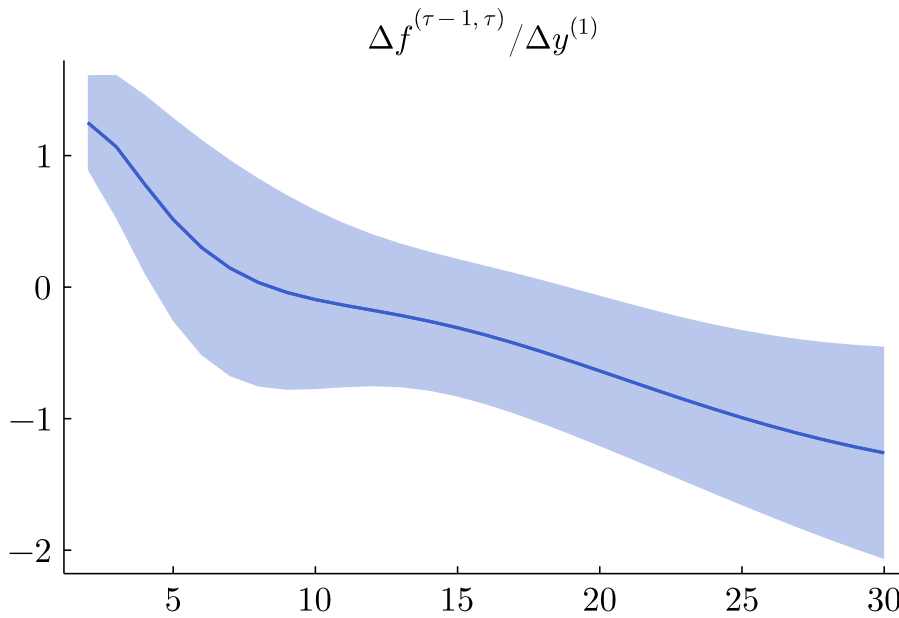


Figure 11: $\Delta f_t^{(\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise (nominal)

Notes: at each integer between 2 and 30 on the x-axis, we plot coefficients and 90% confidence interval using $\Delta f_t^{(\tau)}$ as the outcome variable. Confidence interval based on robust standard errors.

regress the change in one-year nominal forward rate paying between 2 and 30 years on the predicted change in the one-year nominal yield in the second stage.¹⁸

We use Gurkaynak et al. (2006)'s interpolated nominal yield curve to compute yields and forwards at all maturities and horizons at a daily frequency. We use in particular the updated data maintained by the Federal Reserve. We focus on the same January 2004 through December 2016 period used in our analysis of the real yield curve only to maintain comparability with those results. Data for the nominal yield curve is available earlier and we have validated that we obtain similar results over the broader sample.

Figure 11 plots the regression coefficients and associated 90% confidence intervals. Unlike in the case of real forwards, the effect of a monetary tightening on nominal forwards is monotonically declining. Moreover, the effect is economically and statistically significantly negative at long maturities. Table 4 presents the same alternative specifications as in Table 1. As is evident, the same pattern holds across these specifications.

These results are consistent with previous findings in the literature also focused on the nominal yield curve, such as in Gurkaynak et al. (2005b) and Gurkaynak, Sack, and

¹⁸Whether we use the one-year nominal or real yield in the first stage matters little. Both summarize the stance of monetary policy. What does matter is the outcome variable used in the second stage.

Specification	$\Delta f_t^{(5)}$	$\Delta f_t^{(10)}$	$\Delta f_t^{(15)}$	$\Delta f_t^{(20)}$
Baseline	0.51 (0.47)	-0.09 (0.41)	-0.31 (0.31)	-0.64 (0.34)
All FOMC announcements	0.42 (0.26)	-0.09 (0.23)	-0.23 (0.17)	-0.42 (0.19)
Excluding 7/08-6/09	0.10 (0.34)	-0.49 (0.33)	-0.59 (0.43)	-0.84 (0.51)
Excluding announcements with LSAP news	-0.02 (0.30)	-0.52 (0.27)	-0.51 (0.35)	-0.74 (0.40)
Nakamura and Steinsson (2018) IV	0.27 (0.29)	-0.29 (0.29)	-0.37 (0.32)	-0.66 (0.41)
Nakamura and Steinsson (2018) IV, ex. 7/08-6/09	0.91 (0.46)	0.27 (0.40)	-0.12 (0.28)	-0.48 (0.33)

Table 4: $\Delta f_t^{(\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise (nominal)

Notes: robust standard errors provided in parenthesis.

Swanson (2005a). As these papers argue, these patterns are consistent with monetary shocks containing news about the central bank’s long-run inflation target. In particular, if a monetary tightening is associated with news about a lower long-run inflation target, it will lower long-run forward rates. Changes in the long-run inflation target will have no effects on long maturity real forwards, underscoring the importance of focusing on the real yield curve to uncover the effects of monetary shocks on term premia.

With that said, the effects of monetary policy shocks on the nominal yield curve are still critical for the results in our paper because the majority of Treasury securities outstanding are nominal, not real. Hence, the revaluation of nominal bonds, not real bonds, are likely to drive any changes in arbitrageur wealth. In this context, it is important to note that nominal *yields* rise on impact of a monetary easing far out into the yield curve, as shown for our baseline specification in Figure 12, even though long-dated nominal forward rates fall. Similar results are obtained for the alternative specifications described above. We conclude that a monetary tightening will lower the wealth of agents having positive duration in nominal bonds, so long as the duration is not extremely high (above roughly 20 years).

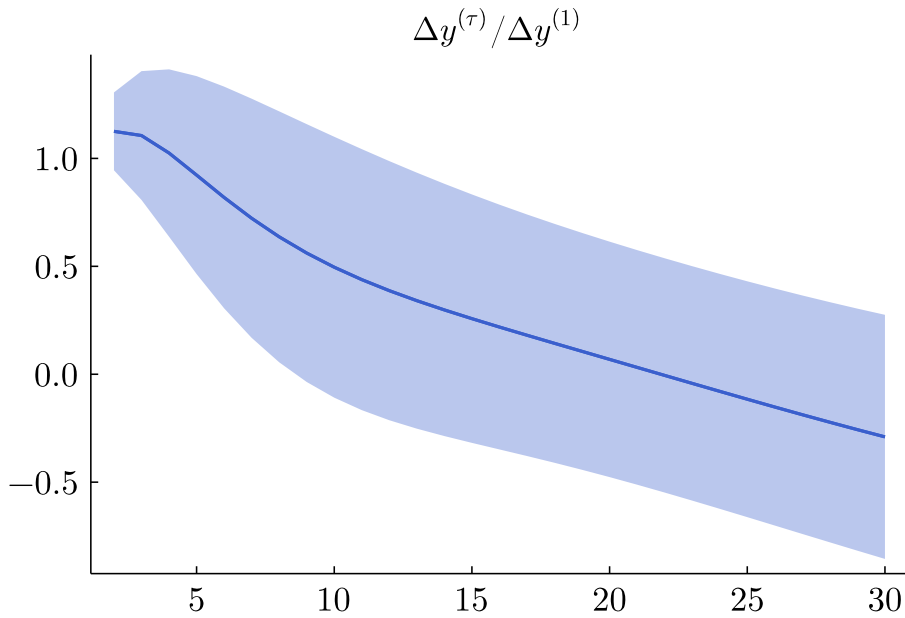


Figure 12: $\Delta y_t^{(\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise (nominal)

Notes: at each integer between 2 and 30 on the x-axis, we plot coefficients and 90% confidence interval using $\Delta y_t^{(\tau)}$ as the outcome variable. Confidence interval based on robust standard errors.

C Equilibrium and solution of full model

We now characterize arbitrageurs' optimality conditions in the full model and provide more details on our computational algorithm.

C.1 Arbitrageurs' optimality

Given a conjectured equilibrium pricing function

$$P_t^{(\tau)} = P^{(\tau)}(r_t, \beta_t, W_t),$$

Ito's Lemma implies

$$dP_t^{(\tau)} = \omega_t^{(\tau)} P_t^{(\tau)} dt + \eta_{r,t}^{(\tau)} P_t^{(\tau)} dB_{r,t} + \eta_{\beta,t}^{(\tau)} P_t^{(\tau)} dB_{\beta,t} \quad (30)$$

for some coefficients $\omega_t^{(\tau)}$, $\eta_{r,t}^{(\tau)}$, and $\eta_{\beta,t}^{(\tau)}$ which we have expressed relative to $P_t^{(\tau)}$ without loss of generality. Defining the portfolio shares

$$\chi_t^{(\tau)} \equiv \frac{x_t^{(\tau)}}{w_t},$$

we can thus write the arbitrageur problem (3)-(4) as maximizing

$$v_t(w_t) = \max_{\{\{\chi_{t+s}^{(\tau)}\}\}} E_t \int_0^\infty \exp(-\xi s) \log w_{t+s} ds$$

subject to

$$\begin{aligned} dw_t = & \left[r_t w_t + \int_0^\infty \chi_t^{(\tau)} w_t \left(\omega_t^{(\tau)} - r_t \right) d\tau \right] dt \\ & + \left[\int_0^\infty \chi_t^{(\tau)} w_t \eta_{r,t}^{(\tau)} d\tau \right] dB_{r,t} + \left[\int_0^\infty \chi_t^{(\tau)} w_t \eta_{\beta,t}^{(\tau)} d\tau \right] dB_{\beta,t}. \end{aligned}$$

The associated Hamilton-Jacobi-Bellman equation is

$$\begin{aligned} (\rho + \xi)v_t(w_t) = & \frac{\partial v_t(w_t)}{\partial t} + \max_{\{\chi_t^{(\tau)}\}} \log w_t + \left[r_t w_t + \int_0^\infty \chi_t^{(\tau)} w_t \left(\omega_t^{(\tau)} - r_t \right) d\tau \right] \frac{\partial v_t(w_t)}{\partial w_t} + \\ & \frac{1}{2} \left(\left[\int_0^\infty \chi_t^{(\tau)} w_t \eta_{r,t}^{(\tau)} d\tau \right]^2 + \left[\int_0^\infty \chi_t^{(\tau)} w_t \eta_{\beta,t}^{(\tau)} d\tau \right]^2 \right) \frac{\partial^2 v_t(w_t)}{\partial w_t^2}. \quad (31) \end{aligned}$$

The first-order conditions are

$$w_t \left(\omega_t^{(\tau)} - r_t \right) \frac{\partial v_t(w_t)}{\partial w_t} = -w_t^2 \left(\int_0^\infty \chi_t^{(s)} \left[\eta_{r,t}^{(\tau)} \eta_{r,t}^{(s)} ds + \eta_{\beta,t}^{(\tau)} \eta_{\beta,t}^{(s)} \right] ds \right) \frac{\partial^2 v_t(w_t)}{\partial w_t^2} \quad (32)$$

for each $\tau \in (0, \infty)$.

Now conjecture that the value function satisfies

$$v_t(w_t) = \frac{1}{\xi} \log w_t + v_t,$$

where v_t does not depend on the arbitrageur's level of wealth. It follows that

$$\frac{\partial v_t(w_t)}{\partial w_t} = \frac{1}{\xi w_t},$$

$$\frac{\partial^2 v_t(w_t)}{\partial w_t^2} = -\frac{1}{\xi w_t^2}.$$

Substituting into (32), it follows

$$\omega_t^{(\tau)} - r_t = \int_0^\infty \chi_t^{(s)} \left[\eta_{r,t}^{(\tau)} \eta_{r,t}^{(s)} ds + \eta_{\beta,t}^{(\tau)} \eta_{\beta,t}^{(s)} \right] ds \quad (33)$$

for each $\tau \in (0, \infty)$. An implication is that the arbitrageur's optimal portfolio shares $\chi_t^{(\tau)}$ do not depend on w_t . Substituting these into (31), on the left-hand side we have

$$\log w_t + \xi v_t,$$

and on the right-hand side we have

$$\begin{aligned} \frac{\partial v_t}{\partial t} + \log w_t + \frac{1}{\xi} \left[r_t + \int_0^\infty \chi_t^{(\tau)} \left(\omega_t^{(\tau)} - r_t \right) d\tau \right] \\ - \frac{1}{2} \frac{1}{\xi} \left(\left[\int_0^\infty \chi_t^{(\tau)} \eta_{r,t}^{(\tau)} d\tau \right]^2 + \left[\int_0^\infty \chi_t^{(\tau)} \eta_{\beta,t}^{(\tau)} d\tau \right]^2 \right). \end{aligned}$$

Canceling $\log w_t$ on both sides, (31) becomes

$$\begin{aligned} \xi v_t = \frac{\partial v_t}{\partial t} + \frac{1}{\xi} \left[r_t + \int_0^\infty \chi_t^{(\tau)} \left(\omega_t^{(\tau)} - r_t \right) d\tau \right] \\ - \frac{1}{2} \frac{1}{\xi} \left(\left[\int_0^\infty \chi_t^{(\tau)} \eta_{r,t}^{(\tau)} d\tau \right]^2 + \left[\int_0^\infty \chi_t^{(\tau)} \eta_{\beta,t}^{(\tau)} d\tau \right]^2 \right). \end{aligned}$$

Since nothing in this partial differential equation depends on w_t , the conjectured form of the value function is satisfied, with v_t solving the above equation.

Finally, since $\chi_t^{(\tau)}$ does not depend on arbitrageurs' individual wealth, aggregation implies

$$\chi_t^{(\tau)} = \frac{X_t^{(\tau)}}{W_t},$$

so that (33) can be written

$$\omega_t^{(\tau)} - r_t = \frac{1}{W_t} \int_0^\infty X_t^{(s)} \left[\eta_{r,t}^{(\tau)} \eta_{r,t}^{(s)} ds + \eta_{\beta,t}^{(\tau)} \eta_{\beta,t}^{(s)} \right] ds.$$

Given (30) together with the evolution of aggregate arbitrageur wealth (5), this can

more intuitively be written

$$E_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right) - r_t dt = \frac{1}{W_t} \int_0^\infty X_t^{(s)} Cov_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds$$

as in the main text.

C.2 Solution algorithm

We now provide more details on our computational algorithm.

Given (6), (7), (21), and (22), Ito's Lemma and $d\tau = -dt$ implies that

$$\begin{aligned} \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} &= \frac{1}{P_t^{(\tau)}} \left[P_{r,t}^{(\tau)} \kappa_r (\bar{r} - r_t) + P_{W,t}^{(\tau)} \omega_t + P_{\beta,t}^{(\tau)} \kappa_\beta (\bar{\beta} - \beta_t) - P_{\tau,t}^{(\tau)} \right. \\ &\quad \left. + \frac{1}{2} P_{rr,t}^{(\tau)} \sigma_r^2 + \frac{1}{2} P_{WW,t}^{(\tau)} (\eta_{r,t}^2 + \eta_{\beta,t}^2) + \frac{1}{2} P_{\beta\beta,t}^{(\tau)} \sigma_\beta^2 + P_{rW,t}^{(\tau)} \sigma_r \eta_{r,t} + P_{\beta W,t}^{(\tau)} \sigma_\beta \eta_{\beta,t} \right] dt \\ &\quad + \frac{1}{P_t^{(\tau)}} \left(P_{r,t}^{(\tau)} \sigma_r + P_{W,t}^{(\tau)} \eta_{r,t} \right) dB_{r,t} + \frac{1}{P_t^{(\tau)}} \left(P_{\beta,t}^{(\tau)} \sigma_\beta + P_{W,t}^{(\tau)} \eta_{\beta,t} \right) dB_{\beta,t}, \end{aligned}$$

where we write $P_{x,t}^{(\tau)}$ and $P_{xx,t}^{(\tau)}$ to denote the first- and second-order partial derivatives of $P^{(\tau)}(r_t, \beta_t, W_t)$ with respect to a generic variable x , and we write $\omega_t = \omega(r_t, \beta_t, W_t)$ and analogously for $\eta_{r,t}$ and $\eta_{\beta,t}$. It follows that

$$\begin{aligned} E_t \left(dP_t^{(\tau)} \right) &= \left[P_{r,t}^{(\tau)} \kappa_r (\bar{r} - r_t) + P_{W,t}^{(\tau)} \omega_t + P_{\beta,t}^{(\tau)} \kappa_\beta (\bar{\beta} - \beta_t) - P_{\tau,t}^{(\tau)} \right. \\ &\quad \left. + \frac{1}{2} P_{rr,t}^{(\tau)} \sigma_r^2 + \frac{1}{2} P_{WW,t}^{(\tau)} (\eta_{r,t}^2 + \eta_{\beta,t}^2) + \frac{1}{2} P_{\beta\beta,t}^{(\tau)} \sigma_\beta^2 + P_{rW,t}^{(\tau)} \sigma_r \eta_{r,t} + P_{\beta W,t}^{(\tau)} \sigma_\beta \eta_{\beta,t} \right] dt \end{aligned}$$

and

$$\begin{aligned} Cov_t \left(dP_t^{(\tau)}, dP_t^{(s)} \right) &= \left(P_{r,t}^{(\tau)} \sigma_r + P_{W,t}^{(\tau)} \eta_{r,t} \right) \left(P_{r,t}^{(s)} \sigma_r + P_{W,t}^{(s)} \eta_{r,t} \right) dt \\ &\quad + \left(P_{\beta,t}^{(\tau)} \sigma_\beta + P_{W,t}^{(\tau)} \eta_{\beta,t} \right) \left(P_{\beta,t}^{(s)} \sigma_\beta + P_{W,t}^{(s)} \eta_{\beta,t} \right) dt. \end{aligned}$$

Plugging both into (19), we obtain the partial differential equation

$$\begin{aligned} &\left[P_{r,t}^{(\tau)} \kappa_r (\bar{r} - r_t) + P_{W,t}^{(\tau)} \omega_t + P_{\beta,t}^{(\tau)} \kappa_\beta (\bar{\beta} - \beta_t) - P_{\tau,t}^{(\tau)} \right. \\ &\quad \left. + \frac{1}{2} P_{rr,t}^{(\tau)} \sigma_r^2 + \frac{1}{2} P_{WW,t}^{(\tau)} (\eta_{r,t}^2 + \eta_{\beta,t}^2) + \frac{1}{2} P_{\beta\beta,t}^{(\tau)} \sigma_\beta^2 + P_{rW,t}^{(\tau)} \sigma_r \eta_{r,t} + P_{\beta W,t}^{(\tau)} \sigma_\beta \eta_{\beta,t} - r_t \right] dt \end{aligned} \quad (34)$$

$$\begin{aligned}
&= \frac{1}{W_t} \left[\left(P_{r,t}^{(\tau)} \sigma_r + P_{W,t}^{(\tau)} \eta_{r,t} \right) \int_0^\infty \left(\alpha(s) \log \left(P_t^{(s)} \right) + \theta_0(s) \right) \frac{1}{P_t^{(\tau)}} \left(P_{r,t}^{(s)} \sigma_r + P_{W,t}^{(s)} \eta_{r,t} \right) ds \right. \\
&+ \left(P_{\beta,t}^{(\tau)} \sigma_\beta + P_{W,t}^{(\tau)} \eta_{\beta,t} \right) \int_0^\infty \left(\alpha(s) \log \left(P_t^{(s)} \right) + \theta_0(s) \right) \frac{1}{P_t^{(\tau)}} \left(P_{\beta,t}^{(s)} \sigma_\beta + P_{W,t}^{(s)} \eta_{\beta,t} \right) ds \\
&+ \beta_t \left[\left(P_{r,t}^{(\tau)} \sigma_r + P_{W,t}^{(\tau)} \eta_{r,t} \right) \int_0^\infty \theta_1(s) \frac{1}{P_t^{(\tau)}} \left(P_{r,t}^{(s)} \sigma_r + P_{W,t}^{(s)} \eta_{r,t} \right) ds \right. \\
&\left. + \left(P_{\beta,t}^{(\tau)} \sigma_\beta + P_{W,t}^{(\tau)} \eta_{\beta,t} \right) \int_0^\infty \theta_1(s) \frac{1}{P_t^{(\tau)}} \left(P_{\beta,t}^{(s)} \sigma_\beta + P_{W,t}^{(s)} \eta_{\beta,t} \right) ds \right] dt.
\end{aligned}$$

(20), (22), and the method of undetermined coefficients imply

$$\omega_t = \xi (\bar{W} - W_t) + W_t r_t + \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \left(\mu_t^{(\tau)} - r_t \right) d\tau \quad (35)$$

$$\eta_{r,t} = \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \frac{1}{P_t^{(\tau)}} \left(P_{r,t}^{(\tau)} \sigma_r + P_{W,t}^{(\tau)} \eta_{r,t} \right) d\tau \quad (36)$$

$$\eta_{\beta,t} = \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \frac{1}{P_t^{(\tau)}} \left(P_{\beta,t}^{(\tau)} \sigma_\beta + P_{W,t}^{(\tau)} \eta_{\beta,t} \right) d\tau. \quad (37)$$

We numerically solve the system (34)-(37) as follows. For clarity we now drop the time subscript t and explicitly write all functions as stationary in the state variables r , β and W . We start with an initial guess $\omega(r, \beta, W) = \eta_r(r, \beta, W) = \eta_\beta(r, \beta, W) = 0$. We then solve for $P^{(\tau)}(r, \beta, W)$ using forward differences in maturity, replacing $P_{\tau,t}^{(\tau)}$ in equation (34) by its approximation

$$P_{\tau,t}^{(\tau)} \approx \frac{P^{(\tau+\Delta\tau)}(r, \beta, W) - P^{(\tau)}(r, \beta, W)}{\Delta\tau}.$$

Given that approximation, we rearrange equation (34) to solve for $P^{(\tau+\Delta\tau)}(r, \beta, W)$ only in terms of $P^{(\tau)}(r, \beta, W)$ and its non-time derivatives. Starting at the known boundary $\tau = 0$, we derive those non-time derivatives using collocation and iterate forward in maturity. We use Chebyshev interpolation on sparse Smolyak grids to approximate the bond price function at each time step in the three non-time dimensions (see Judd, Maliar, Maliar, and Valero (2014)). After deriving the initial solution for $P^{(\tau)}(r, \beta, W)$ we update our guess on $\omega(r, \beta, W)$, $\eta_r(r, \beta, W)$ and $\eta_\beta(r, \beta, W)$ and iterate until convergence on (35)-(37).

After solving the model, we can simulate an unanticipated monetary shock by

introducing a shift m_t to the level of the short-term rate, which occurs at time t_m and reverts back by time T_m . Formally

$$\tilde{r}_t = r_t + m_t \mathbb{I}\{t \in [t_m, T_m]\}$$

with

$$dm_t = -\kappa_m m_t dt.$$

The first order condition of the arbitrageurs is now

$$E_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right) - \tilde{r}_t dt = \frac{1}{W_t} \int_0^\infty X_t^{(s)} Cov_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds.$$

We expand this equation using similar steps as those discussed for the general solution. The method of undetermined coefficients now implies

$$\begin{aligned} \omega_t &= \xi (\bar{W} - W_t) + W_t \tilde{r}_t + \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \left(\mu_t^{(\tau)} - \tilde{r}_t \right) d\tau, \\ \eta_{r,t} &= \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \frac{1}{P_t^{(\tau)}} \left(P_{r,t}^{(\tau)} \sigma_r + P_{W,t}^{(\tau)} \eta_{r,t} \right) d\tau, \\ \eta_{\beta,t} &= \int_0^\infty \left(\alpha(\tau) \log \left(P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \frac{1}{P_t^{(\tau)}} \left(P_{\beta,t}^{(\tau)} \sigma_\beta + P_{W,t}^{(\tau)} \eta_{\beta,t} \right) d\tau. \end{aligned}$$

Because the price function is now non-stationary we adopt a variation of the approach used for the general solution. We start from the terminal date T_m when we know the price function from the general solution and use forward differences in maturity (backward differences in time), replacing $P_{\tau,t}^{(\tau)}$ by

$$P_{\tau,t}^{(\tau)}(r_t, \beta_t, W_t, m_t) \approx \frac{P_t^{(\tau)}(r_t, \beta_t, W_t, m_t) - P_{t-\Delta\tau}^{(\tau+\Delta\tau)}(r_t, \beta_t, W_t, m_t)}{\Delta\tau}$$

In each time t , we can then rearrange this equation and solve for $P_{t-\Delta\tau}^{(\tau+\Delta\tau)}(r_t, \beta_t, W_t, m_t)$ as a function of known or previously computed variables. We then update the initial guess for ω_t , $\eta_{r,t}$, and $\eta_{\beta,t}$ and iterate until convergence. On impact of the monetary shock at time t_m , we isolate the unexpected change in price and use only this unexpected component to evaluate the evolution of wealth.